ELSEVIER

Contents lists available at ScienceDirect

## Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima



# Refractive index of silica aerogel: Uniformity and dispersion law

T. Bellunato<sup>a,\*</sup>, M. Calvi<sup>a,b</sup>, C. Matteuzzi<sup>a</sup>, M. Musy<sup>a</sup>, D.L. Perego<sup>a</sup>, B. Storaci<sup>c</sup>

<sup>a</sup> Istituto Nazionale di Fisica Nucleare, Sezione di Milano-Bicocca, Italy

<sup>b</sup> Università degli Studi di Milano-Bicocca, Italy

<sup>c</sup> CERN, CH-1211, Genève, Switzerland

#### ARTICLE INFO

Available online 19 July 2008

*Keywords:* Cherenkov detectors Silica aerogel

#### ABSTRACT

Two methods for the measurement of the uniformity of the refractive index n within a single block of silica aerogel are described. One is based on the deflection of a laser beam induced by transverse index gradients. The second exploits the Cherenkov effect, measuring the emission angle of photons radiated by 500 MeV electrons traversing the aerogel. The beam can scan the full aerogel surface providing information on point to point variations of n.

The measurement of the dispersion law  $n(\lambda)$  is also reported. An Xe lamp coupled to a diffraction grating provides the monochromatic source. The index for each  $\lambda$  is measured by the prism method at a corner of an aerogel sample. A Sellmeier functional form for  $n(\lambda)$  is assumed, and the parameters best fitting the experimental data are given.

© 2008 Elsevier B.V. All rights reserved.

#### 1. Introduction

Silica aerogel is enjoying an increasing popularity as a radiator for Cherenkov detectors. The impressive improvements in the optical quality of large monolithic blocks over the last few years allow to fully exploit its unique properties as a tunable refractive index, low-density solid. We devised a set of experimental procedures to characterise the optical properties of aerogel, with focus on the uniformity of the index of refraction within a large sample [1] and on the identification of the most appropriate functional form for the dispersion law  $n(\lambda)$  [2]. The aim of this work is to provide as realistic as possible information to be included in the simulation for a detector in the design phase or for understanding the performance of an existing one. All the measurements reported here have been performed on hygroscopic silica aerogel produced by the Boreskov Institute of Catalysis in Novosibirsk for the LHCb experiment [3].

The refractive index has been measured with the prism method which is shown in Fig. 1. Assuming the index of refraction of air  $n_{air} = 1$ , simple calculations in the classical geometrical optics framework give the following formula:

$$\theta = \Phi + \arcsin\left\{n\sin\left[\alpha - \arcsin\left(\frac{\sin\Phi}{n}\right)\right]\right\} - \alpha.$$
(1)

Here,  $\theta$  is the deflection angle,  $\Phi$  is the incident angle on the aerogel surface, *n* is the refractive index of the solid and  $\alpha$  is the angle between the two adjacent sides of the block ( $\alpha \simeq 90^{\circ}$ ).

\* Corresponding author.

#### 2. Uniformity of the refractive index

What makes silica aerogel particularly appealing to RICH designers is that its refractive index can be chosen in the range 1.008–1.1 [4]. The manufacturer can tune the material density  $\rho$ , and n and  $\rho$  are related by

$$n(\lambda) = 1 + k(\lambda)\rho \tag{2}$$

where k is a wavelength-dependent coefficient.

Local density inhomogeneities lead to point-to-point variations of the refractive index within the monoliths. These variations contribute to the Cherenkov angle  $\theta_{\rm C}$  uncertainty in RICH detectors. For LHCb aerogel, the maximum allowed variation is  $\sigma(n-1)/(n-1) = 1\%$ , corresponding, for n = 1.030, to  $\sigma(\theta_{\rm C}) \simeq 1.17$  mrad.

A method to measure refractive index inhomogeneities, shown in Fig. 2, consists of a laser beam directed perpendicular to the aerogel surface. The deviation angle  $\delta$  is proportional to the refractive index gradient,  $dn/dy = n \cdot \delta/t$ , where *t* is the thickness of the block. Scanning  $\delta(y)$  along the *y* direction, the integrated variation can be determined:

$$\Delta n(y) = \frac{n}{t} \cdot \int_{y_0}^{y} \delta(y) \, \mathrm{d}y. \tag{3}$$

This effect is widely exploited in gradient index optics and a full derivation is given in Ref. [1]. The biggest limitation of this method is that only one wavelength at a time is used, and there is a certain arbitrariness in extrapolating the result to the wavelength range relevant for the Cherenkov emission.

