

Hadron Structure from Deep-Inelastic Scattering Experiments

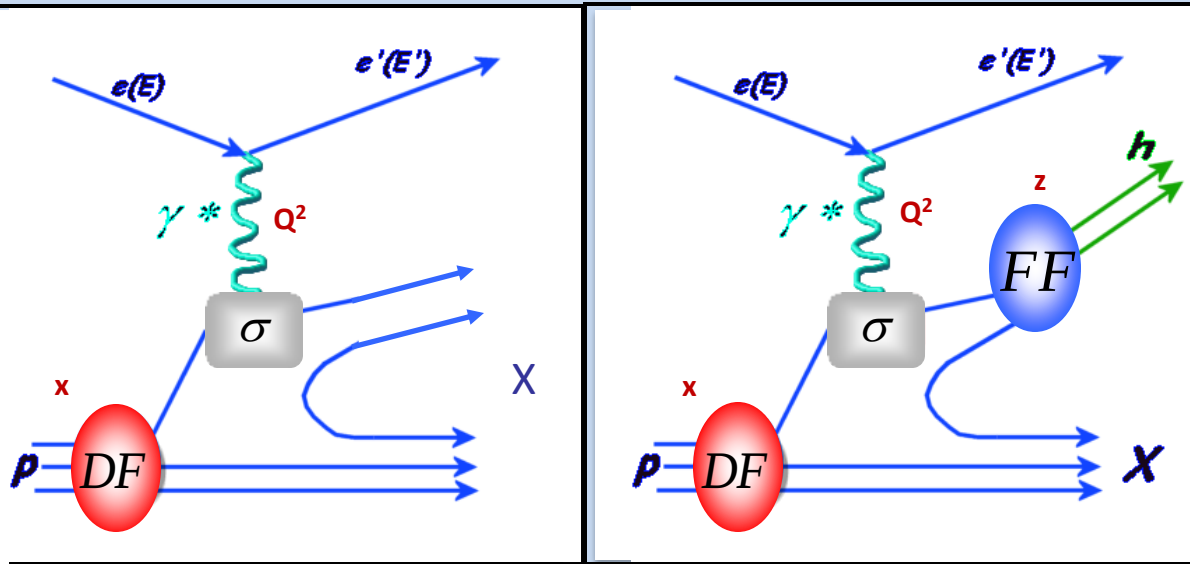
Marco Contalbrigo – INFN Ferrara

GGI School – Frontiers in Nuclear and Hadronic Physics - Firenze, 28 February 2025

Deep-Inelastic Scattering

Inclusive

Semi-inclusive



SFs (x, Q^2)

DFs (x, Q^2) + FF(z, Q^2)

Structure functions
(unpolarized, helicity)

Parton distributions + fragmentation

$$D_u^{p+}(z) > D_u^{p-}(z)$$

Sum over quark charges

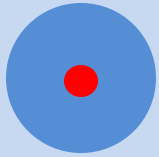
Flavor sensitivity

$$d^2s \propto \sum_q e_q^2 q(x)$$

$$d^3s^h \propto \sum_q e_q^2 q(x) D_q^h(z)$$

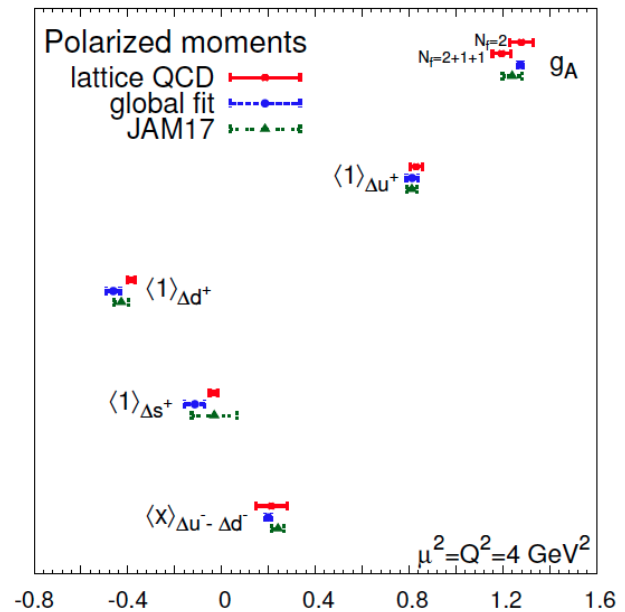
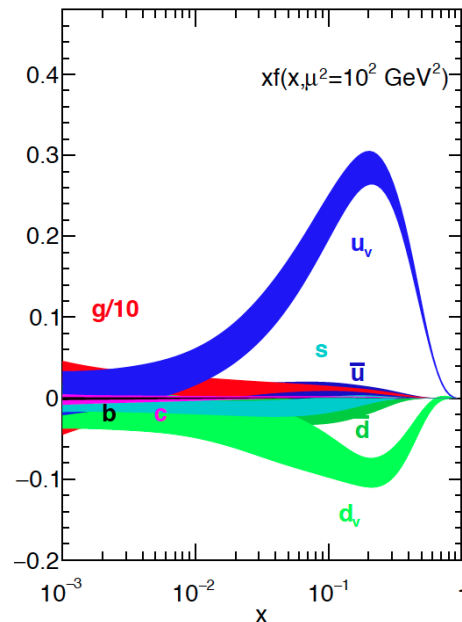
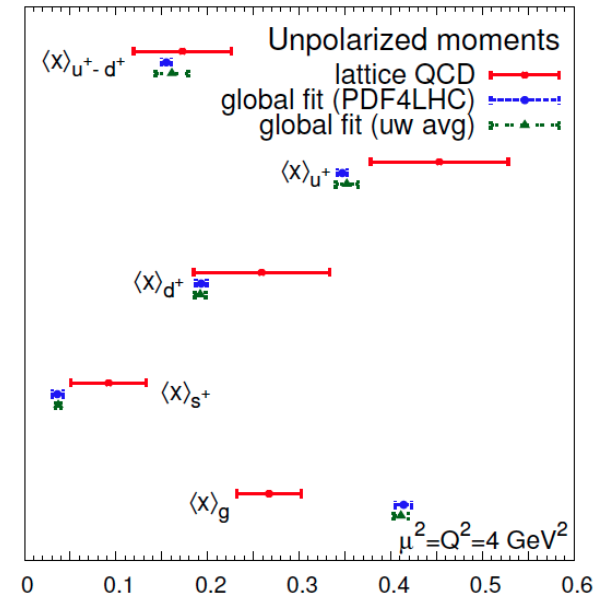
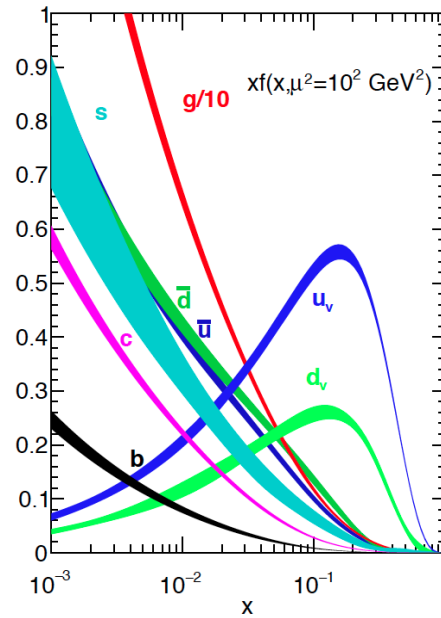
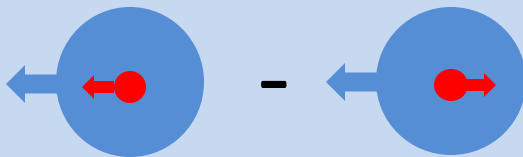
Parton Content

Unpolarized moments



H-W Lin++ [1711.07916]

Polarized (helicity) moments



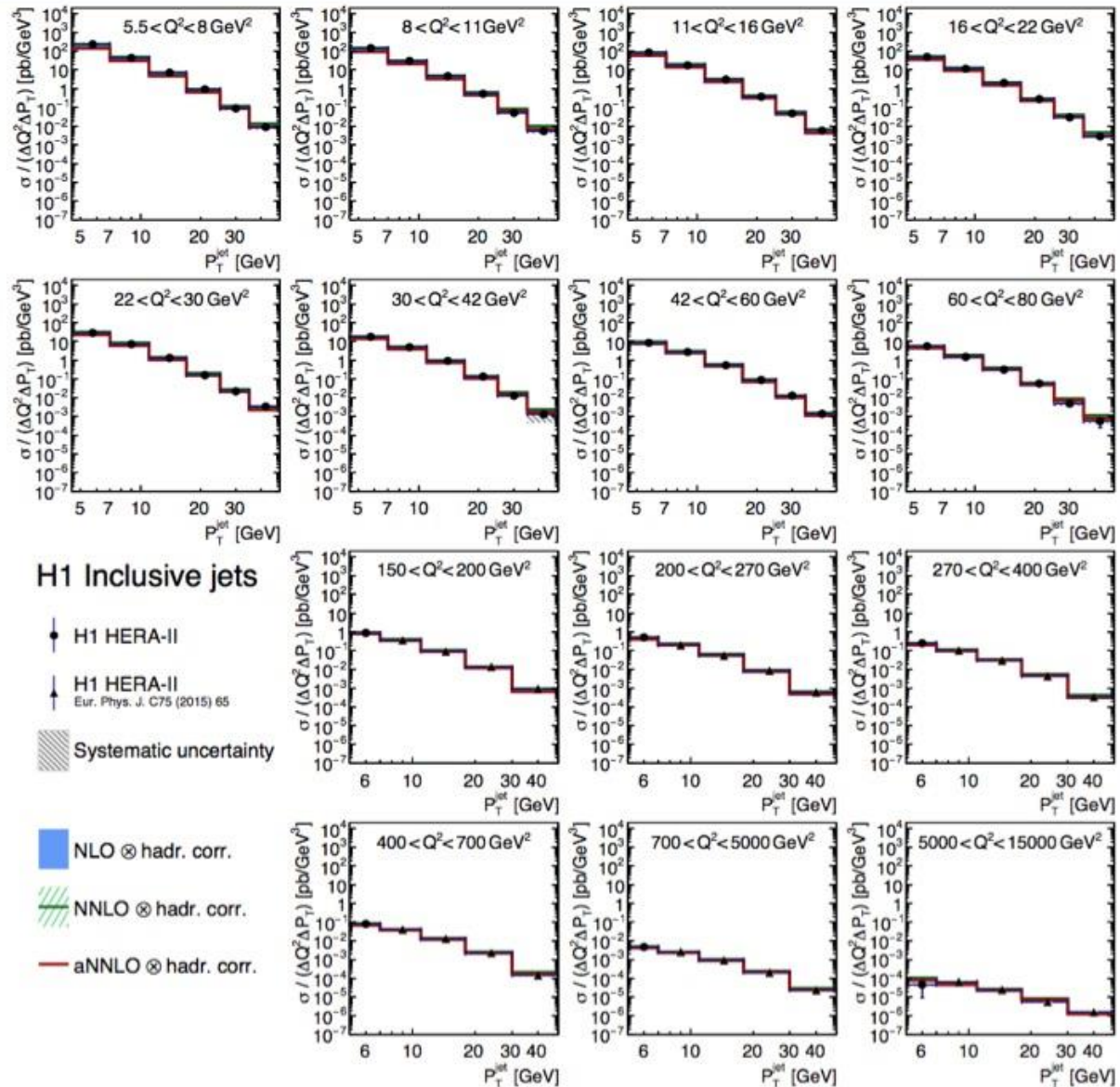
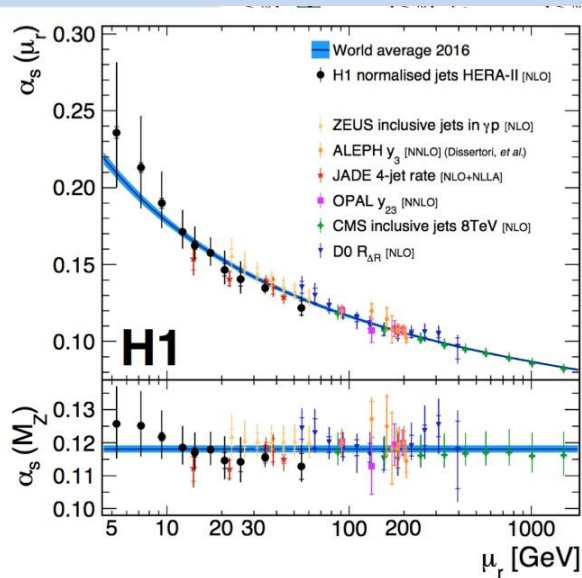
High-Energy e-p

Good perturbative description
(hard gluon emission)

$$p_T > 5 \text{ GeV} \quad Q^2 > 5 \text{ GeV}^2$$

Part in a $p_T \ll Q$ TMD regime

H1 [arXiv: 1611.03421]



Non Perturbative Physics

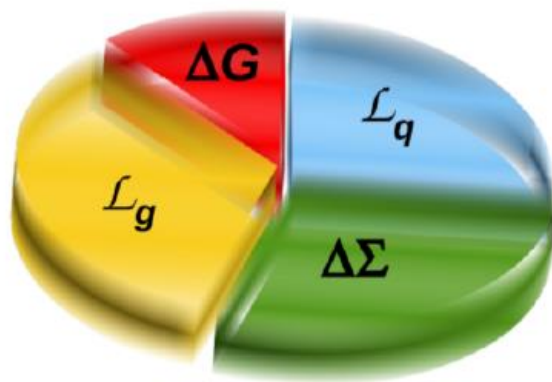
pQCD

Can QCD be a precision science ?

Should not be confused with pQCD, which already can,
but is not touching the intimate nature of the strong interaction

Proton Spin Budget

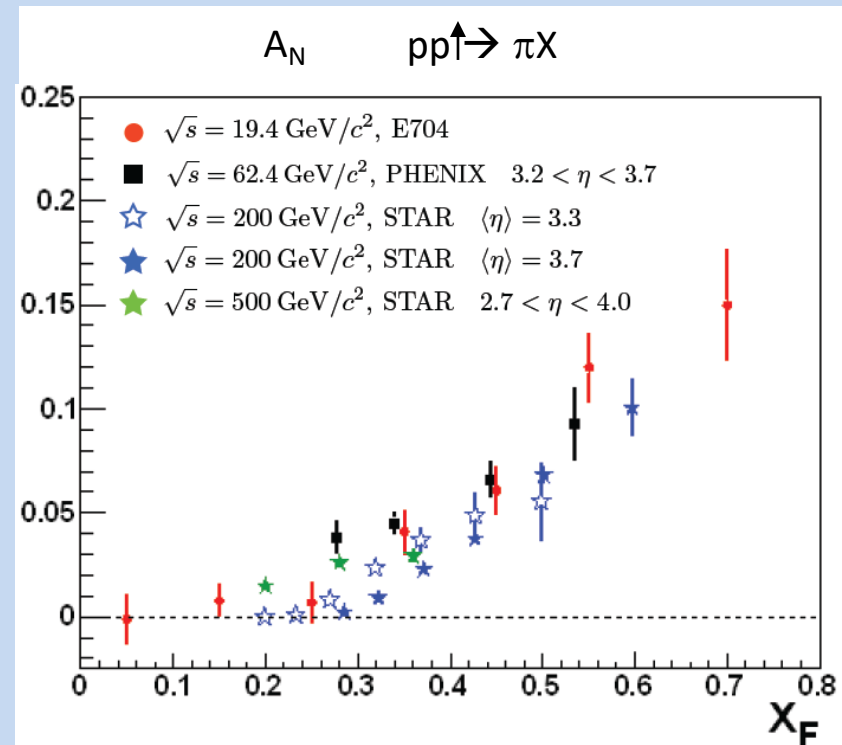
■ Gluon Spin ■ Gluon angular momentum
■ Quark Spin ■ Quark Angular Momentum



$$\frac{1}{2} = \frac{1}{2} \sum_f (q_f^+ - q_f^-) + L_q + \Delta G + L_g$$

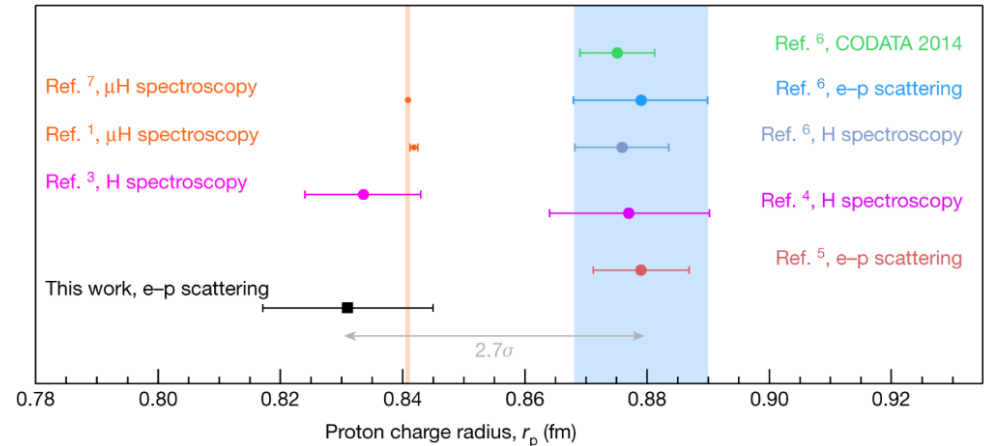
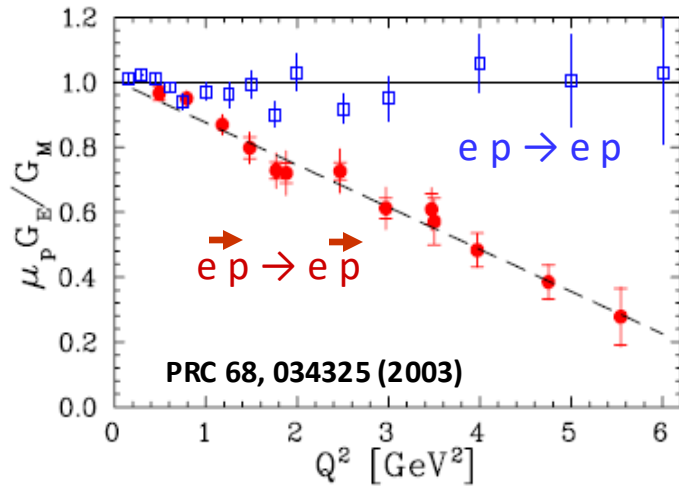


Single Spin Asymmetries



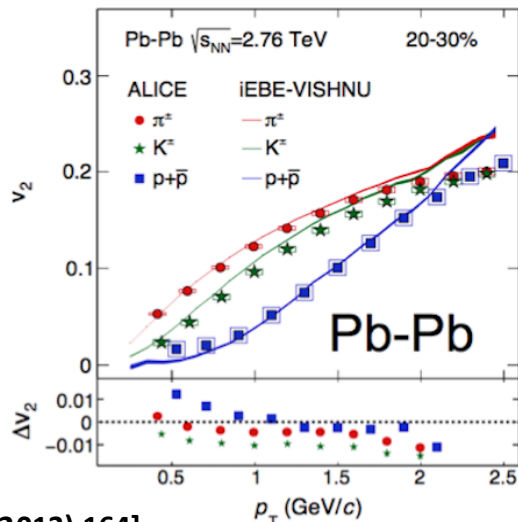
Still Surprising Proton

Do we control the proton form factor and radius ?



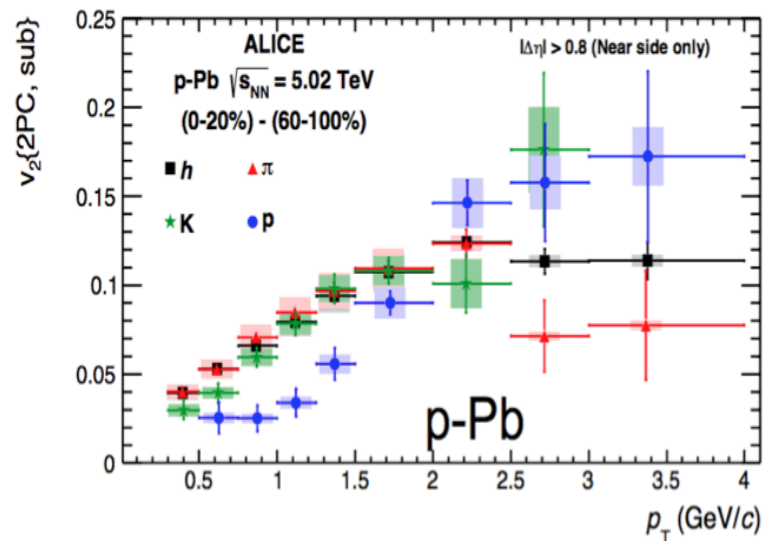
Nature 575 147 (2019)

Is there a collective motion in small systems ?



