# Masses and the Higgs Mechanism

Elementary Particle Physics

Strong Interaction Fenomenology

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# The Higgs Mechanism

The assumption is made that the universe is filled with a spin-zero field, called a Higgs field, which is a doublet in $SU(2)$ and with a nonzero $U(1)$ hypercharge, but a singlet in color space . The gauge bosons and fermions can interact with this field, and in this interaction they acquire mass.

The $SU(2)$ and $U(1)$ quantum numbers of the ground state (i.e. the vacuum are non-zero, so the $SU(2)$ and $U(1)$ symmetries are effectively broken (spontaneously brokedn symmetries).

In this context the masses of the $W^{\pm}$ and $Z^0$ bosons can be calculated in terms of measurable parameters.

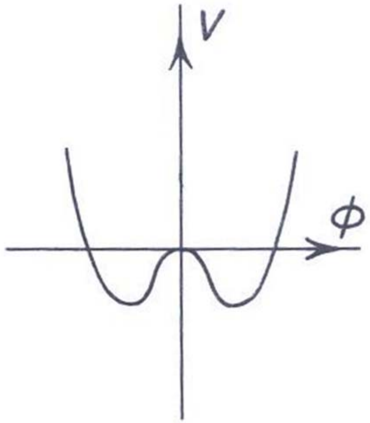
# Spontaneous Symmetry breaking

$$\mathcal{L} = T - V = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right)$$

- We can require $\lambda>0$ in order that the potential be bounded below as $\phi \to \infty$.
- The theory is invariant under $\phi \to -\phi$.
- To find the spectrum it is necessary:
  - to find the minimum of the potential (ground state $\to$ vacuum)
  - to expand the fields around their value at the minimum and determine the excitations
- The $\phi^4$ term represents an interaction of strength $\lambda$.
- Higher powers of $\phi$ would lead to infinities in physical quantities and must therefore be excluded (the theory would not be renormalizable).

If $\mu^2>0$ the vacuum corresponds to $\phi=0$, which minimizes the potential.
In that case $\mu^2$ can be interpreted as (mass)$^2$.
If, however, $\mu^2<0$ we find the minimum of the potential by setting:

$$\frac{\partial V}{\partial \phi} = 0 \quad \Rightarrow \quad \phi\left(\mu^2 + \lambda\phi^2\right) = 0$$




Minimum kinetic energy: $\phi$=constant.
The choice $\phi$=0 is not a minimum of the potential.

$$\phi = \pm\sqrt{\frac{-\mu^2}{\lambda}} \equiv v$$

$v$ is called the vacuum expectation value of $\phi$. The field $\phi$ is called a Higgs field.

To determine the particle spectrum we study the theory in the region of the minimum.

$$\phi(x) = v + \eta(x)$$

We expand around $\eta$=0.
We could also have chosen $\phi(x) = -v + \eta(x)$, but the physics conclusions would be the same, since the theory is symmetric under $\phi \to -\phi$.

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \eta \partial^\mu \eta\right) - \left\{\frac{1}{2}\mu^2\left[v^2 + 2\eta v + \eta^2\right] + \frac{1}{4}\lambda\left[v^4 + 4v^3\eta + 6v^2\eta^2 + 4v\eta^3 + \eta^4\right]\right\}$$

$$= \frac{1}{2}\left(\partial_\mu \eta \partial^\mu \eta\right) - \left\{\frac{v^2}{2}\left(\mu^2 + \frac{1}{2}\lambda v^2\right) + \eta v\left(\mu^2 + \lambda v^2\right) + \frac{\eta^2}{2}\left(\mu^2 + 3\lambda v^2\right) + \lambda v \eta^3 + \frac{1}{4}\lambda\eta^4\right\}$$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \eta \partial^\mu \eta\right) - \left\{\lambda v^2 \eta^2 + \lambda v \eta^3 + \frac{1}{4}\lambda \eta^4\right\} + \text{costante}$$

interactions

$$m_\eta^2 = 2\lambda v^2 = -2\mu^2$$

- The two descriptions of the theory in terms of $\phi$ or $\eta$ must be equivalent.
- It is essential to perturb around the minimum to have a convergent description.
- The scalar particle described by the theory with $\mu^2<0$ is a real scalar, with a mass obtained by its self-interaction with other scalar, because at the minimum of the potential there is a non-zero expectation value $v$.
- There is no trace of the original reflection symmetry. The symmetry was broken when a specific vacuum was chose ($+v$ rather than $-v$): the vacuum does not have the symmetry of the original Lagrangian, so the solutions do not.
- This is called a spontaneous symmetry breaking.

