Properties and interactions of neutrinos

• Neutrino properties
• Natural and artificial neutrino sources
• Neutrino interactions
• Methods to detect neutrinos and antineutrinos of astrophysical interest
• Experiments in underground laboratories
Why do we study neutrinos in this course?

- Neutrinos are produced in several reactions: an important example is \( p+p \rightarrow d+e^-+\nu_e \), the starting reaction in the pp chain for the fusion of 4 protons into a helium nucleus within stars.
- Therefore, neutrinos are markers of nuclear reactions occurring inside stars. Furthermore, they are also indicators of nuclear processes occurring in the early moments of the universe, in the explosion of supernovae, and inside Earth.
- The most important feature of neutrinos is their weak interaction with matter: for this reason they manage to “escape” from the stars (sun, supernovae) and from the matter of primordial universe, practically without alteration and so providing “first-hand” information on the conditions of matter and universe at the instant of their production.
- Only if we know the properties of neutrinos we can use them as probes.
- In the last decade there have been enormous advances in the understanding of neutrinos, in particular with respect to oscillations (changing of flavour).
- This also has implications on neutrino the masses.
- Now that neutrinos are "calibrated" we can use them as astrophysical probes.
A short history of neutrinos

1898 Discovery of the radioactivity
1926 Problems with beta radioactivity
1930 Pauli invents the neutrino particle
1932 Fermi baptizes the neutrino and builds the theory of weak interaction
1946 Pontecorvo elaborates a program for neutrino detection
1956 First observation of the neutrinos by an experiment
1957 Pontecorvo formulates the hypothesis of neutrino oscillation
1962 Discovery of an other type of neutrino: $\nu_\mu$
1970 Davis experiment opens the solar neutrino puzzle
1974 Discovery of neutral currents thanks to the neutrinos
1987 Neutrinos from SN 1987A
1991 LEP experiments show that there are only three light neutrinos
1992 Missing solar neutrinos confirmed by GALLEX and SAGE
2000 $\nu_\tau$ observed
2001 SNO closes the solar neutrino puzzles, by directly proving the transmutation of solar neutrinos
2002 KamLAND observes transmutation of man made (reactor) neutrinos
2005 KamLAND observes antineutrinos from the Earth
2007 Borexino at LNGS detects Solar Be neutrinos
Neutrino properties

- Neutrinos are particles which have only electro-weak and gravitational interactions;
- they are leptons with no electric charge and spin $\frac{1}{2}$
- We know three types (flavours) of neutrinos, each connected with a charged lepton.
- We define neutrino with electron flavour a particle which is produced in $\beta^+$ decays together with positrons, and the same for the other types.
- Each neutrino has a corresponding antineutrino which has opposed leptonic family number and global lepton number.
- The three neutrinos, in doublets with the corresponding charged leptonsss complete the three families (or generations), similar to the three doublets which constitute the three families of quarks.

<table>
<thead>
<tr>
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Elementary Particles

- Quarks: $u$, $c$, $t$, $d$, $s$, $b$, $g$, $\gamma$
- Leptons: $e^-$, $\mu^-$, $\tau^-$, $e^+$, $\mu^+$, $\tau^+$
- Force Carriers: $W$, $Z$
Neutrino masses

The table shows results of experiments performed with nuclei and accelerators which study the kinematics of processes where neutrinos are produced.

All experiments only provide upper limits, i.e. the results are consistent with the fact that neutrino masses are equal to zero.

- As a consequence of CPT theorem, particle and antiparticle have the same mass, therefore the limits are valid both for neutrinos and antineutrinos, and we can obtain one from the other.
- The limit of the electron antineutrino mass is derived from the spectrum in the tritium decay (when anti-$\nu_e$ are produced); the limit on muon (anti)neutrino is obtained from charged pion decay at rest.
- It’s important to know that for neutrinos produced by more energetic processes the information on the mass is less accurate.
- The experiments on the oscillation of neutrinos made in the last decade have shown that *:
  - At least two masses must be different from zero.
  - The differences among masses are at the most of order 0.1 eV.

The information on the masses can thus be summarized as:

$$m_i < 2 \text{ eV}, \delta m_{ij} < 0.1 \text{ eV}$$

*this topic will discuss in the chapter on neutrino oscillations*
Stability of neutrinos

- As far as we know, neutrinos are stable.
- Indeed, observations to detect a possible neutrino decay give only lower limits on the lifetime.
- Suppose that neutrino has a finite lifetime \( \tau \).
- In the laboratory frame, the neutrino time decay is \( t = \gamma \tau = \frac{E}{mc^2} \tau \), where \( E \) is the laboratory frame energy.
- Decay length is \( l_{\text{dec}} = vt \approx c \left( \frac{E}{mc^2} \right) \tau \).
- If we observe that neutrinos move a distance \( L \) without decaying, this means \( L < l_{\text{dec}} \) ossia \( \tau/(mc^2) > L/(cE) \).
- In nuclear reactors antineutrinos with energies \( E \approx 3 \text{ MeV} \) are produced end their number does not decreases on lengths \( L \approx 1 \text{ km} \), which implies \( \tau/(mc^2) > 3 \times 10^{-12} \text{ s/eV} \), much less restrictive than the limit quoted in the upper table.
- If we suppose that photons are produced in the neutrino decay, then, we expect, after a length \( L \), a flux of photons equal to \( \Phi(\gamma) = \Phi(\nu) \frac{L}{l_{\text{dec}}} \).
- In this way, attributing the observed photons (background) at the decay of the neutrino, we obtain the limits, shown in the table, using Reactors, Sun, Accelerators.
- Similarly for the masses, the process of oscillations says that if neutrinos decay, their velocities decay are comparable, and so the result, obtained for electric neutrino, can be used for each type:

\[
\tau/(mc^2) > 7 \times 10^9 \text{ s/eV}
\]

