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THE PROBABILITY OF MUON STICKING TO HELIUM IN THE MUON-CATALYZED FUSION $dt\mu \to \mu\text{-}^4He + n$

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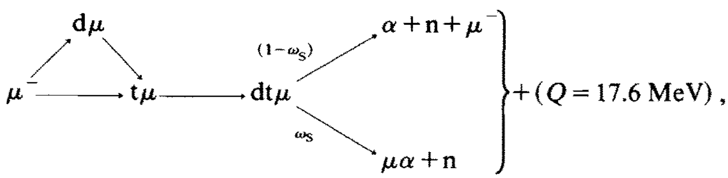
Abstract: In the sudden perturbation approximation, the probability of muon sticking to helium, ω_S^0, is found to equal 0.848×10^{-2} in the reaction $dt\mu \to \mu\alpha + n$. In calculations we have used accurate wave functions of the mesic molecule $dt\mu$ obtained in the adiabatic representation of the three-body problem. Corrections to the sudden approximation do not exceed 3%. In view of a subsequent shaking-off of muons during deceleration of $\mu\alpha$, the resulting sticking probability ω_S equals 0.58×10^{-2}.

1. Introduction

Beginning from 1977 the phenomenon of muon catalysis of nuclear fusion in deuterium–tritium mixture has been intensively investigated in many laboratories of the world, both theoretically and experimentally [for reviews, see refs. [1-4]]. The bottleneck of the chain of muon-catalyzed fusion,



(1)

is the loss of muons in the ($\mu\alpha + n$) channel where the muon sticks to the α-particle. The problem of muon sticking was first discussed by Zeldovich [5]) and Jackson [6]) in 1957. A more careful analysis was performed a few years ago [7-8]), still in the framework of the Born–Oppenheimer approximation and for the case of an infinite rate of nuclear fusion, the latter having been treated in a sudden approximation (BO+S method). The estimated accuracy of these calculations was about 10%*.

In a recent paper [10]), the muon wave function of the $dt\mu$ mesic molecule is calculated with the Monte Carlo method without approximations characteristic for the BO method. The values of the sticking coefficients obtained are lower by 25% than those previously calculated in the BO+S method.

Only recently has some reliable experimental information on muon sticking in the reactions $dd\mu \to \mu\text{-}^3\text{He} + n$ [ref. [11])] and $dt\mu \to \mu\text{-}^4\text{He} + n$ [ref. [12])] become available. The experimental values of muon sticking probabilities appeared somewhat smaller than the theoretical estimates within the BO+S method. In the $dd\mu$ case the experimental accuracy achieved is at the level of a few percent, and it is expected that, in the near future, experimental data of comparable or better accuracy will be available for muon sticking following other nuclear reactions (pd, dt, etc.).

In the present situation it seems reasonable and timely to perform an analysis of the approximations used in the BO+S method and to develop a regular method for a more accurate calculation of muon sticking probabilities in various reactions. These are the aims of the present paper.

The BO+S method involves some approximations, each of them having a corresponding smallness parameter such as the ratio of muon to nucleon mass, m_μ / m_N, the ratio of the characteristic range of nuclear interaction to the muon Bohr radius, R_N / a_μ, etc.

We show in sect. 2 that corrections due to the finite nucleus mass are most important, and, hence, a muon wave function more precise than the BO one is needed.

In sect. 3 a calculation of the muon sticking probability based on the exact muon wave function is presented. The corrections to the sudden approximation are evaluated in sects. 4 and 5. Our results are summarized and discussed in sect. 6.

Only the case of dt fusion is considered here, mainly due to the intense interest in this specific problem. The method developed, however, holds for nuclear reactions involving other hydrogen isotopes [13]). The comparison between theoretical and experimental determination of the sticking probability in all these cases (dd, pd, pt, etc.) is equally important for the solving of the sticking problem in the dt reaction and allows one to gain information which is, in a sense, complementary to that provided by studying the dt reaction.

* A recent calculation [9]) within the BO method is irrelevant, since as the initial-state muon wave function that of the $dt\mu$ mesic molecule at the equilibrium internuclear distance $R = R_0$ was used, instead of the $R \to 0$ limit, as is necessary.

2. Analysis of the BO+S method approximations

The sticking probability ω_S which is actually measured (effective sticking probability) can be expressed as

$$\omega_S = \omega_S^0(1-\gamma)\,, \tag{2}$$

where ω_S^0 is the probability that the muon is bound to the α-particle just after fusion and γ is a shaking-off, or reactivation, coefficient, i.e. the probability that during the slowing-down of the $\mu\alpha$ atom produced the muon is shaken off as a result of stripping reactions during collisions with target nuclei. The BO+S method for the calculation of ω_S^0 relies on the following main assumptions:

(a) The muon follows adiabatically the motion of the two nuclei until fusion occurs.

(b) Fusion is an extremely sudden process, i.e. its time-scale is very short in comparison with any other time relevant to the process.

(c) Any effect associated with finite nuclear size and finite radius of nuclear forces is neglected: dt fusion occurs when the two nuclei are at zero separation, the muon moving in the Coulomb field of a point-like nucleus with the charge and mass of the compound.

