The Standard Model Lagrangian

Elementary Particle Physics
Strong Interaction Fenomenology

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Dirac Formalism

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0$$

$$j^{\mu} = \overline{\psi}\gamma^{\mu}\psi$$

Conserved Current

$$\gamma^{i} = \begin{pmatrix} 0 & -\sigma_i \\ \sigma_i & 0 \end{pmatrix} \qquad \gamma^{0} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \Longrightarrow \qquad \gamma^{5} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} \qquad \mathcal{L} = \overline{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi$$

Gauge Invariance

$$\Psi' = U\Psi$$

$$\mathcal{D}^\mu = \partial^\mu - igA^\mu$$

$$\mathcal{D}'^\mu \Psi' = U\left(\mathcal{D}^\mu \Psi\right)$$

$$A^{\mu'} = -\frac{i}{g}\left(\partial^\mu U\right)U^{-1} + UA^\mu U^{-1}$$

Abelian

$$\mathcal{D}^\mu = \partial^\mu - ig_2 \frac{\vec{\tau}}{2} \cdot \vec{W}^\mu$$

$$\delta W_i^\mu = \frac{1}{g_2}\partial^\mu \theta_i - \varepsilon_{ijk}\theta_j W_k^\mu$$

Non Abelian

Gauge invariance requires the introduction of vector bosons, which act as quanta of new interactions.

In gauge theories the symmetries prescribe the interactions.

The Symmetries of the Standard Model

- *U(1)* invariance. All particles appear to have this kind of invariance, related to electromagnetism. It requires a vector boson, B^{μ}, whose connection with the photon will be determined later.
- *SU(2)* invariance. Non abelian gauge invariance (electroweak isospin). It requires three vector bosons, W_i^{μ}, one for each generator of *SU(2)*. The physical W particles have definite electromagnetic charges.

$$W^+ = \left(-W^1 + iW^2\right)/\sqrt{2} \quad W^- = \left(-W^1 - iW^2\right)/\sqrt{2} \quad W^0 = W^3$$

- *SU(3)* invariance. It requires eight vector bosons, G_a^{μ}, the gluons, the quanta of the strong interaction, described by Quantum ChromoDynamics (QCD).

The Lagrangian

- In order to obtain the Standard Model Lagrangian we start from the free particle Lagrangian and replace the ordinary derivative by the convariant derivative. It will contain two parts:

  $\mathcal{L}_{\text{gauge}}$ kinetic energies of the gauge fields

  $\mathcal{L}_{\text{ferm}}$ covariant derivative $\rightarrow$ fermion kinetic energies

- Next we must specify the particles and their transformation properties under the three internal symmetries.
- Notation

Leptons

$$e_R^- = P_R \psi_{e^-} \qquad e_L^- = P_L \psi_{e^-}$$

Left-handed and right-handed particles behave differently under electroweak *SU(2)* transformations: the electrons R are *SU(2)* singlets, whereas the electrons L are put in doublets together with the L neutrinos.

e_R^- *SU(2)* singlet $\qquad L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ *SU(2)* doublet

- Rotations in electroweak *SU(2)* turn L electrons in L neutrinos and vv.
- Ordinary spin: raising and lowering operators (vectors).
- Strong isospin: pions (vectors).
- Weak isospin: the *W* bosons connect the members of an electroweak doublet
- e_R is not connected to any other state by electroweak transitions.
- $p,q,r=1,2$ e.g.: L_p $L_1=\nu_{eL}$, $L_2=e^-_L$.

Quarks

$$Q_{L\alpha} = \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix}_L \qquad\qquad d_{R\alpha}, u_{R\alpha}$$

- the index α describes how the quark transforms under color $SU(3)$.
- The basic representation is a triplet: α,β,γ = *1,2,3* or r,g,b.
- Color (e.g. r) and anticolor (e.g. $\bar{r}$). Singlet ($r\bar{r}+ g\bar{g}+ b\bar{b}$)
- All leptons are color singlets.
- All quarks are color triplets.
- The gluons generate the transitions from one color to another: they are the quanta of the strong interaction, but unlike photons they carry color charge.
- There are eight “bi-colored” gluons (e.g. $b\bar{g}$): octet representation of color $SU(3)$.