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Neutrino magnetic moment

- Neutrinos have electro-weak interactions, therefore it makes sense to study their e.m properties.
- Neutrinos electric charge is equal to zero, as far as we know.
- A particle, in order to have a magnetic magnetic dipole moment, must have mass and spin differs from zero because:
  - An object with magnetic moment generates a magnetic field and so it carries energy connected with the field, different from zero.
  - In its rest frame, a privileged direction (which points to the magnetic dipole) must exist. This can be given only by the spin, \( \mu = \kappa \sigma \).
- The Standard Model of electro-weak interactions predicts : \( \mu / \mu_B \approx 10^{-19} \text{ m} / \text{ [1 eV]} \)
- Alternative theories predict larger values; the observation of their predictions would be an indication of physics beyond the standard model.
- Nowadays, more stringent limits in experiments derive from reactors (\( \mu / \mu_B < 10^{-10} \)), instead astrophysical arguments (cooling time of white dwarfs, red giants brightness) can provide limits more stringent by 1-2 orders of magnitude.

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$Z_0$ width and the number of families: three and no more than three

- We observe the $Z_0$ decay in $(l^+l^-)$, and $(q\text{ anti-}q)$ because in these processes one has charged particles leaving tracks in the detector.
- The decay $Z_0 \rightarrow \nu +\text{anti-}\nu$ exists but it doesn’t leave track in the detector.
- Its existence can be inferred from the contribution to the global width and this fact determines the number of neutrino types.
- LEP results are consistent with three types of neutrinos, and so they exclude additional families*.

*Unless the new neutrino mass is $m_\nu>M_Z/2=45$ GeV
Natural neutrino sources
Artificial sources of (anti)neutrinos: nuclear reactors

- **Typical commercial reactors** have thermal powers of 3GW and burn uranium $^{235}\text{U}$
- On average every nuclear fission produces $\Delta=200\text{MeV}$ and a typical reactor produces $10^{20}$ fis/s
- One can understand that the average number of neutrinos per fission is 6. Indeed, in the neutron induced fission,

$$92\text{U}^{235} + n \rightarrow X_1 + X_2 + 2n$$

the distribution of fission products has a peak around $A=94$ and $A=140$; for these mass number stables nuclei are $^{40}\text{Zr}^{94}$ and $^{58}\text{Ce}^{140}$. To reach these nuclei with total charge $40+58=98$, starting from 92, 6 protons must be transformed into neutrons, so we need 6 beta decays, with 6 antineutrinos.

- Therefore a reactor with thermal power of 3GW produce isotropically $L_\nu \approx 6 \times 10^{20}$ anti $\nu$/s.
- The flux at 10 m is

$$\Phi = 5 \times 10^{13} \text{ anti } \nu /\text{cm}^2/\text{s}$$
Spectrum of reactor antineutrinos

- Neutrinos from fission carry away on average 1.6 MeV, which means that some 10 MeV per fission is not transformed in heat, but in neutrinos.
- The most abundant neutrinos are those from $^{235}\text{U}$ fission with thermal neutrons, but are also important those from $^{238}\text{U}$, as well as from two plutonium isotopes, $^{239}\text{Pu}$ and $^{241}\text{Pu}$, produced by the pattern shown below.

\[
^{238}\text{U}(n, \gamma)^{239}\text{U} \xrightarrow{T_{1/2}=23.5 \text{ min}}^{239}\text{Np} \quad E_{\text{max}}=1.265 \text{ MeV}
\]

\[
E_{\text{max}}=0.722 \text{ MeV}
\]

\[
T_{1/2}=2.357 \text{ d}
\]

\[
^{239}\text{Pu}(n, \gamma)^{240}\text{Pu}(n, \gamma)^{241}\text{Pu}.
\]

The figure shows the various component of the spectrum, with their uncertainties, and the product with the cross section of the typical reaction used for detection (threshold :1.8 MeV )

\[
\bar{\nu}_e + p \rightarrow e^- + n.
\]

Note the peak around 4 MeV, that is the most probable energy of detected neutrinos.
Neutrinos beams from accelerators

- Since the 60-70’s, we can extract neutrinos beam, principally muonic neutrinos.
- A proton beam impinges onto a target where pions are abundantly produced by strong interaction.
- Charged pions are focused by means of a magnetic horn and afterwards, they enter a “decay pipe” where decay occurs.
- The principal decay mode is
  \[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
  and similarly for \( \pi^- \)
- At the end of decay pipe, muons are stopped in cement block, and only neutrinos are left.
- Therefore, to produce a neutrinos beam, three elements are necessary (Target, horn, Decay pipe)
Targets