For the case of the dt system this means that the initial muon wave function $\Psi_{in}(\boldsymbol{r})$ just before fusion is the wave function of μ-^{5}He in the 1s state with a point-like nucleus,

$$\Psi_{in}^{BO}(\boldsymbol{r}) = \varphi_{1s}(\boldsymbol{r})\,. \tag{3}$$

The amplitude of a muon sticking to the α-particle in the $(\mu\alpha)$ nl state, F_{nl}, is simply expressed as the overlap between Ψ_{in} and the wave function Ψ_f of the $\mu\alpha$ mesic atom in the nl state, moving with velocity $\boldsymbol{V}$, the value of which is defined by the energy release $Q = 17.6$ MeV in reaction (1)*:

$$\Psi_f = e^{im_\mu \boldsymbol{V}\cdot\boldsymbol{r}}\,\Psi_{nl}(\boldsymbol{r})\,, \qquad \boldsymbol{q} = m_\mu \boldsymbol{V}\,, \tag{4}$$

$$F_{nl} = \int d^3r\, \Psi_{nl}^*(\boldsymbol{r})\, e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\, \Psi_{in}(\boldsymbol{r})\,. \tag{5}$$

In refs. [7,8]) partial ω_{nl} and total ω_S^0 sticking probabilities were calculated with the formulae

$$\omega_{nl} = |F_{nl}|^2\,, \qquad \omega_S^0 = \sum \omega_{nl}\,, \tag{6}$$

and the function $\Psi_{in}(\boldsymbol{r}) = \Psi_{in}^{BO}(\boldsymbol{r})$ was used.

Concerning the three main approximations, clearly (a) is exact in the limit that the relative motion of the two nuclei is extremely slow in comparison with the muon's. The relevant smallness parameter equals the ratio of the muon mass m_μ to

* In ref. [8]) the final-state wave function was taken as $\Psi_f = e^{im\boldsymbol{V}\cdot\boldsymbol{r}}\,\Psi_{nl}(\boldsymbol{r})$, where $m = m_\mu m_\alpha (m_\mu + m_\alpha)^{-1}$ is the reduced mass of the $(\mu\alpha)$ atom. It is shown in sect. 5 that a straightforward consideration leads to formula (5).

the reduced mass of the two nuclei m_N,

$$\kappa = m_\mu / m_N . \tag{7}$$

For the dt system one has $\kappa = 0.09$, whereas for the dd and pd systems $\kappa = 0.11$ and 0.15, respectively. The relevant parameter $\tilde{\kappa}$ can also be defined as the ratio of the velocity of the nuclei V_N to the muon velocity $V_\mu = \alpha c$,

$$\tilde{\kappa} \approx V_N / V_\mu . \tag{8}$$

Near the equilibrium position of the nuclei, V_N can be estimated from the relation between the vibrational energy of the nuclei and the muon energy,

$$E_{vib} \approx m_N V_N^2 \simeq (m_\mu / m_N)^{1/2} m_\mu V_\mu^2 , \tag{9}$$

from which

$$\tilde{\kappa} \simeq (m_\mu / m_N)^{3/4} . \tag{10}$$

In this case one obtains $\tilde{\kappa}_{dt} = 0.15$, $\tilde{\kappa}_{dd} = 0.19$, $\tilde{\kappa}_{pd} = 0.26$. These and similar estimates enable one to evaluate the expected contribution from the corrections to the BO approximation at the 10-20% level.

Approximation (b) is based on the fact that the lifetime $\tau_N \simeq 10^{-20}$ s of the compound nucleus $^5He^*$, which can be accepted as a timescale, is small compared to the characteristic time of muon motion in a mesic atom ($\tau_\mu \simeq 10^{-19}$ s). The corresponding parameter is

$$\eta = \tau_N / \tau_\mu \simeq 0.1 . \tag{11}$$

The effects due to the finite size of the nuclei depend on the parameter

$$\xi = R_N / a_\mu \simeq 0.03 , \tag{12}$$

where R_N and a_μ are the characteristic nuclear scale and muon orbit radius. The BO+S method corresponds to the case $\kappa = \eta = \xi = 0$. We now calculate the corrections to the BO+S method due to the finite values of these parameters.

3. The calculation of the muon sticking probability ω_S^0

We calculate the muon sticking probability ω_S^0 without the Born-Oppenheimer approximation, but using an exact dtμ mesic molecule wave function. We will remain in the framework of the sudden approximation and neglect the dimensions of the nuclei and the range of nuclear forces in comparison with the mesic molecular dimension.

In the sudden approximation the sticking probabilities are still defined with formulae (5) and (6), where, however, the muon initial wave function $\Psi_{in}(\boldsymbol{r})$ at the moment of fusion is expressed via the dtμ mesic molecule wave function $\Psi^{Jv}(\boldsymbol{r}, \boldsymbol{R})$ in a rotational-vibrational state (Jv) in the following way:

$$\Psi_{in}(\boldsymbol{r}) = \lim_{R \to 0} \frac{\Psi^{Jv}(\boldsymbol{r}, \boldsymbol{R})}{\left\{ \int d^3 r |\Psi^{Jv}(\boldsymbol{r}, \boldsymbol{R})|^2 \right\}^{1/2}} , \tag{13}$$

R being the distance between d and t in the dtμ mesic molecule. The wave functions $\Psi^{Jv}(\boldsymbol{r}, R)$ have been calculated in refs. [14,15]) with algorithms [16,17]) for the numerical solution of the Coulomb three-body problem in the adiabatic representation [18]). The $R \to 0$ asymptotic of the mesic molecule wave functions has been found in ref. [19]). For J-states it is

$$\Psi^{Jv}(\boldsymbol{r}, R) \underset{R\to 0}{\simeq} A^{Jv} R^J \left\{ \sum_{N=1}^{\infty} a_N^{Jv} \phi_{N0}(\boldsymbol{r}) + \int_0^{\infty} \mathrm{d}k\, a_k^{Jv} \phi_{k0}(\boldsymbol{r}) \right\}, \tag{14}$$

i.e. in this limit the variables r and R which describe the motion of the muon and the nuclei are factorized. Here, $\phi_{N0}(\boldsymbol{r})$ and $\phi_{k0}(\boldsymbol{r})$ are discrete and continuous spectra wave functions of the mesic atom in the $l = 0$ state with nucleus charge $Z = 2$ and mass of the tμ mesic atom [18]) (namely, these functions being involved in the adiabatic representation of the dtμ mesic molecule wave function). Coefficients A^{Jv}, a_N^{Jv} and a_k^{Jv} are known from the numerical solution of the Coulomb three-body problem [18,19]) and are normalized according to

$$\sum_{N=1}^{\infty} |a_N^{Jv}|^2 + \int_0^{\infty} \mathrm{d}k |a_k^{Jv}|^2 = 1 . \tag{15}$$