ALICE [PLB 726 (2013) 164]

$$\frac{dN}{d\varphi} = \frac{N_0}{2\pi} (1 + 2v_1 \cos(\varphi - \Psi_1) + 2v_2 \cos[2(\varphi - \Psi_2)] + \dots)$$



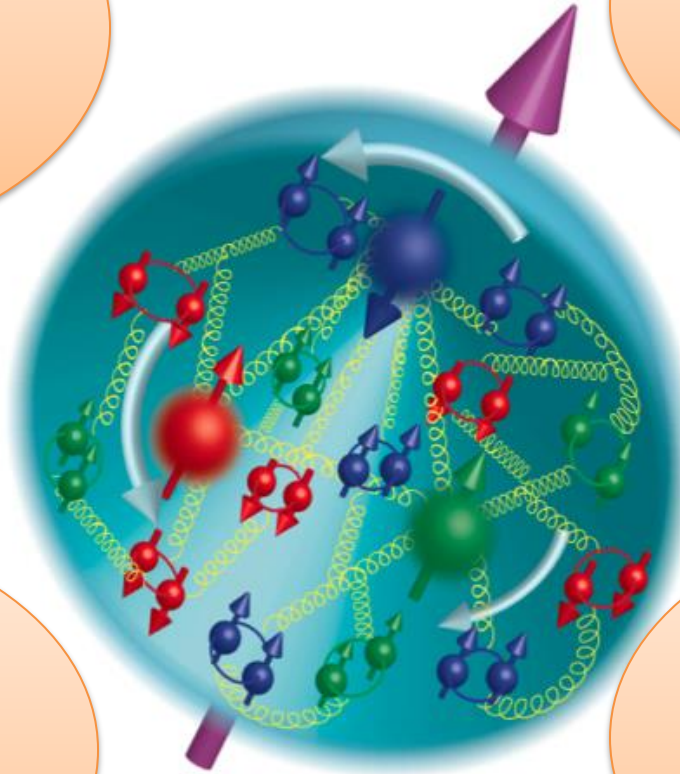
The QCD View

Dynamic Spin

- Parton polarization
- Orbital motion
- Form Factors
- Magnetic Moment

Parton Correlations

- dPDFs
- Short range
- MPI



Hadronization

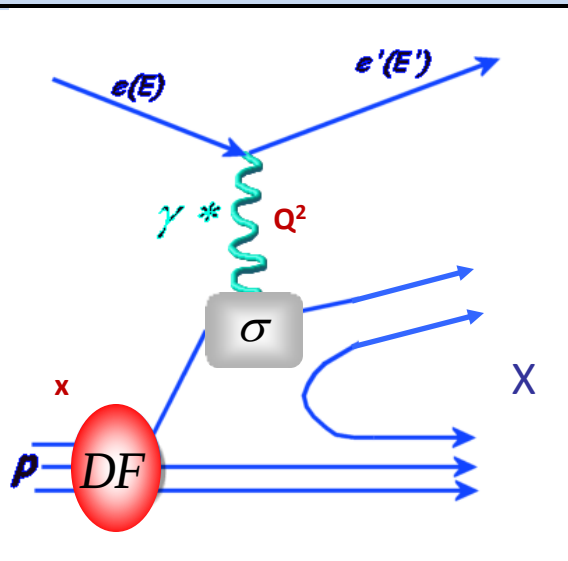
- Spin-orbit effects
- Parton energy loss
- Jet quenching

Color charge density

- Nucleon tomography
- Diffractive physics
- Gluon saturation
- Color force

Deep-Inelastic Scattering

Inclusive



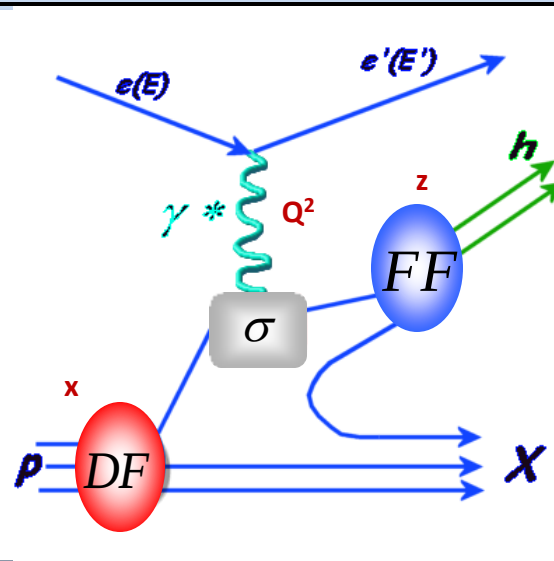
SFs (x, Q^2)

Structure functions
(unpolarized, helicity)

Sum over quark charges

$$d^2 s \mu \sum_q e_q^2 q(x) F_2$$

Semi-inclusive



DFs (x, Q^2) + FF(z, Q^2)

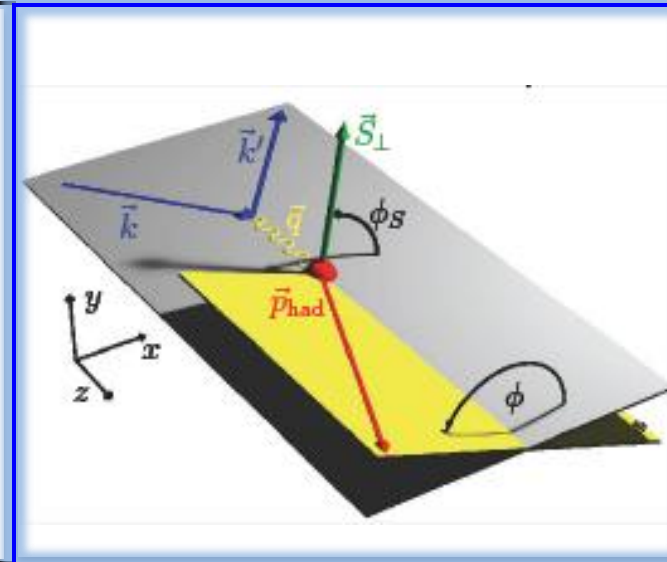
Parton distributions + fragmentation

$$D_u^{p+}(z) > D_u^{p-}(z)$$

Flavor sensitivity

$$d^3 s^h \mu \sum_q e_q^2 q(x) D_q^h(z)$$

Semi-inclusive



TMDs ($x, z, P_{h\perp}, Q^2$)

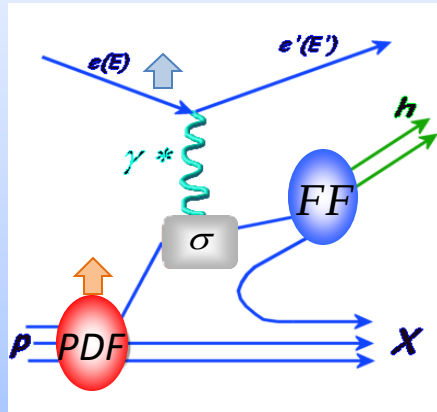
Transverse momentum
dependent parton functions

Spin-Orbit effects

$$d^6 s^h \mu \sum_q e_q^2 q(x, k_T) \ddot{A} D_q^h(z, p_T)$$

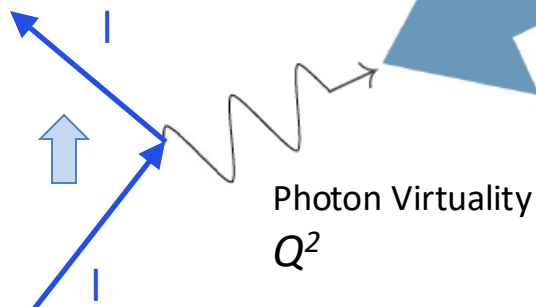
Rich and Involved phenomenology !!

The 3D Nucleon Structure from SIDIS

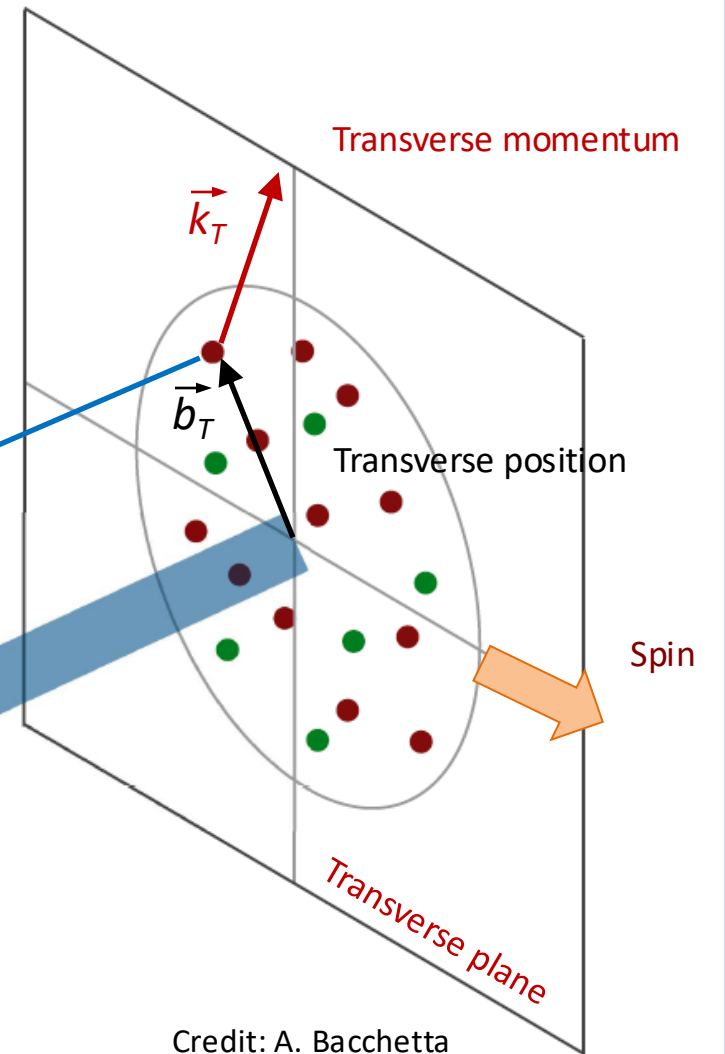


Confinement Scale

Longitudinal momentum
 $k^+ = xP^+$



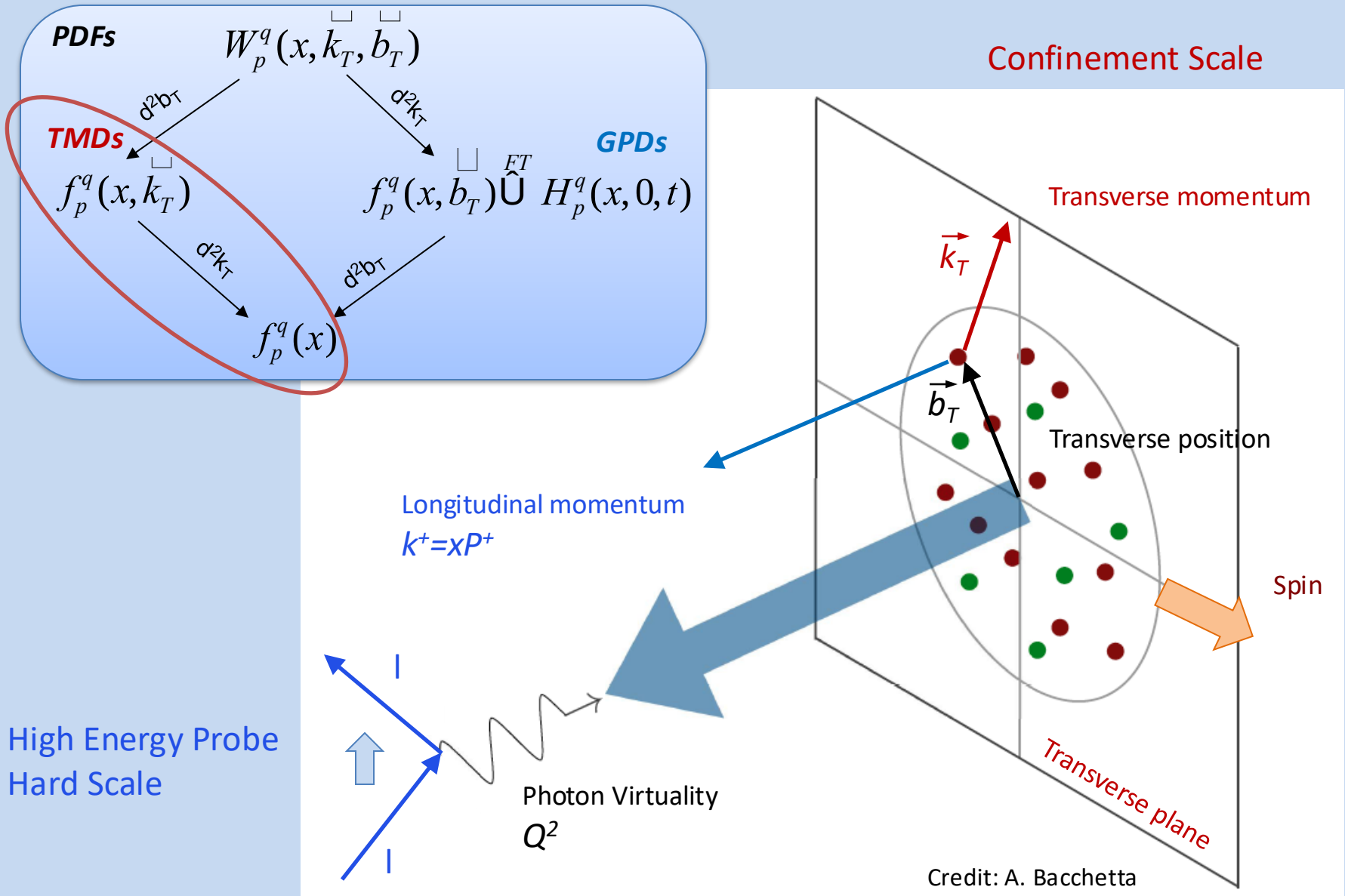
Photon Virtuality
 Q^2



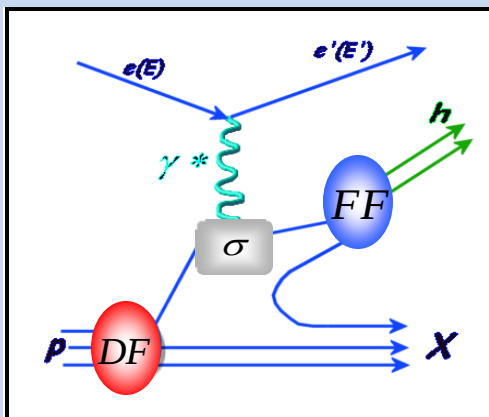
Credit: A. Bacchetta

High Energy Probe
Hard Scale

The 3D Nucleon Structure from SIDIS

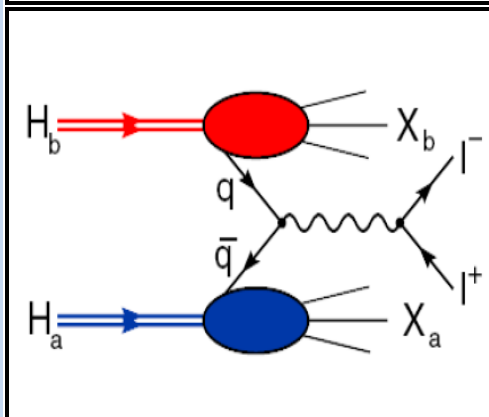


Physics Channels



SIDIS: rich phenomenology, the most explored so far

$$\text{SIDIS} \quad s^{ep \rightarrow ehX} = \int_q \text{DF} \otimes s^{eq \rightarrow eq} \otimes \text{FF}$$

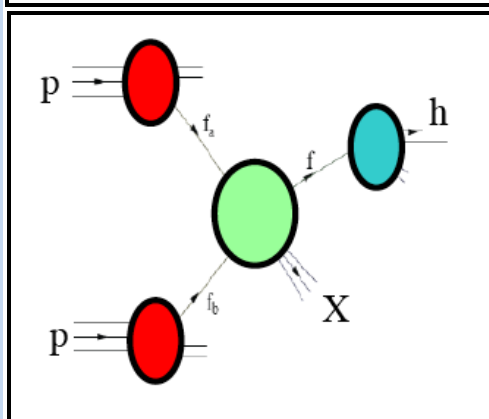


e^+e^- colliders: powerful fragmentation laboratories

$$e^+e^- \quad s^{ee \rightarrow hhX} = \int_q s^{qq \rightarrow ee} \otimes \text{FF} \otimes \text{FF}$$

DY: challenging for experiments (only unpolarized so far)

$$\text{DY} \quad s^{pp \rightarrow eeX} = \int_q \text{DF} \otimes \text{DF} \otimes s^{qq \rightarrow ee}$$

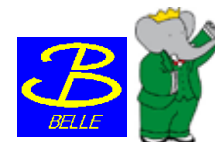


Hadron reactions: challenging for theory (ISI + FSI)

$$\text{pp} \quad s^{pp \rightarrow hX} = \int_q \text{DF} \otimes \text{DF} \otimes s^{qq \rightarrow qq} \otimes \text{FF}$$



Jefferson Lab



BES III



Fermilab

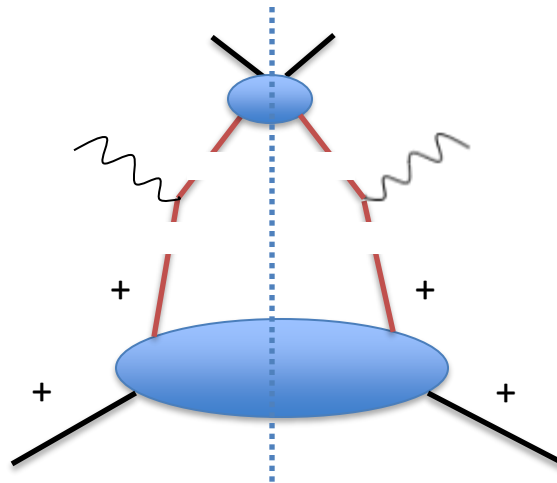
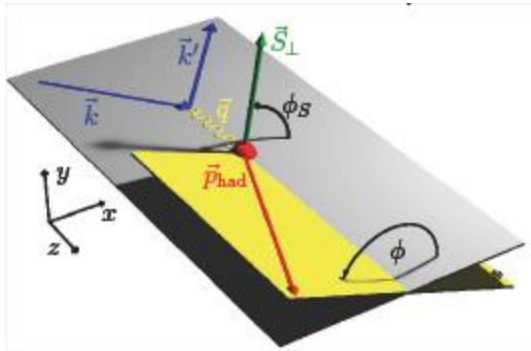
BROOKHAVEN
NATIONAL LABORATORY



BROOKHAVEN
NATIONAL LABORATORY



$$\frac{d^6 S}{dx dQ^2 dz dP_h df df_S} = \mu \left[\begin{aligned} & \frac{1}{2} F_{UU}^{LT} + e \cos(2f) F_{UU}^{\cos(2f)} + S_L \frac{1}{2} F_{UL}^{\sin(2f)} \\ & + S_T \frac{1}{2} F_{UT}^{\sin(f-f_S)} + e \sin(f+f_S) F_{UT}^{\sin(f+f_S)} + e \sin(3f-f_S) F_{UT}^{\sin(3f-f_S)} \\ & + S_L \frac{1}{e} \sqrt{1-e^2} F_{LL} + S_T \frac{1}{e} \sqrt{1-e^2} \cos(f-f_S) F_{LT}^{\cos(f-f_S)} + O\left(\frac{1}{Q}\right) \end{aligned} \right]$$



Quark fragmentation

TMD Factorization
holds for $p_T \ll Q$

Quark parton distribution

Wide kinematic coverage is needed to resolve the convolution

$$F_{UU} = f \otimes D = \int d^2 p_T d^2 k_T d^2 \mathbf{p}_T \delta^2(\mathbf{P}_h - \mathbf{z} \mathbf{k}_T - \mathbf{p}_T) w(\mathbf{k}_T, \mathbf{p}_T) f^q(x, k_T^2) D^q(z, p_T^2)$$

$$\begin{aligned}
 \frac{d^6 S}{dx dQ^2 dz dP_{h_1} df df_S} = & \mu_e^2 F_{UU}^{LT} + e \cos(2f) F_{UU}^{\cos(2f)} + S_L e \sin(2f) F_{UL}^{\sin(2f)} \\
 & + S_T e \sin(f - f_S) F_{UT}^{\sin(f-f_S)} + e \sin(f + f_S) F_{UT}^{\sin(f+f_S)} + e \sin(3f - f_S) F_{UT}^{\sin(3f-f_S)} \\
 & + S_L / e \sqrt{1 - e^2} F_{LL}^{\hat{g}_1} + S_T / e \sqrt{1 - e^2} \cos(f - f_S) F_{LT}^{\hat{g}_{1T}} + O\left(\frac{1}{Q^2}\right)
 \end{aligned}$$

Access to independent correlators bringing information on the confinement dynamics

$$f(x) \longrightarrow f(x, k_T)$$

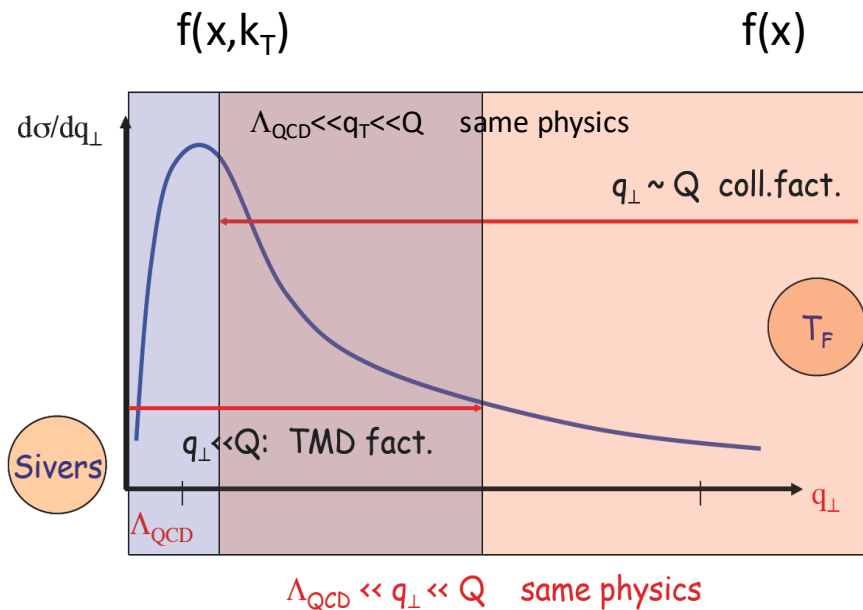
$$D(x) \longrightarrow D(z, p_T)$$

nucleon polarisation	quark polarisation			
	N/q	U	L	T
	U	f_1		$h_1^\perp$
	L		g_1	$h_{1L}^\perp$
	T	$\hat{f}_{1T}$	$\hat{g}_{1T}$	$h_1, \hat{h}_{1T}$

quark polarisation			
N/q	U	L	T
U	D_1		$\hat{H}_1$

SIDIS Cross-Section and TMDs

$$\begin{aligned}
 \frac{d^6 S}{dx dQ^2 dz dP_{h_1} df df_s} &= \mu_{LT} \left[\overset{f_1}{e} F_{UU} + e \cos(2f) F_{UU}^{\cos(2f)} \right] + S_L \left[\overset{h_{1L}}{e} \sin(2f) F_{UL}^{\sin(2f)} \right] \\
 &+ S_T \left[\overset{f_{1T}}{e} \sin(f - f_s) F_{UT}^{\sin(f-f_s)} + e \sin(f + f_s) F_{UT}^{\sin(f+f_s)} + e \sin(3f - f_s) F_{UT}^{\sin(3f-f_s)} \right] \\
 &+ S_L / e \sqrt{1 - e^2} F_{LL} + S_T / e \sqrt{1 - e^2} \cos(f - f_s) F_{LT}^{\cos(f-f_s)} + O\left(\frac{1}{Q}\right)
 \end{aligned}$$



TMD Factorization:

Holds for $p_T \ll Q$

--> Proper domain of phenomenological fits ?

