PART III

# Magnetic Field

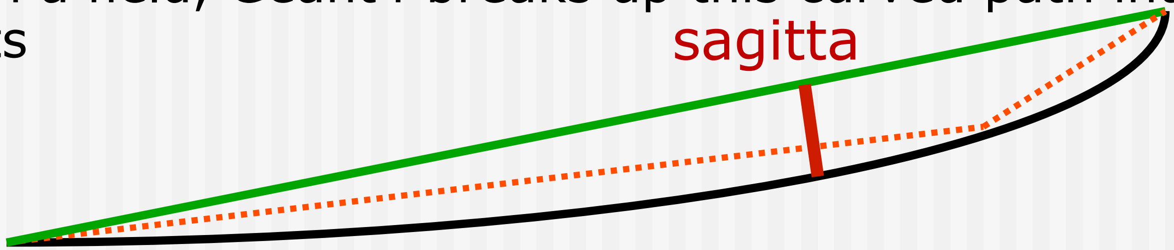
- *Field Propagation & accuracy*
- *Global & Local Field*
- *Tunable parameters*
- *Field Integration*

# Field Propagation

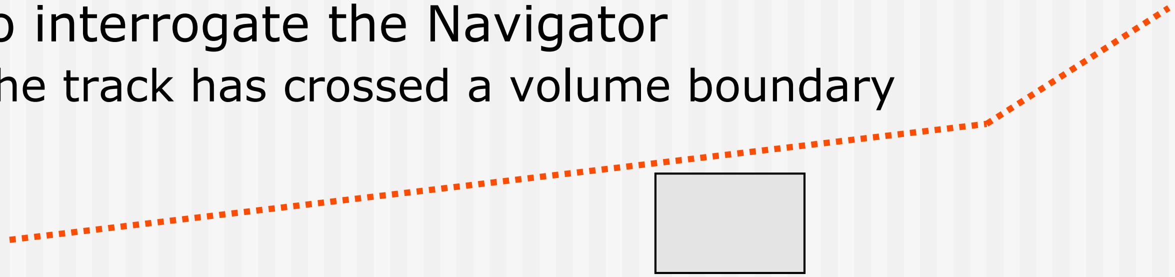
- In order to propagate a particle inside a field (e.g. magnetic, electric or both), we integrate the equation of motion of the particle in the field
- In general this is best done using a **Runge-Kutta** (RK) method for the integration of ordinary differential equations
  - Several RK methods are available
- In specific cases other solvers can also be used:
  - In a uniform field, using the known analytical solution
  - In a nearly uniform but varying field, with RK+Helix

# Chords

- Once a method is chosen that allows Geant4 to calculate the track's motion in a field, Geant4 breaks up this curved path into linear chord segments