From (13)–(15) it follows that the normalized muon initial-state function has the form*

$$\Psi_{\mathrm{in}}(\boldsymbol{r}) \equiv \Psi_{\mathrm{in}}^{Jv}(\boldsymbol{r}) = \sum_N a_N^{Jv} \phi_{N0}(\boldsymbol{r}) + \int \mathrm{d}k\, a_k^{Jv} \phi_{k0}(\boldsymbol{r}) . \tag{16}$$

The muon wave functions at the moment of fusion, i.e. the coefficients a_N^{Jv} and a_k^{Jv}, depend, though weakly, on the mesic molecular state (Jv) (see table 1). The probabilities P_{Jv} of the nuclear reaction from the various rotational–vibrational states (Jv) are known from cascade calculations [21]):

$$P_{Jv} = \begin{cases} 0.84, & (Jv) = (01) \\ 0.16, & (Jv) = (00) \\ 0.00, & J \neq 0 . \end{cases} \tag{17}$$

The probability of fusion from rotational states with $J \neq 0$ being negligibly small**, the muon sticking should be considered for $J = 0$ states only.

Sticking coefficients ω_{nl}^{Jv} are calculated with a formula analogous to (6),

$$\omega_{nl}^{Jv} = |F_{nl}^{Jv}|^2 , \tag{18}$$

* Taking into account the nuclear dt interaction [see ref. [20])] essentially influences the coefficient A^{Jv} only; coefficients a_N are practically unchanged and coefficients a_k are changed noticeably for momenta $k \gg R_N^{-1} \gg 1$ only (R_N being the range of the nuclear dt interaction).

** The mesic molecule dtμ is formed in state $(Jv) = (11)$ [ref. [22])], but for all states with $J \neq 0$ its de-excitation rates significantly exceed the rates of nuclear fusion [21]).

TABLE 1

Form factors $F_N(1s)$ and $F_k(1s)$ and coefficients a_N and a_k of decompositions (16) [a])

N	$F_N(1s)$ $(\times 10^{-1})$	(a_N) $(J=0, v=1)$	(a_N) $(J=v=0)$	$a_N F_N / a_1 F_1$ $(J=v=0)$
1	0.9312	0.9831	0.9827	1
2	0.3514	−0.0733	−0.0747	−0.0287
3	0.1936	−0.0293	−0.0297	−0.0063
4	0.1263	−0.0173	−0.0175	−0.0024
5	0.0905	−0.0124	−0.0126	−0.0012
k	$F_k(1s)$ $(\times 10^{-1})$	a_k $(\times 10^{-1})$	a_k $(\times 10^{-1})$	$a_k F_k / a_1 F_1$ $(\times 10^{-1})$
0.2	0.2273	−0.2702	−0.2740	−0.0623
0.4	0.3223	−0.3610	−0.3659	−0.1179
0.6	0.3964	−0.4039	−0.4090	−0.1621
0.8	0.4605	−0.4140	−0.4190	−0.1929
1.0	0.5188	−0.4017	−0.4061	−0.2106
1.2	0.5737	−0.3755	−0.3793	−0.2176
1.4	0.6265	−0.3420	−0.3451	−0.2162
1.6	0.6784	−0.3060	−0.3086	−0.2094
1.8	0.7301	−0.2705	−0.2726	−0.1990
2.0	0.7822	−0.2372	−0.2389	−0.1869
2.5	0.9174	−0.1680	−0.1690	−0.1550
3.0	1.0636	−0.1187	−0.1193	−0.1268
3.5	1.2215	−0.0847	−0.0851	−0.1040
4.0	1.3831	−0.0615	−0.0618	−0.0854
5.0	1.5997	−0.0343	−0.0344	−0.0550
6.0	1.3295	−0.0205	−0.0205	−0.0273
7.0	0.7081	−0.0130	−0.0130	−0.0092
8.0	0.3143	−0.0087	−0.0087	−0.0028
9.0	0.1461	−0.0060	−0.0060	−0.0008
10	0.0748	−0.0043	−0.0043	−0.0003

[a]) The values are calculated at $V=5.843$, $q=(m_\mu/m)V=6.063$, $m/m_\mu=0.963748$, in units $e=\hbar=m=1$.

where, according to (5) and (16),

$$F_{nl}^{Jv}=\sum_N a_N^{Jv} F_N(nl)+\int_0^\infty \mathrm{d}k\, a_k^{Jv} F_k(nl)\,,$$

$$F_N(nl)=\int \mathrm{d}^3 r\, \Psi_{nl}^*(\boldsymbol{r})\, \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\, \phi_{N0}(\boldsymbol{r})\,,$$

$$F_k(nl)=\int \mathrm{d}^3 r\, \Psi_{nl}^*(\boldsymbol{r})\, \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\, \phi_{k0}(\boldsymbol{r})\,, \tag{19}$$

Here $\boldsymbol{q}=m_\mu \boldsymbol{V}$; $V=5.843$ is the velocity of the $\mu\alpha$ mesic atom calculated within a relativistic kinematics for the process $\mathrm{dt}\,\mu \to \mu\alpha+\mathrm{n}$. [A more accurate definition [10]) of $\boldsymbol{V}$ is relevant, because the value ω_S is extremely sensitive to $\boldsymbol{V}$.]

Coefficients a_N^{Jv} and a_k^{Jv} for the states $J=0$ are presented in table 1 and shown in fig. 1b.

