NEUTRINOS FROM SAN MARCO AND BELOW

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ABSTRACT

Order of magnitude estimates of radiogenic heat and antineutrino production are given, using the San Marco cathedral as an example. Prospects of determining the radiogenic contribution to terrestrial heat by detection of antineutrinos from natural radioactivity (geoneutrinos) are discussed. A three kton scintillator detector in three years can clearly discriminate among different models of terrestrial heat production. In addition, the study of geoneutrinos offers a possibility of improving the determination of neutrino mass and mixing, by exploiting the knowledge of Th/U abundance in the Earth.

1. A few facts about San Marco

After the visit of the San Marco cathedral, accompanied by a most interesting historical and artistic overview, nothing should be added but our gratitude to Milla Baldo Ceolin for organizing this most interesting and timely conference and for spicing it with exceptional events. Physicists however are tenacious, and we cannot resist adding a few additional information, some of these a touristic guide will never tell you.

First of all, San Marco contains radioactive materials. The cathedral mass being in the range of 100 kton, we expect it contains about 100 kg of Uranium, as the typical Uranium abundance in rocks is in the range of one part per million, however with large variations. We also expect some 400 kg of Thorium, since almost everywhere in the solar system (meteorites, Moon, Venus and also Earth) the typical abundance ratio is $Th/U \simeq 4$. In addition, there are about 100 Kg of ${}^{40}K$, corresponding to the typical ratio $K/U \simeq 10^4$ which one finds in Earth rocks and to the natural abundance ${}^{40}K/K = 1.2 \cdot 10^{-4}$.

San Marco is also a heat source. In fact, each decay chain releases energy over long time scales (*e.g.* for $^{238}U \Delta = 52$ MeV and $\tau_{1/2} = 4.5$ Gyr), the total heat flow being:

$$H = 9.5M(U) + 2.7M(Th) + 3.6M(^{40}K)$$
(1)

with masses in 100 kg and H in mW. This gives 24 mW for San Marco, really a very weak heat source, although it does not matter in these sunny days.

More interesting to people attending this conference, San Marco is an anti-neutrino source. Each decay chain releases antineutrinos together with heat, with a well fixed ratio. (e.g. $^{238}U \rightarrow ^{206}Pb + 8^{4}He + 6\bar{\nu} + 52$ MeV). The antineutrino luminosity is:

$$L = 7.4M(U) + 1.6M(Th) + 27M(^{40}K)$$
(2)

where again M is in 100 Kg and L in $10^9 \bar{\nu}/s$. This gives about $4 \cdot 10^{10} \bar{\nu}/s$ for San Marco. Antineutrinos from the progenies of Uranium $(E_{max} = 3.3 MeV)$ and Thorium $(E_{max} = 2.2 MeV)$ can be detected by means of inverse beta decay reaction:

$$\bar{\nu} + p \to n + e^+ - 1.804 \, MeV \tag{3}$$

At least in principle, the two components can be discriminated, due to the different end points. Unfortunately, antineutrinos from β decay of ${}^{40}K$ are below the threshold for (3), whereas neutrinos from electron capture are obscured by the Sun. The cross section of (3) for 2.5 MeV antineutrinos ($\sigma \simeq 5 \cdot 10^{-44} cm^2$) corresponds to an interaction length in water $\lambda = 7 \cdot 10^{18}$ m. There is plenty of water near the cathedral and San Marco square ($S \simeq 10^4 \text{ m}^2$) is often covered with Acqua Alta (high water), a 10 cm height corresponding roughly to 1 kton, the size of KamLAND. Should Acqua Alta reach the clock of Torre dell'Orologio, the size of Superkamiokande is reached. With a height of 100m a megaton detector is obtained, the cathedral being now deeply submerged. A pointlike source emitting $10^{10}\bar{\nu}/s$ with E=2.5 MeV at the center of a megaton water sphere will produce one event every four years. Also in view of the environmental impact of such a project, better we look at some other direction.

