A Charge Breeder for the SPES Project: theoretical approach and simulations

Il anno Dottorato XXVII ciclo
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2013 ACTIVITY

- Numerical Simulation: benchmark for CB, new Extraction System
- Experimental Activity within EMILIE Project
- Other Activities (conference, other projects)
THE CHARGE BREEDER

1+ BEAM: $E = (V + \Delta V)$ keV

Inside the plasma:
- Elastic collisions between charged particles
- Ionization/Charge Exchange

CB: +V kV

Focusing
Deceleration

Magnetic field

potential

$\varepsilon_{N+} \tau_{cb}$
CB SIMULATIONS: Slowing Down Of Ions In A Plasma

- **THEORY:** Chandrasekhar (Interaction of stars, Principles of Stellar Dynamics)
- **APPLICATION TO CHARGED PARTICLES:** Spitzer (Physics of Fully Ionized Gases)

Long Range Elastic Collisions → Many Small Deflections → Broadening of the Velocity Distribution

- On Average

![Graph showing initial, after N encounters, and after 10N encounters velocity distributions](image)
CB SIMULATIONS: Slowing Down Of Ions In A Plasma

FOKKER PLANK EQUATION

\[
\left( \frac{\partial}{\partial t} \right)_v f(v, t) = -\frac{\partial}{\partial v} \cdot \left\{ f(v, t) \langle \frac{\Delta v}{\Delta t} \rangle \right\} + \frac{1}{2} \sum_{i,j} \frac{\partial}{\partial v_i} \frac{\partial}{\partial v_j} \left\{ f(v, t) \langle \frac{\Delta v_i \Delta v_j}{\Delta t} \rangle \right\}
\]

AVERAGING OVER A PLASMA WITH A MB DISTRIBUTION

DYNAMICAL FRICTION

\[
\langle \Delta w_\parallel \rangle = -\frac{A_D}{C_s^2} \left( 1 + \frac{m}{m_s} \right) G \left( \frac{w}{C_s} \right)
\]

DIFFUSION IN VELOCITY SPACE

\[
\frac{\langle \Delta w_\parallel^2 \rangle}{w} = \frac{A_D}{w} G \left( \frac{w}{C_s} \right)
\]

\[
\frac{\langle \Delta w_{\perp}^2 \rangle}{w} = \frac{A_D}{w} \left\{ \Phi \left( \frac{w}{C_s} \right) - G \left( \frac{w}{C_s} \right) \right\}
\]

CHARACTERISTIC TIMES

\[
\tau_s = -\frac{w}{\langle \Delta w_\parallel \rangle} = \frac{wC_s^2}{(1 + m/m_s)A_D G(w/C_s)}
\]

\[
\tau_E = \frac{E^2}{\langle (\Delta E)^2 \rangle} = \frac{w^3}{4A_D G(w/C_s)}
\]

\[
\tau_D = \frac{w^2}{\langle (\Delta w_{\perp})^2 \rangle} = \frac{w^3}{A_D (\Phi(w/C_s) - G(w/C_s))}
\]

WHERE

\[
A_D = 2\Gamma n_s = \frac{(Z_s Z')^2 e^4 n_s \ln \Lambda}{2\pi \epsilon_0^2 m^2}
\]

\[
\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp \left( -y^2 \right) \, dy
\]

\[
G(x) = -\frac{1}{2} \ln \left( \frac{\Phi(x)}{x} \right) - \Phi(x) - x \Phi'(x)
\]

\[
x = w/C_s,
\]

\[
C_s = \sqrt{\frac{KT_s}{m_s}}
\]
CB SIMULATIONS: Slowing Down Of Ions In A Plasma

FOKKER PLANK EQUATION

\[ \left( \frac{\partial}{\partial t} \right) f(v,t) = -\frac{\partial}{\partial v} \cdot \left\{ f(v,t) \left( \frac{\Delta v}{\Delta t} \right) \right\} + \frac{1}{2} \sum \frac{\partial}{\partial v_i} \frac{\partial}{\partial v_j} \left\{ f(v,t) \left( \frac{\Delta v_i}{\Delta t} \frac{\Delta v_j}{\Delta t} \right) \right\} \]