- In all neutrinos beams, the first step is the production of secondaries \((\pi^\pm, \kappa^\pm)\) through interaction of the primary beam of protons on target.
- The target consist of a set of cylindrical bars a few centimeters thick, separated by layers of air to minimize the absorption of secondaries by the target.
- The geometry is optimized to reduce as much as possible mechanical and thermal stress due to the intensity of primary beam.
- The material currently used is beryllium.
HORNs: Focusing of secondary particles

- Horn is a magnetic lens.
- In the horn the particles are deflected by a radial magnetic field produced by two coaxial cables through which flow currents of equal magnitude but opposite directions.
- We can deduce the internal profile of the horn imposing condition of parallel emission.
An example: the beam of KEK for K2K

The K2K, KEK-to-Kamioka, experiment is the first accelerator-based long-baseline experiment [2]. Its primary goal is to give a definite answer concerning the existence of neutrino oscillation found in atmospheric neutrino observations. A conventional beam is produced at KEK, and the far detector is Super-Kamiokande (SK) located 250 km away from KEK. The experiment started in 1999 and is on going.

The proton beam is extracted from the 12-GeV proton synchrotron (PS) in a single turn with a 1.1 μs width every 2.2 s. The design intensity is $6 \times 10^{12}$ protons on target/pulse. Every beam spill is stamped with time measured by the global positioning system (GPS) with an accuracy of $<200$ ns [3]. The production target is a 66 cm long Al rod. Its diameter is 2 cm for runs in June 1999, and 3 cm since November 1999. Positive pions are focused by two electromagnetic horns [4]. Both horns are operated by a pulsed current of $\sim 1$ ms width and 200 kA peak for the June 1999 run and 250 kA peak for runs since November 1999. The length of the decay pipe is 200 m. A beam dump at the end of the decay pipe is 3-m thick from the target, there is a hole in which front neutrino
**Interactions of neutrinos**

- The detection of neutrinos is based on their interactions which electrons, nucleons or nuclei in the target.
- We recall that there are two types of weak interactions, the charged current and the neutral current, respectively mediated by W and Z bosons, whose properties will be summarized in the following slides.
- Next, we will estimate cross section of neutrinos with different constituents of the target.
- An important point to remember is that the neutrinos energies of astrophysical interest are in the order of MeV, much smaller than the natural scale of weak interactions, given by the masses of the W and Z, order 100 GeV.
- As a first approximation, all the scattering amplitudes are of the order of $A \approx G_F$ where $G_F = 10^{-5} \text{ m}^{-2}$ is the constant that characterizes the low-energy weak processes.*
- An other important point is that the neutrino wavelength $\lambda = h/p$, is of the order of 200 fm, so it is small compared to the size of the atom, but large compared to those of the nucleus. For this reason, astrophysical neutrinos:
  a) distinguish the electrons from atomic nuclei
  b) don’t distinguish nucleons inside to nuclei

* I’m using $h/2\pi = c = 1$, so that the scattering amplitudes have dimensions $E^{-2}$
The general framework of the weak charged current process

- We can define weak processes of charged current all those processes (= collisions or decays) which occur though emission or absorption of real and virtual W.
- They are all described in terms of combinations of vertices of the fundamental type (Wqq'), (Wll') where at each vertex:
  - i) electric charge is conserved
  - ii) barion number is conserved
  - iii) lepton number is conserved

- Weak interactions have the same strength for each family of leptons.

- Bosons W^± (spin 1, M_W = 80.4 GeV) are therefore the “mediators” charged current of weak interactions, as photons are mediators of e.m interactions.

- Unification of weak and e.m. interactions means that, at energies of the order of M_W, the probability to emit or to adsorb W is comparable with to emit or adsorb photons.

- At low energy, weak interactions appear “weaker” than electromagnetic ones because the emission/adsorption of virtual W can occur during (much) shorter time, obtained by energy-time uncertainty principle.
Processes involving leptons and hadrons

- Similar to the leptons, we can use the same concepts for the hadrons and introduce possible virtual processes which involve $W$:
  
  \[
  q(-1/3) \rightarrow W^- + q(2/3) \\
  q(2/3) \rightarrow W^+ + q(1/3)
  \]

- You can easily reinterpret all the processes concerning weak interactions studied at the beginning, in terms of emission and absorption of virtual $W$ between leptons and quarks. For example:

  1) the pion decay $\pi^+ = (u, \text{antid})$,
  
  \[
  \pi^+ \rightarrow \mu^+ + \nu_\mu
  \]

  can be treated as the annihilation of quark pair $(u, \text{anti-d})$ in $W^+$ (virtual) which decays into muon and neutrino:
  
  \[
  \pi^+ = u + \text{anti-d} \rightarrow W^+ \rightarrow \mu^+ + \nu_\mu
  \]

  2) In $\beta^-$ decay,
  
  \[
  n \rightarrow p + e + \text{anti-}\nu_e
  \]

  inside a neutron $n = (u,d,d)$ a quark $d \rightarrow u + W^-$ and then $W^- \rightarrow e + \text{anti-}\nu_e$. Hence the result is:
  
  \[
  n = (u,d,d) \rightarrow (u,u,d) + e + \text{anti-}\nu_e = p + e + \text{anti-}\nu_e
  \]
Z\(^0\) and weak interactions of neutral current