E-mail address: tito.bellunato@mib.infn.it (T. Bellunato).

<sup>0168-9002/\$ -</sup> see front matter  $\circledcirc$  2008 Elsevier B.V. All rights reserved. doi:10.1016/j.nima.2008.07.072



**Fig. 1.** Set-up of the prism method measurement of the silica aerogel refractive index *n*.



Fig. 2. Set-up for the laser beam method used to study the refractive index uniformity.

An alternative method was therefore developed, based on the use of a charged particle beam of velocity  $\beta \approx 1$ . This method exploits the Cherenkov effect itself, and it is therefore appropriate to study the influence of refractive index variations convoluted with the emission spectrum on the Cherenkov angle reconstruction performance.

A table-top RICH detector has been designed and built, under the acronym "APACHE" for Aerogel Photographic Analysis by CHerenkov Emission. A light tight anodised aluminium vessel filled with dry N<sub>2</sub> provides the housing for the aerogel, a spherical mirror and the ILFORD<sup>®</sup> HP5 Plus 400 black and white film used as photon detector. The spectral sensitivity of the film is flat for  $400 < \lambda < 630$  nm and rapidly falls off outside this range. The setup, sketched in Fig. 3, has been installed at the 500 MeV electron beam of the DAΦNE Beam Test Facility in Frascati (BTF) [5]. The electron beam, producing saturated Cherenkov rings in the aerogel, is used to scan the index of refraction by varying the entrance point of the beam in the aerogel block. This is achieved by moving the aerogel only, leaving the rest of the set-up and the beam settings unaffected.

The GEANT4 [6] simulation toolkit has been used to study the systematics involved in data processing and analysis. All the details of the APACHE configuration have been implemented in a dedicated complete simulation. Fig. 4 is an example of a simulated run and a scanned image of a film of an actual APACHE run, both displaying the aerogel ring and the smaller nitrogen ring.

Data analysis has been performed on digitized images. From the corresponding absolute position in space of a pixel in the scanned image, one can infer its Cherenkov emission angle and build the corresponding distribution. The angle corresponding to



Fig. 3. A schematic view of the APACHE set-up.



**Fig. 4.** On the left, simulation of the photon distribution on the film for a definite refractive index. On the right, the measured distribution for an unknown refractive index.



Fig. 5. An interpolated colour map showing the measured deviation from the average Cherenkov angle for each beam entrance point, represented as a white dot.

the maximum of the distribution is computed to within an uncertainty of 0.3 mrad and compared for 25 entrance points of the electron beam inside an LHCb aerogel sample. The mean deviation from the average value of the Cherenkov angle is 0.9 mrad, corresponding to a variation  $\sigma(n-1)/(n-1) = 0.76\%$ , well within the LHCb specifications, as shown in Fig. 5.

#### 3. Dispersion law

The dispersion law of aerogel can be parameterised by a multipole Sellmeier formula which can be written as a function of the wavelength  $\lambda$ :

$$n^{2}(\lambda) - 1 = \frac{a_{0}\lambda^{2}}{\lambda^{2} - \lambda_{0}^{2}} + \frac{a_{1}\lambda^{2}}{\lambda^{2} - \lambda_{1}^{2}} + \cdots$$
 (4)

Refractive indices at several wavelengths have been measured using the experimental arrangement shown in Fig. 1. A monochromator coupled to an Xe UV-vis lamp has been used as light source. It selects wavelengths from 200 to 900 nm, with a resolution of 1 nm. The quality of the spot exiting the tile is strongly influenced by diffusion from imperfections on the side surface. In order to cope with this problem, a good-quality area was selected as the beam entrance point, and measurements were performed at a fixed incident angle  $\Phi$ . The angle  $\Phi$  has been estimated by measuring the displacement at  $\lambda = 632.8 \text{ nm}$ , corresponding to n = 1.0283, as previously measured with a laser beam. For each selected wavelength  $\lambda$ , the value of the refractive index  $n(\lambda)$  has been obtained by solving numerically Eq. (1). The evaluation of the uncertainty  $\sigma_n$  has been performed using a Monte Carlo simulation in which all variables in Eq. (1) are normally distributed around their measured values.

Data and the fit to Eq. (4) are shown in Fig. 6. The results for a single-pole expansion give  $a_0 = 0.05639 \pm 0.00004$  and  $\lambda_0 = 83.22 \pm 1.25$  nm. An attempt to fit experimental data with a two-pole Sellmeier formula in the range 350–700 nm gives two superimposed poles.