- In the Standard Model there are no R neutrinos:
  - Experimentally ony ν_L are observed.
  - Neutrino masses are very small (but non-zero, oscillations).
  - If right-handed neutrinos ν_R exist either they are very heavy or they interact very weakly.
- R and L fermions were put in different electroweak *SU(2)* multiplets: that implies a violation of parity, since clearly the theory is not invariant under the reversal of the component of spin in the direction of motion. This is the way in which parity violation is described in the standard model, but it is not explained in a fundamental sense.
- The same theory can be applied to the two other fermion families: (ν_μ, μ, c, s) and (ν_τ, τ, t, b).
  - The universe consists of fermions from the first generation.
  - The other families are produced in high-energy cosmic ray collisions and in particle accelerators.
  - No reason has been found for the existence of three families of particles with identical quantum numbers and interactions.

The Quark and Lepton Lagrangian

$$\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi \rightarrow \bar{\psi}\gamma^{\mu}\mathcal{D}_{\mu}\psi$$

$$\mathcal{D}_{\mu} = \partial_{\mu} - ig_1 \frac{Y}{2} B_{\mu} - ig_2 \frac{\tau^i}{2} W_{\mu}^i - ig_3 \frac{\lambda^a}{2} G_{\mu}^a$$

- B_{μ} is the spin-one field needed to maintain the $U(1)$ gauge invariance. g_1 is the coupling strength (to be measured experimentally. Y is the generator of $U(1)$, transformations, a constant, but in principle different for the different fermions.
- Analogous remarks describe the SU(2) and SU(3) terms. We introduce 3 and, respectively, 8 vector bosons which are needed to maintain the local gauge invariance. $\tau^i W_{\mu}^i = \tau^1 W_{\mu}^1 + \tau^2 W_{\mu}^2 + \tau^3 W_{\mu}^3$
- $\mathcal{D}_{\mu}$ gives a zero result when it acts on a term of different matrix form. For example $\tau^i W^i$ is a 2×2 matrix in $SU(2)$ and it gives zero acting on e_R, u_R, d_R.

$$\mathcal{L}_{\text{ferm}} = \sum_f \bar{f}\gamma^{\mu}\mathcal{D}_{\mu} f \qquad f = L, e_R, Q_L, u_R, d_R$$

Gauging the Global Symmetries

Dirac kinetic energy Lagrangian for the first generation:

$$\mathcal{L} = \bar{e}_R \gamma^\mu \partial_\mu e_R + \bar{e}_L \gamma^\mu \partial_\mu e_L + \bar{\nu}_L \gamma^\mu \partial_\mu \nu_L$$

We put e_L and ν_L in a doublet, e_R in a singlet.

$$L \rightarrow e^{i\vec{\tau}\cdot\vec{\theta}/2} L$$

$$e_R \rightarrow e_R$$

Global *SU(2)* symmetry

$$L \rightarrow e^{i\beta} L$$

$$e_R \rightarrow e^{i\beta'} e_R$$

Global *U(1)* symmetry

We make these symmetries locals by introducing potentials W_i^μ and B^μ. and by replacing ∂^μ with the covariant derivative $\mathcal{D}^\mu$. Thus we obtain the same result. Some attempts to extend the Standard Model proceed along these lines, by adding particles and symmetries and then gauging the symmetries.

The Electroweak Lagrangian

- Since the term ∂^μ is always present we will not write it.
- All the calculations in $SU(2)$ will be done only for the leptons.
- Since the color labels of the quarks do not operate in the $U(1)$ or $SU(2)$ space, quarks will behave the same way as leptons for $U(1)$ and $SU(2)$ interactions.