- The boson Z\(^0\) (m=91.2 GeV S=1, neutral) decays into pairs of leptons and into pairs of quark antiquark, with comparable width.
- Decay width is the same for each family both of leptons and quarks.
- The fundamental vertices involving Z\(^0\) have properties similar to those for W: in particular electric charge, baryonic number and leptonic number are conserved.
- Like W, then also Z\(^0\) can be emitted or adsorbed during real and virtual processes.
- This is a further class of interactions which have Z\(^0\) as the mediator, and which originate new processes, forbidden by charged currents interactions.
- For example, it produces elastic collisions \(\nu_\mu + e \rightarrow \nu_\mu + e\), that aren’t mediated by charged current.
Scattering amplitudes and cross sections of two-body processes

- We are interesting in two body processes
  \[A + B \rightarrow C + D,\]
  where A…D, are leptons or quarks (the last ones confined inside nucleons or nuclei)

- Differential cross section with respect to momentum transfer, \(d\sigma/dt\), is given by
  \[d\sigma/dt = |\mathcal{A}|^2,\]
  where \(\mathcal{A}\) is the scattering amplitude and the Mandelstam variable \(t\) is defined by
  \[t = (P_A - P_C)^2 - (E_A - E_C)^2\]
  - Note that \(t\) is a relativistic invariant.
  - By the quadri-momentum conservation law we can also write:
    \[t = (P_B - P_D)^2 - (E_B - E_D)^2\]
  - Note that \(d\sigma/dt\) is a relativistic invariant because “\(\sigma\)” describes a transverse dimension, Lorentz invariant, and \(t\) is defined so that it is a relativistic invariant.
  - Using natural units (\(\hbar = c = 1\)) everything can be measured in terms of a power of energy. In this way\([l] = [E]^{-1}, [p] = [E]\) and so
    \[d\sigma/dt = [E]^{-4}, \quad [\mathcal{A}] = [E]^{-2}\]
  - A cross section which is 1\(\text{GeV}^{-2}\) in natural units, in traditional units becomes:
    \[(\hbar c)^2 \ 1\text{GeV}^{-2} = 1/25 \text{ fm}^2 = 4 \times 10^{-28} \text{ cm}^2\]
Scattering amplitudes and cross sections of low-energy weak processes

- We are interested in two body decay processes \( A+B \rightarrow C+D \), where \( A \ldots D \) are leptons or quarks (the last ones confined in nucleons or nuclei).
- The scattering amplitude \( \mathcal{A} \) is obtained by combining (=sewing) all vertices allowed by the theory (i.e. for CC and NC) that have external legs corresponding to particles \( A, B, C \) and \( D \).
- A value \( \mathcal{A}_i \) is obtained for each of the resulting graphs and the scattering amplitude is the linear sum:
  \[
  \mathcal{A} = \mathcal{A}_1 + \mathcal{A}_2 + \ldots
  \]
- Therefore, the differential cross section
  \[
  \frac{d\sigma}{dt} = |\mathcal{A}|^2 = |\mathcal{A}_1 + \mathcal{A}_2 + \ldots|^2,
  \]
  contains interference terms when more than one graph contributes.
- Generally scattering amplitudes are functions of \( s \) and \( t \).
- In the low energy limit, the amplitudes of weak processes tend to the limit \( \mathcal{A} = \zeta(A,B,C,D) G_F \) where \( \zeta \) is a numeric factor of order unity which depends on the type of particles involved (and on nuclear structure) and \( G_F = 10^{-5} \text{ GeV}^{-2} \) is the Fermi costant, so that:
  \[
  \mathcal{A} \approx G_F
  \]
Neutrino-electron scattering

• There are important differences with respect to the neutrino flavour.

• For muon (or tau) neutrino only neutral current contributes to the amplitude
  \[ A = A_{NC} \]

• For electron neutrino, the same graph exists, with the same value due to the universality of weak interactions, but also charged current graphs appear so that:
  \[ A = A_{NC} + A_{+CC} \]

• Similarly, for electron antineutrinos we have both a charged current term and neutral current term.

• This means that the cross sections are different in the various cases.
Neutrino-electron scattering: generalities

- We want to study elastic collision
  \[ \nu + e \rightarrow \nu + e \]
of a neutrino beam hitting electrons of a target, which we can think at rest.
- After collision, electrons recoil, and we can detect them in various ways, considering that they are charged particles, which produce ionization and/or emit Cerenkov radiation (if their velocity is greater than that of light in the medium).
- “The detection” of neutrinos consists thus in the observation of the signal produced by electrons.
- Independently of type of neutrino or antineutrino, kinematical expressions are always the same as for the Compton effect,
  \[ \gamma + e \rightarrow \gamma + e \]
as long as we neglect the neutrino mass (\( \leq \text{eV} \)) with respect to neutrino energy (\( \approx \text{MeV} \))
- On the other hand, the value of the cross section depends on the neutrino flavour, as discussed in the previous slide.
Kinematic of neutrino-electron scattering (1)

- We consider the elastic collision
  \[ \nu + e \rightarrow \nu + e \]
of a neutrino with energy \( E_\nu \) impinging onto an electron at rest.

- We know that all kinematic quantities are fixed when we fix scattering angle.