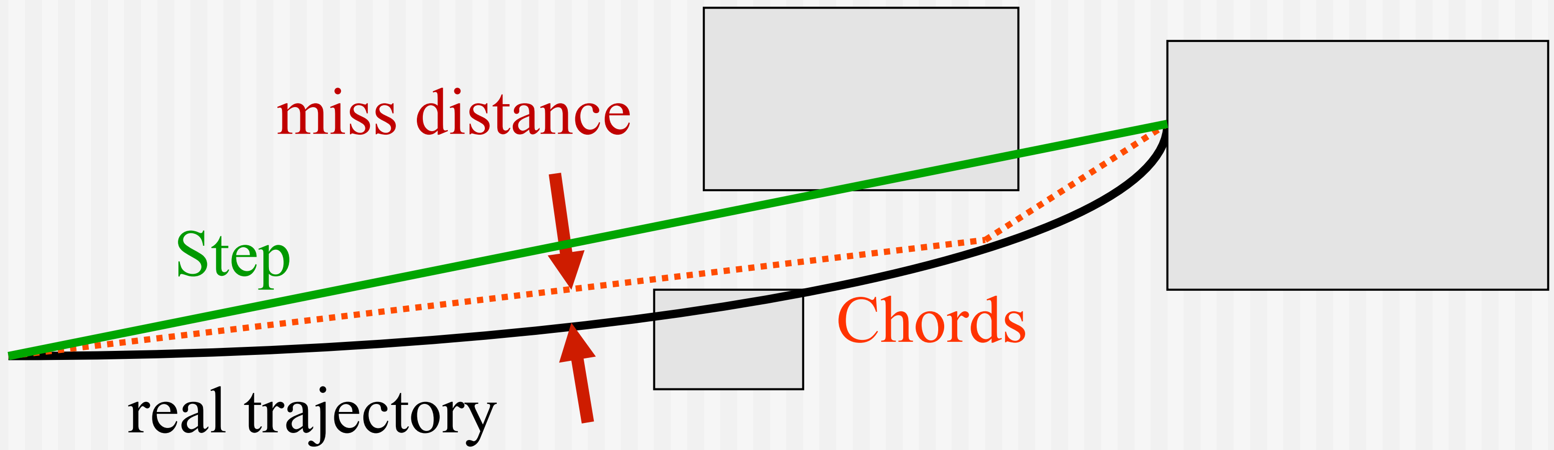



- The chord segments are determined so that they closely approximate the curved path; they're chosen so that their sagitta is small enough
  - The *sagitta* is the maximum distance between the curved path and the straight line
  - Small enough: is smaller than a user-defined maximum
- Chords are used to interrogate the Navigator
  - to see whether the track has crossed a volume boundary



# Intersection accuracy

- The accuracy of the volume intersection can be tuned
  - by setting a parameter called the "miss distance"
    - The *miss distance* is a measure of the error resolution by which the chord may intersect a volume
    - Default *miss distance* is 0.25 mm
    - Setting small *miss distance* may be highly CPU consuming
- One step can consist of more than one chord
  - In some cases, one step consists of several turns



# How to set a Magnetic Field …

- Magnetic field class
  - Uniform field :

    **G4UniformMagField** class object
  - Non-uniform field :

    Concrete class derived from **G4MagneticField**
- Set it to **G4FieldManager** and create a Chord Finder

```
G4FieldManager* fieldMgr =
  G4TransportationManager::GetTransportationManager()
          ->GetFieldManager();
fieldMgr->SetDetectorField(magField);
fieldMgr->CreateChordFinder(magField);
```

# Global & Local Fields

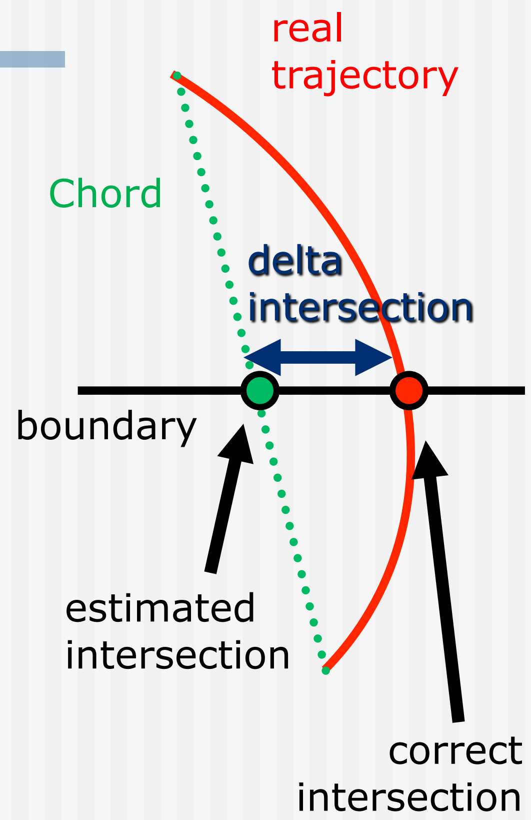
- One field manager is associated with the 'world'
- Other volumes/regions in the geometry can override this
  - An alternative field manager can be associated with any logical volume
    - The field must accept position in global coordinates and return field in global coordinates
  - The assigned field is propagated to all the daughter volumes

```
G4FieldManager* localFieldMgr = new G4FieldManager(magField);
logVolume->setFieldManager(localFieldMgr, true);
```

  where '**true**' makes it *push* the field to all the daughter volumes, unless a daughter has its own field manager.
- It is possible to customise the field propagation classes
  - Choosing an appropriate stepper for the field
  - Setting precision parameters