2. The sources of terrestrial heat

Earth re-emits in space the radiation coming from the sun $(K_{\odot} = 1.4 \text{ kW/m}^2)$ adding to it a a tiny flux of heat produced from its interior ($\Phi \simeq 80 mW/m^2$). By integrating this latter over the Earth surface one gets a flow $H_E = 40TW$, the equivalent of some 10⁴ nuclear power plants. The origins of terrestrial heat are not understood in quantitative terms. In 1980 J. Verhoogen concluded a review of terrestrial heat sources by saying ¹): "...what emerges from this morass of fragmentary and uncertain data is that radioactivity itself could possibly account for at least 60 per cent if not 100 per cent of the Earth's heat output. If one adds the greater rate of radiogenic heat production in the past, possible release of gravitational energy (original heat, separation of the core...) tidal friction ... and possible meteoritic impact ... the total supply of energy may seem embarassingly large...". One can appreciate the complexity of the problem by comparing Sun and Earth energy inventories. In fact, a constant heat flow H can be sustained by an energy source U for an age t provided that $U \ge Ht$. For sustaining the sun over an age $t_{solar} = 4.5 \cdot 10^9$ yr, gravitational and chemical energies are short by a factor 10^2 and 10^6 respectively and only nuclear energy can succeed, as beautifully demonstrated by Gallium experiments in the nineties. On the other hand, the terrestrial heat can be sustained over geological times by any energy source, be it nuclear, gravitational or chemical.

Observational data on the amounts of Uranium, Thorium and Potassium in Earth interior are rather limited, since only the crust and the upper part of the mantle are accessible to geochemical analysis. As U, Th and K are lithofile elements, they accumulate in the continental crust (CC). Estimates for the Uranium mass in the crust are in the range^{2,3)}

$$M_c(U) = (0.2 - 0.4)10^{17} kg \quad . \tag{4}$$

Concentrations in the mantle are much smaller, however the total amounts are comparable due to the much larger mass of the mantle. Estimates for the whole mantle are in the range¹⁾

$$M_m(U) = (0.4 - 0.8)10^{17} \, kg \quad . \tag{5}$$

One has to remark, however, that these estimates are much more uncertain than for the crust as they are obtained by: i)collecting data for upper mantle $(h_u = 600 km)$, ii)extrapolating them to the completely unexplored lower mantle $(h_l = 3000 km)$.

Concerning the abundance ratios, one has generally $Th/U \simeq 4$, consistent with the meteoritic value. A remarkable exception is the oceanic crust where $Th/U \simeq 2$, however both U and Th abundances are an order of magnitude smaller with respect to CC, which is also much thicker.

Concerning Potassium, generally one finds $K/U \simeq 10,000$. Earth looks thus significantly impoverished in Potassium with respect to Carbonaceous Chondrites, and also to other meteorites. This has long been known as the Potassium problem^{4,5)}. In fact, elements as heavy as Potassium should not have escaped from a planet as big as Earth. It has been suggested that at high pressure Potassium behaves as a metal and thus it could have been buried in the Earth core, where it could provide the energy source of the terrestrial magnetic field. However, Potassium depletion is also observed in Moon and Venus rocks. The most reasonable assumption is that it volatized in the formation of planetesimals from which Earth has accreted.

In conclusion, the determination of the Uranium, Thorium and Potassium in the Earth is an important scientific problem, as it can fix the radiogenic contribution to terrestrial heat production.

3. Antineutrinos from below

"If there are more things in heaven and Earth than are dreamt of in our natural philosphy, it is partly because electromagnetic detection alone is inadequate". With these words in 1984, Krauss, Glashow and Schramm⁶) proposed a program of antineutrino astronomy and geophysics, which could open vast new windows for exploration above us and below. Now that we understand the fate of neutrinos it is time to tackle the program (including Earth energetics, a detailed study of the solar core, neutrinos from past supernovae...) Determination of the radiogenic contribution to terrestrial heat production is the first step.