- Using energy and momentum conservation, we obtain

  (1) \( E_\nu + m = \epsilon + E \)
  (2) \( E_\nu = p \cos \theta + \pi_\parallel \)
  (3) \( 0 = p \sin \theta + \pi_t \)

From (2) and (3) we obtain

\[ \epsilon^2 = \pi_\parallel^2 + \pi_t^2 = (E_\nu - p \cos \theta)^2 + p^2 \sin^2 \theta \]
and therefore

(4) \( \epsilon^2 = E_\nu^2 + p^2 - 2E_\nu p \cos \theta \)

- From (1) we have:
  (5) \( \epsilon^2 = (E_\nu + m - E)^2 \)

- From (4) and (5) we can eliminate the final neutrino energy \( \epsilon \) :
  (6) \( (E_\nu + m - E)^2 = E_\nu^2 + p^2 - 2E_\nu p \cos \theta \)

- Expressing in terms of electron kinetic energy \( T \), we have

\[ T = E - m ; \quad p = [T(T+2m)]^{1/2} \]
and so from (6) we express the relation between kinetic energy \( T \) and scattering angle \( \theta \):

\[ T = 2m E_\nu^2 \cos^2 \theta / [(E_\nu + m)^2 - E_\nu^2 \cos^2 \theta] \]
this is the kinematic relation which fixes the electron energy as a function of scattering angle.
**Kinematic of neutrino-electron scattering (2)**

- The relation between energy and angle implies that the kinetic energy is maximum when $\theta = 0$,
  \[ T_{\text{max}} = \frac{E_\nu}{(1 + \frac{m}{2} E_\nu)} \]

- Note that the electron, due to quadri-momentum conservation, never takes all the energy of the neutrino.

- Increasing the angle, $T$ decreases and reaches $T=0$ when $\theta = \pi/2$.

- The Mandelstam’s variable $t$ can be expressed in terms of kinematic quantities of the electron:
  \[ t = p^2 - (m-E)^2 = p^2 - T^2 = 2mT \]

- Therefore, $t$ is linear with electron kinetic energy. The maximum value corresponds to $t_{\text{max}} = \frac{2mE_\nu}{(1 + \frac{m}{2} E_\nu)}$.
Neutrino-electron scattering: cross section

- We know that the scattering amplitudes are $\mathcal{A} \approx G_F$ and so $d\sigma/dt \approx G_F^2$.
- We write $d\sigma/dt = \zeta G_F^2$ and we study the consequences.
- The total elastic cross section is
  \[ \sigma = G_F^2 t_{\text{max}} = \zeta G_F^2 2mE_\nu /[(1+m/2E_\nu)] \]
- If neutrinos energies are large with respect to $m$, we can neglect the second term in the denominator, so the cross section increases linearly with neutrino energy.
  
  Numerically:
  \[ \sigma = \zeta G_F^2 2mE_\nu = \zeta 10^{-16} \text{ GeV}^{-2} (E_\nu / 1\text{MeV}) = 4 \times 10^{-44} \text{ cm}^2 (E_\nu / 1\text{MeV}) \]
- In the table we can find cross sections relative to various types of neutrinos. We note that the interaction of $e^- \nu_e$ have larger cross section, about six times, than that of $\nu_\mu$ or $\nu_\tau$ (which have equal cross sections).
- If the beam is formed by $\nu_e e^- \nu_\mu$, electrons detection is six times more sensitive to $\nu_e$ compared to $\nu_\mu$. 

<table>
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<th>Elastic scattering</th>
<th>[ \sigma_{\nu_e e^- \nu_e e^-} = \frac{G_F^2}{\pi} \left[ \left( \frac{1}{2} + \xi \right)^2 + \frac{1}{3} \xi^2 \right] ]</th>
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<td>$\sigma_{\nu_e e^- \nu_e e^-} = \frac{G_F^2}{\pi} \left[ \frac{1}{2} \left( \frac{1}{2} + \xi \right)^2 + \frac{1}{3} \xi^2 \right]$</td>
<td>$\approx 1.6 \times 10^{-49} \text{ m}^2 \left( \frac{E_\nu}{1\text{MeV}} \right)$</td>
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<tr>
<td>$\sigma_{\nu_\mu e^- \nu_\mu e^-} = \frac{G_F^2}{\pi} \left[ \left( \frac{1}{2} - \xi \right)^2 + \frac{1}{3} \xi^2 \right]$</td>
<td>$\approx 1.3 \times 10^{-49} \text{ m}^2 \left( \frac{E_\nu}{1\text{MeV}} \right)$</td>
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$\xi = \sin^2 \theta_W \approx 0.23$
Neutrino-electron scattering: electron energy distribution

- We know that scattering amplitudes are $\mathcal{A} \approx G_F$ and so $d\sigma/dt \approx G_F^2$.

- For simplicity*, let us put:
  
  $d\sigma/dt = G_F^2$

  and we study consequences.

- Since $t = 2mT$, we have $d\sigma/dT = 2mG_F^2$

  This means that the kinetic energy distribution of scattered electrons is uniform within the kinematic interval.

- In the liquid scintillator detectors (e.g. Borexino at GS, KamLAND in Japan) neutrinos are detected by measuring the energy of the scattered electron, which is transmitted to the medium as scintillation light.

- The figure shows, as an example, the expected signal from scattering on electrons of monocromatic neutrinos.