Here and below, unless otherwise stated, the following units are used: $e=\hbar=m=1$, and $m=m_\mu m_t/(m_\mu+m_t)$, where $m_\mu=105.66$ MeV, $m_t=2808.94$ MeV are the muon and tritium masses, respectively. Form factors $F_N(1\mathrm{s})$ and $F_k(1\mathrm{s})$ calculated with formulae (19) are displayed in fig. 1a. They satisfy the closure relation

$$\sum_N |F_N(1\mathrm{s})|^2+\int_0^\infty \mathrm{d}k\,|F_k(1\mathrm{s})|^2=(1+(qa)^2)^{-1}\,, \tag{20}$$

where $a=(m_\mu+m_\alpha)/2m_\mu m_\alpha$.

Although state $N=1$ dominates in the normalization condition (15) of the mesic molecule wave function ($|a_1|^2\simeq 0.97$), its contribution to (20) is about 10% only. Hence, despite the smallness of a_N and a_k, their inclusion essentially decreases the value of ω_S^{BO} calculated in the BO approximation due to the destructive interference

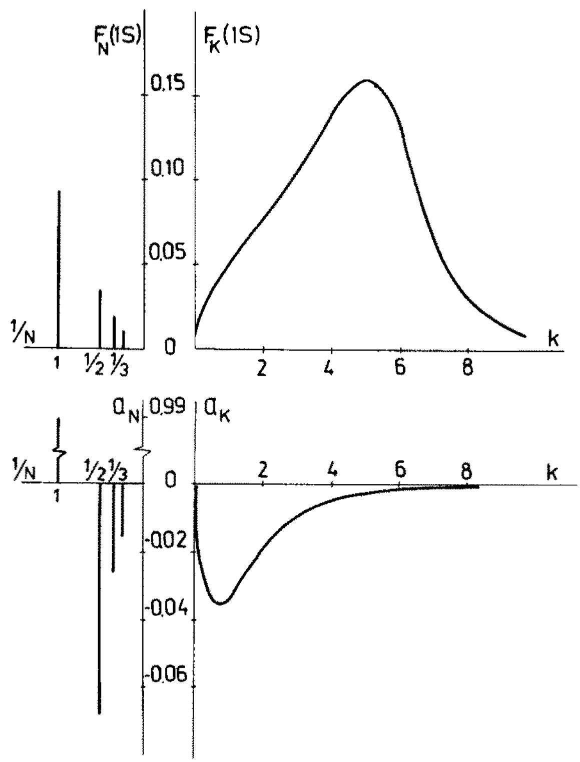


Fig. 1. Form factors $F_N(1\mathrm{s})$ and $F_k(1\mathrm{s})$ (a) calculated with formula (19), coefficients a_N and a_k (b) of the mesic molecule $\mathrm{dt}\mu$ wave function $\Psi^{00}(\boldsymbol{r},\boldsymbol{R})$ decomposition (14).

with the $N=1$ term. Thus the probability ω_{1s} decreases from $\omega_S^{BO}=0.903\times10^{-2}$ for the wave function $\varphi_{1s}(\boldsymbol{r})$ of the μ-^{5}He atom to $\omega_{1s}=0.867\times10^{-2}$ for the μ-^{3}He atom wave function $\phi_{1s}(\boldsymbol{r})$. Including discrete spectrum states $N>1$ in decomposition (16) decreases ω_{1s} to 0.770×10^{-2} and, finally, taking the continuum into account diminishes it to $\omega_{1s}=0.653\times10^{-2}$. About 23% of muons are captured in excited nl states.

The probability of a muon sticking to excited states nl with $n\geqslant5$ was calculated with the following formula [13]):

$$\omega_S(n)=\sum_{l=0}^{n-1}\omega_S(nl)=(4\zeta/n)^3\frac{[(1-\zeta/n)^2+(qa)^2]^{n-3}}{[(1+\zeta/n)^2+(qa)^2]^{n+3}}$$
$$\times\left\{[(1-\zeta)(1-\zeta^2/n^2)+(1+\zeta)(qa)^2]^2+\frac{4\zeta^2}{3(1-1/n^2)(qa)^2}\right\}, \qquad (21)$$

where

$$\zeta=\frac{m_\alpha(m_\mu+m_{^5\mathrm{He}})}{m_{^5\mathrm{He}}(m_\mu+m_\alpha)}.$$

The resulting initial sticking probability

$$\omega_S^0=\sum_{J\upsilon}\sum_{nl}P_{J\upsilon}\omega_{nl}^{J\upsilon}=0.848\times10^{-2} \qquad (22)$$

TABLE 2

Sticking probabilities $\omega_{nl}^{J\upsilon}$ ($\times10^{-2}$) for the reaction $(\mathrm{dt}\mu)^{J\upsilon}\to(\mu\text{-}^4\mathrm{He})_{nl}+\mathrm{n}$ from the states $(J\upsilon)=(01)$ and (00)

nl	ω_{nl}^{01}	ω_{nl}^{00}	ω_{nl}^{AC} [a])	ω_{nl}^{BO} [b])
1s	0.6526	0.6502	0.689	0.9030
2s	0.0937	0.0934	0.099	0.1287
2p	0.0239	0.0238	0.024	0.0321
3s	0.0285	0.0284	0.030	0.0391
3p	0.0086	0.0086	0.009	0.0115
3d	0.0003	0.0003		0.0003
4s	0.0122	0.0121	0.013	0.0166
4p	0.0038	0.0037		0.0051
4d+4f	0.0003	0.0003		0.0003
$\sum_{n=5}\sum_{l=0}^{n-1}$	0.0242	0.0241	0.031	0.0278
total	0.848	0.845	0.895 ± 0.004	1.164
		$\omega_S^0=0.848$		

[a]) Data from ref. [10]). Values ω_{4p}^{AC} and ω_{3d}^{AC} are included in the sum over the higher states.

[b]) The listed values are calculated from formulae (5) and (6) with functions (3). The contribution from $n\geqslant5$ states was found with formula (21).

is smaller by 27% than the value $\omega_S^{BO}=1.164\times10^{-2}$ calculated within the Born–Oppenheimer approximation*). The ω_{nl}^{00} calculated here reasonably coincide with the ω_{nl}^{AC} from ref. [10]) obtained with the Monte Carlo method (they are listed in the third column of table 2). Such an agreement evidences the correctness and high accuracy of both methods. From table 2 one can also see that the ω_{nl}^{0v} practically do not depend on the vibrational state v of the dtμ mesic molecule.