One can build several models for the radiogenic heat production. Since for any element there is a well fixed ratio heat/(anti)neutrinos each model also provides a prediction for the antineutrino luminosities, the basic equations being (1) and (2), where the same numerical coefficients can be used when masses are in units of 10^{17} kg, powers are in TW and luminosities in $10^{24}/s$. At this level, everything is fixed in terms of three inputs. The range of plausible models is covered in fig.1, which deserves the following comments:

i)In a simple chondritic model one assumes that Earth is obtained by assembling together the same material as we find in these meteorites, without loss of heavy enough elements. The amounts of U, Th and K are determined from their ratio to Si in meteorites, by rescaling to the known abundance of this latter element in the Earth. This simple model easily accounts for 3/4 of H_E , mainly supplied from ${}^{40}K$, however it implies $K/U = 7 \cdot 10^4$, a factor seven larger than the value observed in Earth rocks.

ii) In the standard model of geochemists, the so called Bulk Silicate Earth (BSE) model, with $K/U = 1 \cdot 10^4$, the radiogenic production is about one half of the terrestrial heat flow, being supplied mainly from U and Th.

iii) A fully radiogenic model, where the BSE abundances are rescaled imposing by imposing $H_{rad} = 40$ TW, is not excluded by observational data.

In all models, see fig.2, antineutrino production is dominated by ${}^{40}K$ decays. Th and U anti-neutrino luminosities are in the range $(10 - 20) \cdot 10^{24}/s$. By dividing over the earth surface, one gets fluxes of order 10^6 cm⁻²s⁻¹, in the same range as that of solar boron neutrinos.

Clearly, in order to estimate fluxes at a specific site one needs assumptions about



Figure 1: Contributions to terrestrial heat production (TW), as estimated in different models of Earth interior. CON= chondritic, BSE=bulk silicate Earth, RAD= fully radiogenic. The non radiogenic contribution is indicated as NR.

the distribution of the radioactive materials, the total amounts being an insufficient information. In a paper submitted on December 2002 we provided estimates corresponding to different models for several sites⁷). The numbers of predicted events for Kamland (normalized to an exposure of $0.14 \cdot 10^{32} p \cdot yr$, a detection efficiency $\epsilon = 78\%$ and a survival probability $P_{ee} = 0.55$) are 3.5, 4 and 6 for the chondritic, BSE and fully radiogenic model respectively.

The paper ended with: "The determination of the radiogenic component of terrestrial heat is an important and so far unanswered question...., the first fruit we can get from neutrinos, and Kamland will get the firstlings very soon".

A few days later the first results from Kamland appeared⁸⁾, containing a first glimpse of Earth interior in addition to important information on neutrino oscillation. Out of a total of 32 counts, in the prompt energy region below 2.6 MeV, 20 events are associated with antineutrinos from reactors and 3 correspond to the estimated background. The remaining 9 counts are the first indication of geo-neutrinos! Clearly the uncertainty is large, since the expected statistical fluctuations are about $\sqrt{32}$. Within this error, the result is consistent with any model for the radiogenic contribution to terrestrial heat, from 0 to 100 TW. One has to remind that data have been collected in just six months and that significant accumulation can be achieved



Figure 2: Neutrino luminosity. Antineutrinos production rates from U, Th and K according to different models of Earth interior. In the last column neutrinos from e.c. of ${}^{40}K$ are shown. Units are 10^{24} s⁻¹.

in the future.

4. Prospects for measuring terrestrial heat with geoneutrinos

The first Kamland results really open a new window on Earth interior. A natural question is thus: what can be learnt on heat production from geoneutrino measurements?