# Complex Scalar Field

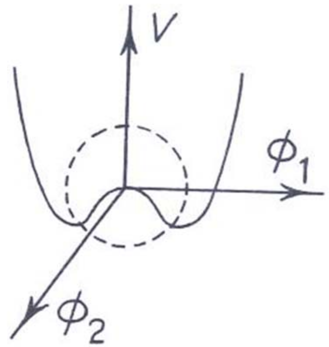
$$\phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \qquad \mathcal{L} = \left(\partial_\mu \phi\right)^* \left(\partial^\mu \phi\right) - \mu^2 \phi^* \phi - \lambda\left(\phi^* \phi\right)^2$$

$$\phi \to \phi' = e^{i\chi}\phi$$ Global $U(1)$ symmetry

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi_1\right)^2 + \frac{1}{2}\left(\partial_\mu \phi_2\right)^2 - \frac{1}{2}\mu^2\left(\phi_1^2 + \phi_2^2\right) - \frac{\lambda}{4}\left(\phi_1^2 + \phi_2^2\right)^2$$

For $\mu^2>0$ the minimum is at the origin
For $\mu^2<0$ the minimum is along a circle of radius $v$.

$$\phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda}$$




We choose $\phi_1=v$, $\phi_2=0$

$$\phi(x) = \frac{v + \eta(x) + i\rho(x)}{\sqrt{2}}$$

Mexican Hat Potential

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \rho)^2 + \frac{1}{2}(\partial_\mu \eta)^2 + \mu^2\eta^2 + \lambda v(\eta\rho^2 + \eta^3) - \frac{\lambda}{2}\eta^2\rho^2 - \frac{\lambda}{4}\eta^4 - \frac{\lambda}{4}\rho^4 + \text{costante}$$

$$m_\eta^2 = 2|\mu^2|$$

- the $\eta$ field corresponds to a massive particle
- The terms $\rho^2$ vanish, thus $\rho$ has zero mass: Goldstone boson !
- Goldstone theorem: whenever a continuous global symmetry is spontaneously broken the spectrum contains a massless, spin-zero boson.
- In this case the global $U(1)$ symmetry is broken because we had to choose a particular point on the circle to expand around.
- The potential is a minimum along a circle: radial excitations correspond to massive particles (curvature of the potential), whereas along the circle the potential is flat, hence the massless excitation.

# The Abelian Higgs Mechanism

Let us require local gauge invariance:

$$\phi(x) \to \phi'(x) = e^{i\chi(x)}\phi(x) \qquad \partial_\mu \to \mathcal{D}_\mu = \partial_\mu - igA_\mu$$

$$A_\mu \to A'_\mu = A_\mu - \frac{1}{g}\partial_\mu \chi(x)$$

$$\mathcal{L} = (\mathcal{D}_\mu \phi)^* (\mathcal{D}^\mu \phi) - \mu^2 \phi^* \phi - \lambda(\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

- For $\mu^2>0$ this describes the interaction of a scalar particle of mass $\mu$ with the electromagnetic field $A_\mu$.
- There is no mass term for $A_\mu$.
- This Lagrangian contains four independent fields: the two real scalars $\phi_1$ and $\phi_2$ and the two transverse polarization states of the massless vector boson $A_\mu$ .
- Also in this case let us look for solutions for $\mu^2<0$.

$$\phi(x) = \eta(x) e^{-i\rho(x)} \qquad \phi(x) = \frac{v + h(x)}{\sqrt{2}}$$

$\eta, \rho$ and $h$ real, using a suitable $\chi$.

$$\mathcal{L} = \frac{1}{2}\left[\left(\partial^{\mu} + igA^{\mu}\right)(v+h)\right]\left[\left(\partial_{\mu} - igA_{\mu}\right)(v+h)\right]$$

$$- \frac{v^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$= \frac{1}{2}\left(\partial_{\mu}h\right)\left(\partial^{\mu}h\right) + \frac{1}{2}g^2v^2A_{\mu}A^{\mu} - \lambda v^2h^2 - \lambda v h^3$$

$$- \frac{\lambda}{4}h^4 + g^2vhA_{\mu}A^{\mu} + \frac{1}{2}g^2h^2A_{\mu}A^{\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