Not trivial gauge invariance

--> Sign change from SIDIS to DY ($f_{1T}^\perp$, $h_{1L}^\perp$)

Peculiar Q^2 evolution (~~DGLAP~~)

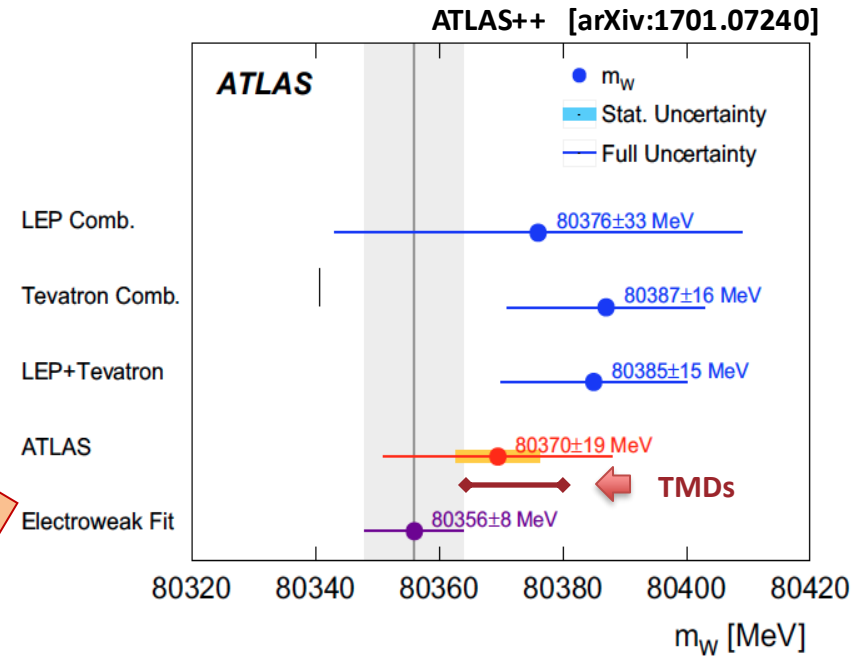
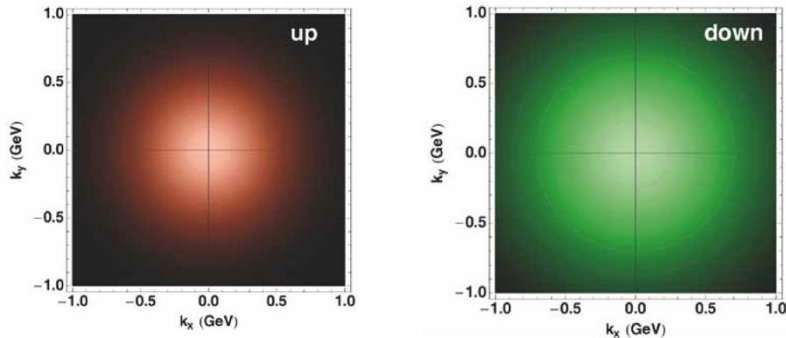
--> Non-perturbative inputs from data

Parton Number Density

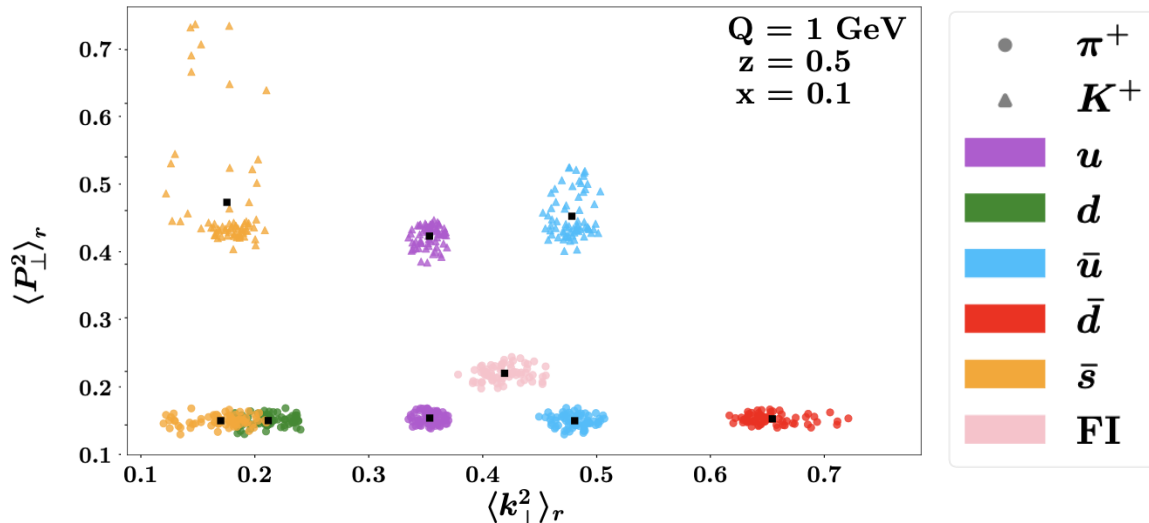


Unpolarized TMDs

$$\langle P_h^2 \rangle = z^2 \langle k_T^2 \rangle + \langle p_T^2 \rangle$$



A. Bacchetta++ [arXiv:2405.13833]



A. Bacchetta++ [arXiv:1807.02101]

MeV

$$m_W = 80370 \pm 7 \text{ (stat.)}$$

$$\pm 11 \text{ (exp. syst.)}$$

$$\pm 14 \text{ (mod. syst.)}$$

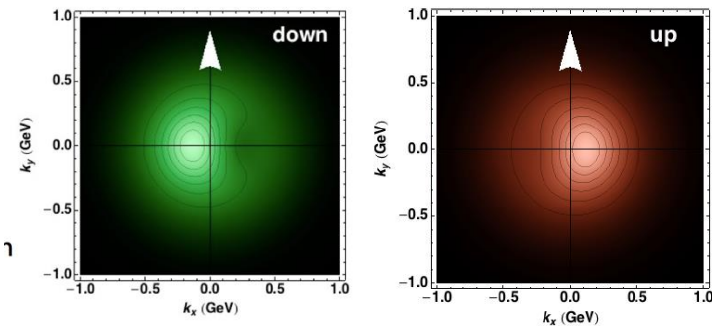
$$+9 / -6 \text{ (TMDs)}$$

Spin-Orbit Effect



Spin-Orbit Effect: Sivers

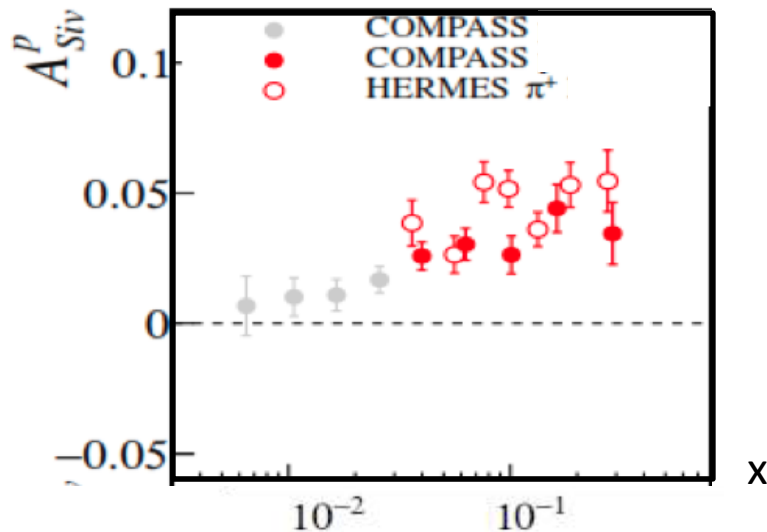
$$S_{UT}^{\sin(f-f_S)} \mu f_{1T}^{\wedge} \ddot{A} D_1$$



Sivers from polarized SIDIS

HERMES [arXiv:0906.3918]

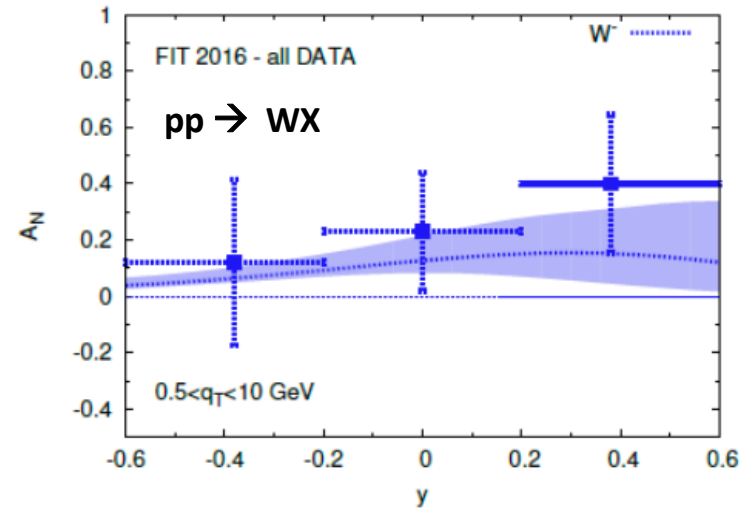
COMPASS [arXiv:1205.5122]



(-1)

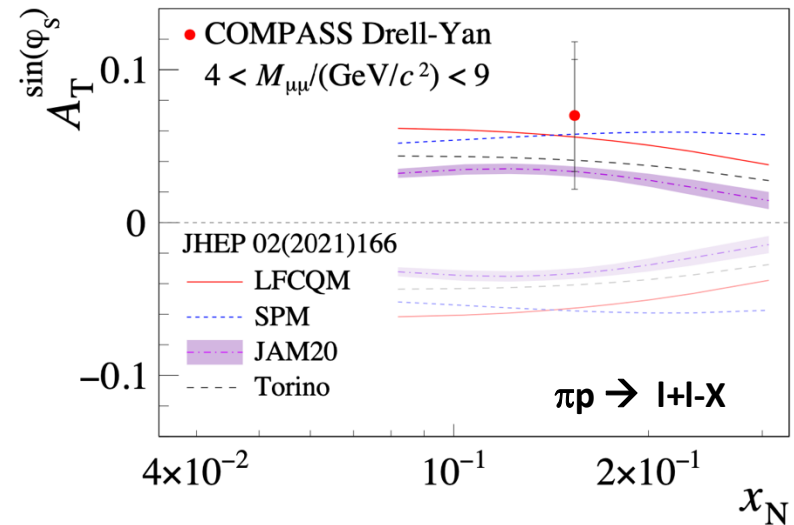


STAR [arXiv: 1511.06003]



Sivers from hadronic reactions

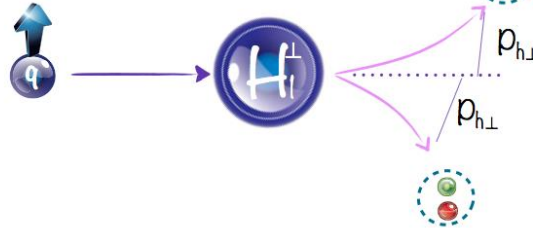
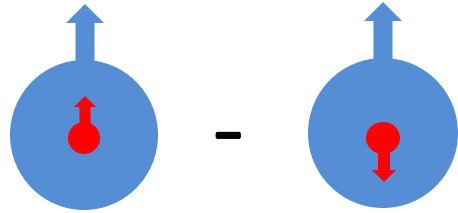
COMPASS [arXiv: 2312.17379]



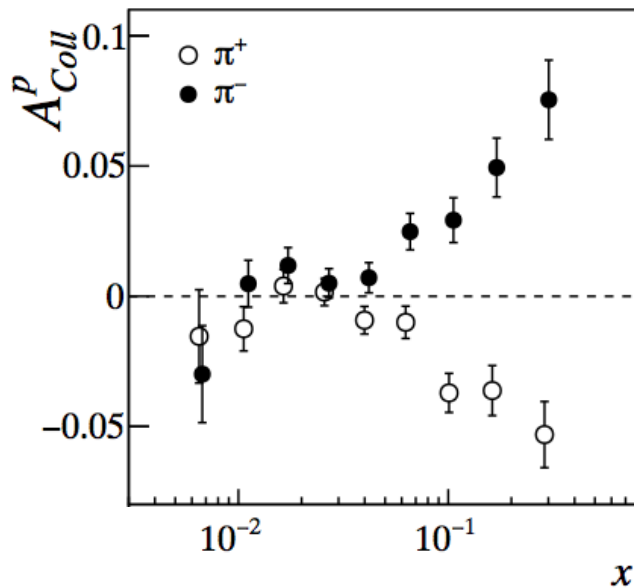
Spin-Orbit Effect: Collins

$$S_{UT}^{\sin(f+f_S)} \mu h_1 \hat{A} H_1^{\wedge}$$

$$Q^2 \sim 5-7 \text{ GeV}^2$$

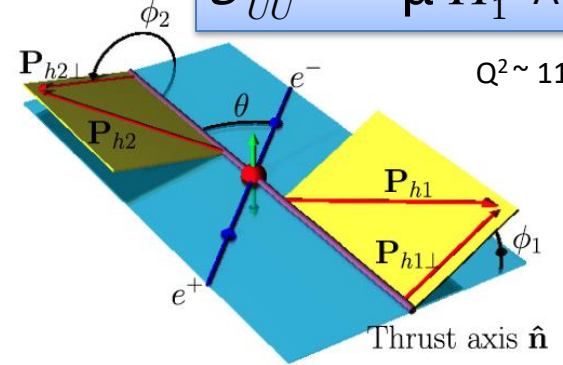


HERMES [arXiv 0408013]
 HERMES [arXiv 0906.3918]
 COMPASS [arXiv 1005.5609]
 COMPASS [arXiv 1408.4405]

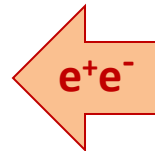
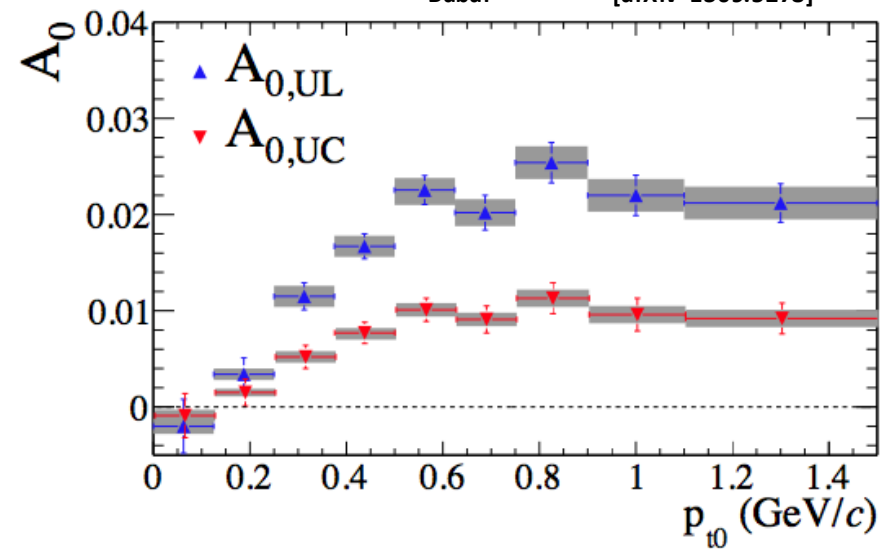


$$S_{UU}^{\sin(f_1+f_2)} \mu H_1^{\wedge} \hat{A} H_1^{\wedge}$$

$$Q^2 \sim 110 \text{ GeV}^2$$



Belle [talk at DIS2014]
 BESIII [arXiv 1507.06824]
 Babar [arXiv 1309.5278]

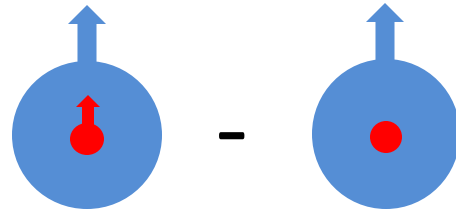
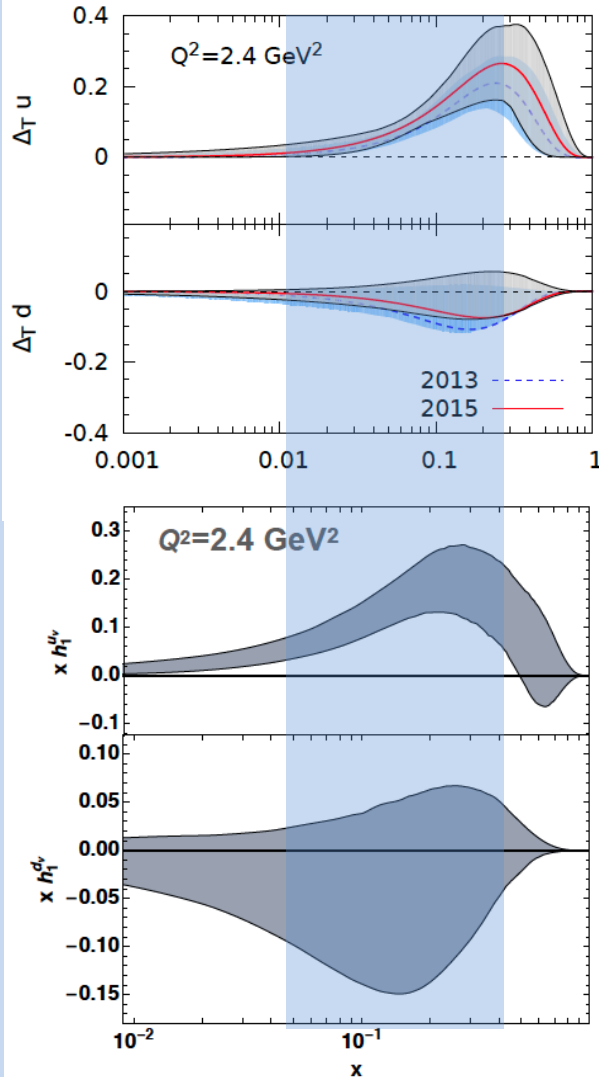


Parton Polarization



Transversity and Tensor Charge

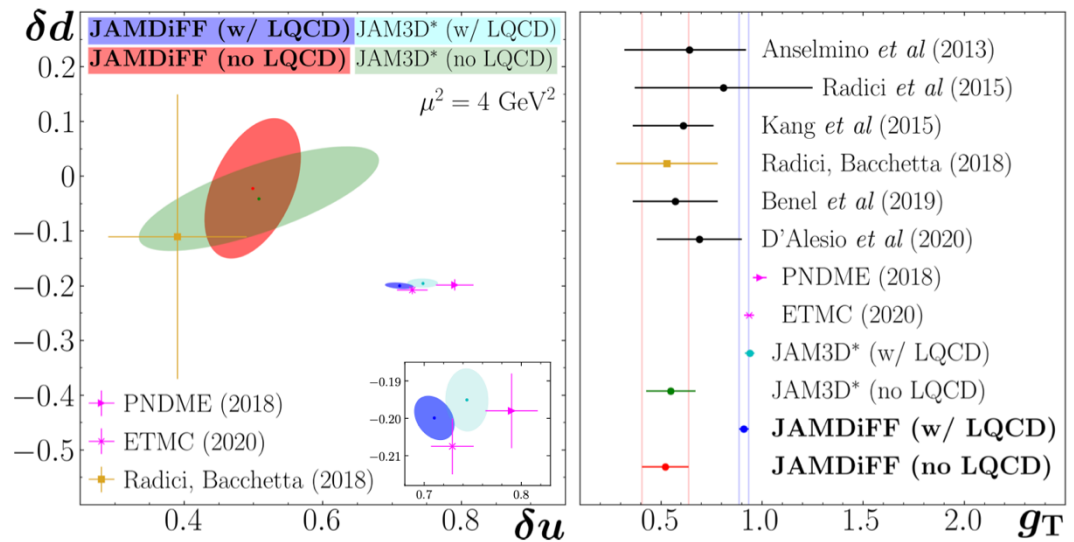
Distributions:



Charges:

$$\delta q \equiv \int_0^1 dx [\Delta_T q(x) - \Delta_T \bar{q}(x)]$$

JAM, 2306.12998

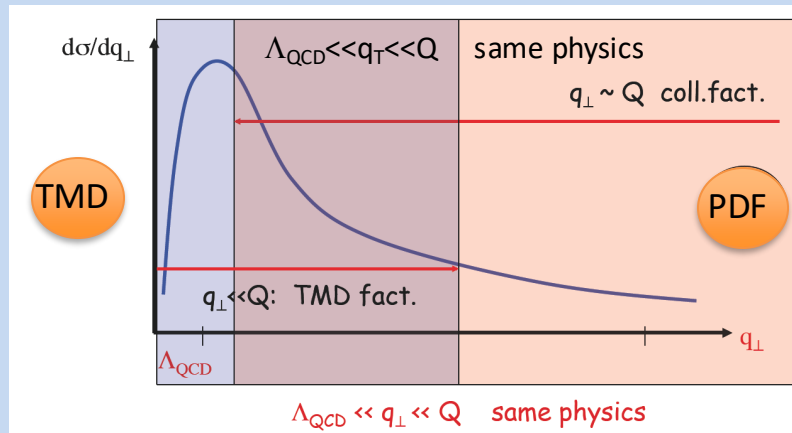
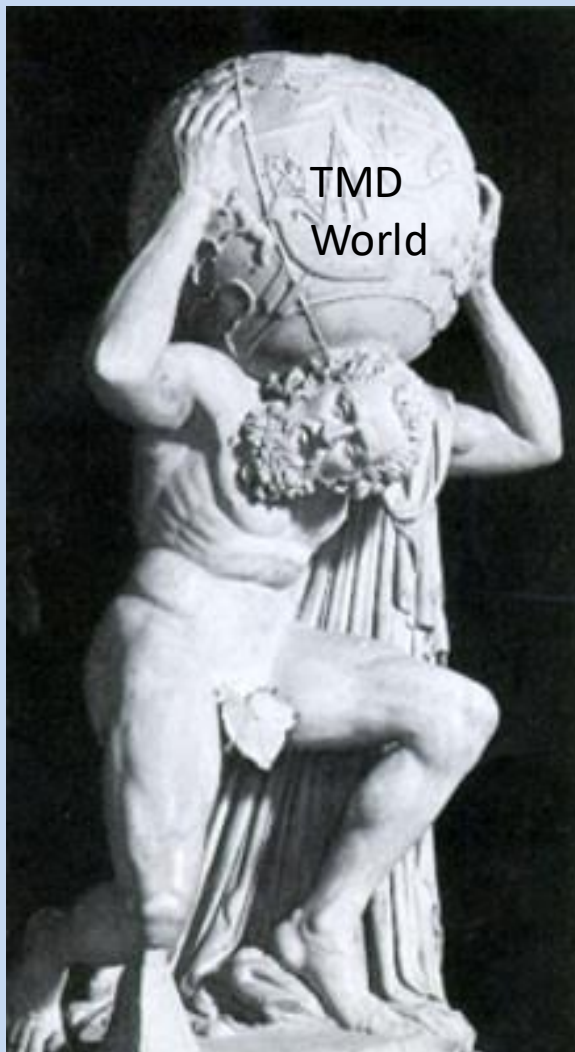