Let us consider Uranium geoneutrinos, since expected event numbers are higher and energies can be chosen in a range where background is negligible. The basic equations relating event number N(U), Uranium mass in the crust $M_c(U)$, in the mantle $M_m(U)$, and Uranium contribution to heat production rate H(U) are:

$$H(U) = h[M_c(U) + M_m(U)]$$
(6)

$$N(U) = n_c M_c(U) + n_m M_m(U).$$
 (7)

The first equation simply adds the contribution of crust and mantle, the proportionality constant being $h = 9.5 T W / 10^{17}$ kg. The different coefficients n_c and n_m reflect the different distances of the detector from sources in the crust and in the mantle. They also depend on the detector characteristics and on neutrino properties: as a normalization we shall consider an exposure of $10^{32}p \cdot yr^{\rm a}$, a 100% detection efficiency and a neutrino survival probability $P_{ee} = 0.55$.

Assuming a uniform distribution in the mantle one has, independently of the site, $n_m = 18/(10^{17} \text{kg})$. On the other hand, n_c depends on the location. Our preliminary

^a1kton of mineral oil contains $0.86 \cdot 10^{32}$ free protons.

calculations, based on a crustal map of the whole Earth, give $n_c = 52/(10^{17} \text{kg})$ and $n_c = 63/(10^{17} \text{kg})$ for the Kamioka mine and the Gran Sasso respectively.

There are two unknowns in eqs.(6) and (7), $M_c(U)$ and $M_m(U)$. From a single experiment one cannot extract both (in principle this could be achieved with two experiments, at markedly different geological sites, i.e., with very different values of n_c).

One has to remind that $M_m(U)$ has been estimated by extrapolating to the whole mantle rather uncertain data obtained from the upper mantle. A reasonable program is thus: i)use available geochemical information to fix $M_c(U)$ and ii) use geo-neutrinos to get information on $M_m(U)$, and thus on H. In this way, from the above equations one gets:

$$H(U) = (h/n_m)[N(U) - (n_c - n_m)M_c(U)]$$
(8)

A significant experiment should be capable of discriminating among models of heat production. In the BSE model one expects H(U) = 8.6 TW. Predictions for the Uranium contribution are between 6 and 16 TW, the lower (upper) value corresponding to the chondritic (fully radiogenic) model. A significant experiment should thus provide a measurement with a (1σ) error of about 2-3 TW.

By considering the full range of estimated Uranium mass in the crust as a 3σ interval, one has $M_c(U) = (0.30 \pm 0.07)10^{17}$ kg. From eq.(8), which tells how to get H(U) once that N(U) has been measured with geoneutrinos, one derives the uncertainty on the extracted value of H(U):

$$\Delta H(U) = (\Delta H_1^2 + \Delta H_2^2)^{1/2} \quad , \tag{9}$$

where:

$$\Delta H_1 = (h/n_m) \Delta N(U) \quad , \tag{10}$$

and

$$\Delta H_2 = h \frac{(n_c - n_m)}{n_m} \Delta M_c(U) \quad . \tag{11}$$

Geoneutrino events are obtained from the counts number C after subtracting reactor R and background B events, so that:

$$\Delta N = C^{1/2} = (N + R + B)^{1/2} \quad . \tag{12}$$

As demonstrated by KamLAND, background can be negligible in the energy region where most of U-events occur. The expectations for various experiments at different sites, collected in table 1, suggest the following comments:

i)KamLAND can reach an exposure corresponding to $10^{32}p \cdot yr$ (and full efficiency) in a rather short time. With this exposure, unless the nearby reactors power is reduced, the total uncertainty on H(U) would $\Delta H(U) \simeq 8$ TW, dominated by fluctuations of reactor events. A ten times large exposure (achievable in several years, with KamLAND, or in a much shorter time using a mineral oil detector with

Superkamiokande size) becomes most interesting. One can estimate an accuracy of about 3 TW even with the present reactor power. This would really provide significant information on the radiogenic heat production. Note however that some 270 Uranium events will have to be extracted from a total of about 2000 counts and that in the present calculation accidental background has been neglected.