* Remember that there are different numerical factors related to different types of neutrino.
Neutrino-electron scattering: electron angular distribution

- We have just observed that, except for numerical factors
  \[ \frac{d\sigma}{dT} = 2mG_F^2 \]
- So the relation between energy and angle, allowing us to obtain differential cross section \( \frac{d\sigma}{d\cos\theta} \), by means of a change of variable. Putting \( x = \cos\theta \) and \( \mu = m/E_\nu \), we find:
  \[ \frac{d\sigma}{dx} = x \left[ \frac{(1+\mu)^2+x^2}{[(1+\mu)^2-x^2]} \right] 24m^2G_F^2 \]
  Note that the function is monotonically increasing in the interval from 0 to 1.
- This means that electron angular distribution has a forward (\( x=1, \) i.e. \( \theta=0 \)) maximum, i.e. The angular distribution has a forward peak with respect to the incident beam.
- On the other hand, electrons direction is correlated with the direction of the incident neutrino.

- This effect is used in the Cerenkov radiation detectors (as Kamiokande and Superkamiokande) to reconstruct the scattered electron direction and, in this way, to obtain the direction of the incident neutrino.
Interaction of anti-neutrinos on protons

• The classical reaction for the detection of few MeV antineutrinos is the so called inverse beta decay:

\[ \text{(1) } \overline{\nu}_e + p \rightarrow n + e^+ \]

The process is described by the CC graph shown in figure.

• Reaction (1) has a threshold at

\[ \Delta = m_p - m_n - m_e = 1.8 \text{ MeV} \]

• Note that anti-\(\nu_\mu\) and anti-\(\nu_\tau\) cannot produce this reaction, since in the final state one should have a \(\mu\) or a \(\tau\), see figure, but this requires hundreds of MeV. It follows that these anti-neutrinos are sterile in the energy range of interest to us.

• Note that the threshold allows detection of anti-neutrinos produced in the decay chain of U and Th.
Cross section of antineutrinos on protons

- The recoiling nucleon has a little energy, therefore (almost) all the available energy is carried by the positron, which carries kinetic energy $T = (E_n - \Delta)$ and momentum

$$p = [T(T + 2m)]^{1/2} = [(E_n - \Delta)((E_n - \Delta + 2m)]^{1/2}$$

- A part from a numerical factor we have

$$d\sigma/dt = G_F^2 \text{ and so}$$

$$\sigma = G_F^2 (t_{\text{max}} - t_{\text{min}})$$

- This difference can be calculated comparing $t$ in the range from $\theta=0$ to $\theta=\pi$ and so we can find

$$\sigma = 4G_F^2 E_\nu p = 4E_\nu [(E_\nu - \Delta)((E_\nu - \Delta + 2m)]^{1/2}$$

The cross section, shown in the figure, goes to zero at the threshold energy of $\Delta = m_p - m_n - m_e = 1.8 \text{ MeV}$

- Note that this cross section, for energy of few MeV, is of the order $\sigma \approx G_F^2 E_\nu^2$ and so it is larger than that for neutrinos on electrons, $\sigma \approx G_F^2 mE_\nu$
Detection of antineutrinos

• The principle of the experiment is still the one used by Reines and Cowan, described in the previous year and in the appendix.

• In a liquid scintillator one measures the energy released by the positron during its slowing down and in the annihilation. This is the “prompt” signal.

• The neutron thermalizes and then it is captured by a nucleus (e.g., \( n + p \rightarrow d + \gamma \)), and one detects the \( \gamma \). This is the “delayed” component of the signal, see the figure related to the distribution of events in KamLAND.

• Is essential to have this double signal, in order to eliminate the background, which is always most important in experiments with neutrinos, characterized by a low number of events.

\[
\begin{align*}
1) & \quad \text{anti-}v_e + p \rightarrow n + e^+ \\
2) & \quad e^+ + e^- \rightarrow 2\gamma \\
3) & \quad n+^{A}Z \rightarrow ^{A+1}Z + \gamma
\end{align*}
\]
Deuterium, a versatile nucleus for detection of neutrinos and anti-neutrinos of astrophysical interest

- Deuterium $d=(p,n)$ is a versatile nucleus, because it can cause various interactions, both for neutrinos (and antineutrons):
  CC: $\nu_e + d \rightarrow e + p + p$
  NC: $\nu_x + d \rightarrow \nu_x + p + n$
- Note that, at energy of few MeV, the CC reaction can be only induced by electron neutrinos, while the NC reaction occurs for each type of neutrino, with the same cross section.
- So a measurement of the ratio of charged current events to neutral current events means a measurement of the ratio of $\nu_e$ flux to the total (=any flavour) neutrino flux.
- Note that it’s also possible to have charged current interactions relative for electron anti-neutrinos.