The *U(1)* Terms

$$-\mathcal{L}_{\text{ferm}}\left(U(1),\text{leptoni}\right)=\bar{L}\,i\gamma^{\mu}\left(ig_1\frac{Y_L}{2}B_{\mu}\right)L+\bar{e}_R i\gamma^{\mu}\left(ig_1\frac{Y_R}{2}B_{\mu}\right)e_R$$

$$\bar{L}\gamma^{\mu}L=\bar{\nu}_L\gamma^{\mu}\nu_L+\bar{e}_L\gamma^{\mu}e_L$$

$$\mathcal{L}_{\text{ferm}}\left(U(1),\text{leptoni}\right)=\frac{g_1}{2}\left[Y_L\left(\bar{\nu}_L\gamma^{\mu}\nu_L+\bar{e}_L\gamma^{\mu}e_L\right)+Y_R\bar{e}_R\gamma^{\mu}e_R\right]B_{\mu}$$

The *SU(2)* Terms

$$-\mathcal{L}_{\text{ferm}}\left(SU(2),\text{leptoni}\right) = \bar{L}\, i\gamma^{\mu}\left(ig_2\frac{\tau^i}{2}W^i_{\mu}\right)L$$

$$= -\frac{g_2}{2}\left(\bar{\nu}_L \quad \bar{e}_L\right)\gamma^{\mu}\begin{pmatrix} W^3_{\mu} & W^1_{\mu} - iW^2_{\mu} \\ W^1_{\mu} + iW^2_{\mu} & -W^3_{\mu}\end{pmatrix}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$$

$$= -\frac{g_2}{2}\left(\bar{\nu}_L \quad \bar{e}_L\right)\gamma^{\mu}\begin{pmatrix} W^0_{\mu} & -\sqrt{2}W^+_{\mu} \\ -\sqrt{2}W^-_{\mu} & -W^0_{\mu}\end{pmatrix}\begin{pmatrix}\nu_L \\ e_L\end{pmatrix}$$

$$= -\frac{g_2}{2}\left(\bar{\nu}_L \quad \bar{e}_L\right)\gamma^{\mu}\begin{pmatrix} W^0_{\mu}\nu_L - \sqrt{2}W^+_{\mu}e_L \\ -\sqrt{2}W^-_{\mu}\nu_L - W^0_{\mu}e_L\end{pmatrix}$$

$$= -\frac{g_2}{2}\left[\bar{\nu}_L\gamma^{\mu}\nu_L W^0_{\mu} - \sqrt{2}\bar{\nu}_L\gamma^{\mu}e_L W^+_{\mu} - \sqrt{2}\bar{e}_L\gamma^{\mu}\nu_L W^-_{\mu} - \bar{e}_L\gamma^{\mu}e_L W^0_{\mu}\right]$$

The Neutral Current

Electromagnetic interaction of particles of charge Q:

$$\mathcal{L}_{\mathrm{EM}} = QA_\mu \left[\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R\right]$$

There are terms involving neutrinos $\left(-\frac{g_1}{2} Y_L B_\mu - \frac{g_2}{2} W_\mu^0\right)\bar{\nu}_L \gamma^\mu \nu_L$

We assume the the electromagnetic field A_μ is the orthogonal combination:

$$A_\mu \propto g_2 B_\mu - g_1 Y_L W_\mu^0 \qquad Z_\mu \propto g_1 Y_L B_\mu + g_2 W_\mu^0$$

$$A_\mu = \frac{g_2 B_\mu - g_1 Y_L W_\mu^0}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \qquad Z_\mu = \frac{g_1 Y_L B_\mu + g_2 W_\mu^0}{\sqrt{g_2^2 + g_1^2 Y_L^2}}$$