# Tunable Parameters

- In addition to the "miss distance" there are two more parameters which can be set in order to adjust the accuracy (and performance) of tracking in a field
  - Such parameters govern the accuracy of the intersection with a volume boundary and the accuracy of the integration of other steps
- The "delta intersection" parameter is the accuracy to which an intersection with a volume boundary is calculated.
  - This parameter is especially important because it is used to limit a bias that the algorithm (for boundary crossing in a field) exhibits
  - The intersection point is always on the 'inside' of the curve. By setting a value for this parameter that is much smaller than some acceptable error, one can limit the effect of this bias



# Tunable Parameters

- The “delta one step” parameter is the accuracy for the endpoint of 'ordinary' integration steps, those which do not intersect a volume boundary
  - It is a limit on the estimation error of the endpoint of each physics step
- Parameters “delta intersection” and “delta one step” are strongly coupled
  - These values must be reasonably close to each other (within one order of magnitude)
- Parameters can be set by:

```
theChordFinder->SetDeltaChord ( miss_distance );
theFieldManager->SetDeltaIntersection ( delta_intersection );
theFieldManager->SetDeltaOneStep ( delta_one_step );
```

# Imprecisions …

- … are due to approximating the curved path by linear sections (chords)
  - Parameter to limit this is maximum sagitta $\delta_{chord}$
- … are due to numerical integration, 'error' in final position and momentum
  - Parameters to limit are $\varepsilon_{integration}$ max, min
- … are due to intersecting approximate path with the volume boundary
  - Parameter is $\delta_{intersection}$

# Key elements

- Precision of track required by the user relates primarily to:
  - The precision (error in position) $e_{pos}$ after a particle has undertaken track length $s$
  - Precision DE in final energy (momentum) $\delta_E = \Delta E/E$
  - Expected maximum number $N_{int}$ of integration steps
- Recipe for parameters:
  - Set $\varepsilon_{integration\ (min,\ max)}$ smaller than
    - The minimum ratio of $e_{pos}$ / $s$ along particle's trajectory
    - $\delta_E$ / $N_{int}$ the relative error per integration step (in E/p)
  - Choosing how to set $\delta_{chord}$ is less well-defined. One possible choice is driven by the typical size of the geometry (size of smallest volume)