  CC: $\bar{\nu}_e + d \rightarrow e^+ + n + n$

- The versality of deuterium has been used by the SNO experiment, which has given important results on neutrino physics.
CC reactions on heavy nuclei: low energy neutrino detection (pp neutrinos)

- None of the previous processes is suitable for detecting low energy neutrinos such as the pp ones (E<0.4 MeV):
  - the elastic $\nu + e$ scattering has no kinematic thresholds, but at low energies the background becomes dominant
  - all the CC interaction considered so far have a larger threshold.
- Consider an interaction on heavy nucleus where CC interaction can occur
  
  $\nu_e + (Z,A) \rightarrow e + (Z-1,A)$

  Daughter nucleus is instable, and so it decay by means of electron capture
  
  $e + (Z-1,A) \rightarrow \nu_e + (Z,A)$

- It is thus necessary finding a nucleus such that: 1) pp neutrinos are above the threshold for CC ; 2) daughters can be separated from the target and 3) daughters have sufficiently long lifetime so that after the separation one can detect their decay (for example, with a proportional counter).
- An ideal nucleus is $^{71}$Ga, used by Gallex and SAGE
- Note that CC current can be induced only by electron neutrinos.

$^{71}$Ga($\nu_e,e$)$^{71}$Ge ($E_{\text{thr}} = 233$ keV)

EC

$\tau = 16.5$ d

$^{71}$Ga
The underground laboratories

- Modern neutrino experiments are carried out in underground laboratories,
- Since neutrino cross sections are small, the detector has to be shielded from any radiation that could constitute a background.
- The laboratories are hundreds or thousands of meters below the ground, so as to reduce the cosmic radiation for orders of magnitude.
- The largest underground laboratory is at Gran Sasso, below 1000 m of rock (3000 MWe, i.e. meter water equivalent), where cosmic muon flux is reduced by $10^6$ compared to the surface.
- There are other big laboratories in Canada (Sudbury Neutrino Observatory, SNO), Japan (Kamioka) and smaller one in other countries.
Principal underground experiments

- Davis (Homestake, USA) (1964-1995), radiochemical method Cl-Ar, first detection of solar Boron neutrinos,
- Kamiokande and Superkamiokande (1985 – ongoing), water Cerenkov detector, first real time and directional detection of solar Boron neutrinos. It also detected anti-neutrinos from supernova SN 1987A.
- Gallex (LNGS) and Sage (Russia) (1992-2005) radiochemical method Ga-Ge, measurement of low energy neutrinos (pp)
- Borexino (LNGS), ongoing, 300 ton of ultrapure liquid scintillator. measurement of solar Berillium neutrinos and geoneutrinos
- Luna (LNGS), ongoing: measurement of cross sections of astrophysical interest.
- SNO: (2001 -2008) heavy water detection, first appearance experiment, proofs neutrino oscillations, with solar boron neutrinos
- KamLAND: (2002 -ongoing) 1000 ton of liquid scintillator, confirmation of neutrinos oscillations by means of reactors, first detection of geoneutrinos
Decalogue

1) There are three neutrinos families
2) The neutrinos are stable, with masses not exceeding eV.
3) The main artificial sources of neutrinos are reactors (anti-$\nu_e$ with energies of few MeV) and accelerators (mainly $\nu_\mu$, with energy of the order of hundreds MeV).
4) Neutrinos and antineutrinos have charged and neutral current interactions.
5) At MeV energy, all cross sections are of order $\frac{d\sigma}{dt} \approx G_F^2$
6) Considering astrophysical processes, muonic and tauonic neutrinos are sterile relative to charged current processes.
7) Scattering of neutrinos on electrons is directional; cross section are largest for $\nu_e$
8) Beta inverse process is the classical reaction to detect anti-$\nu_e$ of few MeV; it produces positrons with kinetic energy $T = E - \Delta$, with approximatively isotropic distribution and neutrons, these latter detected by means of delayed capture.
9) On deuterium nuclei, CC processes can occur induced by $\nu_e$, but also NC processes, where all types of neutrinos are active.
10) We need CC reaction on nuclei with low threshold, like $^{71}$Ge, for detection of pp neutrinos.
Appendix

• Scintillator
• Cerenkov radiation
• Reines and Cowan experiment
• Two antineutrinos experiment
A scintillator is a material which exhibits the property of luminescence when excited by ionizing radiation. Luminescent materials, when struck by an incoming particle, absorb its energy and scintillate, i.e. reemit the absorbed energy in the form of a small flash of light, typically in the visible range. (Throughout this article, the word “particle” will be used to mean “ionizing radiation” and can refer to either charged particulate radiation such as electrons and heavy charged particles, or to uncharged radiation such as photons and neutrons, provided that they have enough energy to induce ionization.) If the reemission occurs promptly, i.e. within the ~10^-8s required for an atomic transition, the process is called (or more precisely related to) fluorescence.

A scintillation detector or scintillation counter is obtained when a scintillator is coupled to an electronic light sensor such as a photomultiplier tube (PMT) or a photodiode. PMTs absorb the light emitted by the scintillator and reemit it in the form of electrons via the photoelectric effect. The subsequent multiplication of those electrons (sometimes called photo-electrons) results in an electrical pulse which can then be analyzed and yield meaningful information about the particle that originally struck the scintillator. Vacuum photodiodes are similar but do not amplify the signal while silicon photodiodes accomplish the same thing directly in the silicon.

The first use of a scintillator dates back to an experiment in 1903 where Sir William Crooks observed a ZnS screen struck by α-particles.