Terms involving electrons: $\bar{e}_L \gamma^\mu e_L \left(-\frac{g_1}{2} Y_L B_\mu + \frac{g_2}{2} W^0_\mu \right) + \bar{e}_R \gamma^\mu e_R \left(-\frac{g_1}{2} Y_R B_\mu \right)$

$$B_\mu = \frac{g_2 A_\mu + g_1 Y_L Z_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \qquad W^0_\mu = \frac{-g_1 Y_L A_\mu + g_2 Z_\mu}{\sqrt{g_2^2 + g_1^2 Y_L^2}}$$

$$-A_\mu \left\{ \bar{e}_L \gamma^\mu e_L \left[\frac{g_1 g_2 Y_L}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \right] + \bar{e}_R \gamma^\mu e_R \left[\frac{g_1 g_2 Y_R}{2\sqrt{g_2^2 + g_1^2 Y_L^2}} \right] \right\}$$

$$-Z_\mu \left\{ \bar{e}_L \gamma^\mu e_L \left[\frac{g_1^2 Y_L^2 - g_2^2}{2\sqrt{g_2^2 + g_1^2 Y_L^2}} \right] + \bar{e}_R \gamma^\mu e_R \left[\frac{g_1^2 Y_R Y_L}{2\sqrt{g_2^2 + g_1^2 Y_L^2}} \right] \right\}$$

The term in A_μ must be the usual electromagnetic current.
The term in Z_μ can be an additional interaction, to be checked experimentally.

$$-e = \frac{g_1 g_2 Y_L}{\sqrt{g_2^2 + g_1^2 Y_L^2}} \qquad -e = \frac{g_1 g_2 Y_R}{2\sqrt{g_2^2 + g_1^2 Y_L^2}}$$

$$Y_R = 2Y_L$$

$$Y_L = -e\frac{\sqrt{g_2^2 + g_1^2 Y_L^2}}{g_1 g_2}$$

We can choose Y_L=-1, since any change in Y_L can be absorbed by a redefinition of g_1.

$$Y_L = -1 \quad \Rightarrow \quad e = \frac{g_1 g_2}{\sqrt{g_2^2 + g_1^2}}$$

The theory we have been writing can be interpreted to contain the usual electromagnetic interaction, plus an additional neutral current interaction with Z_μ for both electrons and neutrinos.

Define:

$$\sin\theta_W = \frac{g_1}{\sqrt{g_2^2 + g_1^2}}$$

$$\cos\theta_W = \frac{g_2}{\sqrt{g_2^2 + g_1^2}}$$

θ_W weak mixing angle (Weinberg angle)

$$g_1 = \frac{e}{\cos\theta_W}$$

$$g_2 = \frac{e}{\sin\theta_W}$$

g_1 and g_2 are written in terms of the known e $(e^2/4\pi \approx 1/137)$ and the electroweak mixing angle, which needs to be measured or calculated some other way.

$$\sin^2\theta_W \approx 0.23$$

ν-Z Coupling

$$-\frac{\sqrt{g_2^2+g_1^2}}{2} Z_\mu \bar{\nu}_L \gamma^\mu \nu_L = -\frac{g_2}{2\cos\theta_W} Z_\mu \bar{\nu}_L \gamma^\mu \nu_L$$

$\dfrac{g_2}{2\cos\theta_W}$ quantity to be associated to each ν_L-Z vertex.
"electroweak charge" of the left-handed neutrino.

$$\sqrt{g_2^2+g_1^2} = \left[\frac{e^2}{\cos^2\theta_W} + \frac{e^2}{\sin^2\theta_W}\right]^{1/2}$$

$$= \left[\frac{e^2}{\cos^2\theta_W \sin^2\theta_W}\right]^{1/2}$$

$$= \frac{e}{\cos\theta_W \sin\theta_W}$$

e-Z Coupling

$$-Z_\mu \left\{ \bar{e}_L \gamma^\mu e_L \left[\frac{g_1^2 - g_2^2}{2\sqrt{g_2^2 + g_1^2}} \right] + \bar{e}_R \gamma^\mu e_R \left[\frac{g_1^2}{\sqrt{g_2^2 + g_1^2}} \right] \right\}$$