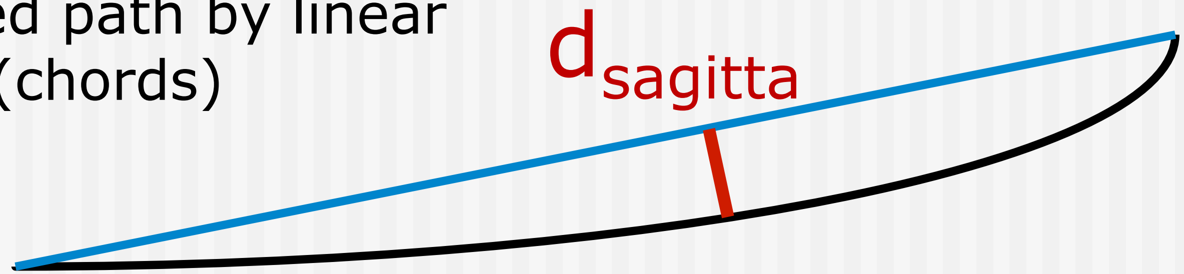
# Where to find the parameters …

| Parameter | Name | Class | Default value |
|---|---|---|---|
| $\delta_{miss}$ | `DeltaChord` | `G4ChordFinder` | `0.25 mm` |
| $d_{min}$ | `stepMinimum` | `G4ChordFinder` | `0.01 mm` |
| $\delta_{intersection}$ | `DeltaIntersection` | `G4FieldManager` | `1 micron` |
| $\varepsilon_{max}$ | `epsilonMax` | `G4FieldManager` | `0.001` |
| $\varepsilon_{min}$ | `epsilonMin` | `G4FieldManager` | $5\ 10^{-5}$ |
| $\delta_{one\ step}$ | `DeltaOneStep` | `G4FieldManager` | `0.01 mm` |

# Volume miss error

- Due to the approximation of the curved path by linear sections (chords)

$d_{sagitta} < \delta_{chord}$




Parameter $\delta_{chord}$ value

- Parameter $\delta_{chord}$ = maximum sagitta
- Effect of this parameter as $\delta_{chord} \longrightarrow 0$

$$s_{1step}^{propagator} \sim (8\,\delta_{chord}\,R_{curv})^{1/2}$$

so long as $s^{propagator} \Leftarrow s^{phys}$ and $s^{propagator} > d_{min}(integr)$

# Integration error

Due to error in the numerical integration (of equations of motion)

Parameter(s): $\varepsilon_{integration}$

- The size $s$ of the step is limited so that the estimated errors of the final position $\Delta r$ and momentum $\Delta p$ are both small enough:

$$\max( \| \Delta\vec{r} \| / s \ , \ \|\Delta\vec{p}\| / \|\vec{p}\| ) < \varepsilon_{integration}$$

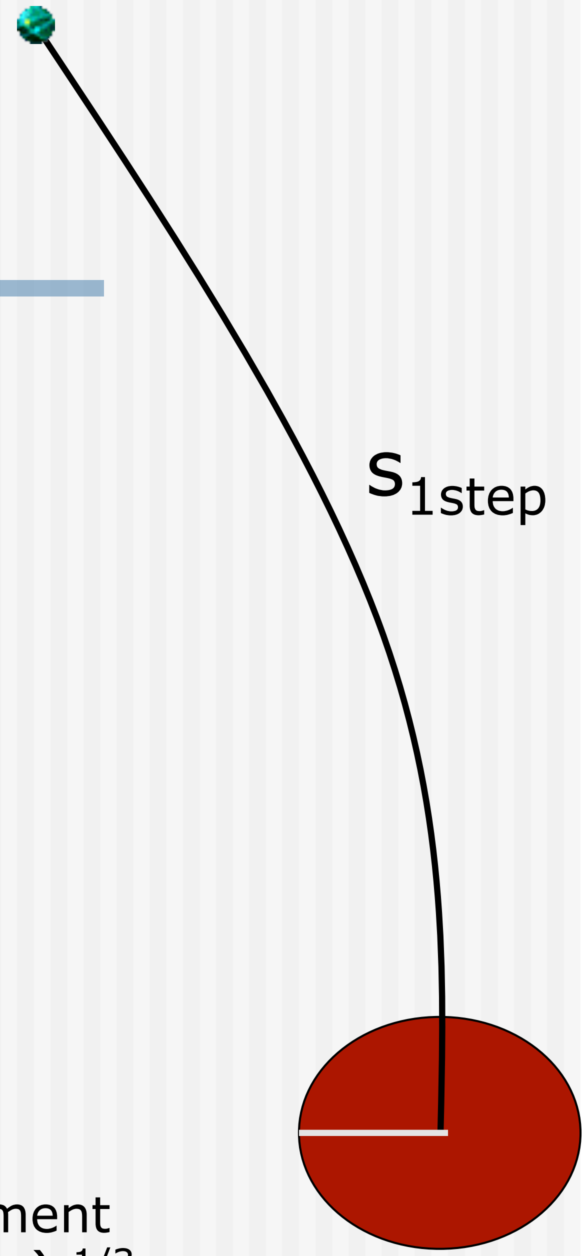
- For ClassicalRK4 Stepper

$$s_{1step}^{integration} \sim (\varepsilon_{integration})^{1/3}$$

for small enough $\varepsilon_{integration}$

- The integration error should be influenced by the precision of the knowledge of the field (measurement or modeling ).