4. Corrections to the sudden approximation

In this section we perform a more detailed (as compared to sect. 2) analysis of the sudden approximation. We continue to neglect nuclear finite-size effects (see sect. 5). However, we start from a more accurate definition for the sticking probability of muons to helium.

Expressions (5), (6) and (18) for ω_{nl}^{Jv} are simplified versions of the general definition

$$\omega_{nl}^{Jv}=\lambda_{nl}^{Jv}/\lambda^{Jv}, \tag{23}$$

where λ_{nl}^{Jv} is a partial rate of the nuclear reaction

$$(\mathrm{d}t\mu)^{Jv}\xrightarrow{\lambda_\beta^{Jv}}(\mu\alpha)_\beta+\mathrm{n}, \tag{24}$$

$(\mu\alpha)_\beta$ being the system produced in the discrete spectrum state $\beta=(nl)$, and

$$\lambda^{Jv}=\sum_{\beta=\{nl,k\}}^{v}\lambda_\beta^{Jv} \tag{25}$$

is the total rate of nuclear fusion (1) from mesic molecular states (Jv).

We now estimate the uncertainties appearing due to the transition from (23) to (18). The partial rate λ_β^{Jv} is defined, in first-order perturbation theory in terms of the fusion amplitude T, by the formula

$$\lambda_\beta^{Jv}=|T|^2p_\beta\left|\int \mathrm{d}^3r\,\Psi_\beta^*(\boldsymbol{r})\,\mathrm{e}^{-i\boldsymbol{q}_\beta\cdot\boldsymbol{r}}\,\Psi^{Jv}(\boldsymbol{r},0)\right|^2. \tag{26}$$

(In what follows $J=0$, and indices Jv are omitted.) Here $\boldsymbol{p}_\beta$ is the momentum of the $(\mu\alpha)_\beta$ system in the c.m.s.,

$$p_\beta=\left[\frac{2(Q-\mathscr{E}_\beta)m_\mathrm{n}(m_\mu+m_\alpha)}{m_\mathrm{n}+m_\mu+m_\alpha}\right]^{1/2}, \tag{27}$$

$Q=17.6$ MeV is the energy release in reaction $\mathrm{d}t\mu\to\alpha+\mathrm{n}+\mu$, $\mathscr{E}_\beta$ is the energy of $(\mu\alpha)_\beta$, and

$$\boldsymbol{q}_\beta=\frac{m_\mu}{m_\mu+m_\alpha}\boldsymbol{p}_\beta. \tag{28}$$

* The difference between this value and those presented in refs. [7,8]) is due to the contribution from the states nl with $n\geq4$.

From (23), (25) and (26) follows the standard expression (18) for the sticking probability ω_{nl}, provided the dependence of $\boldsymbol{q}$ and $\boldsymbol{p}$ on the final state β in formula (26) is neglected and the closure relation

$$\sum_{\beta} \Psi^*_\beta(\boldsymbol{r}')\Psi_\beta(\boldsymbol{r}) = \delta(\boldsymbol{r}'-\boldsymbol{r})$$

applied.

In order to estimate the accuracy of this approximation we put $p_\beta = p_0 + \delta p_\beta$, then

$$\omega_\beta(p_\beta) = \omega_\beta(p_0) + \delta\omega_\beta, \qquad \lambda_\beta = \lambda_\beta(p_0) + \delta\lambda_\beta, \qquad \lambda = \lambda(p_0) + \delta\lambda,$$

$$\frac{\delta\omega_\beta}{\omega_\beta} = \frac{\delta\lambda_\beta}{\lambda_\beta} - \frac{\delta\lambda}{\lambda}. \tag{29}$$

The main contribution to the total rate λ comes from the final states β, where the muon is a spectator, i.e. at $\beta = k$, $|\boldsymbol{k}-\boldsymbol{q}| \lesssim 1/a_\mu$. The average energy carried away by muon is about $\mathscr{E}_c \simeq 10$ keV, the total rate λ being proportional to the phase-space volume of relative nα motion,

$$\lambda = |T|^2 p \int |\Psi^{J\upsilon}(\boldsymbol{r}, 0)|^2 \, d^3 r. \tag{30}$$

Here momentum $\boldsymbol{p}$ is defined up to terms $\sim \mathscr{E}_c/Q$ with the formula

$$p = \left(\frac{2Qm_n m_\alpha}{m_n + m_\alpha}\right)^{1/2}. \tag{31}$$

In contrast to λ, partial rates λ_β quickly vary with momentum $\boldsymbol{p}_\beta$. For instance, the partial rate for $\beta = 1$s is

$$\lambda_{1s} \sim \frac{p_{1s}}{(1+(\frac{1}{2}q_{1s}a)^2)^4} \sim p_{1s}^{-7}, \qquad p_{1s} \gg 1. \tag{32}$$

Let $p_0 = p_{1s}$, then the relative errors of partial fusion rates are minimal and can be estimated with the formula

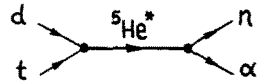
$$\left|\frac{\delta\lambda_\beta}{\lambda_\beta}\right| \simeq \frac{1}{\lambda_\beta}\left|\frac{d\lambda_\beta}{dp_\beta}\right| \delta p_\beta \lesssim 7\frac{\delta p_\beta}{p_0} \simeq 7\frac{\mathscr{E}_c}{2Q} \lesssim 2\times 10^{-3}. \tag{33}$$

The relative error in total rate, according to (30), is

$$\frac{\delta\lambda}{\lambda} \simeq \frac{|p-p_0|}{p} \simeq \frac{m_\mu m_n}{2m_\alpha(m_\alpha + m_\mu)} \simeq 3\times 10^{-3}. \tag{34}$$

Thus, the uncertainties in ω_S^0, caused by the neglected dependence of the mesic atom $(\mu\alpha)_{nl}$ momentum on the state nl in which it is produced, do not exceed 0.5%.