There is now a mass term for the gauge boson !!!

$$\boxed{M_A = gv}$$

The Higgs boson $h$ with mass $\sqrt{2\lambda v^2}$

- La Lagrangian is gauge invariant, but the vacuum is not.
- The number of independent fields is still 4: the Higgs boson and the three polarization states of the massive gauge boson.
- The Goldstone boson of the previous case (global $U(1)$) symmetry) has become the longitudinal polarization state of the gauge boson.
- Higgs Mechanism.
- The Higgs boson should exist as a physical particle.
- The gauge boson mass is fixed if $g^2$ and $v$ are known, but the mass of the Higgs boson $h$ depends on the unknown parameter $\lambda$.

# The Higgs Mechanism in the Standard Model

In the standard model the Higgs field is a doublet in *SU(2)*

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix}$$

$$\phi^{+} = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \qquad \phi^{0} = \frac{\phi_3 + i\phi_4}{\sqrt{2}}$$

Complex fields

$$\mathcal{L} = \left(\partial_{\mu}\phi\right)^{\dagger}\left(\partial^{\mu}\phi\right) - \mu^{2}\phi^{\dagger}\phi - \lambda\left(\phi^{\dagger}\phi\right)^{2}$$

$$\phi^{\dagger}\phi = \begin{pmatrix} \phi^{+*} & \phi^{0*} \end{pmatrix}\begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix} = \phi^{+*}\phi^{+} + \phi^{0*}\phi^{0}$$

$$\phi^{\dagger}\phi = \frac{\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2}{2}$$

Let us study the potential: $V(\phi) = \mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$

$V(\phi)$ is invariant under the local gauge transformation:

$$\phi(x) \to \phi'(x) = e^{i\vec{\alpha}(x)\cdot\vec{\tau}/2}\phi(x)$$

For $\mu^2 < 0$ $V(\phi)$ has a minimum at:

$$\phi^\dagger \phi = \frac{-\mu^2}{2\lambda} = \frac{v^2}{2}$$

Let us choose a direction: $\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ $\quad \phi_3 = v, \phi_1 = \phi_2 = \phi_4 = 0$

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We look for equations satisfied by H(x). This choice is always possible thanks to the local gauge invariance.

The original ( $O(4)$ ) symmetry was:

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \text{invariante}$$

By choosing a direction we have three broken global symmetries so three massless Goldstone bosons, which will become the longitudinal parts of $W^\pm$ and $Z^0$.

The electric charge, the weak isospin eigenvalue $T_3$ and the $U(1)$ hypercharge $Y_H$ of the Higgs field are related by:

$$Q = T_3 + \frac{Y_H}{2}$$

We have $Y_H$=1.

The choice of $\phi^0$ as the component which gets a vacuum expectation value ensures the conservation of electric charge.

If the vacuum $\phi_0$ is invariant under some subgroups of the original $SU(2)\times U(1)$ any gauge bosons associated with that subgroup will still be massless.

privo di massa. Since only one component of the Higgs doublet gets a vacuum expectation value the $SU(2)$ symmetry is broken. Since $Y_H\neq 0$ then also the $U(1)$ symmetry is broken. However if we operate with the electric charge $Q$:

$$Q\phi_0 = \left(T_3 + \frac{Y}{2}\right)\phi_0$$

the vacuum is invariant under a transformation $\phi_0 \to \phi_0' = e^{i\alpha(x)Q}\phi_0 = \phi_0$

$U'(1)$ transformation corresponding to the electromagnetic interaction: the photon remains massless.

$$\mathcal{D}_\mu = \partial_\mu - i g_1 \frac{Y}{2} B_\mu - i g_2 \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu$$

When $\phi$ gets a vacuum expectation value, proceeding as before the Lagrangian contains extra terms:

$$\phi^\dagger \left( i g_1 \frac{Y}{2} B_\mu + i g_2 \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu \right)^\dagger \left( i g_1 \frac{Y}{2} B^\mu + i g_2 \frac{\vec{\tau}}{2} \cdot \vec{W}^\mu \right) \phi$$