$$N_{steps} \sim (\varepsilon_{integration})^{-1/3}$$




$\Delta\vec{r}$

# Integration error - 2

- $\varepsilon_{integration}$ is currently represented by 3 parameters

*Defaults*

  - **`epsilonMin`**, a minimum value (used for big steps) — *$0.5*10^{-7}$*
  - **`epsilonMax`**, a maximum value (used for small steps) — *0.05*
  - **`DeltaOneStep`**, a distance error (for intermediate steps) — *0.25 mm*

$$\varepsilon_{integration} = \delta_{\text{one step}} / s_{\text{physics}}$$

- Determining a reasonable value
  - Suggested to be the minimum of the ratio (accuracy/ distance) between sensitive components, …
- Another parameter

*Default*

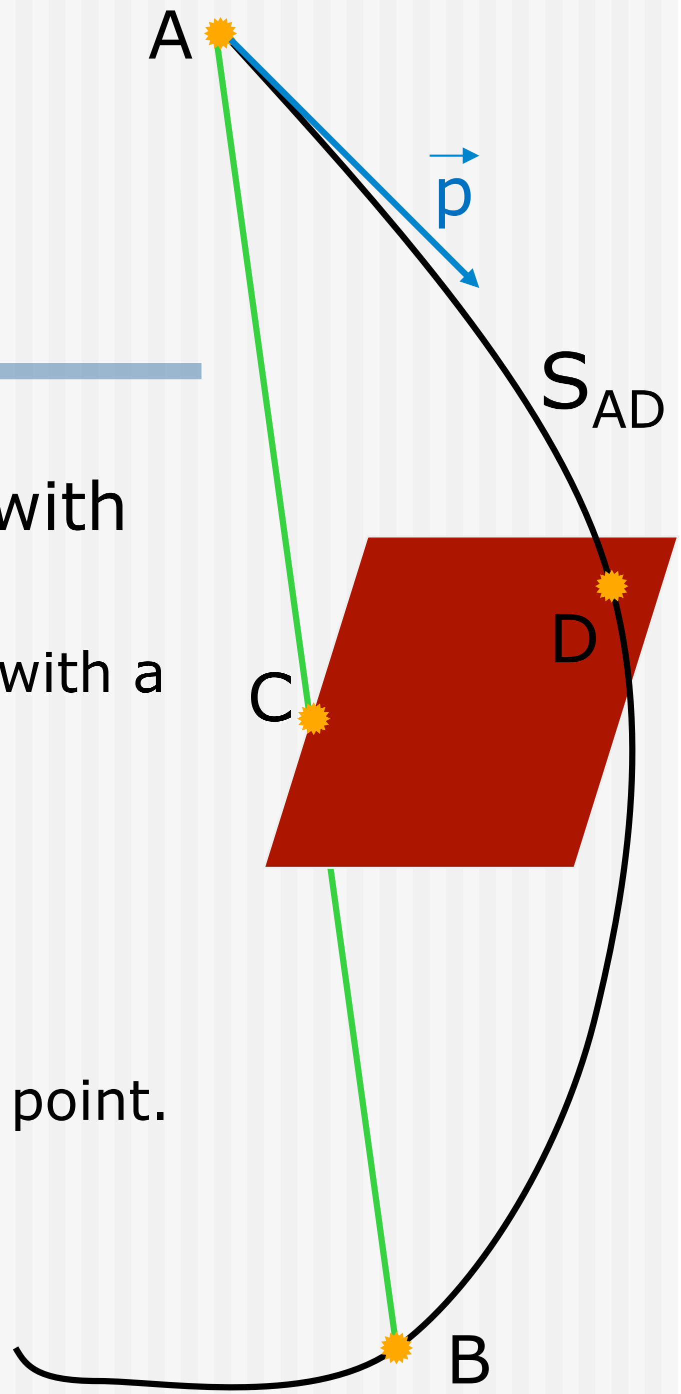
  - $d_{min}$ is the minimum step of integration — *0.01 mm*