$$\frac{g_1^2 - g_2^2}{2\sqrt{g_2^2 + g_1^2}} = \frac{e^2}{2\sqrt{g_2^2 + g_1^2}} \left(\frac{1}{\cos^2 \theta_W} - \frac{1}{\sin^2 \theta_W} \right)$$

$$= \frac{e}{\cos\theta_W \sin\theta_W} \left(-\frac{1}{2} + \sin^2 \theta_W \right)$$

e_L Coupling

$$\frac{g_1^2}{\sqrt{g_2^2 + g_1^2}} = -\frac{e^2}{\cos^2 \theta_W} \frac{\cos\theta_W \sin\theta_W}{e}$$

$$= \frac{e}{\cos\theta_W \sin\theta_W} \left(\sin^2 \theta_W \right)$$

e_R Coupling

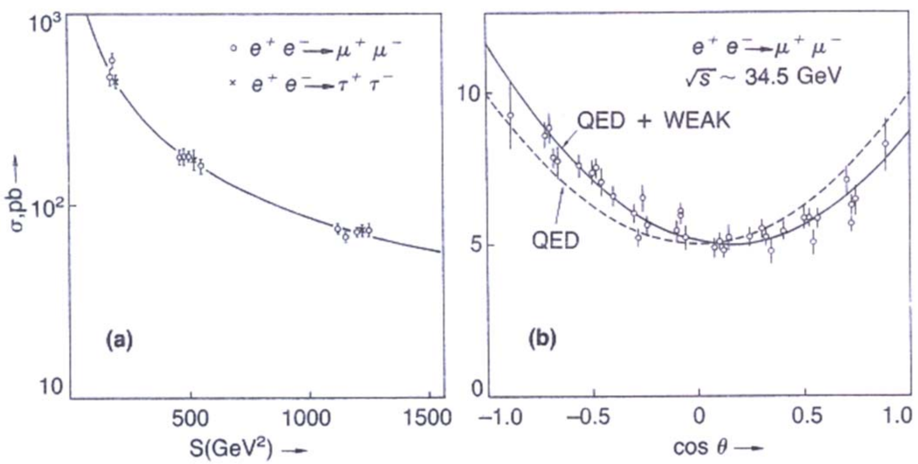
$$\frac{e}{\cos\theta_W \sin\theta_W}\left(T_3^f - Q_f \sin^2\theta_W\right)$$

This expression gives the electroweak charge of any fermion, i.e. the strength of the coupling to the Z.

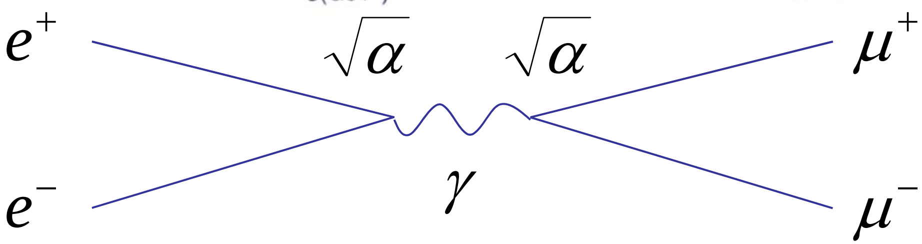
T_3^f is the eigenvalue of T_3 for any fermion f.
For a singlet ($f=e_R,u_R,d_R$ ecc) $T_3^f = 0$.
For the upper member of a doublet ($f=\nu_L,u_L$ ecc) $T_3^f = +1/2$.
For the lower member of a doublet ($f=e_L,d_L$ ecc) $T_3^f = -1/2$.
Q_f is the electric charge of the fermion in units of e:
$Q_e=-1.\ Q_\nu=0,\ Q_u=2/3,\ Q_d=-1/3)$