We now return to the effects of the finite rate of nuclear fusion. In order to take them into account, the energy dependence of the nuclear reaction amplitude has to be considered. By using a simple resonance model to describe the nuclear interaction in the coupled dt and nα channels, the T-matrix is given by

$T=$ 

$$=\frac{V_1|^5\mathrm{He}\rangle\langle^5\mathrm{He}|V_2}{E-E_\mathrm{R}+\frac{1}{2}i\Gamma}$$

Fig. 2.

The description of the nuclear reaction $(\mathrm{dt}\mu)^{Jv}\to(\mu\alpha)_{nl}+\mathrm{n}$ results in a three-coupled-channels problem ($\mu+\mathrm{d}+\mathrm{t}\to\mu+\alpha+\mathrm{n}$ and $\mu+{}^5\mathrm{He}^*$)*. The amplitude M_{nl}^{Jv} for this reaction can be represented by the graph shown in fig. 3.

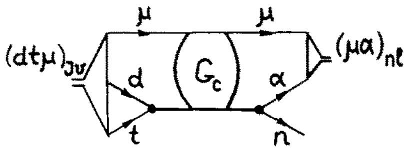


Fig. 3. The amplitude for the reaction $(\mathrm{dt}\mu)^{Jv}\to(\mu\alpha)_{nl}+\mathrm{n}$ in the three-coupled-channels problem.

The diagram involves the propagator of the muon G_C in the Coulomb field of a point-like nucleus with charge $Z=2$ and mass $m_\mathrm{d}+m_\mathrm{t}$, which can be conveniently written by using the spectral representation

$$\langle \boldsymbol{r}'|G_\mathrm{C}|\boldsymbol{r}\rangle=\sum_i\frac{\varphi_i^*(\boldsymbol{r}')\varphi_i(\boldsymbol{r})}{E-\mathscr{E}_i}. \tag{35}$$

By neglecting effects connected with the finite fusion radius, the dtμ mesic molecule wave function in the $R\to0$ limit is decomposed over the basis of the Coulomb functions involved in the spectral representation (35) of G_C,

$$\Psi^{Jv}(\boldsymbol{r},\boldsymbol{R})\underset{R\to0}{=}\chi^{Jv}(R)\left\{\sum_N b_N^{Jv}\varphi_{N0}(\boldsymbol{r})+\int_0^\infty \mathrm{d}k\, b_k^{Jv}\varphi_{k0}(\boldsymbol{r})\right\}, \tag{36}$$

i.e. over functions $\varphi_{N0}(\boldsymbol{r})$ and $\varphi_{k0}(\boldsymbol{r})$ of discrete and continuous spectra of the μ-^{5}He mesic atom. Note that coefficients b_N^{Jv} and b_k^{Jv} in decomposition (36) slightly differ from coefficients a_N^{Jv} and a_k^{Jv} in (16). From (35)-(36) we obtain (an integral over k is included in a sum over N)

$$M_\beta^{Jv}=\chi^{Jv}(0)T(E_1)\sum_N\tilde{b}_N^{Jv}\int\mathrm{d}^3r\,\Psi_{nl}^*(\boldsymbol{r})\,\mathrm{e}^{-i\boldsymbol{q}_\beta\cdot\boldsymbol{r}}\varphi_{N0}(\boldsymbol{r}), \tag{37}$$

* We are using here a simplified version of a model developed in refs. [20,23]) which fits all the experimental data on the dt reaction in the resonance $^5\mathrm{He}^*(\frac{3}{2}^+)$ region.

where

$$T(E)=\langle \alpha, \mathrm{n};\ k_2|T|\ \mathrm{d},\mathrm{t};\ k_1\rangle$$

$$\tilde{b}_N^{J\upsilon}=b_N^{J\upsilon}\frac{T(E_N)}{T(E_1)},\qquad E=\frac{k_1^2}{2M_1}=\frac{k_2^2}{2M_2}-Q,\qquad E_N=E-\mathscr{E}_N,\tag{38}$$

and $N=1$ corresponds to the 1s state.

Should the energy dependence of the T-matrix be neglected, expression (37) reduces to the result of the sudden approximation (5), (16), the only difference, however, being that functions $\varphi_{nl}(\boldsymbol{r})$ of the μ-^{5}He mesic atoms are involved instead of μ-^{3} He ones $\phi_{nl}(\boldsymbol{r})$. It is the first term of decomposition (37) that corresponds to the Born–Oppenheimer approximation.

Note that the energy dependence of the T-matrix influences only the non-adiabatic corrections to the muon wave function. In other words, corrections to the value ω_S^0 calculated in the sudden approximation, are rather small, because they change only the contribution from non-adiabatic corrections to the BO approximation.

From (38) it follows that

$$\tilde{b}_N=b_N\left(1+\frac{(\mathscr{E}_1-\mathscr{E}_N)(E_R+\frac{1}{2}i\Gamma)}{E_R^2+\frac{1}{4}\Gamma^2}\right).\tag{39}$$

All the non-adiabatic corrections $\delta\omega_S^0$ are linear in the real parts of coefficients $\tilde{b}_N$ (see sect. 3); hence, corrections $\Delta\omega_S^0$ due to the energy dependence of T-matrix are

$$\Delta\omega_S^0=\delta\omega_S^0\frac{\mathscr{E}_c E_R}{E_R^2+\frac{1}{4}\Gamma^2}=10^{-1}\times\delta\omega_S^0.\tag{40}$$

Since $\delta\omega_S^0/\omega_S^0=0.23$, corrections $\Delta\omega_S^0$ to the sudden approximation value do not exceed 3%, i.e. they are essentially smaller than one would expect from the naive arguments of sect. 2.