# Intersection error

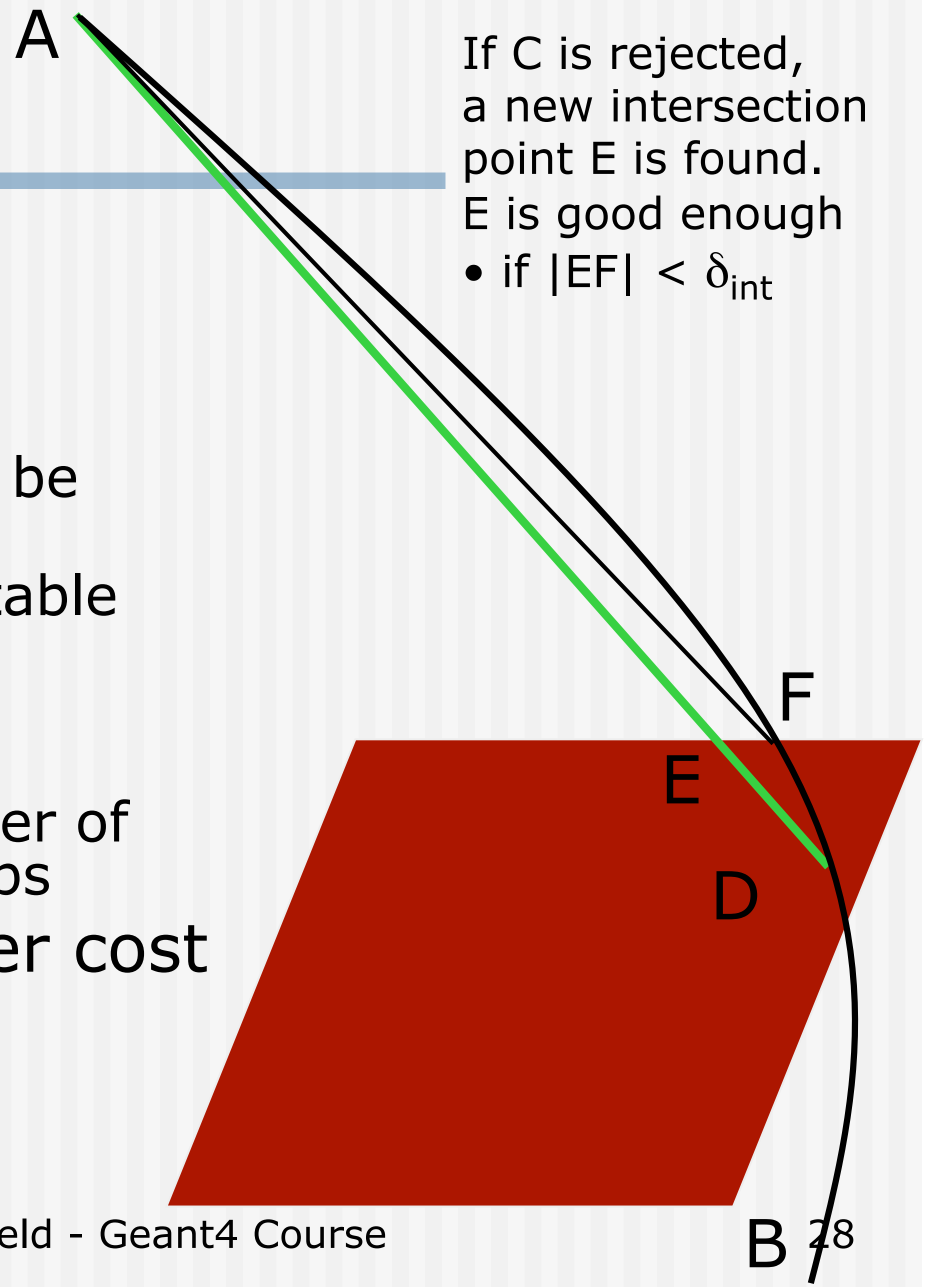
- In intersecting approximate path with volume boundary
  - In trial step AB, intersection is found with a volume at C
  - Step is broken up, choosing D, so