Connected to Beyond Standard Model physics:

- Electric Dipole Moment of particles
- Tensor coupling (beyond V-A)

TMD Parton Correlators

Beauty and complexity of the
unique strong-interacting world



$f(x) \rightarrow f(x, k_T)$

quark polarisation

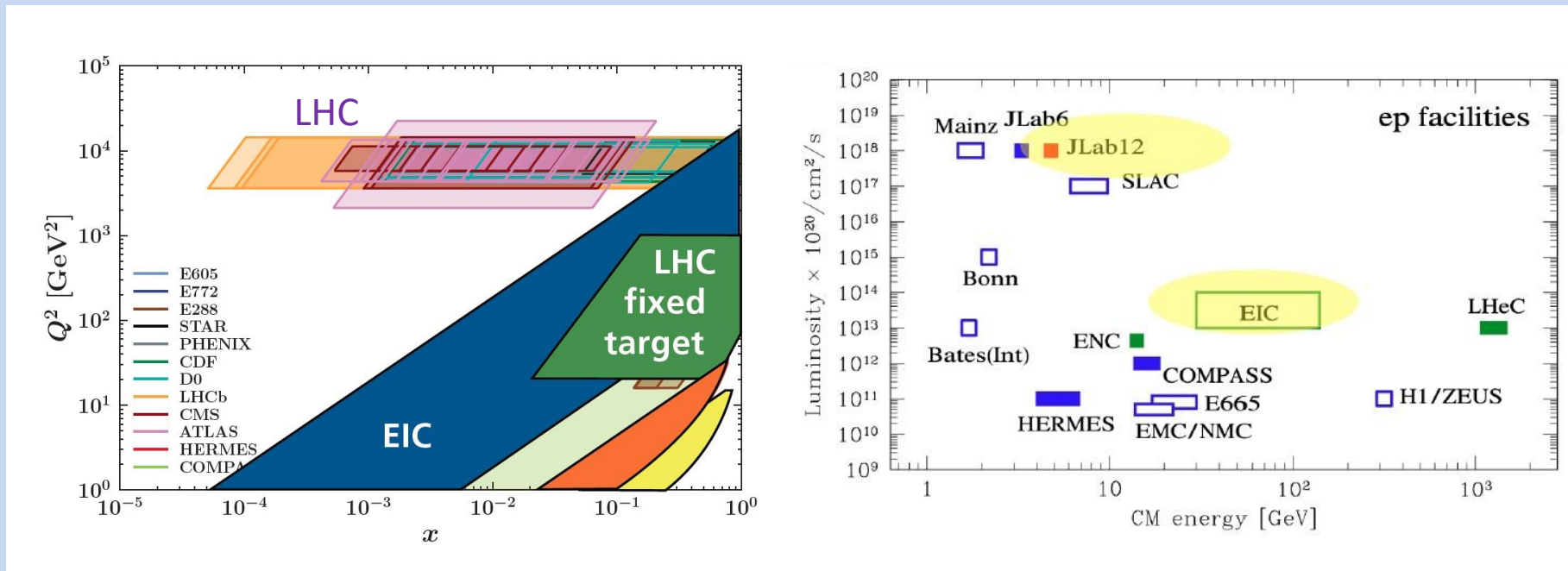
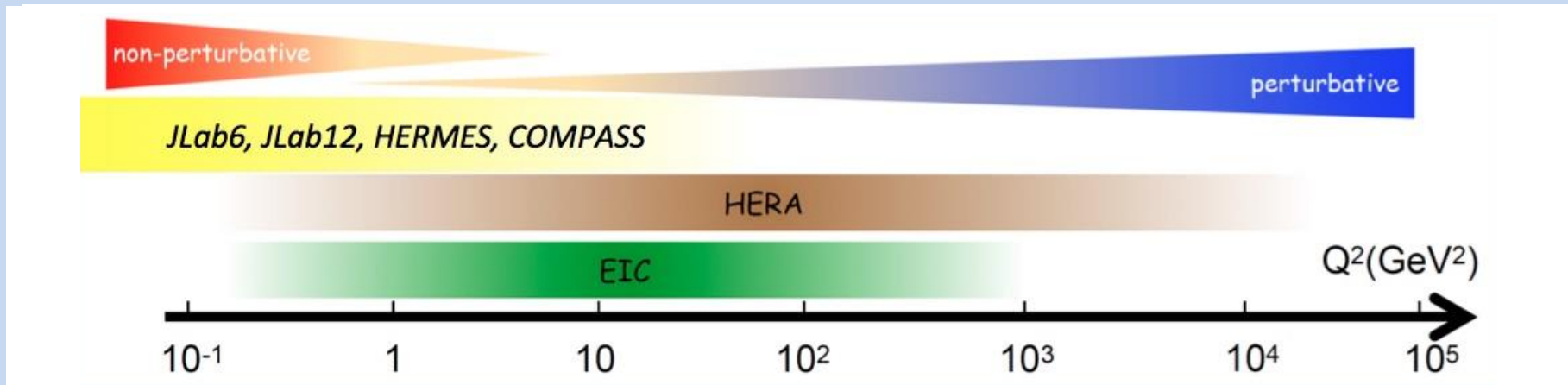
	N/q	U	L	T
nucleon polarisation	U	f_1		$h_1^{\perp}$
	L		g_1	$h_{1L}^{\perp}$
	T	$\hat{f}_{1T}$	$\hat{g}_{1T}$	$h_1, \hat{h}_{1T}$

quark polarisation

	N/q	U	L	T
	U	D_1		$\hat{H}_1$

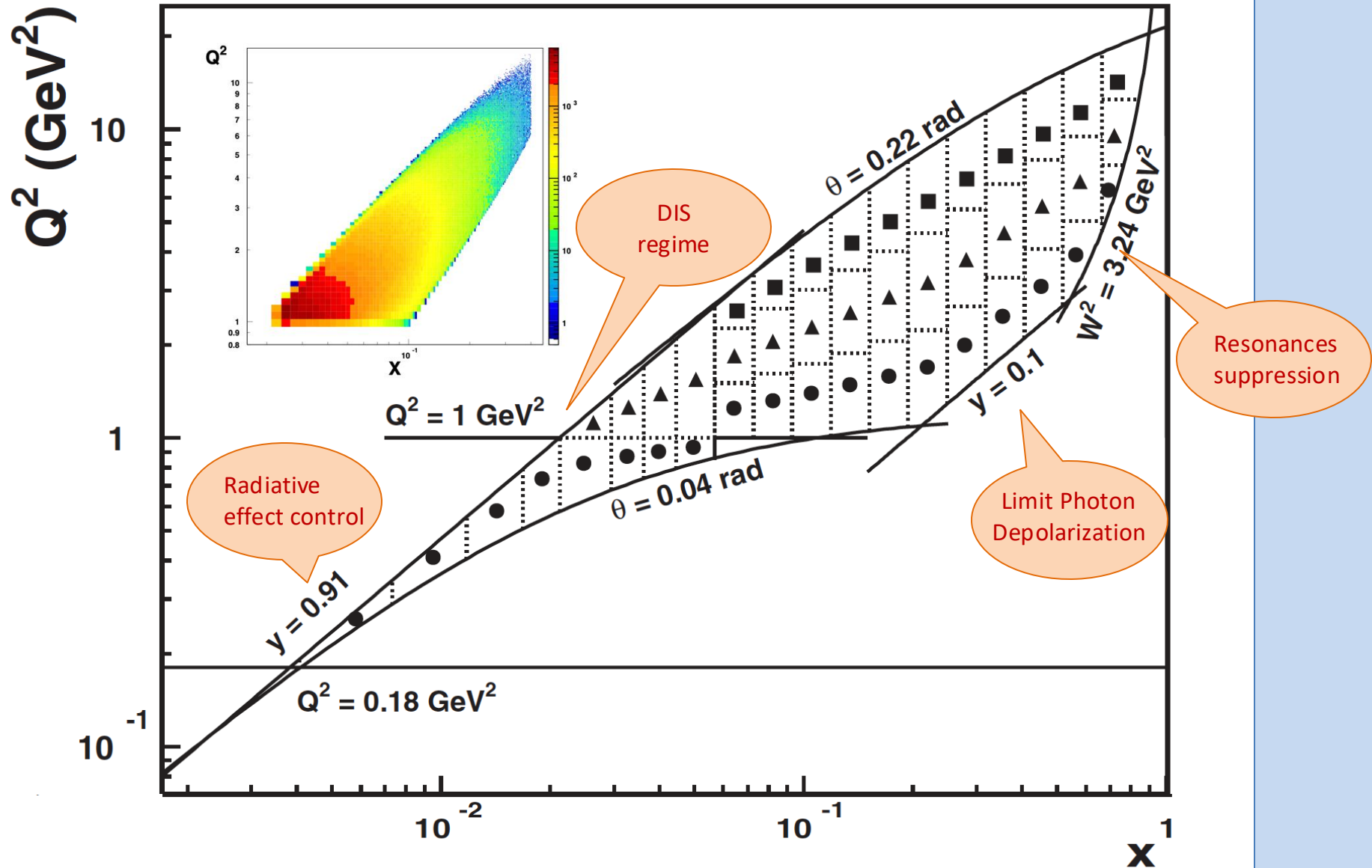
TMD Baseline

Energy range matching perturbative and non-perturbative regimes



$k = (E, \vec{k}), k' = (E', \vec{k}')$	4-momenta of the initial and final-state leptons
θ, ϕ	Polar and azimuthal angle of the scattered lepton
$P^{\text{lab}}(M, 0)$	4-momentum of the initial target nucleon
$q = k - k'$	4-momentum of the virtual photon
$Q^2 \equiv -q^2 \stackrel{\text{lab}}{=} 4EE' \sin^2 \frac{\theta}{2}$	Negative squared 4-momentum transfer
$\nu \equiv \frac{P \cdot q}{M} \stackrel{\text{lab}}{=} E - E'$	Energy of the virtual photon
$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M\nu}$	Bjorken scaling variable
$y \equiv \frac{P \cdot q}{P \cdot k} \stackrel{\text{lab}}{=} \frac{\nu}{E}$	Fractional energy of the virtual photon
$W^2 = (P + q)^2 = M^2 + 2M\nu - Q^2$	Squared invariant mass of the photon-nucleon system
$p = (E_h, \vec{p})$	4-momentum of a hadron in the final state
$z = \frac{P \cdot p}{P \cdot q} \stackrel{\text{lab}}{=} \frac{E_h}{\nu}$	Fractional energy of the observed final-state hadron
$x_F = \frac{p_{CM}^{\parallel}}{ \vec{q} } \stackrel{\text{lab}}{\simeq} \frac{2p_{CM}^{\parallel}}{W}$	Longitudinal momentum fraction of the hadron

Electron Kinematic Range

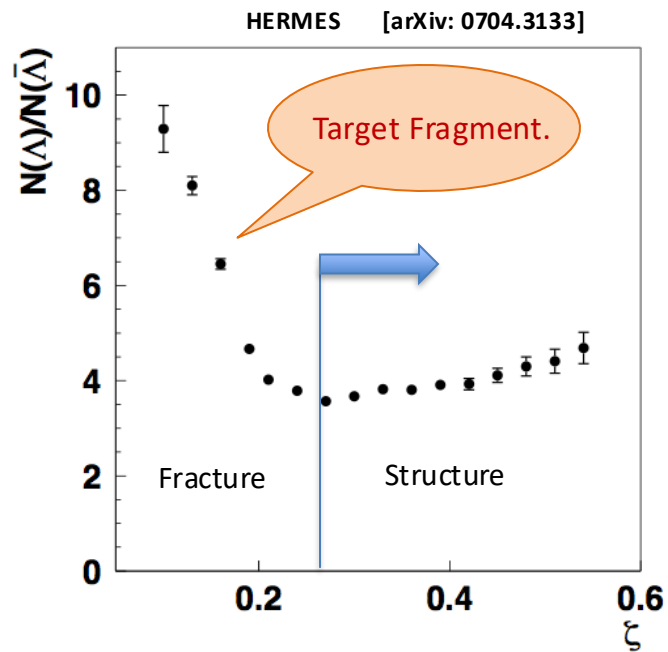


Proper Fragmentation Range

How to ensure we are not in target fragmentation ?

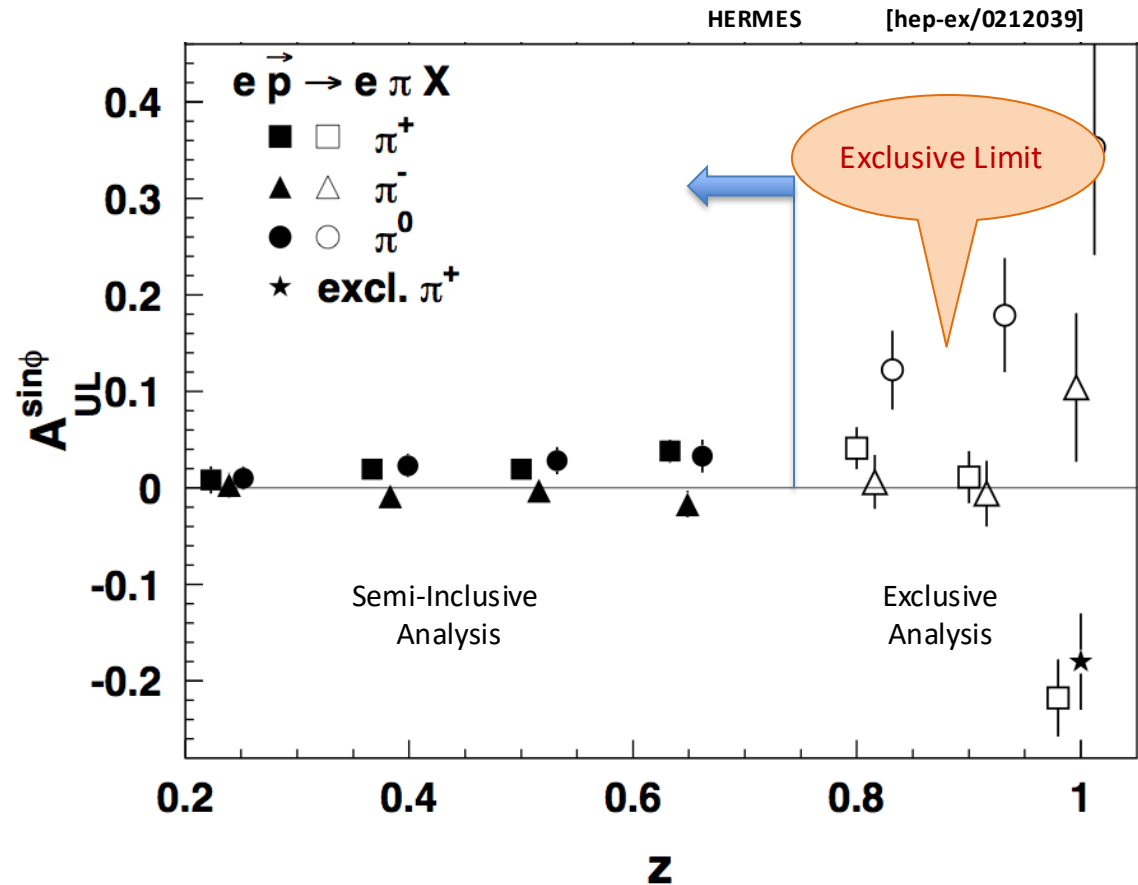
When the exclusive is no more part of the semi-inclusive ?

Inclusive $\bar{\Lambda}/\Lambda$ yield
at $W \sim 5$ GeV (HERMES)



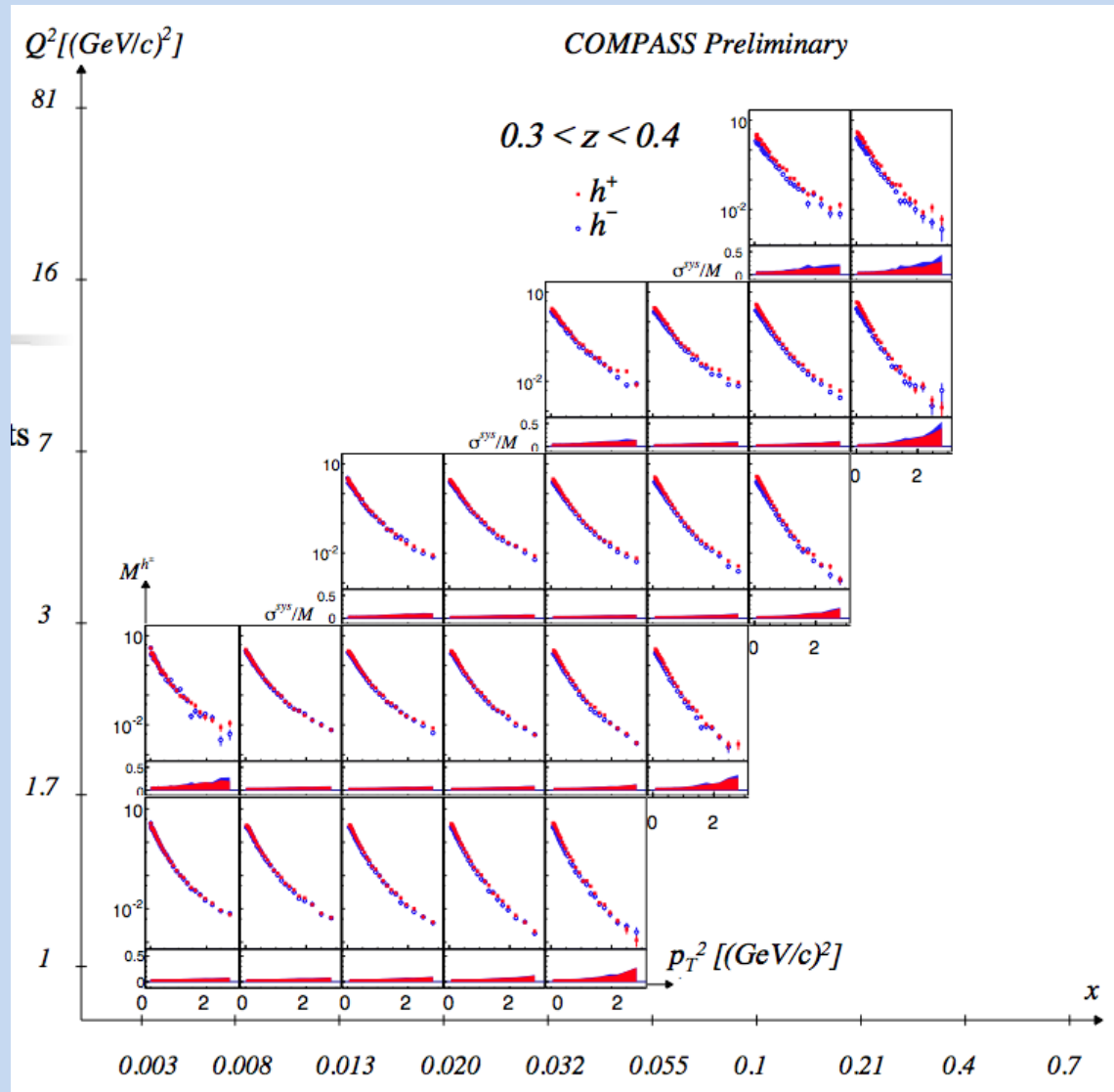
$$\zeta \equiv (E_{\Lambda} + p_{z\Lambda}) / (E_e + p_e)$$

Highly correlated to z , x_F



Multi-D Investigation

Achieve the maximum phase space coverage with:



Disentanglement

- ✓ kinematic dependences
- ✓ dynamical regimes
 - twist in Q^2
 - perturbative in p_T