Today, scintillation detectors are used in a wide array of applications, including fundamental research in particle and nuclear physics, oil exploration, field spectrometry, container and baggage scanning, health physics, space physics, industrial gauging, and medical diagnostics and therapy (PET, SPECT, therapy imaging, etc…).
Cerenkov radiation*

- Čerenkov radiation (also spelled Cerenkov or Cherenkov) is electromagnetic radiation emitted when a charged particle (such as an electron) passes through a medium at a speed greater than the speed of light in that medium. The characteristic "blue glow" of nuclear reactors is due to Čerenkov radiation. It is named after Russian scientist Pavel Alekseyevich Čerenkov, the 1958 Nobel Prize winner who was the first to characterise it rigorously…

- A common analogy is the sonic boom of a supersonic aircraft or bullet. The sound waves generated by the supersonic body do not move fast enough to get out of the way of the body itself. Hence, the waves "stack up" and form a shock front. In a similar way, a charged particle can generate a photoni shock wave as it travels through an insulator.

- In the figure, the particle (red arrow) travels in a medium with speed $v$ and we define the ratio between the speed of the particle and the speed of light as $\beta = \frac{v}{c}$ where $c$ is speed of light. $n$ is the refractive index of the medium and so the emitted light waves (blue arrows) travel at speed $v_{em} = \frac{c}{n}$.

- Radiation is emitted along a cone with aperture given by $\cos \theta = 1/n\beta$

* vedi Cerenkov radiation Wiki
**β inverse process and the antineutrinos detection**

- The detection of antineutrinos, i.e. the products of their interactions, was performed for the first time in 1956, observing a process that is essentially the inverse of the neutron β decay and still is the classic way to study these particles

\[ \text{(1)} \quad \text{anti-} \nu_e + p \rightarrow n + e^+ \]

- The positron, the light particle in the final state, brings with it (almost) all the available energy, its kinetic energy being \( T_e = E_{\nu} + m_p - m_n - m_e = E_{\nu} - 1.8 \text{MeV} \). The process is therefore possible for antineutrinos with \( E_{\nu} > 1.8 \text{MeV} \).

- In the target, the positron slows down and annihilates on an electron,

\[ \text{(2)} \quad e^+ + e^- \rightarrow 2 \gamma \]

this provides a first signal: two photons in opposite directions and each with \( E = m_e \).

- The neutron is slowed down to thermal energies (in time \( \Delta t \) of the order of tens of μs in a liquid). If the target, \(^A\text{\,Z}\), is an absorber of neutrons, the neutron can be identified by the γ emitted by the capture:

\[ \text{(3)} \quad n + ^A\text{\,Z} \rightarrow ^{A+1}\text{\,Z} + \gamma. \]

- The presence of this signal, delayed with respect to annihilation, is a distinctive feature of the interaction of antineutrinos: we have lots of background counts (due to cosmic and / or natural radioactivity) corresponding to (2) or (3), but a lot less if we require both (2) and (3).
Reines and Cowan experiment

• As a source of anti-\(v_e\) they used a nuclear fission reactor, where - on average - six anti-\(v_e\) for each fission are produced. The energy spectrum is continuous, with a maximum around 5 MeV. Outside the core of a power reactor there are neutrino fluxes of order \(\Phi \approx 10^{13} \text{ cm}^{-2} \text{ s}^{-1}\).

• The target contained 200 liters of water with \(\approx 10^{28}\) “free” protons.

• A cadmium salt was dissolved in water. Cadmium is a nucleus with large cross section for neutron capture, so one can detect neutrons using (3).

• The target was surrounded by liquid scintillator which was coupled with photomultipliers in order to detect \(\gamma\) from annihilation (2) and from capture (3).

• The detector was located about a dozen meters from the reactor core and a dozen metres underground, to get a screen from cosmic rays.

• Reines and Cowan, selected the events which featured both (2) that (3).

• From the data we can derive the cross section \(\sigma\) of (1), since \(\nu = \varepsilon \Phi \sigma N_p\): 
\[
\sigma = \frac{\nu}{\varepsilon \Phi N_p} \approx 10^{-43} \text{ cm}^2
\]
Two neutrinos experiment

- The $\pi^+$ meson decays mainly in $\pi^+ \rightarrow \mu^+ + \nu_\mu$ where we have denoted with $\nu_\mu$ the neutrino produced together with $\mu^+$.
- This is not the same state that accompanies the $e^+$, that is $\nu_e$. If so ($\nu_\mu = \nu_e = \nu$) in a subsequent collision with nuclei neutrinos should induce reactions $\nu + Z \rightarrow (Z+1) + e$.
- Ledermann, Schwartz and Steinberger observed that the neutrinos associated with $\mu^+$ produce the reaction $\nu + Z \rightarrow (Z+1) + \mu$ but not $\nu + Z \rightarrow Z+1 + e$. 

![Diagram of a proton beam and detector](image-url)
Lepton family numbers

• In 1975 the charged lepton $\tau$ was discovered and in 2000 reactions induced by neutrinos $\nu_\tau$ on nuclei were observed.

• The picture that emerges is the conservation of lepton family number, defined for each family $\alpha = e, \mu, \tau$ as $L_\alpha = 1$ for $l^+\alpha$ and $\nu_\alpha$, 0 for the other families, the opposite for antiparticles.

• Clearly $L = L_e + L_\mu + L_\tau$.

• The conservation of the family numbers imply the conservation of lepton number, but the converse is not true: the lepton number can be conserved but the family numbers may be violated.