**AVERAGING OVER A PLASMA WITH A MB DISTRIBUTION**

**DYNAMICAL FRICTION**

\[ \langle \Delta w_\parallel \rangle = -\frac{A_D}{C_s^2} \left( 1 + \frac{m}{m_s} \right) \]

**DIFFUSION IN VELOCITY**

\[ \langle (\Delta w_\parallel)^2 \rangle = \frac{A_D}{w} G \left( \frac{w}{C_s} \right) \]

\[ \langle (\Delta w_\perp)^2 \rangle = \frac{A_D}{w} \left\{ \Phi \left( \frac{w}{C_s} \right) - G \left( \frac{w}{C_s} \right) \right\} \]

**WHERE**

\[ \tau_D = \frac{w^2}{A_D \left( \Phi \left( \frac{w}{C_s} \right) - G \left( \frac{w}{C_s} \right) \right)} = \frac{w^3}{A_D \left( \Phi \left( \frac{w}{C_s} \right) - G \left( \frac{w}{C_s} \right) \right)} \]

\[ x = \frac{w}{C_s}, \quad C_s = \sqrt{\frac{2KT_s}{m_s}} \]

Thermalization

Friction

Diffusion

WHERE

\[ \tau_D = \frac{Z_s Z'_s}{2\pi e^2 n_s \ln \Lambda} \]

\[ (x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp \left( -y^2 \right) dy \]

\[ \frac{1}{2} \frac{d}{dx} \left( \frac{\Phi(x)}{x} \right) = \frac{\Phi(x) - x \Phi'(x)}{2x^2} \]
**BENCHMARK**

**INJECTED IONS**
- $Z = 6$
- $M = 132$
- $V_x = V_y = 0$
- $V_z = 3.4 \times 10^3 \text{ m/s} \Rightarrow x = 0.985$

**PLASMA IONS**
- $<Z> = 3.55$
- $M = 16$
- $n = 7 \times 10^{15} \text{ ioni/m}^3$
- $K_T = 1 \text{ eV}$

**INTERACTING WITH**

**CHARACTERISTIC TIMES**
- $\tau_s = 15.323 \mu\text{s}$
- $\tau_D = 46.305 \mu\text{s}$
- $\tau_E = 33.287 \mu\text{s}$
FIRST APPROACH

**Integration of Equation of Motion**

- \( v = v_0 + a \cdot T_{\text{step}} \)
- \( x = x_0 + v \cdot T_{\text{step}} \)

**Continuous Friction**

- \( a = -v/\tau_S \)

**Monte Carlo treatment of Diffusive processes**

- \( \tau_D \rightarrow P_{\text{rot}} \)
- \( \tau_E \rightarrow P_{\text{diff}} \)

**Characteristics of the Simulation**

- \( N = 10000 \)
- \( T_{\text{step}} = 3 \cdot 10^{-10} \text{s} \)
- \( \text{Int Time} = 1.26 \cdot 10^{-4} \text{s} \)

**Comparison with random numbers**

- \( \sigma = \sqrt{\frac{KT}{M}} \)

**90° Rotation of \( v \)**

- Random Kick to \( v \):
  \( v = v + N(0, \sigma) \)
**FIRST APPROACH**

**Integration of Equation of Motion**

\[ v = v_0 + a \cdot T_{step} \]

\[ x = x_0 + v \cdot T_{step} \]

**Characteristics of the Simulation**

- \( N = 10000 \)
- \( T_{step} = 3 \times 10^{-10} \) s
- \( \text{Int Time} = 1.26 \times 10^{-4} \) s

**Continuous Friction**

\[ a = \frac{-v}{\tau} \]

**Monte Carlo treatment of Diffusive processes**

\[ \tau_D \rightarrow P_{rot} \]

\[ \tau_E \rightarrow P_{diff} \]

**Compared with random numbers**

- 90° Rotation of \( v \)
- Random Kick to \( v \)

\[ v = v + N(0, \sigma) \]

**Distribution of speed after 54\mu s**

**TOO MANY SLOW PARTICLES!!!!**

**Same picture for x, y and z**
SECOND APPROACH

Integration of Equation of Motion

\[ v = v_0 + a \times T_{\text{step}} \]

\[ x = x_0 + v \times T_{\text{step}} \]

Characteristics of the Simulation

\[ N = 10000 \]

\[ T_{\text{step}} = 1 \times 10^{-10} \text{ s} \]

\[ \text{Int Time} = 2.88 \times 10^{-4} \text{ s} \]

\[ V_0 = 3.4 \times 10^3 \text{ m/s} \]