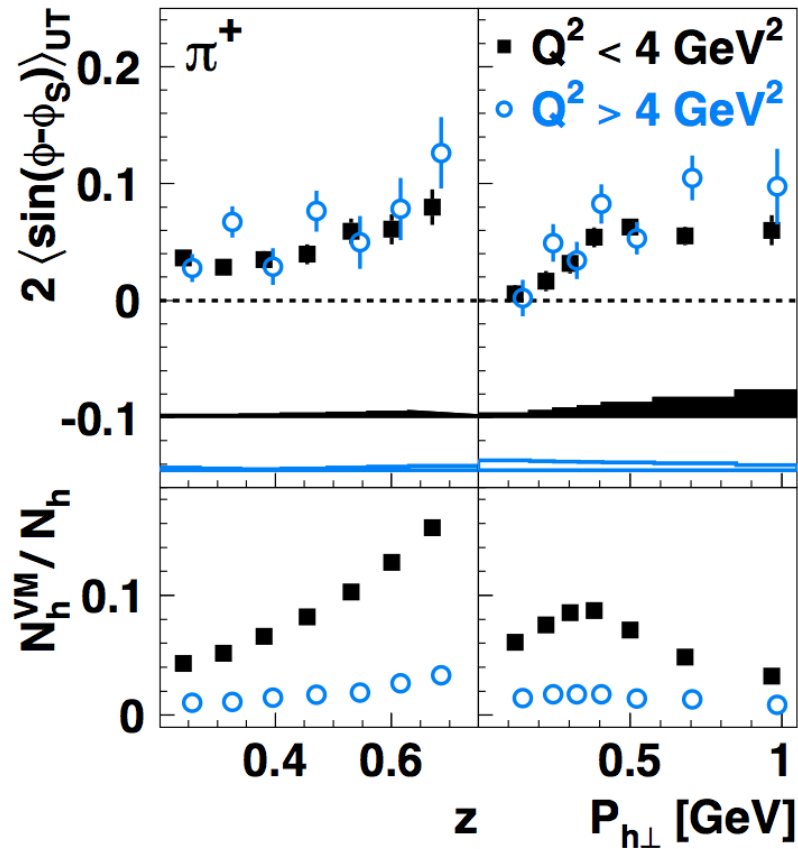
Asymmetries so far used to suppress systematics effects

$$A_{LL} = \frac{S^+ - S^-}{S^+ + S^-}$$

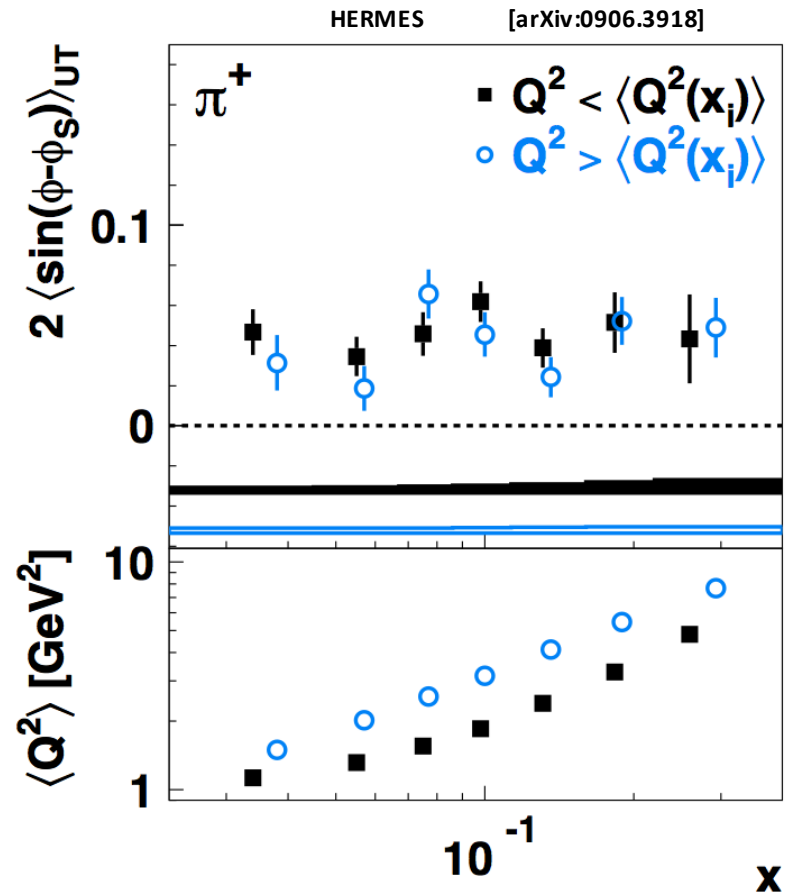
$$A_{LL} = \frac{1}{fP_T P_B} \frac{N^+ - N^-}{N^+ + N^-}$$

They suppress also physics (i.e. evolution)

Test of exclusive channel influence

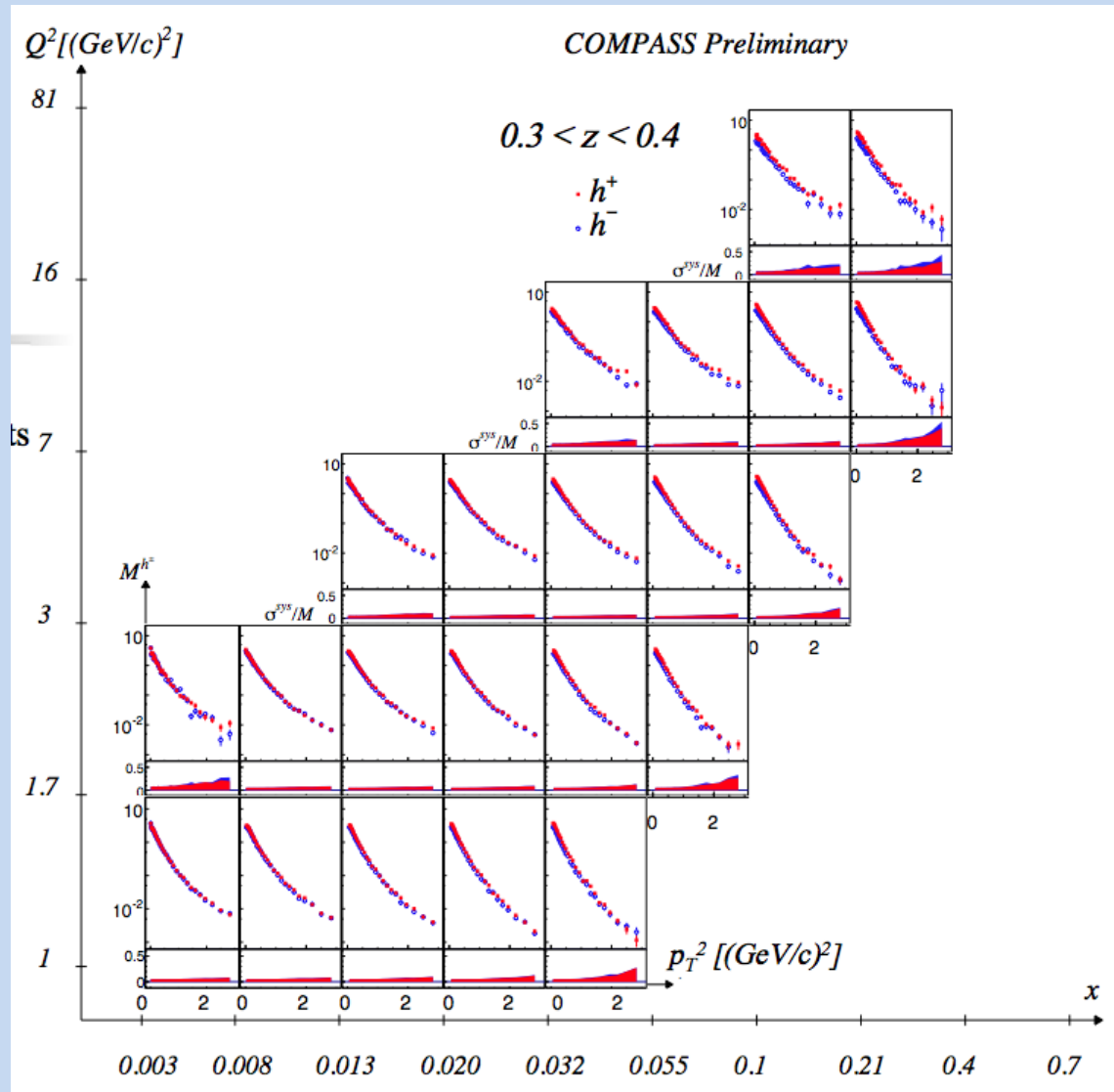


Test of higher-twist contribution



Multi-D Investigation

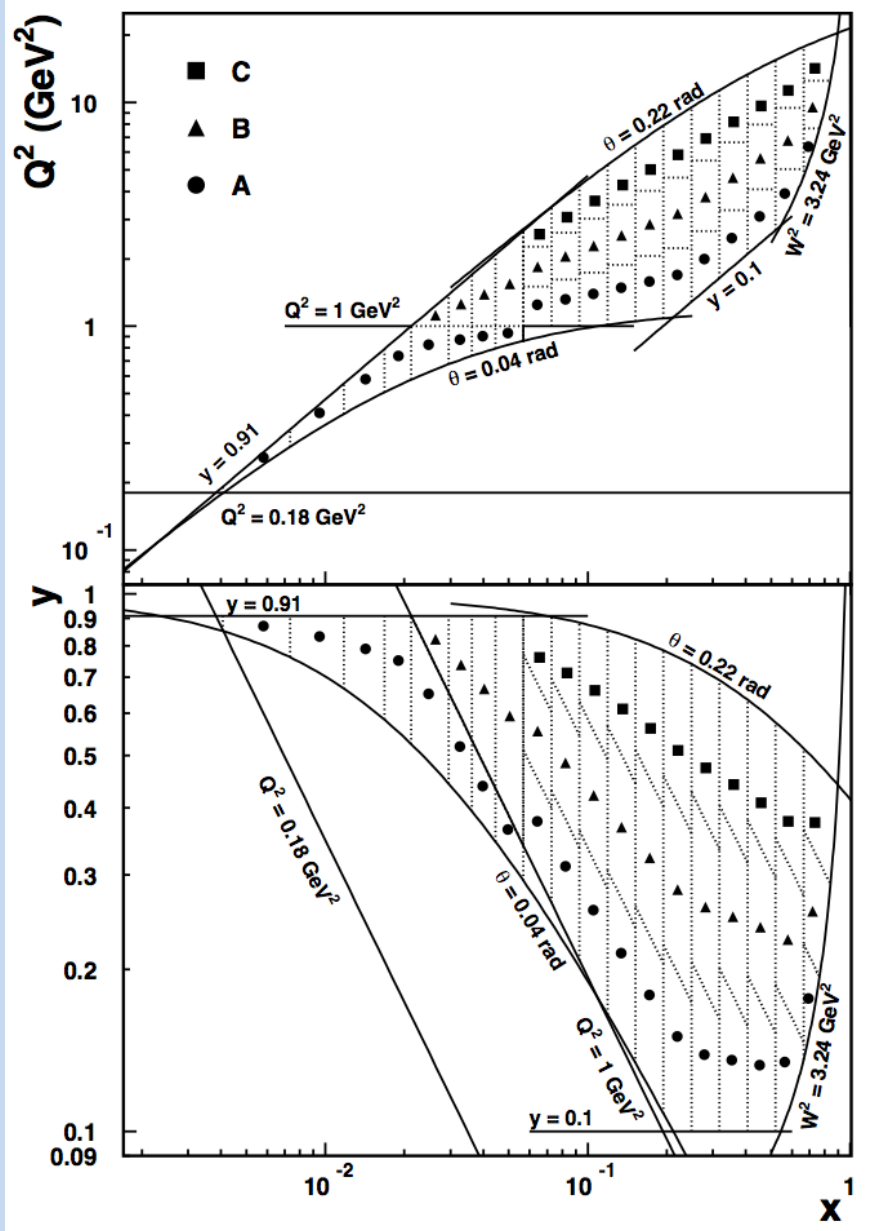
Achieve the maximum phase space coverage with:



Disentanglement

- ✓ kinematic dependences
- ✓ dynamical regimes
 - twist in Q^2
 - perturbative in p_T
- ✓ kinematical dilutions
- ✓ instrumental dilutions
- radiative smearing & x-talk
- acceptance effects

Analyzing Power



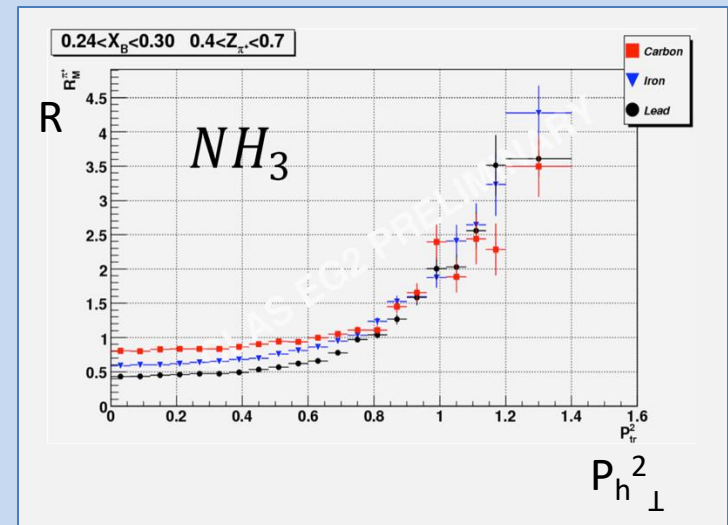
Analyzing power matters

Goes like y

$$A_{LL} \propto \frac{\sqrt{1 - \epsilon^2} F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}}$$

Nuclear effects from real (⁶LiD, NH₃, ³He) targets

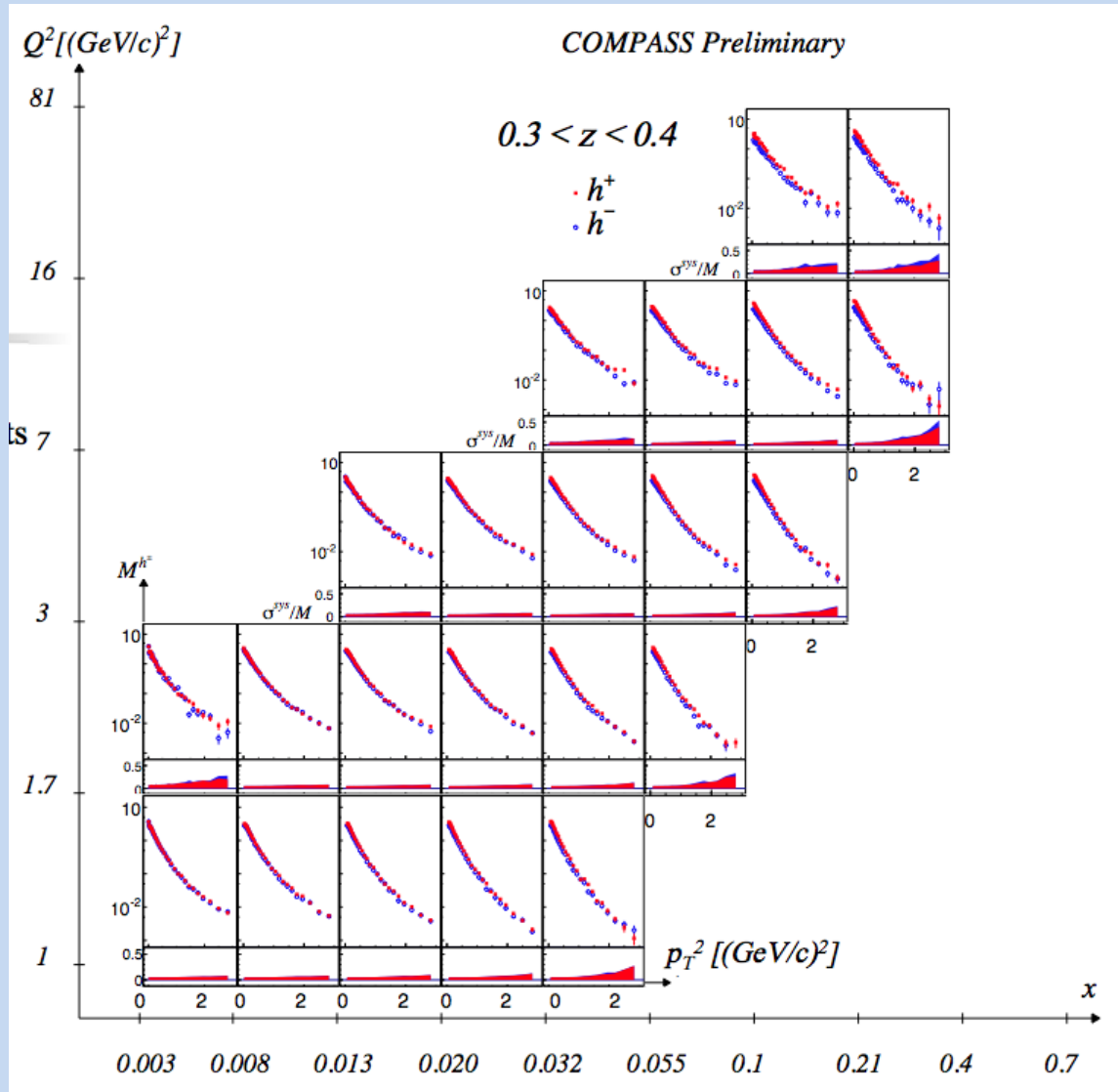
$$A_{LL} = \frac{1}{f P_T P_B} \frac{N^+ - N^-}{N^+ + N^-}$$



Dilution goes with $\frac{N}{\sqrt{N}}$
Precision improves with $\frac{N}{\sqrt{N}}$

Multi-D Investigation

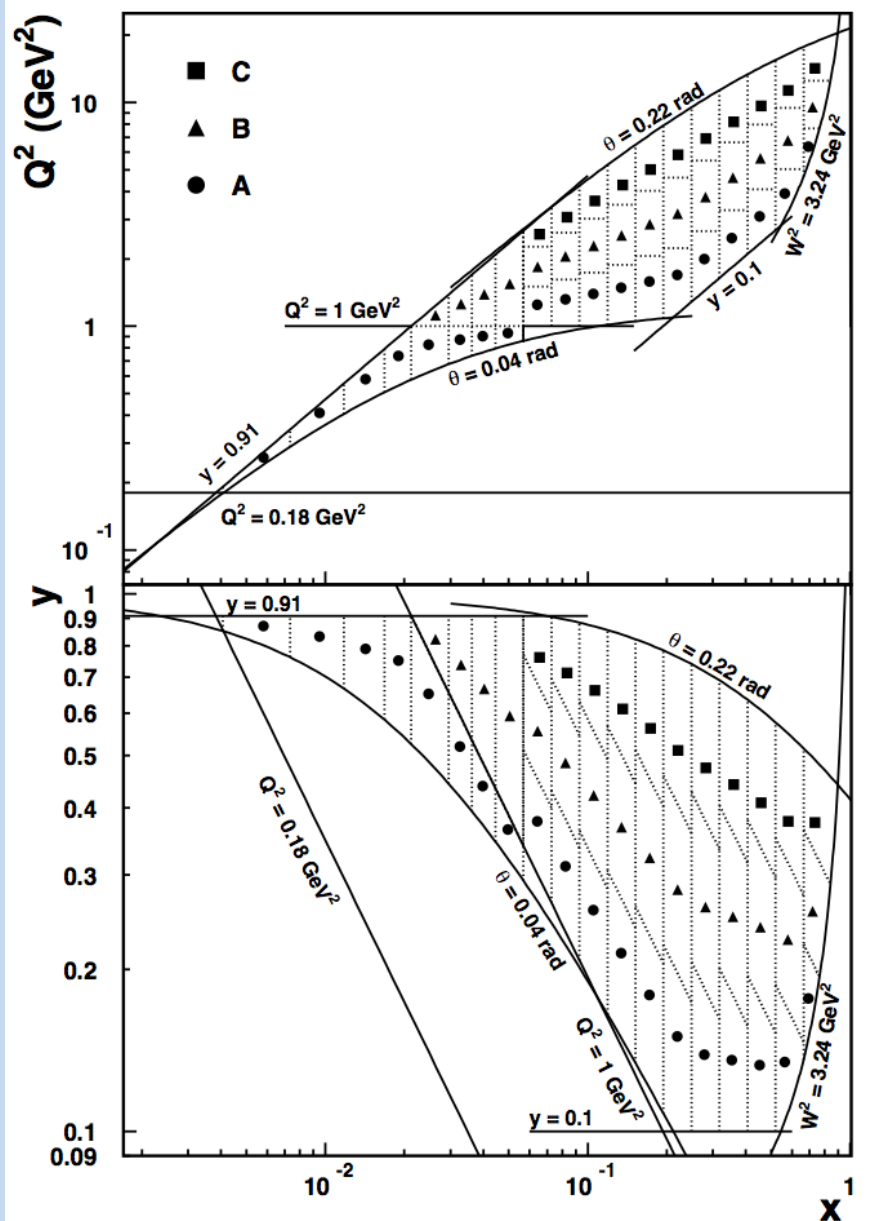
Achieve the maximum phase space coverage with:



Disentanglement

- ✓ kinematic dependences
- ✓ dynamical regimes
 - twist in Q^2
 - perturbative in p_T
- ✓ kinematical dilutions
- ✓ instrumental dilutions
- ✓ radiative smearing & x-talk
- acceptance effects

Smearing Effect



What about event migration ?