5. Other corrections

In this section we consider corrections to $\omega_{nl}^{J\upsilon}$ arising from the finite nuclear size and finite range of nuclear forces, as well as some other corrections, in particular, those due to the influence of the nuclear final- and initial-state interaction on the muon wave function.

First of all, we take into account the fact that an α-particle is produced in the fusion reaction not exactly in the c.m.s. of nuclei (as has been assumed in formula (6)), but is somehow spread. To do this, we calculate amplitude M_{nl} of the nuclear reaction (24), which corresponds to the diagram in fig. 3. Vertex form factors for ^{5}He$^*\to$ d + t and ^{5}He$^*\to$ n + α in the coordinate representation

$$\xi_1(\boldsymbol{R})=\langle \mathrm{d}t|V_1|^5\mathrm{He}^*\rangle,$$

$$\xi_2(\boldsymbol{R})=\langle \mathrm{n}\alpha|V_2|^5\mathrm{He}^*\rangle,\tag{41}$$

are localized inside $R \leqslant R_N$, $R_N \simeq 4$ fm being the characteristic size of the $^5He^*$ resonance. For the sake of simplicity we write the amplitude M_{nl} for the mesic molecule wave function in the Born-Oppenheimer approximation $\Psi^{BO}(\boldsymbol{r}, \boldsymbol{R}) \underset{R\to 0}{\simeq} \chi(\boldsymbol{R})\varphi_{1s}(\boldsymbol{r})$ (it will be clear from the following that using the exact wave function does not influence the results):

$$M_{nl}^{Jv} = (E - E_R + \tfrac{1}{2}i\Gamma)^{-1} \int d^3R\, d^3R'\, d^3r\, \chi^{Jv}(\boldsymbol{R})\xi_1(\boldsymbol{R}) \times \psi_{nl}^*(\boldsymbol{\rho})\, e^{-i\boldsymbol{Q}\cdot\boldsymbol{\mathcal{R}}}\, \xi_2(\boldsymbol{R}')\varphi_{10}(\boldsymbol{r})\,. \tag{42}$$

Here $\boldsymbol{R}$ and $\boldsymbol{R}'$ are the relative coordinates of nuclei in initial and final states, $\boldsymbol{\rho}$ is the muon coordinate with respect to the α-particle, $\boldsymbol{\mathcal{R}}$ is the $(\mu\alpha)$ atom coordinate with respect to the neutron, and $\boldsymbol{Q} = (m_\mu + m_\alpha)\boldsymbol{V}$ is the $\mu\alpha$ atom momentum in the c.m.s. Using relations (see fig. 4)

$$\boldsymbol{\rho} = m_n\boldsymbol{R}'/(m_n + m_\alpha) + \boldsymbol{r}\,,$$

$$\boldsymbol{\mathcal{R}} = \boldsymbol{\rho} m_\mu/(m_\mu + m_\alpha) - \boldsymbol{R}'\,, \tag{43}$$

we transform (42):

$$M_{nl} = B \int d^3R'\, \xi_2(\boldsymbol{R}')\, e^{i\boldsymbol{Q}\cdot\boldsymbol{R}'} \int d^3r\, \psi_{nl}^*(\boldsymbol{r})\, e^{-im_\mu \boldsymbol{V}\cdot\boldsymbol{r}}\, \varphi_{10}(\boldsymbol{r} - \boldsymbol{x})\,,$$

$$\boldsymbol{x} = \frac{m_n}{m_n + m_\alpha}\boldsymbol{R}'\,, \qquad B = (E - E_R + \tfrac{1}{2}i\Gamma)^{-1} \int d^3R\, \chi(\boldsymbol{R})\xi_1(\boldsymbol{R})\,. \tag{44}$$

Expanding $\varphi_{10}(\boldsymbol{r} - \boldsymbol{x})$ in the small parameter x, we obtain

$$M_{nl} = Bg(\boldsymbol{Q})(F_{nl} + F_{nl}^{(1)} + F_{nl}^{(2)} + \cdots)\,, \tag{45}$$

where $g(\boldsymbol{Q}) = \int d^3R'\, \xi_2(\boldsymbol{R}')\, e^{i\boldsymbol{Q}\cdot\boldsymbol{R}'}$. The leading term F_{nl} in decomposition (45) coincides with the sticking amplitude defined with formula (5), and the corrections are

$$F_{nl}^{(1)}/F_{nl} \simeq i(R_N/5a_\mu)\,,$$

$$F_{nl}^{(2)}/F_{nl} \simeq (R_N/5a_\mu)^2\,, \tag{46}$$

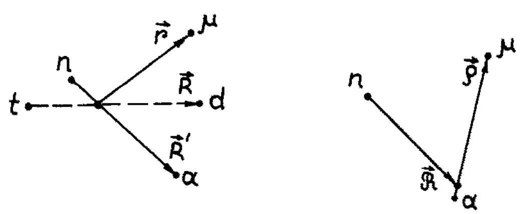


Fig. 4. The relative coordinates of the particles involved in reaction (1).

hence

$$\omega_{nl} = |F_{nl} + F^{(1)}_{nl} + F^{(2)}_{nl}|^2 \simeq |F_{nl} + F^{(2)}_{nl}|^2 + |F^{(1)}_{nl}|^2$$
$$= |F_{nl}|^2(1 + \mathrm{O}((R_\mathrm{N}/5a_\mu)^2)) . \qquad (47)$$

Thus, the corrections to ω_{nl} due to the finite range of nuclear fusion are rather small*:

$$\frac{\delta\omega_{nl}}{\omega_{nl}} \simeq \left(\frac{R_\mathrm{N}}{5a_\mu}\right)^2 \leqslant 3\times 10^{-5} .$$