Implementation of the Langevin Equation*

\[ \dot{v} = v_0 + a \times T_{\text{step}} + Q \]

\[ a = -v/\tau_S \]

\[ Q = \text{random vector chose form:} \]

\[ \phi(Q) = \frac{1}{(2\pi T_{\text{step}})^{3/2}(\Delta w_1)^2(\Delta w_2)^2)} \exp \left( - \frac{Q_3^2}{2(\Delta w_1)^2 T_{\text{step}}} - \frac{Q_4^2 + Q_5^2}{2(\Delta w_2)^2 T_{\text{step}}} \right) \]

Where Axes 3 // v; Axes 1,2 ⊥ v

CB SIMULATIONS: RESULTS

**Friction**
- Particles are damped until $\langle v \rangle = 0$
- An isotropic distribution is reached

**Thermalization**
- Isotropic distribution
  - $1.5\ast KT$
  - $\sqrt{KT/M}$
CB SIMULATIONS: RESULTS

CHECK FOR A MB DISTRIBUTION

ALL THE THREE DISTRIBUTION ARE MB

Vx Distribution

Vz Distribution

Matlab Distribution

Vy Distribution
CB SIMULATIONS: Different Tstep and V\(_z(0)\)

**High Tstep_High v**
- N=10000
- Tstep=1E-10
- \(V_0=3.4E+3\) m/s
- Int Time = 2.88E-4 s

**Small Tstep_High v**
- N=2000
- Tstep=1E-11
- \(V_0=3.4E+3\) m/s
- Int Time = 5.1E-5 s

**Small Tstep_Small v**
- N=2000
- Tstep=1E-11
- \(V_0=3.4E+2\) m/s
- Int Time = 5.1E-5 s

Friction not affected by the change in time step

Depending on initial velocity the average energy decrease or increase

In both velocities regimes the expected isotropic distributions are reached

\[ 1.5*KT = \sqrt{KT/M} \]
• The processes of Friction and Velocity Diffusion are well reproduced

• Particles reach a MB Distribution as predicted

• Depending on the initial energy, ions are cooled or heated.
CB SIMULATIONS: UPGRADES

3D Magnetic field of the Booster

Losses on the Chamber Wall

The code is ready to implement ionization

Application of the Lotz formula for a “two zones” plasma

Part of the plasma chamber between the two maxima

Tables ready for $^{132}\text{Sn}$ up to 30+

Analytic Formulas

\[
B_x = -\frac{x \partial B_z}{2 \partial z} + 2S_{xy}
\]

\[
B_y = -\frac{y \partial B_z}{2 \partial z} + S(x^2 - y^2)
\]

\[S = 617.28\]
CB 4 SPES: NEW EXTRACTION SYSTEM

PHOENIX BOOSTER

Stable or Radioactive 1+ beam

Grounded Electrode

Plasma Chamber and Magnetic System

N+ beam to post-acceleration

Single gap Extraction @ 20kV

3 Elect System @ 40kV (new!)

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<td>Simulated @ 40kV</td>
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New Extraction System

Electrode
CB 4 SPES: Experiment within the EMILIE Project

Influence of 2 Frequency Heating on Ar and Kr efficiencies

Injected Ions influence plasma equilibrium

Not Captured Ions and “In-Flight” Ionization
(Investigated the last week @ LPSC)

Work presented at ICIS’13 and accepted on RSI
OTHER ACTIVITIES:
ICIS’13 CHIBA (JAPAN)

• Complete CB beam line

• Tantalum Liner Technique for Ca Beams

• COOLBEAM (RSI 85, 02B909)

• ESS Extraction System (Accepted on RSI)
OTHER ACTIVITIES: ICIS’13 CHIBA (JAPAN)

- Complete CB beam line
- Tantalum Liner Technique
- COOLBEAM (RSI 85, 02B9)
- ESS Extraction System
OTHER ACTIVITIES: ICIS’13 CHIBA (JAPAN)

- Complete CB beam line
- Tantalum Liner Technique for Ca beams
- COOLBEAM (RSI 85, 02B909)
- ESS Extraction System (accepted on RSI)

Feasible for Rb and Cs (SPES)!!!
THANK YOU