ii)At Gran Sasso, where the reactor antineutrino flux is much smaller, an exposure corresponding to $10^{32}p \cdot yr$ and full efficiency should be reached in a reasonable time by Borexino. The uncertainty would be $\Delta H(U) \simeq 5$ TW. An experiment with ten times more data could reduce the uncertainty to about 2 TW.

iii)As a limiting case, let us consider and ideal site where crust and mantle have the same weight in the determining neutrino events, so that N(U) is proportional to the total Uranium mass in the crust plus mantle. The ideal place is thus such that $n_c = n_m$, so that uncertainty on $M_c(U)$ do not affect the result, see eq.(11). Presumably this means a place in the middle of oceans, far away from the continental crust. This place is presumably also remote from nuclear plants, so that the error on N is essentially due to statistical fluctuations. Already with $10^{32}p \cdot yr$ one can reach an accuracy of about 2 TW.

In conclusion, there are good prospects for reaching an accuracy $\Delta H(U) = (2-3)$ TW and thus fixing an important missing point on the sources of terrestrial heat.

Table 1: The achievable accuracy ΔH on the determination of U contribution to heat flow. Calculation are performed for $10^{32}p \cdot yr$, 100% efficiency and a survival probability $P_{ee} = 0.55$. Background counts are set to zero. In the last two lines, calculations are rescaled for an exposure of $10^{33}p \cdot yr$, the achievable accuracy being now $\Delta H(33)$. Errors on heat flows are in TW.

		Location		Remarks
	Kamioka	Gran Sasso	Ideal	
n_c	52	63	18	
n_m	18	18	18	
R	180 a)	$35 \mathrm{b})$	0	Reactor events:
				a)from KamLAND data
				b)from Raghavan et al. $estimate^{9}$
В	0	0	0	No background is assumed
N(best)	26.4	29.7	16.2	Geo-events from best BSE estimate
				$(M_c(U) = 0.3 \text{ and } M_m(U) = 0.6)$
C = R + B + N(best)	206.4	64.7	16.2	Total counts
$\Delta N = \sqrt{C}$	14.4	8.04	4	Statistical fluctuations only
$\Delta H_1 = (h/n_m)\Delta N$	7.6	4.2	2.1	Error from geo-neutrinos
$\Delta H_2 = h(n_c - n_m)/n_m \Delta M_c$	1.3	1.7	0	Error from crust
${oldsymbol{\Delta}} H = \sqrt{\Delta H_1^2 + \Delta H_2^2}$	7.7	4.6	2.1	Total error
$\Delta H_1(33) = (h/n_m)\Delta N$	2.4	1.3	0.7	for $10^{33}p \cdot yr$
$\Delta H(33) = \sqrt{\Delta H_1^2 + \Delta H_2^2}$	2.7	2.1	0.7	for $10^{33}p \cdot yr$

5. Can we learn on neutrinos from geoneutrinos?

Uncertainties on the *individual* Uranium and Thorium fluxes Φ are large. In practice, one cannot use $\Phi(U)$ and/or $\Phi(Th)$ as additional input for extracting neutrino mass and mixing from the lowest energy region ($E_{vis} < E_{geo} = 2.6$ MeV) in a reactor experiment.

However, one can still gain some information on θ and Δm^2 by observing that the *ratio* of events from U and Th is well constrained:

$$r = N(Th)/N(U) = 0.25 \pm 0.05$$
 (13)

This follows from the fact that the abundance ratio Th/U is well fixed in the solar system, being very similar on meteorites, Venus, Moon as well as on Earth , all this information pointing to a common origin of the solar system.