Consider now the corrections caused by strong interaction between nuclei in initial and final states on the muon wave function. This means that the nuclear reaction amplitude (24) should be calculated not to first order in the nuclear interaction amplitude (as was done when obtaining formula (26)), but exactly. The diagrams where the internuclear interaction is sandwiched by the Coulomb interaction of muon with nuclei should be taken into account. (Examples of such graphs are shown in fig. 5.) As was shown in refs. [20,23]), such diagrams, taken into account when calculating the total rate λ^{Jv} of nuclear reaction, lead to about 5% renormalization of the nuclear amplitude T in formula (26)**. The corrections to the sticking probability ω_{nl} are essentially smaller, since, according to (23), ω_{nl} does not involve the nuclear reaction amplitude. They can be estimated as follows. First, neglect the structure of the ^{5}He* resonance (i.e. consider it interacting with a muon as a point-like charge), then the sum of all diagrams of the type in fig. 5 will coincide with the diagram in fig. 3, the corresponding amplitude having been calculated in sect. 4.

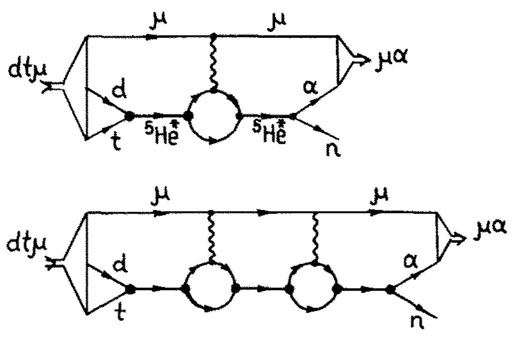


Fig. 5. Example of the diagrams for the reaction $\mathrm{dt}\mu \to \mu\alpha + \mathrm{n}$ with the internuclear interactions sandwiched by the Coulomb interaction of muons with nuclei.

* Using the exact wave function of the dtμ mesic molecule instead of $\Psi^{\mathrm{BO}}(\boldsymbol{r}, \boldsymbol{R})$ influnces only the non-adiabatic corrections from the faraway part of the continuum $k \geqslant R_\mathrm{N}^{-1} \simeq 60$. This, however, is nonessential, because, according to sect. 3, the main contribution to these corrections comes from the smaller momenta $k \leqslant q \simeq 6$, $qR_N \simeq 10^{-1} \gg 1$.

** As is clear from the previous discussion and the form of the diagrams (fig. 5), the influence of the nuclear dt interaction on the muon wave function $\Psi_{\mathrm{in}}(\boldsymbol{r})$ is negligible. In this respect the statements of refs. [27,28]) about the importance of such influence seem strange.

To take into account the structure of $^5He^*$, one should include the $^5He^*$ charge form factor dependence on the transferred momentum $\boldsymbol{q}$. The characteristic value of $\boldsymbol{q} \simeq \boldsymbol{V} m_\mu$ is small compared to the inverse radius of ^{5}He: $qR_N \simeq 10^{-1}$. Corrections to the sticking probability due to the influence of $^5He^*$ resonance structure on the muon wave function are defined by monopole and quadrupole formfactors and are about $(qR_N)^2 \simeq 10^{-2}$, i.e. $\delta\omega_S^0/\omega_S^0 \lesssim 1\%$.

Of the same order of smallness should be the corrections due to the finite sizes of the nuclei, d, t and α. Finally, note the correction to ω_{nl}^0 due to vacuum polarization [29])

$$\frac{\delta\omega_{nl}^0}{\omega_{nl}^0} \approx \frac{q^2}{1+(\frac{1}{2}q)^2}\frac{2\alpha}{3\pi}\ln\left(2\alpha\frac{m_\mu}{m_e}\right) \approx 6\times 10^{-3}\,. \tag{48}$$

6. Concluding remarks

In the present paper the value $\omega_S^0 = 0.848 \times 10^{-2}$ for the probability of muons sticking to helium in reaction (1) is obtained in the framework of the sudden approximation. The uncertainty of this calculation due to uncertainties in the numerical wave function used for the $dt\mu$ mesic molecule, $\Psi^{Jv}(\boldsymbol{r}, \boldsymbol{R})$, is noticeably smaller than the corrections to the sudden approximation, which are, according to our estimations, about 3%.

Our result is in good agreement with ref. [10]), where the sticking probability for the state $(Jv) = (00)$ of the $dt\mu$ mesic molecule has been calculated for the mesic molecule wave function obtained with the Monte Carlo method. As an argument for the correctness of the calculation, we indicate the coincidence between the sticking coefficient $\omega_d = 0.12$ [ref. [13])] for the reaction $dd\mu \to \mu\text{-}^3He + n$ calculated with the same technique and the experimentally measured one $\omega_d = 0.126 \pm 0.004$ [ref. [11])].

The observed value $\omega_S = \omega_S^0(1-\gamma)$ is smaller than the calculated ω_S^0 by $\gamma\omega_S^0$ due to possible stripping processes during slowing down of the $\mu\alpha$ mesic atom in the medium: ionization from the 1s state of $\mu\alpha$ [refs. [7,8])], step-by-step ionization [8,25,26]), muons shaking off the excited states [24]), etc. According to ref. [24]) the resulting sticking coefficient is

$$\omega_S = \omega_S^0\left\{0.96 - 0.03\frac{\varphi}{\varphi+2}\right\}\exp\{-0.26 - 0.07\varphi\}\,. \tag{49}$$

At medium density $\varphi \simeq 1$,

$$\omega_S = 0.58 \times 10^{-2}\,.$$

This value is almost twice less than the first estimates [5,6]) within the Born–Oppenheimer approximation, but still approximately twice greater than the recent experimental values reported by the Idaho–Los Alamos group [12]) at $\varphi \simeq 1$. The reason for such a drastic discrepancy between theory and experiment is still to be understood.

We emphasize, however, that present uncertainties in ω_S are due to the insufficiently well known reactivation coefficient γ, while the initial sticking probability ω_S^0 is now established quite reliably.

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