Assuming aerogel is just a rarefied form of silica,  $a_0$  and the density of the material are linked by the relation [2]

$$\rho(\text{aerogel}) = \frac{a_0(\text{aerogel})}{a_0(\text{SiO}_2)} \cdot \frac{n^2(\text{SiO}_2) + 2}{n^2(\text{aerogel}) + 2} \cdot \rho(\text{SiO}_2).$$
(5)

From the Sellmeier parameters found one can infer a density  $\rho(\text{aerogel}) = (0.158 \pm 0.001) \text{g/cm}^3$ , in coarse agreement with  $\rho_0 = (0.149 \pm 0.004) \text{g/cm}^3$  provided by the manufacturer.

The effects of the dispersion law of the aerogel on the particle identification performances in a RICH detector have been investigated with an adapted version of the simulation package described earlier. For the current studies the photographic film has been replaced with a photon detector whose quantum efficiency has been set to be the same as for the LHCb HPDs [7].

Mixtures of K<sup>-</sup>'s and  $\pi$ -'s (60k particles in total for each run) with a momentum of 10 GeV/c have been generated. Four



**Fig. 6.** Chromatic dispersion  $n = n(\lambda)$  measured in a hygroscopic silica aerogel tile. A one-pole Sellmeier parameterisation has been fitted to experimental data.

different parameterisations of the dispersion law of the aerogel have been used:

- (i)  $n_1 = \text{const.}$ , wavelength-independent;
- (ii)  $n_2 = n_2(\lambda)$ , the one-pole Sellmeier formula measured in this paper;
- (iii)  $n_3 = n_3(\lambda)$  derived from the Clausius–Mossotti equation [8] assuming a simple binary mixture of air and SiO<sub>2</sub>;
- (iv)  $n_4 = n_4(\lambda)$ , a two-pole Sellmeier formula used in the LHCb performance studies in Ref. [3].

The graph of these dispersion curves is shown in Fig. 7; the density-dependent factors have been tuned to match the value n = 1.0283 measured at  $\lambda = 632.8$  nm. Both  $n_3$  and  $n_4$  have been proposed in the past as possible approximations to the aerogel dispersion curve. It is interesting, therefore, to compare the results of the simulation for these two curves and the measured one,  $n_2$ .

For each run and for each kind of particle, the average value of the reconstructed  $\theta_c$  and the standard deviation of the single photon resolution have been determined with a Gaussian fit. Results are listed in Table 1. Apart from the case with constant *n*, which is unphysical, the average  $\theta_c$  values do not differ significantly for the different parameterisations, while the single photon resolutions differ substantially. The particle identification performance of a RICH detector strongly depends upon the latter parameter. The determination of the correct law therefore is a powerful tool in the conceptual design of an aerogel-based Cherenkov detector.



**Fig. 7.** Dispersion laws used in the simulation of Table 1. On the  $n_3$  curve the points correspond to measured values of the refractive index of fused silica.

Table 1

Mean  $\theta_c$  and single photon resolution  $\sigma_{\theta}$  (mrad) for the different dispersion laws and particle species considered in the simulation

	К-		π-	
	$\theta_{\rm C}$ (mrad)	$\sigma_{ heta}~({ m mrad})$	$\theta_{\rm C}$ (mrad)	$\sigma_ heta$ (mrad)
<i>n</i> <sub>1</sub>	229.8	0.6	234.3	0.5
n <sub>2</sub>	231.8	2.8	236.3	2.8
n <sub>3</sub> n <sub>4</sub>	231.9	3.5	236.4	3.5

## 4. Conclusions

The dispersion law of the refractive index of silica aerogel and the uniformity within a single block have been measured. Both information provide useful input in the design stage of a Cherenkov detector with an aerogel radiator, as well as a better understanding of its performance.

### Acknowledgements

We acknowledge financial support from JRA9 of I3HP and by INTAS-5579.

#### References

- [1] T. Bellunato, et al., Nucl. Instr. and Meth. A 556 (2006) 140.
- [2] T. Bellunato, et al., Eur. Phys. J. C 52 (2007) 759.
- [3] The LHCb Collaboration, LHCb reoptimized detector design and performance TDR, CERN/LHCC/2003-030, 2003.
- [4] A. Buzykaev, et al., Nucl. Instr. and Meth. A 379 (1996) 465.
  [5] G. Mazzitelli, et al., Nucl. Instr. and Meth. A 515 (2003) 524.
  [6] S. Agostinelli, et al., Nucl. Instr. and Meth. A 506 (2003) 250.
  [7] T. Gys, Nucl. Instr. and Meth. A 567 (2006) 176.
- [8] J.D. Jackson, Classical Electrodynamics, Wiley, New York, 1999.