- define bins larger than resolution

but keep in mind non-linearities!!!

$$\frac{dx}{x} \mu \frac{1}{y} \frac{dp}{p}$$

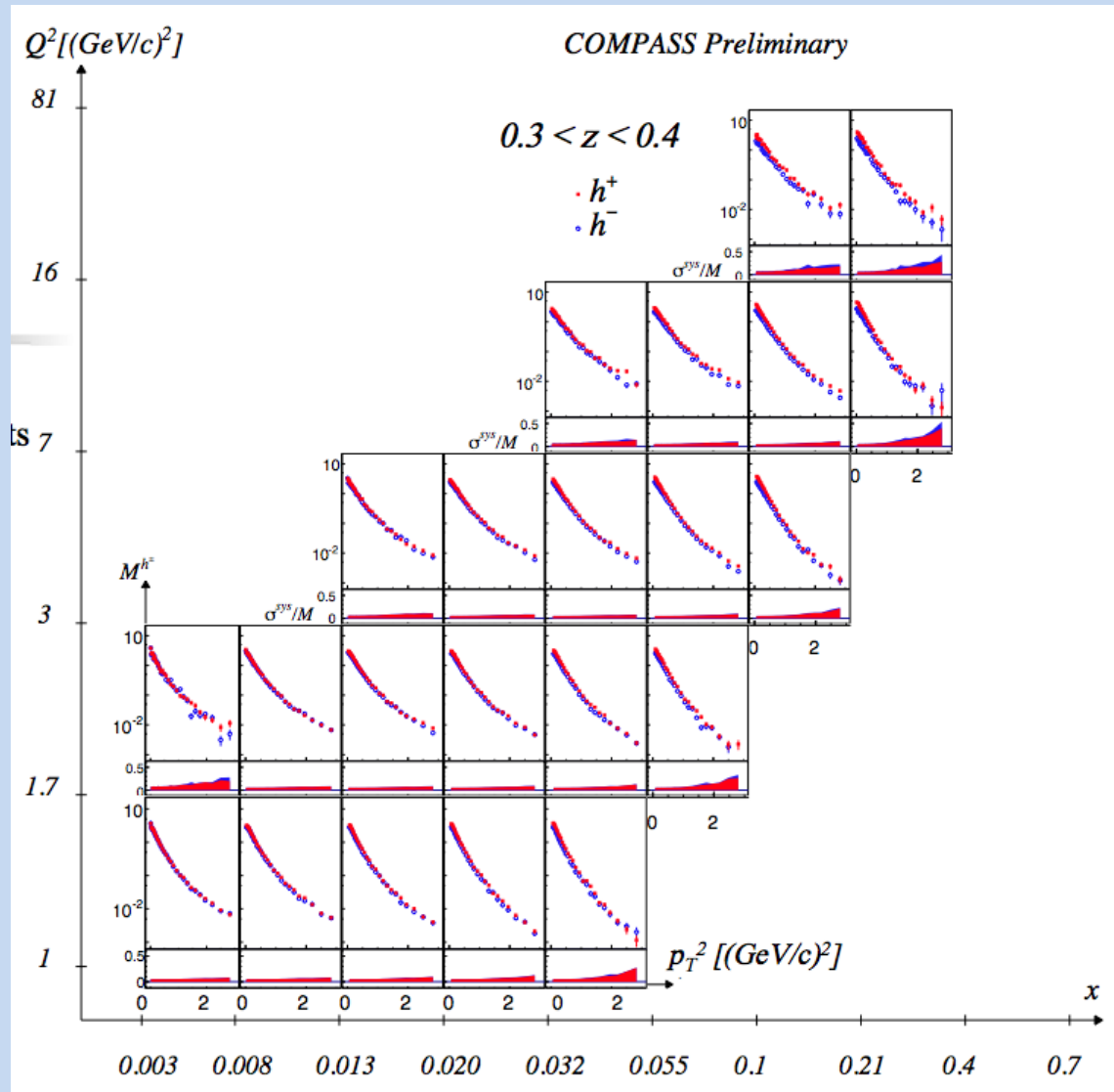
- radiative effects change the kinematics (larger y) and introduce a x -talk between modulations

a full knowledge of the hadronic tensor is in principle required

- unfold smearing and radiative effects introduce a statistical correlation

Multi-D Investigation

Achieve the maximum phase space coverage with:

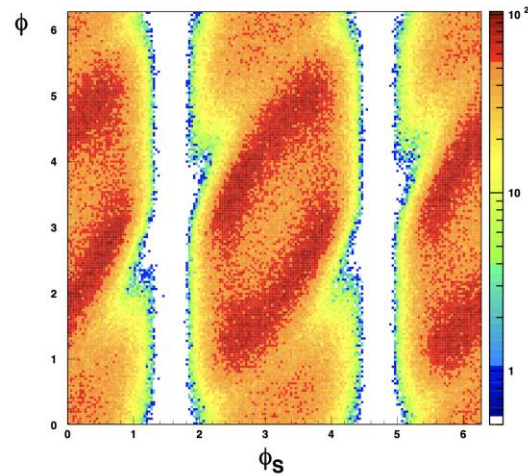
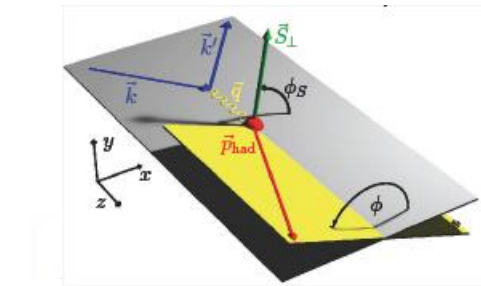


Disentanglement

- ✓ kinematic dependences
- ✓ dynamical regimes
 - twist in Q^2
 - perturbative in p_T
- ✓ kinematical dilutions
- ✓ instrumental dilutions
- ✓ radiative smearing & x-talk
- ✓ acceptance effects

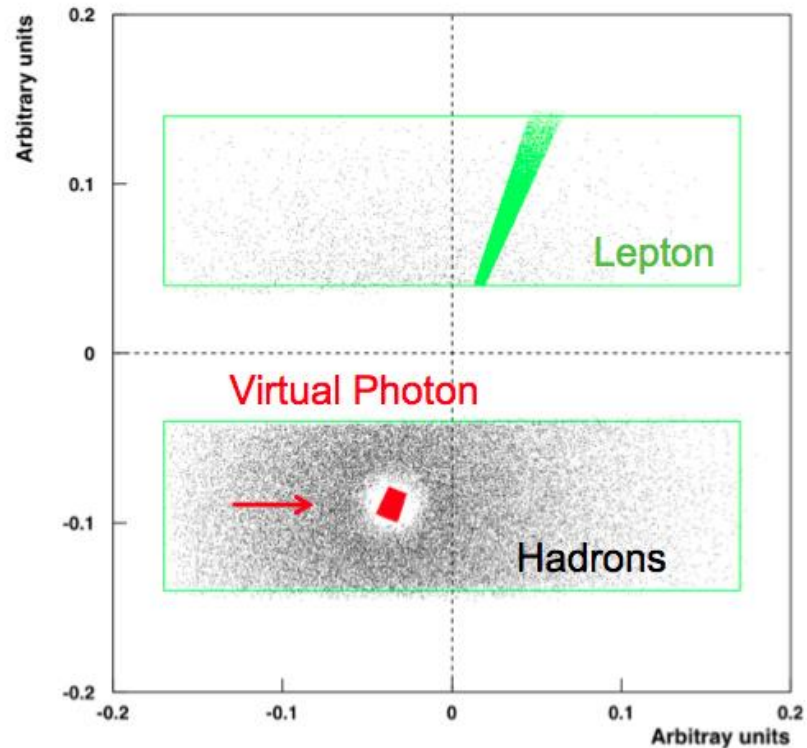
Multi-D Investigation

Do we have enough (ϕ, ϕ_S) coverage in each bin ?



Partial coverage in ϕ at a given detected event kinematics

Acceptance introduce azimuthal modulations in the measured yields !



(x, Q^2)

$(z, P_{h\perp})$

Unbinned Maximum likelihood possible for spin-asymmetries

Next-Gen DIS Facilities: JLab

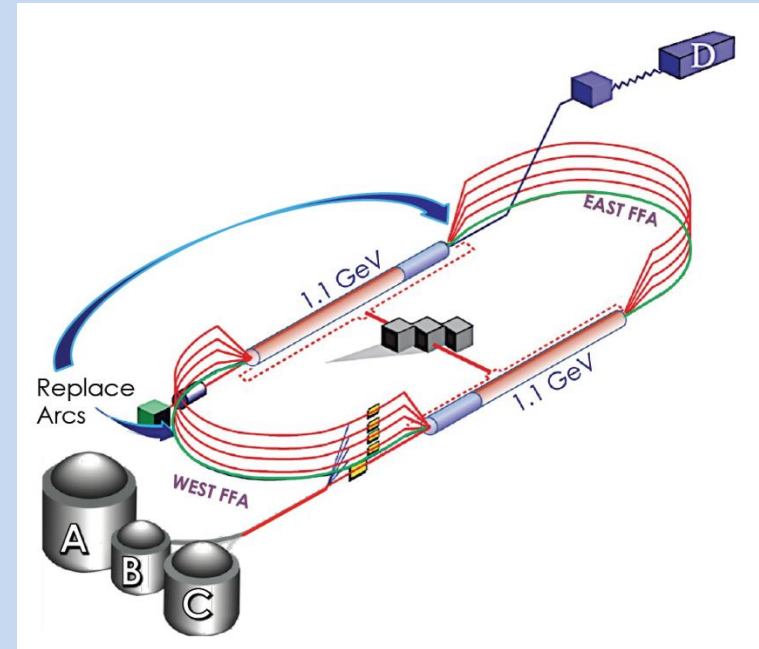
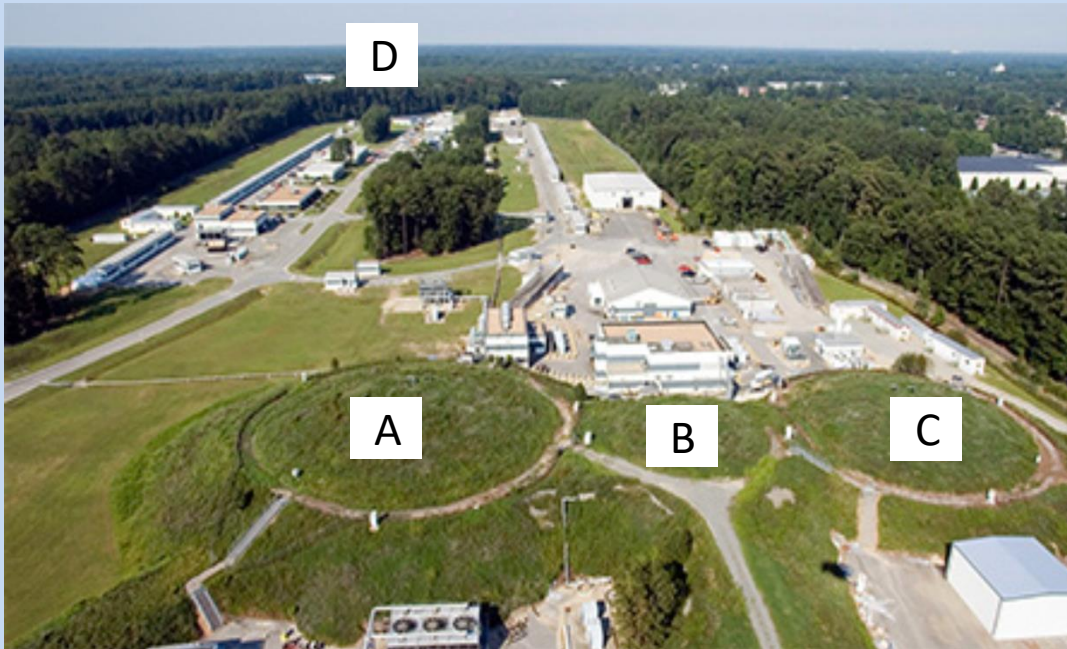
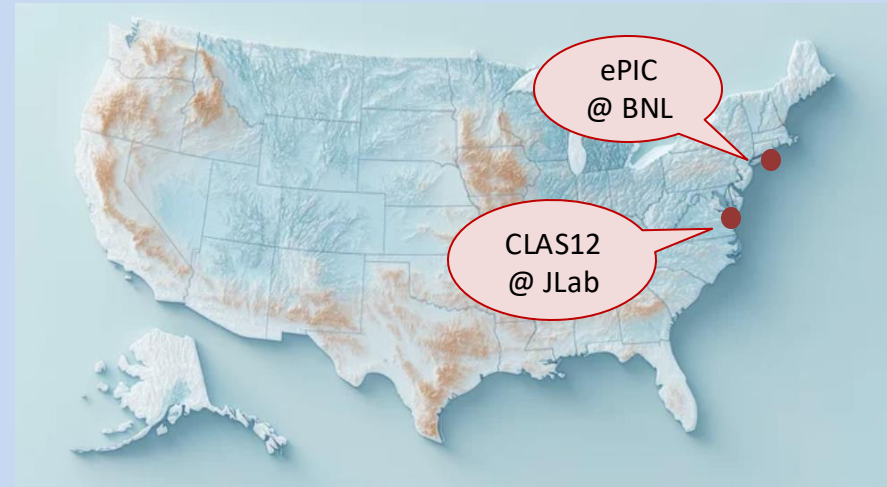
Jefferson Laboratory (JLab) Continuous Electron Beam Accelerator Facility

12 GeV energy

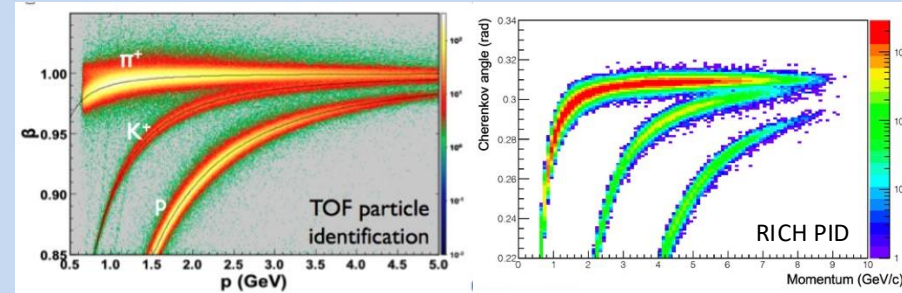
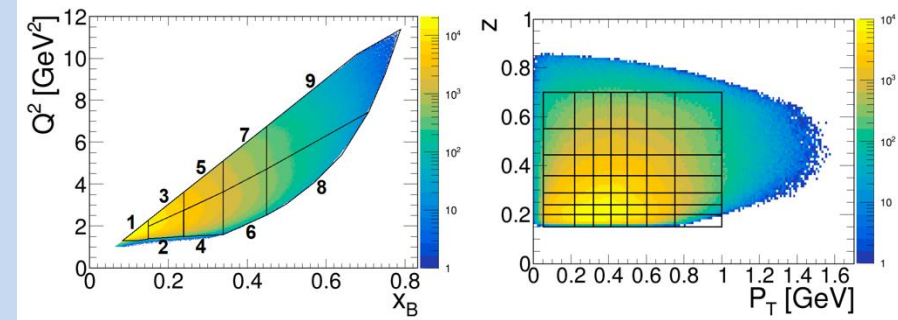
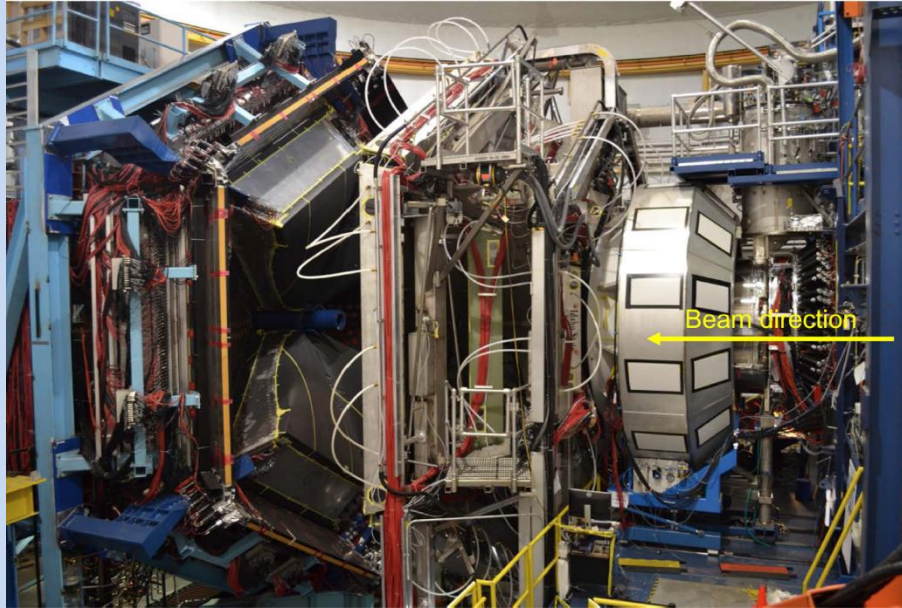
90 μ A Beam Current

85 % Polarization

4 experimental Halls (A-D)

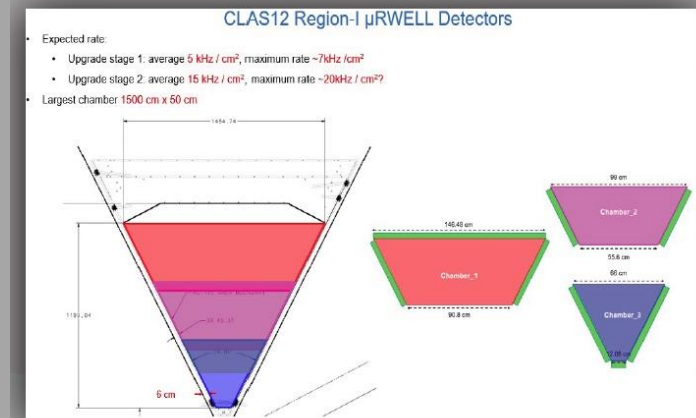


CLAS12 wide coverage, excellent PID, various polarized targets, high luminosity

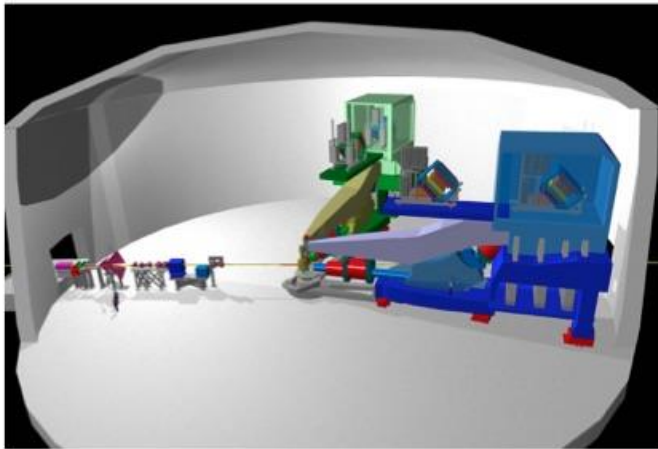


Year	Period	Run	Target	Polarization	Beam
2018	Spring-Fall	RGA	Proton	-	10.6 GeV
	Fall	RGK	Proton	-	6.5-7.5 GeV
2019	Spring	RGA	Proton	-	10.6 GeV
2019	Spring-Fall	RGB	Deuteron	-	10.6 GeV
2020	Spring-Fall	RGF	Deuteron	-	10.6 GeV
2021	Fall	RGM	Nuclear	-	Several GeV
2022	Spring-Fall	RGC	NH ₃ -ND ₃	Longitudinal	10.6 GeV
> 2022		RGH	NH ₃ -ND ₃	Transverse	10.6 GeV
> 2022			³ He	Longitudinal	10.6 GeV
> 2022		RGG	⁷ LiD, ⁶ LiH	Longitudinal	10.6 GeV

Luminosity upgrade Stage-1: $2 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}$ 3 years
 Stage-2: $> 10^{37} \text{ cm}^{-2}\text{s}^{-1}$ 7-10 years



SBS: Spectrometer Pair



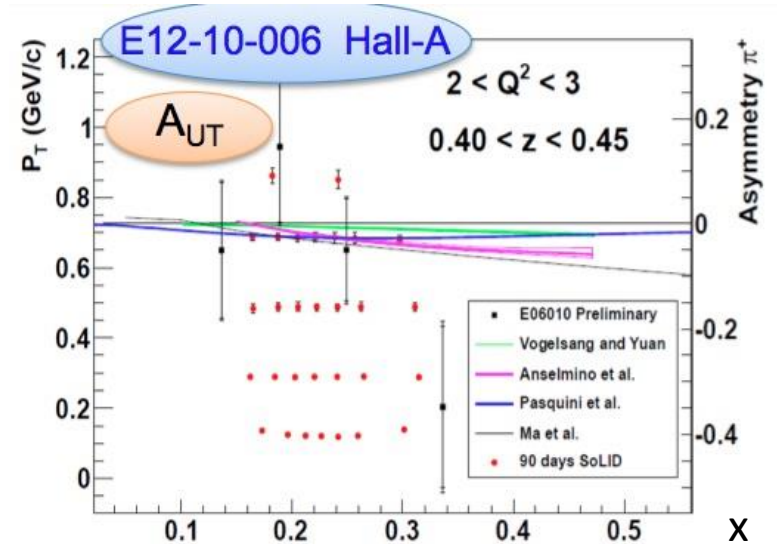
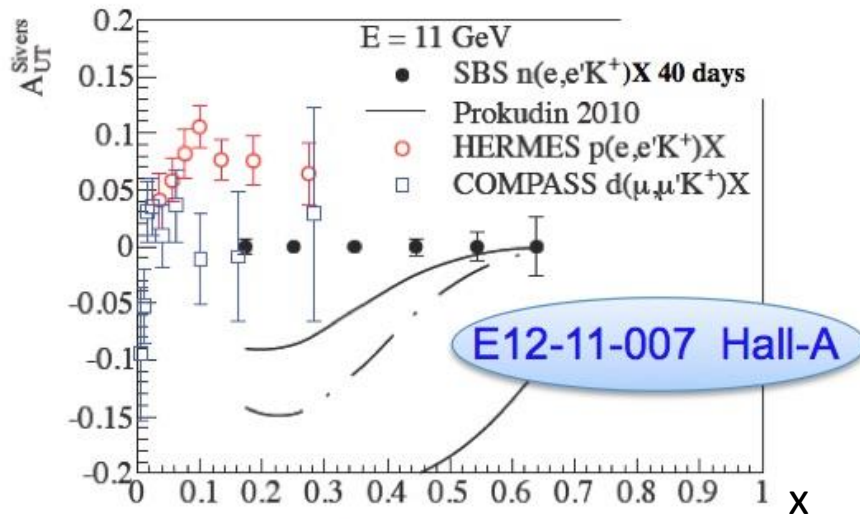
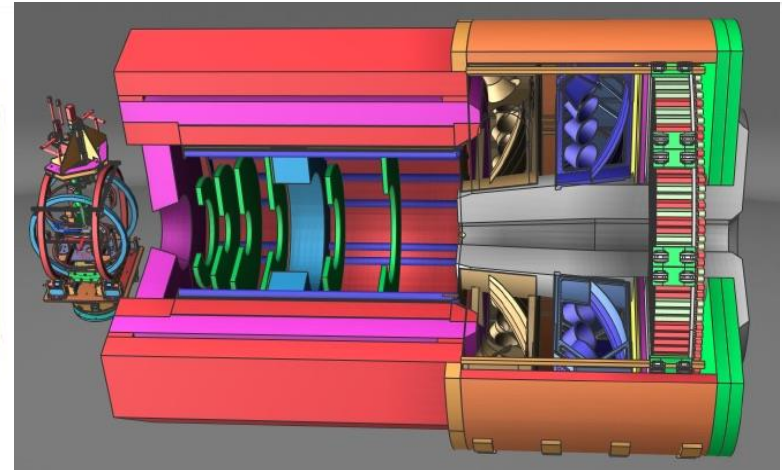
Hall-A:

High-luminosity
 $10^{38} \text{ cm}^{-2} \text{ s}^{-1}$

^3He targets

Wide coverage

SOLID: Large Acceptance Detector



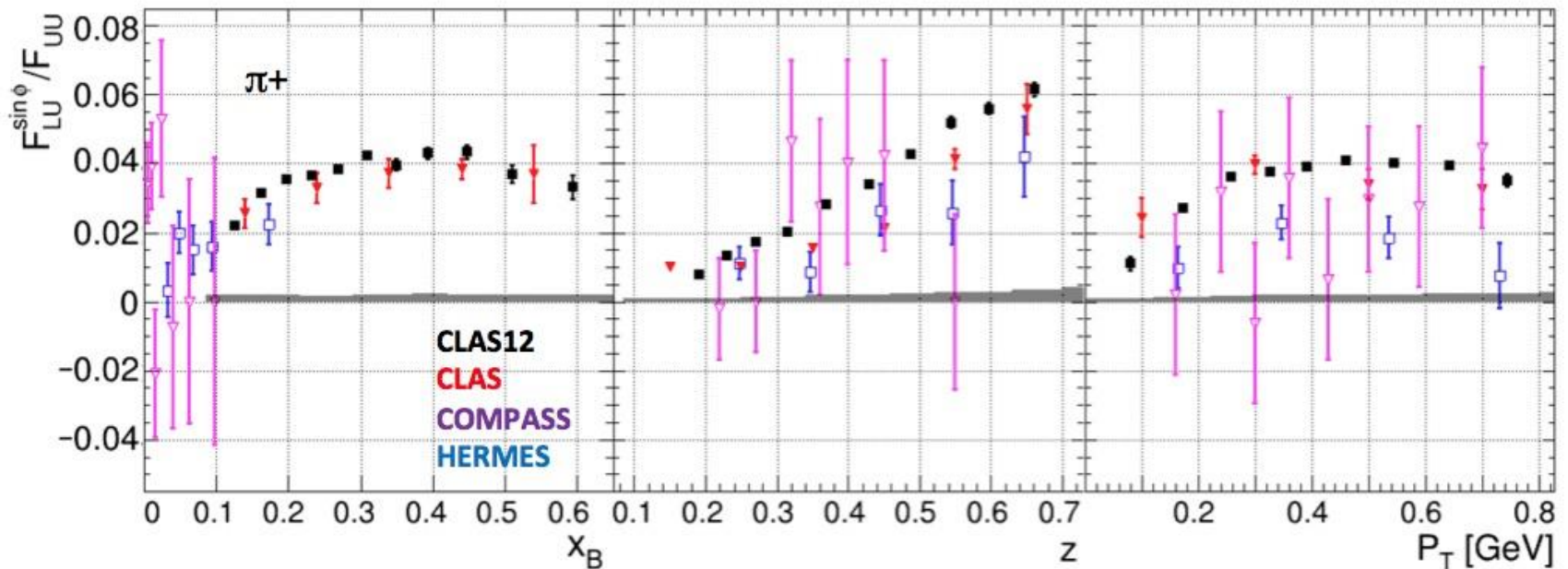
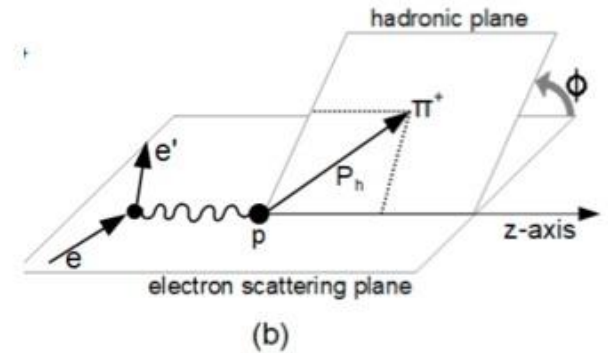
+ precision higher-twist and low p_T physics in Hall-C

CLAS12 proton data (RGA)

S. Diehl et al., e-Print: 2101.03544

$$F_{LU}^{\sin\phi} = \frac{2M}{Q} C \left[-\frac{\hat{h} \cdot k_T}{M_h} \left(x_B e H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{G}^\perp}{z} \right) + \frac{\hat{h} \cdot P_T}{M} \left(x_B g^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{E}}{z} \right) \right]$$

$86.9 \pm 2.6 \%$



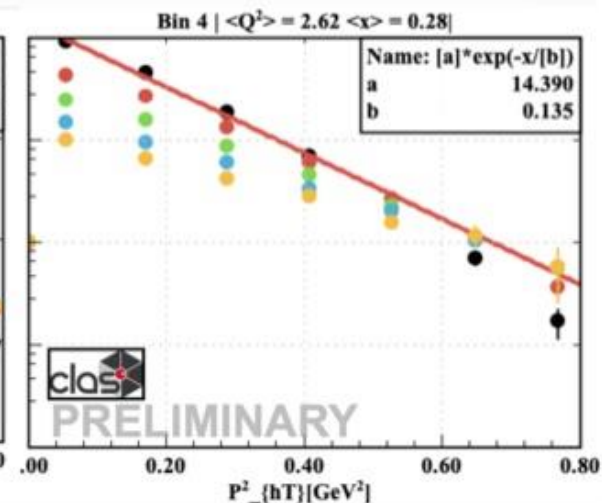
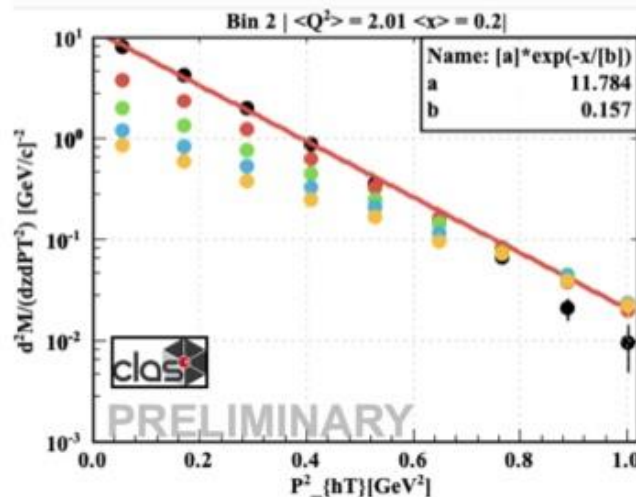
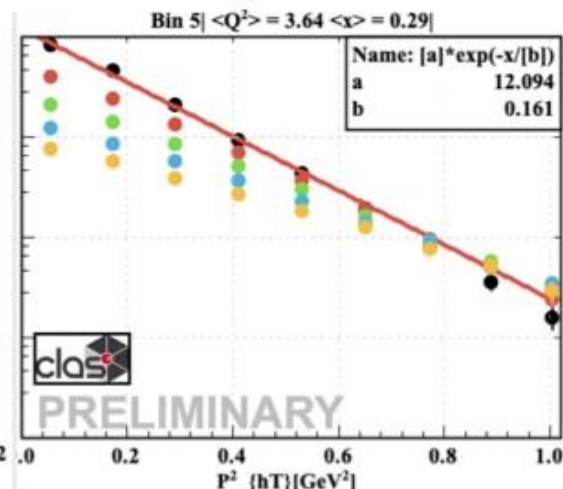
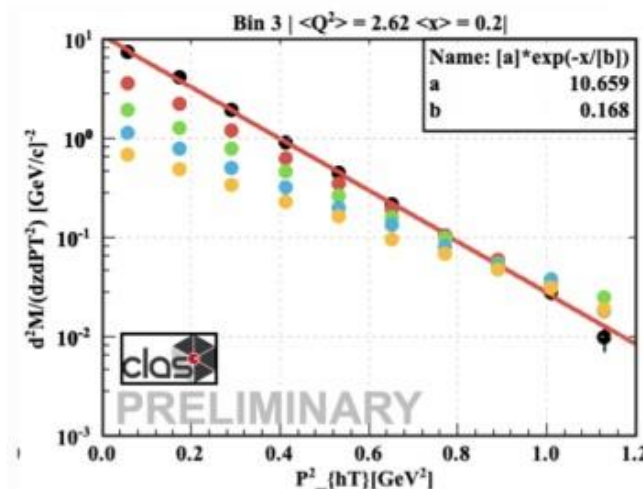
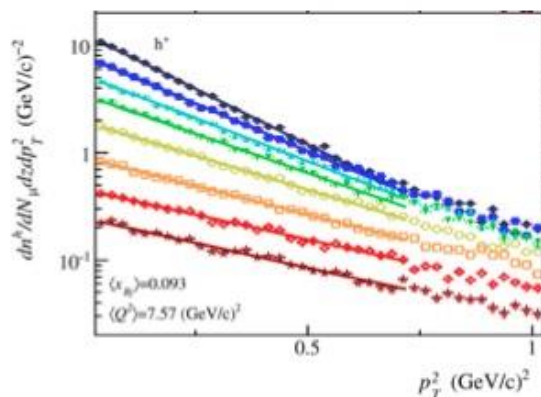
Transverse momentum dependence and phase space

ep → eπ + X

Color legend

- 0.2 < z < 0.3
- 0.3 < z < 0.4
- 0.4 < z < 0.5
- 0.5 < z < 0.6
- 0.6 < z < 0.7

COMPASS, EPJC 73 (2013) 8, 2531



<x> = 0.2

<x> = 0.29

Extend the reach in Q^2 and p_T to exploit an unique facility at the intensity frontier

Energy increase to 20++ GeV

SOLID and CLAS12+

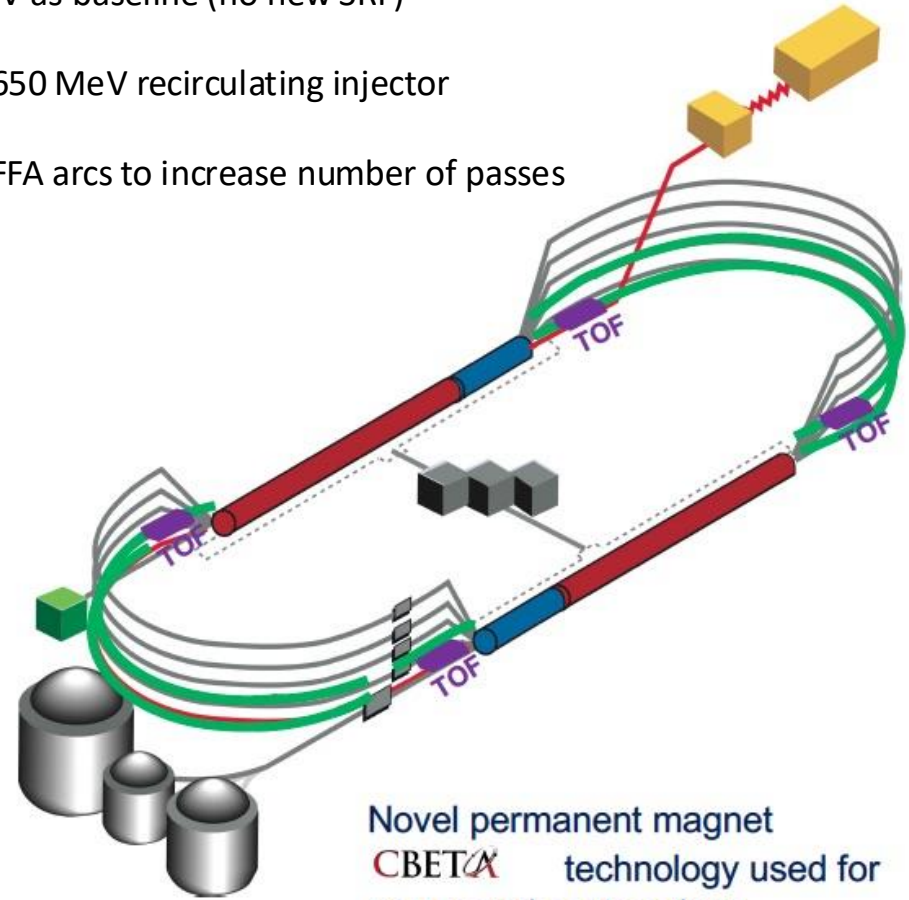
Positron source

12 GeV as baseline (no new SRF)

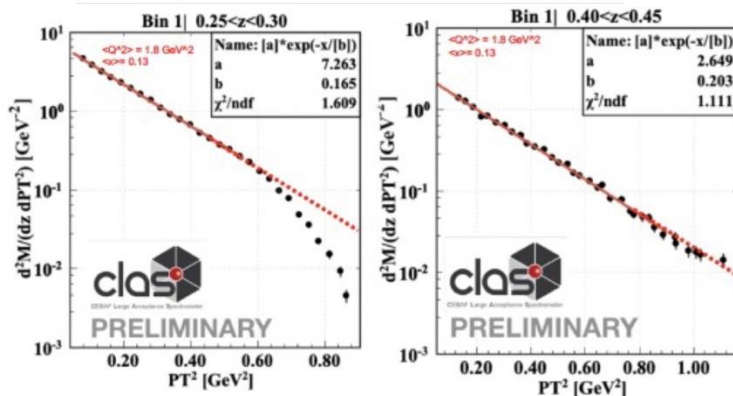
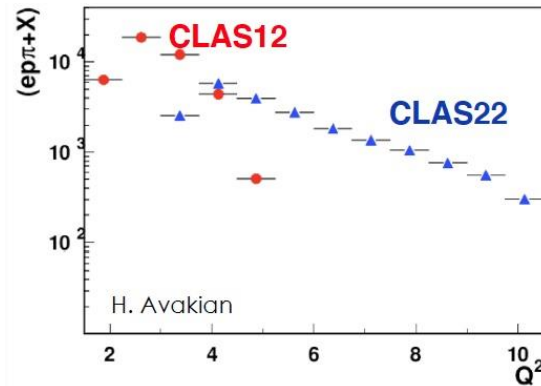
New 650 MeV recirculating injector

New FFA arcs to increase number of passes

650
MeV



Novel permanent magnet
CBETA technology used for
power and cost savings



DIS Facilities: BNL

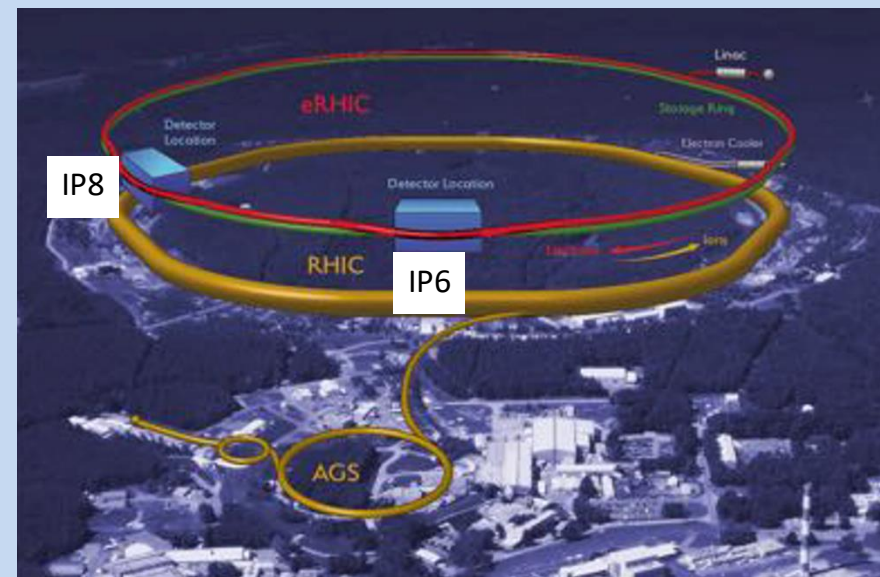
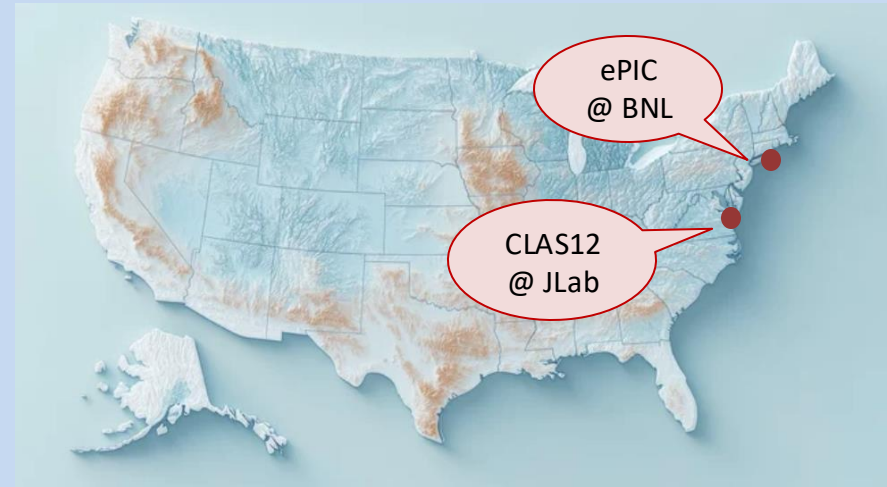
Brookhaven National Laboratory (BNL) Electron-Ion Collider

Hadron Beam 41-275 GeV

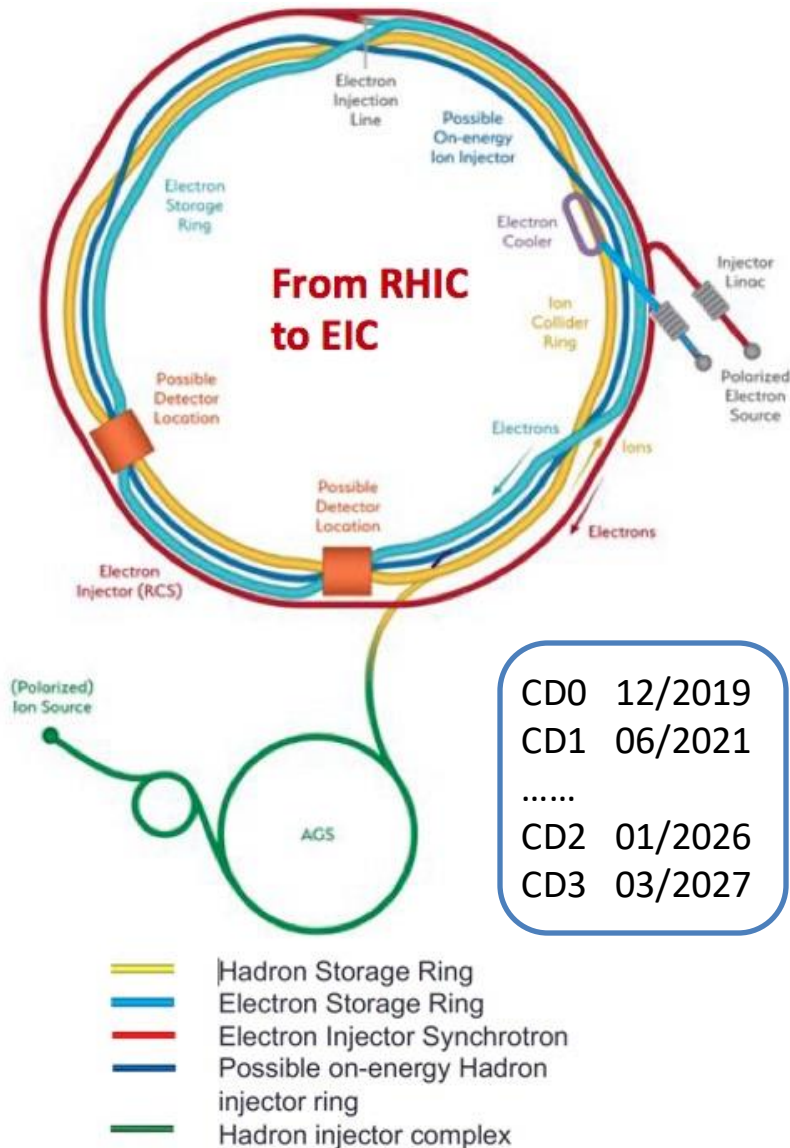
Electron Beam 5 -18 GeV

Polarized Electron and Light Ions

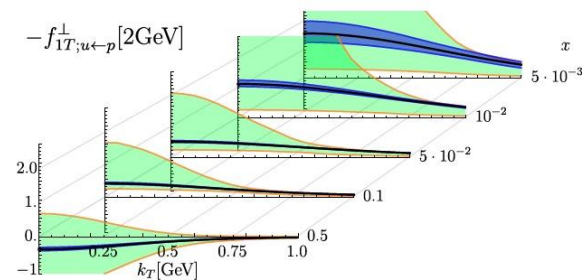
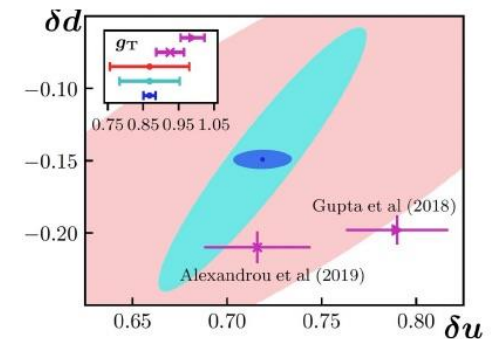
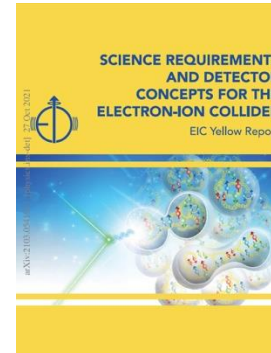
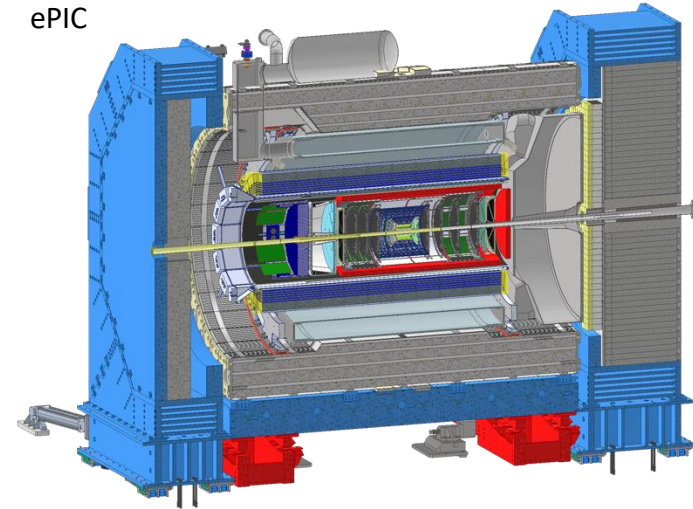
2 Interaction Points (IP6, IP8)