The 20% uncertainty corresponds essentially to uncertainties on the Th/U ratio in the various components of the Earth which contribute appreciably to neutrino production (continental crust and mantle). This uncertainty accounts for the comparison of different estimates and also of different regions of the Earth interior. With respect to the average value Th/U = 3.8 value, the extrema are Th/U = 2 for the oceanic crust (which is however poor in both U and Th and also it is thin, so that it contributes little to neutrino production) and $Th/U \simeq 6$ for the crust estimate by one author, most authors giving values in the range 3.8–4.2. An estimate $Th/U = 3.8\pm0.7$ should be more accurate than 1σ .

The constraint in eq.(13) was derived in 10° assuming a uniform Th/U distribution inside the Earth and assuming that the survival probability of geo-neutrinos reaching the detector is the same, independently of energy and distance. However, as remarked in 7° , the constraint holds within its error even if these simplifying assumptions are relaxed.

Concerning the effect of regional Th/U variations, from ¹¹) it was found that r is changed by less than 2% when the detector is placed at Kamioka, or Gran Sasso, or Tibet (on the top of a very thick continental crust) or at the Hawaii (sitting on the middle of a thin, U- and Th- poor oceanic crust). Coming to the effect of local variations, by assuming that within 100 km from the detector the Uranium abundance is double, Th/U=2, one gets r = 0.22, whereas if its is halved, Th/U=8, one finds r = 0.28. Neutrino oscillations clearly do not affect eq. (7) if the oscillation lengths for both U and Th neutrinos are both very short or very long in comparison with some typical Earth dimension. By numerical calculation, one finds that the effect of finite oscillation lengths does not change r by more than 2% for $\Delta m^2 > 1 \cdot 10^{-5} eV^2$. In conclusion, all these effects are well within the estimated 20% uncertainty on r.

The constraint (13) has been used in (10) in order to extract information from the full data set of KamLAND. A slight preference for the LMA-I solution with respect



Figure 3: The effect of the Th/U constraint on the determination of mass and mixing parameters, from ref.¹⁰

to LMA-II has been found, however it is not statistically significant. On the other hand, a significant reduction of the mixing parameters space has been found, in spite of the limited statistics. This constraint could become really useful when more data are available.

6. Concluding remarks: from neutrons to neutrinos

We all owe very much to Bruno Pontecorvo, the man who addressed the detectability of neutrinos, discussed available sources (Sun, reactors and accelerators), conceived the Cl-Ar method and invented the beautiful phenomenon of neutrino oscillations. There is an additional lesson we can learn from him. In 1941, a few years after the celebrated Rome studies on slow neutrons, Bruno was a research physicist at Well Surveys Inc., and published a paper entitled "Neutron Well Logging - A New Geological Method Based on Nuclear Physics"¹²). He had invented the neutron well log, an instrument still used for the prospection of water and hydrocarbons, see the page of the Society of Professional Well Log Analysts at http://www.spwla.org/. It consists of a neutron source and neutron or gamma detector (well shielded from the rays coming directly from the source) to be placed in the well. As hydrogen atoms are by far the most effective in the slowing down of neutrons, the detected radiation is primarily determined by the hydrogen concentration, i.e. water and hydrocarbons. In this way, the discoveries of the Rome group were applied to the study of quite different problems. Possibly now we have similar opportunities with neutrinos, due to the important achievements of the last few years:

i)The fate of neutrinos is now essentially understood. We don't need anymore to rely on standard solar model calculations or on the comparison among different experiments. SNO has directly observed the transmutation of solar neutrinos and KamLAND has neatly confirmed the result with man made antineutrinos from nuclear reactors.

ii) KamLAND has demonstrated that it can reach purity levels such that detection of geo-neutrinos is feasible, a most impressive experimental result.

All this opens the road for using low energy neutrinos as real probes of nature, be it Earth, Sun, future and past supernovae as well as - may be - the big bang. Measurement of the radiogenic component of terrestrial heat will be the first step. It can be achieved with few kiloton (mineral oil) detectors in a few years, best if far from nuclear reactors.

This goal can be reached without submerging San Marco inside megaton detectors and it will provide a definite answer to an important and long standing scientific problem.

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8. References

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