Using $\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ we obtain:

$$\frac{1}{8} \left| \begin{pmatrix} g_1 B_\mu + g_2 W_\mu^3 & g_2 \left( W_\mu^1 - i W_\mu^2 \right) \\ g_2 \left( W_\mu^1 + i W_\mu^2 \right) & g_1 B_\mu + g_2 W_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$= \frac{1}{8} v^2 g_2^2 \left[ \left( W_\mu^1 \right)^2 + \left( W_\mu^2 \right)^2 \right] + \frac{1}{8} v^2 \left( g_1 B_\mu - g_2 W_\mu^3 \right)^2$$

$$\boxed{\left( \frac{1}{2} v g_2 \right)^2 W_\mu^+ W^{-\mu}}$$

$$\boxed{\frac{1}{2} \left( \frac{1}{2} v \sqrt{g_1^2 + g_2^2} \right)^2 Z_\mu Z^\mu}$$

For a charged boson the mass term would be of the form $m^2W^+W^-$.
Comparing with:

$$\left(\frac{1}{2}vg_2\right)^2 W_\mu^+ W^{-\mu}$$

We obtain:

$$M_W = \frac{vg_2}{2}$$

For a neutral boson the mass term would be of the form $m^2ZZ/2$.

$$M_Z = \frac{1}{2}v\sqrt{g_1^2 + g_2^2}$$

$$M_\gamma = 0$$

$$\frac{M_W}{M_Z} = \cos\theta_W$$

$$\rho = \frac{M_W}{M_Z \cos\theta_W}$$

# Fermion Masses

Interaction Lagrangian of the leptons with the Higgs field:

$$\mathcal{L}_{\text{int}} = g_e \left( \overline{L}\, \phi\, e_R^- + \phi^\dagger\, \overline{e_R^-}\, L \right)$$

Invariant in $SU(2)$. $g_e$ arbitrary constant.

$$\phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$\mathcal{L}_{\text{int}} = \frac{g_e v}{\sqrt{2}} \left( \overline{e_L^-} e_R^- + \overline{e_R^-} e_L^- \right) + \frac{g_e}{\sqrt{2}} \left( \overline{e_L^-} e_R^- + \overline{e_R^-} e_L^- \right) H$$

$H$-$e$ Vertex

$$\frac{g_e}{\sqrt{2}} = \frac{m_e}{v}$$

Mass term for $e$

$$m_e = \frac{g_e v}{\sqrt{2}}$$

$$\mathcal{L}_{\text{int}} = m_e \bar{e} e + \frac{m_e}{v} \bar{e} e H$$

For the neutrino there is no mass term, due to the absence of $\nu_R$. This implies that the neutrino does not interact with the Higgs. If there were a $\nu_R$ it would have $T_3=0$, $Q=0$ and it would not couple to $W^\pm$, $Z^0$, $\gamma$, therefore it would be very difficult to observe.

For quark masses we must take into account also the existence of $u_R$.

$$\psi = \begin{pmatrix} a \\ b \end{pmatrix} \text{ doublet in } SU(2) \quad \Longrightarrow \quad \psi_c = -i\tau_2\psi^* = \begin{pmatrix} -b^* \\ a^* \end{pmatrix} \text{ doublet in } SU(2)$$

$$\phi_c = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} \qquad \phi_c \to \begin{pmatrix} -\frac{v+H}{2} \\ 0 \end{pmatrix}$$

$$\mathcal{L}_{\text{int}} = g_d \overline{Q}_L \phi d_R + g_u \overline{Q}_L \phi_c u_R + \text{h.c.}$$

$$\mathcal{L}_{\text{int}} = m_d \bar{d} d + m_u \bar{u} u + \frac{m_d}{v} \bar{d} d H + \frac{m_u}{v} \bar{u} u H$$

Also in this case the parameters $g_d$ and $g_u$ are arbitrary. Thus masses are included in the Standard Model, but they are not predicted: they must be measured.

This procedure can be repeated for the second and third families.
The Higgs interacts with fermions with a strength proportional to their mass $m_f$, therefore it couples more strongly to the heaviest fermions.