Electron-Ion Collider

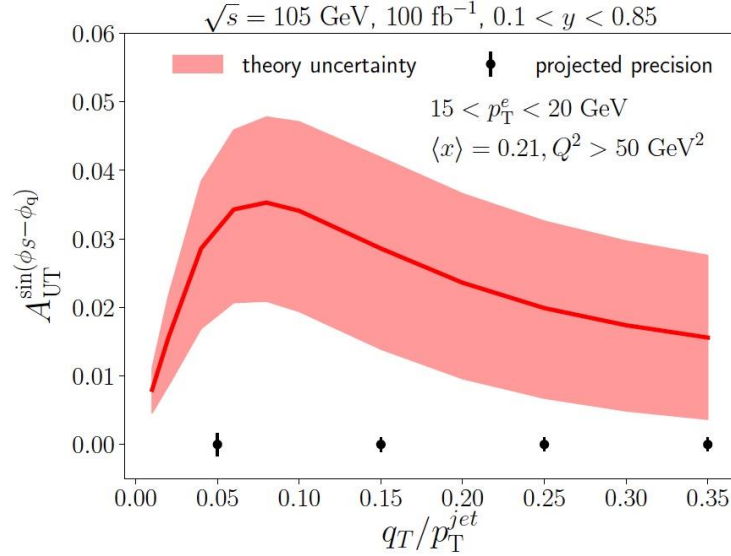


ePIC



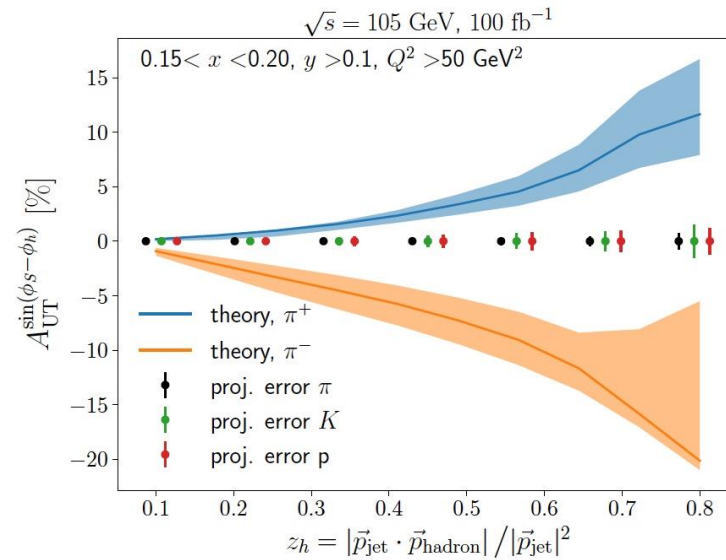
Yellow Report:
arXiv: 2103.05419

Jets Sivers



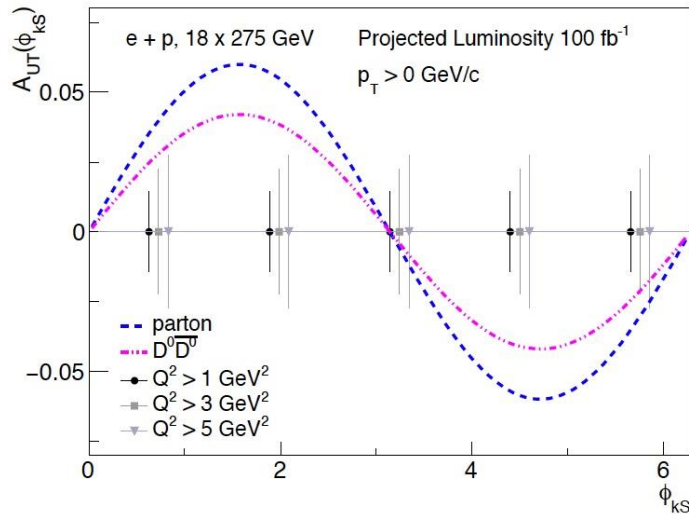
Jets Collins

E.C. Aschenauer ++ [arXiv: 1708.01527]



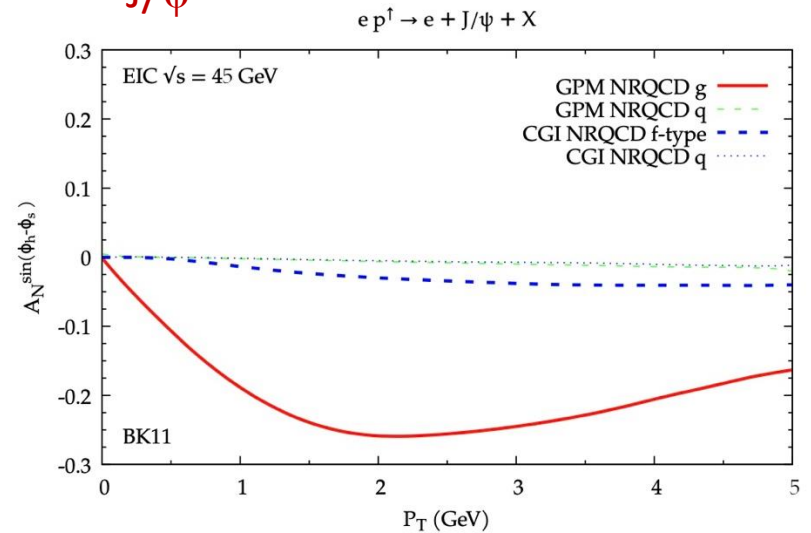
D^0

Gluon TMDs



J/ψ

U. D'Alesio++ [arXiv: 2203.03299]



SIDIS Cross-Section

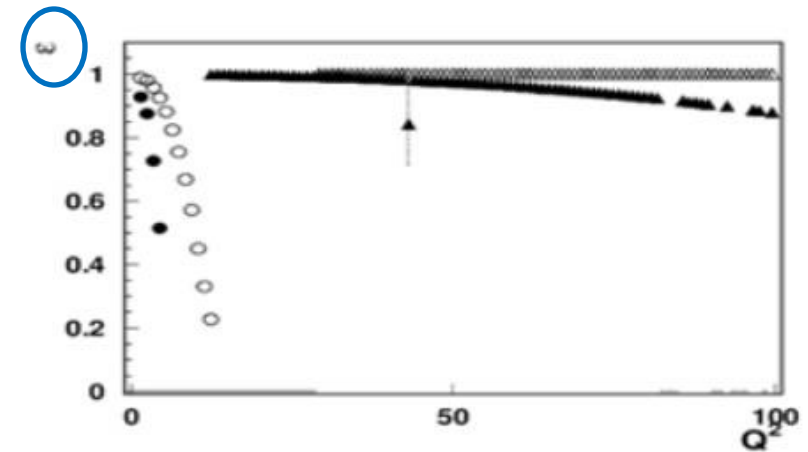
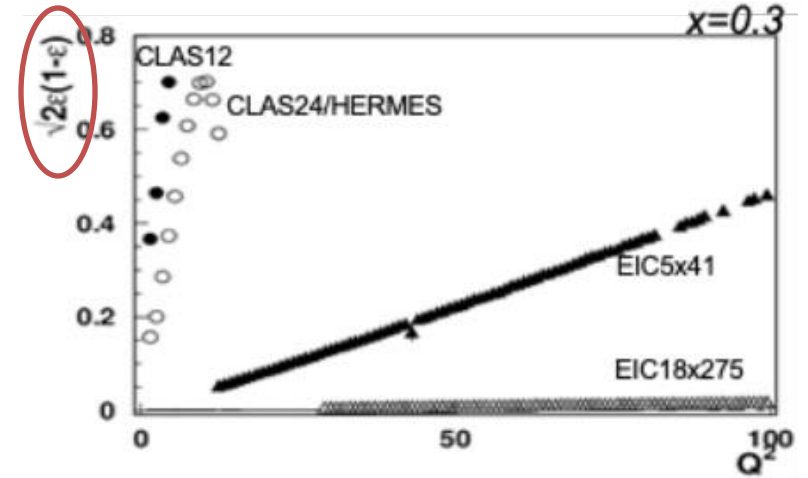
$$\begin{aligned}
 & \frac{d\sigma}{dx dy d\phi_S dz d\phi_h dP_{h\perp}^2} \\
 &= \frac{\alpha^2}{x y Q^2} \frac{y^2}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right. \\
 &+ \lambda_e \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} + S_L \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \\
 &+ S_L \lambda_e \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 &+ S_T \left[\sin(\phi_h - \phi_S) \left(F_{UT,T}^{\sin(\phi_h - \phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) + \varepsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right. \\
 &+ \varepsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} \\
 &+ \left. \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] + S_T \lambda_e \left[\sqrt{1-\varepsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} \right. \\
 &+ \left. \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} - \sqrt{2\varepsilon(1-\varepsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \left. \right\}
 \end{aligned}$$

Sub-Leading Terms

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q} C \left[-\frac{\hat{h} \cdot \mathbf{k}_T}{M_h} \left(x h H_1^\perp + \frac{M_h}{M} f_1 \frac{\tilde{D}^\perp}{z} \right) - \frac{\hat{h} \cdot \mathbf{p}_T}{M} \left(x f^\perp D_1 + \frac{M_h}{M} h_1^\perp \frac{\tilde{H}}{z} \right) \right]$$

TMD Evolution

Evolution kernel (with CS non-perturbative kernel)

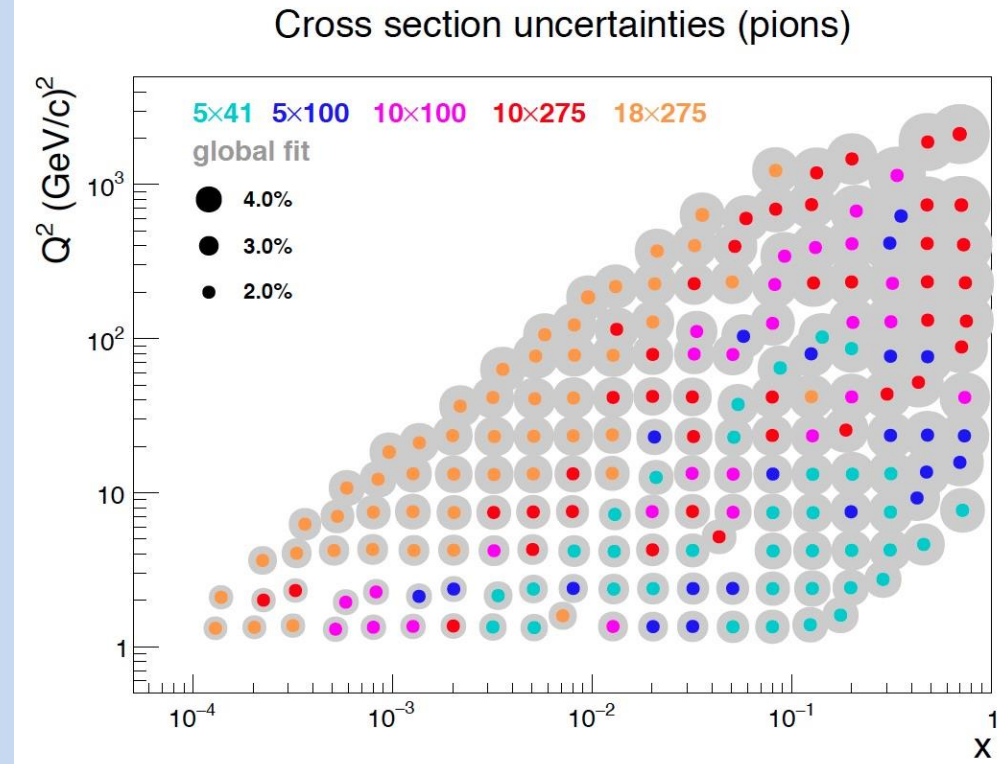
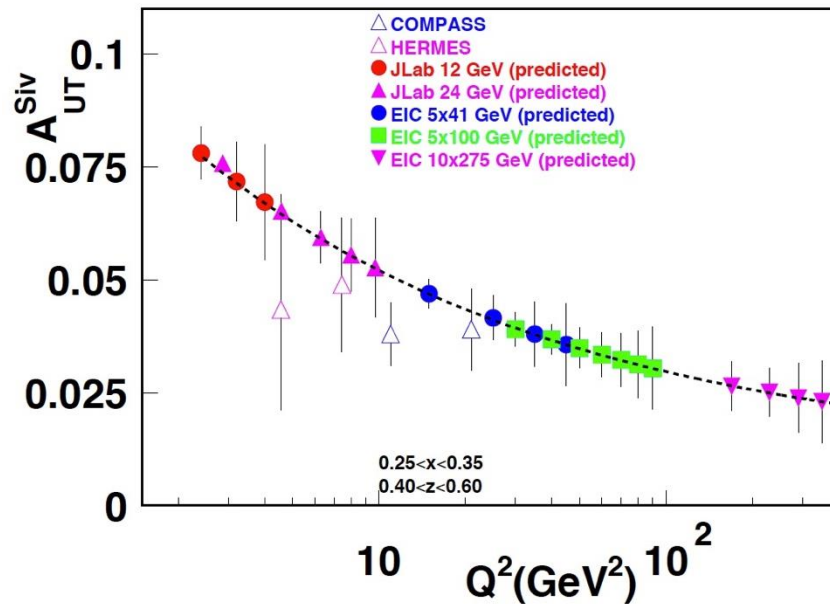


H. Avakian at Transversity 2022

The Q² Game

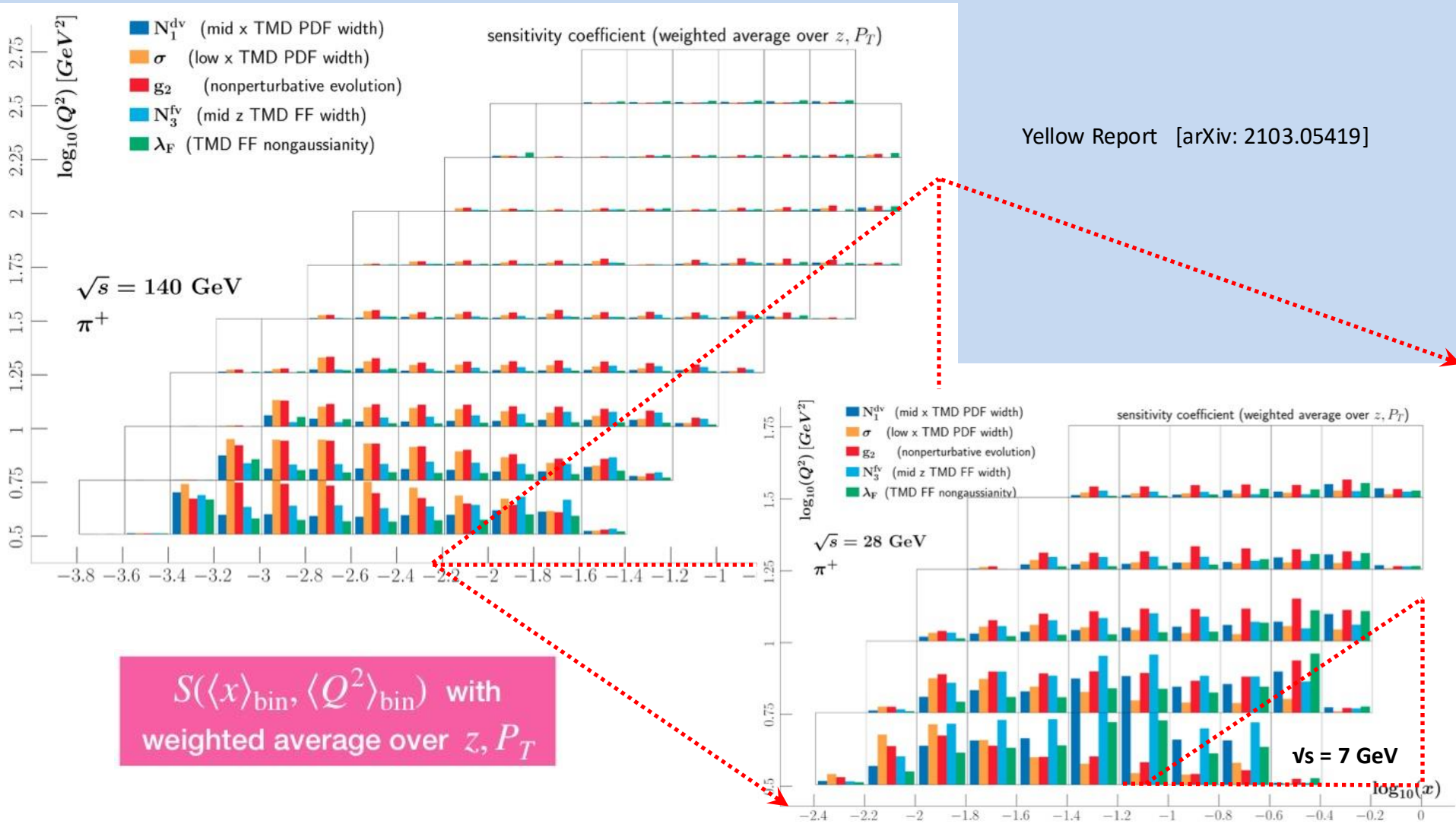
Wide leverage (at given x) to: isolate higher-twists (1/Q suppressed terms)
probe Q² evolution
disentangle x dependence

Keep Q² moderate to: avoid perturbative dilution



TMDs Description

The sensitivity on the relevant parameters changes with center of mass energy

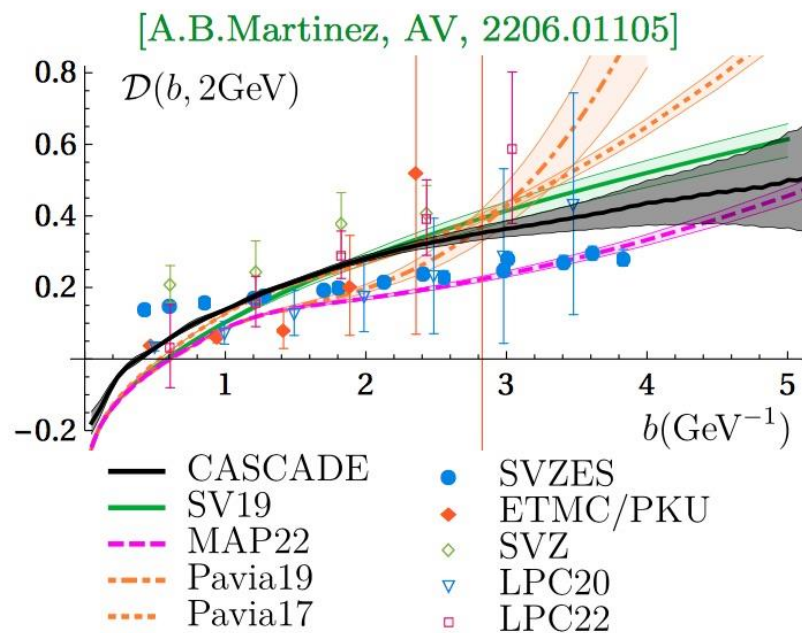


The missing non-perturbative universal piece can be extracted from data

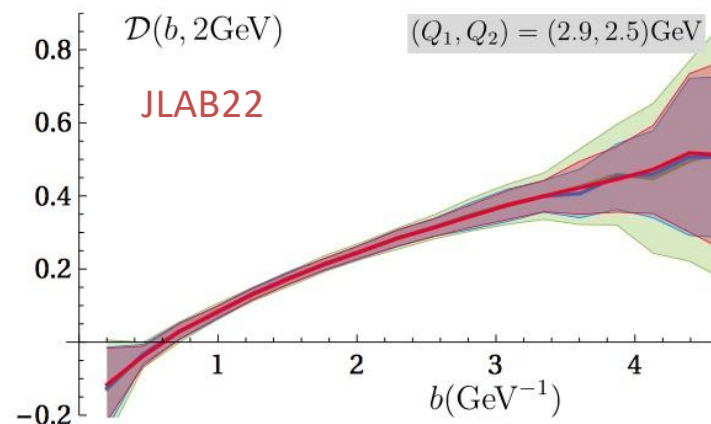
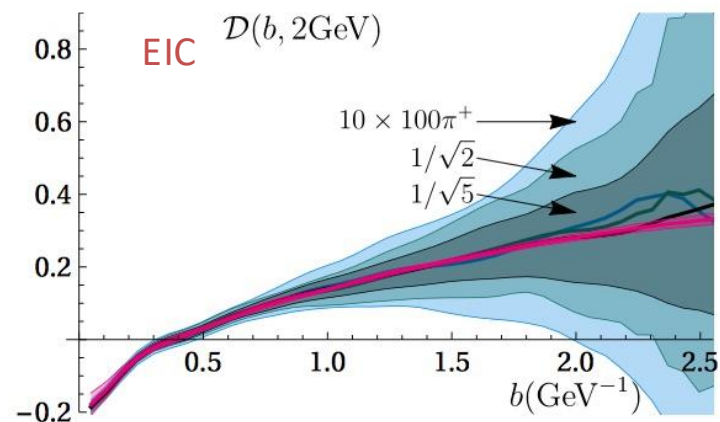
With b as Fourier conjugate of P_T/z

$$F_{UT}^{\sin(\phi_h - \phi_S)} = \sum_q e_q^2 |C_V(Q)|^2 \int \frac{d^2b}{(2\pi)^2} e^{i(b \cdot P_T)/z} R(Q, b, \mu_0) f_{1T}^{\perp q}(x, b; \mu_0) D_1^q(z, b; \mu_0)$$

Collins-Soper non-perturbative evolution kernel



Complementarity in Q^2 and b coverage



A. Vladimirov @ APCTP22 workshop

Conclusions

The last decade provided many evidences that correlation of partonic transverse degrees of freedom in the nucleon do exist and manifest in hadronic interactions

Next step: Moving from phenomenology to rigorous treatment (predictive power)

New data coming from JLab++ at high-luminosity and EIC at high-energy should allow to:

- Constrain models in the valence and sea region
- Test factorization, universality and evolution
- Study higher twist effects
- Investigate non-perturbative to perturbative transition (along P_T)
- Flavor separation via proton and deuteron targets and hadron ID
- Test of Lattice QCD calculations

A comprehensive study provides access to the peculiar dynamics of the QCD confined world