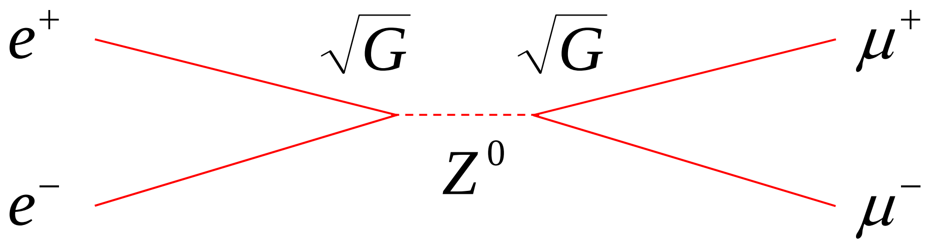
The electroweak theory contains both the electromagnetic interaction, mediated by the photon, and the weak neutral current, mediated by the Z^0, which couples to any fermion with electric charge or electroweak isospin.

The strength of the Z^0 interaction is not intrinsically small, but it gets reduced by the high value of its mass.



The process $e^+e^- \to \mu^+\mu^-$ (or $e^+e^- \to \tau^+\tau^-$) is not purely electromagnetic, but it has a weak component, due to the exchange of a Z^0.





$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}(QED) + \frac{d\sigma}{d\Omega}(weak) + \frac{d\sigma}{d\Omega}(interference)$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\frac{\alpha^2}{s} \qquad\qquad G^2 s \qquad\qquad \alpha G$$

The asymmetry comes from the interference term, the effect is of the order of 10 % for s = 1000 GeV^2.

The Charged Current

The *U(1)* part of the Lagrangian contains only terms diagonal in the fermions, whereas the *SU(2)* part has also non diagonal terms.

$$\mathcal{L}_{\mathrm{ferm}} = \frac{g_2}{\sqrt{2}}\left[\bar{\nu}_L \gamma^\mu e_L W_\mu^+ + \bar{e}_L \gamma^\mu \nu_L W_\mu^-\right]$$ charged current

$$\bar{\nu}_L \gamma^\mu e_L = \frac{1}{2}\bar{\nu}\gamma^\mu\left(1-\gamma^5\right)e$$ V-A interaction

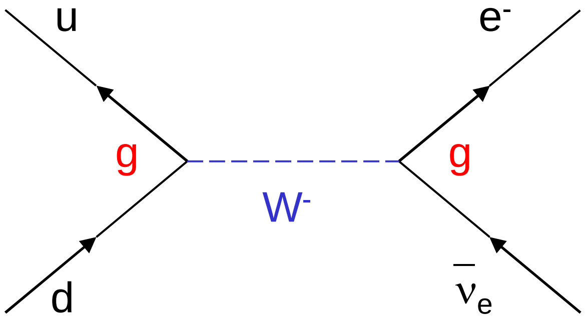
We thus expect $W^{\pm}$ bosons and the associated charged current transitions. The observed charged currents occur with a strength much smaller than one would expect:

$$\frac{\left(g_2/\sqrt{2}\right)^2}{4\pi} = \frac{\left(e^2/4\pi\right)}{2\sin^2\theta_W} \approx \frac{2}{137}$$

Example of a Charge Current: β decay

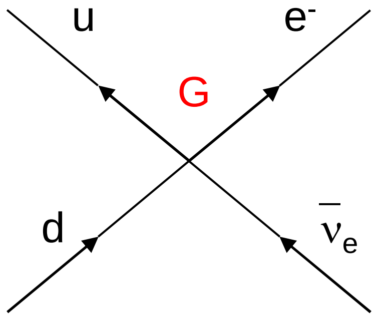
$$n \rightarrow p + e^- + \overline{\nu}_e \qquad \tau = 885.7 \pm 0.8\ s$$

$$\left(d \rightarrow u + e^- + \overline{\nu}_e\right)$$



$M_W = 80.425 \pm 0.038$ GeV/c^2

$q^2 \ll M^2_W$



The interaction is practically pointlike, described by a 4-fermion couplgin

$$G = \frac{g^2}{M_W^2}$$

As is the case with neutral currents, also for charged currents the charged current strenghts are reduced by the high value of the $W^{\pm}$ mass.