    $S_{AD} = S_{AB} * |AC| \; / \; |AB|$

  - If $|CD| < \delta_{intersection}$
    - Then C is accepted as intersection point.
  - So $\delta_{int}$ is a position error/bias



# Intersection error - 2




If C is rejected,
a new intersection
point E is found.
E is good enough
- if |EF| < $\delta_{int}$

- $\delta_{int}$ must be small
  - compared to tracker hit error
  - its effect on reconstructed momentum estimates should be calculated
    - … and limited to be acceptable
- Cost of small $\delta_{int}$ is less
  - than making $\delta_{chord}$ small
  - it is proportional to the number of boundary crossings – not steps
- Quicker convergence / lower cost
  - Possible with optimization

# Customizing field integration

- **Runge-Kutta** integration is used to compute the motion of a charged track in a general field. There are many general steppers from which to choose
  - Low and high order, and specialized steppers for pure magnetic fields
- By default, Geant4 uses the classical fourth-order **Runge-Kutta** stepper (**`G4ClassicalRK4`**), which is general purpose and robust.
  - If the field is known to have specific properties, lower or higher order steppers can be used to obtain the results of same quality using fewer computing cycles
- If the field is calculated from a field map, a lower order stepper is recommended
  - The less smooth the field is, the lower the order of the stepper that should be used
  - The choice of lower order steppers includes the third order stepper (**`G4SimpleHeum`**) the second order (**`G4ImplicitEuler`** and **`G4SimpleRunge`**), and the first order (**`G4ExplicitEuler`**)
    - A first order stepper would be useful only for very rough fields
    - For somewhat smooth fields (intermediate), the choice between second and third order steppers should be made by trial and error

# Customizing field integration

- Trying a few different types of steppers for a particular field or application is suggested if maximum performance is a goal
- Specialized steppers for pure magnetic fields are also available
  - They take into account the fact that a local trajectory in a slowly varying field will not vary significantly from a helix
  - Combining this in with a variation, the Runge-Kutta method can provide higher accuracy at lower computational cost when large steps are possible
- To change the stepper:

```
theChordFinder
   ->GetIntegrationDriver()
   ->RenewStepperAndAdjust( newStepper );
```

# Other types of field

- It is possible to create any specialised type of field:
  - inheriting from **G4VField**
  - Associating an *Equation of Motion* class (inheriting from **G4EqRhs**) to simulate other types of fields
  - Fields can be time-dependent
- For pure electric field:
  - **G4ElectricField** and **G4UniformElectricField** classes
- For combined electromagnetic field:
  - **G4ElectroMagneticField** class
- The *Equation of Motion* class for electromagnetic field is **G4MagElectricField**.

```
G4ElectricField* fEMfield
   = new G4UniformElectricField( G4ThreeVector(0., 100000.*kilovolt/cm, 0.) );
G4EqMagElectricField* fEquation = new G4EqMagElectricField(fEMfield);
G4MagIntegratorStepper* fStepper = new G4ClassicalRK4( fEquation, nvar );
G4FieldManager* fFieldMgr
   = G4TransportationManager::GetTransportationManager()-> GetFieldManager();
fFieldManager->SetDetectorField( fEMfield );
G4MagInt_Driver* fIntgrDriver
   = new G4MagInt_Driver(fMinStep, fStepper, fStepper->GetNumberOfVariables() );
G4ChordFinder* fChordFinder = new G4ChordFinder(fIntgrDriver);
```