The Quark Electroweak Terms

The $SU(2)$ and spin structure of quarks and leptons are the same, consequently all the previous conclusions hold without modifications for the quarks:

- They couple to the same gauge bosons $W^{\pm}$, Z^0, γ.
- Normal electromagnetic coupling to the photon.
- Charged current coupling generating transitions $u_L \leftrightarrow d_L$, but no charged current transitions for u_R *e* d_R.
- Neutral current transitions with a universal strength:

$$\frac{e}{\cos\theta_W \sin\theta_W}\left(T_3^f - Q_f \sin^2\theta_W\right)$$

f	Q	T_3^f
u_L	$+2/3$	$+1/2$
d_L	$-1/3$	$-1/2$
u_R	$+2/3$	0
d_R	$-1/3$	0

The Quark QCD Lagrangian

$$\frac{g_3}{2}\bar{q}_\alpha \gamma^\mu \lambda^a_{\alpha\beta} G^a_\mu q_\beta \qquad \begin{cases} \alpha, \beta = 1,2,3 \\ a = 1,\ldots,8 \end{cases}$$

- It contains only quarks, since leptons have no color charge.
- In the electroweak case the W^i are related to states of electromagnetic charge because of the interaction with the photon. The gluons are electrically neutral, i.e. they have no interaction with the electromagnetic field.
- Since the generators λ^a are not all diagonal, the interaction with a gluon can change the color charge of a quark.
- Gluons and quarks are confined within hadrons.

The Second and Third Families

$$\left.\begin{pmatrix}\nu_e\\ e\end{pmatrix}\atop\begin{pmatrix}u\\ d\end{pmatrix}\right\}\rightarrow\begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}\begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}\atop\begin{pmatrix}c\\ s\end{pmatrix}\begin{pmatrix}t\\ b\end{pmatrix}$$

- All known phenomenology is consistent with the above replacements
- It is unknown whether more families or additional quarks and leptons exist
- All fermions of the three families have been observed experimentally.
- The same set of gauge bosons (γ, $W^{\pm}$, Z^0,g) interacts with all the families of fermions:
  - lepton universality
  - u- and d- universality

The Fermion-Gauge Boson Lagrangian

$$\mathcal{L} = \sum_{f=\nu,e,u,d} eQ_f \left(\bar{f}\gamma^\mu f\right) A_\mu$$

$$+ \frac{g_2}{\cos\theta_W} \sum_{f=\nu,e,u,d} \left[\bar{f}_L \gamma^\mu f_L \left(T_f^3 - Q_f \sin^2\theta_W\right) + \bar{f}_R \gamma^\mu f_R \left(-Q_f \sin^2\theta_W\right)\right] Z_\mu$$

$$+ \frac{g_2}{\sqrt{2}} \left[\left(\bar{u}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu e_L\right) W_\mu^+ + \text{h.c.}\right]$$

$$+ \frac{g_3}{2} \sum_{q=u,d} \bar{q}_\alpha \gamma^\mu \lambda_{\alpha\beta}^a q_\beta G_\mu^a$$

Masses

- For fermions a mass term would be of the form $m \bar{\psi}\psi$.

$$m\bar{\psi}\psi = m\left(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R\right)$$

  Since L fermions are members of an *SU(2)* doublet, whereas the R fermions are singlets, the terms $\bar{\psi}_R\psi_L$ and $\bar{\psi}_L\psi_R$ are not *SU(2)* singlets and would not give and *SU(2)* invariant Lagrangian,
- For gauge bosons mass terms are of the form

$$\frac{1}{2}m_B^2 B^\mu B_\mu$$

  which is clearly not invariant under gauge transformations.

The solution of this problem lies in the Higgs mechanism.