Outline

Unitarity constraints

Triviality constraints

Stability constraints

Metastability constraint

Post LHC phase diagram

ELECTROWEAK VACUUM

Lecture 3: Theoretical Constraints from the Higgs discovery

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Outline

The outline of Lecture 3 is

Theoretical constraints

- Unitarity constraints
- Triviality constraints
- Stability constraints
  - Post LHC stability bounds
- Metastability constraints
  - Thermal corrections
  - Thermal tunneling
  - Constraints
- Post LHC phase diagram

Unitarity constraints

- The longitudinal components of the W and Z bosons (W_L, Z_L) give rise to interesting features
- In the gauge boson rest frame one can define the transverse and longitudinal polarization four–vectors as

$$\epsilon^\mu_{T_1} = (0,1,0,0)\,, \quad \epsilon^\mu_{T_2} = (0,0,1,0)\,, \quad \epsilon^\mu_L = (0,0,0,1)$$

- For a four–momentum $p^\mu = (E,0,0,|\vec{p}|)$, after a boost along the z direction, the transverse polarizations remain the same while the longitudinal polarization becomes

$$\epsilon^\mu_L = \left(\frac{|\vec{p}|}{m_V},0,0,\frac{E}{m_V}\right) \overset{E\gg m_V}{\Longrightarrow} \frac{p_\mu}{m_V}$$

- Since this polarization is proportional to the gauge boson momentum, at very high energies the longitudinal amplitudes will dominate the scattering of gauge bosons

- In processes involving the W_L and Z_L bosons, this would eventually lead to cross sections which increase with the energy which would then violate unitarity at some stage
- We will briefly illustrate this aspect in the following, taking as an example the scattering process $W^+W^- \to W^+W^-$ at high energies, which can violate the unitarity bounds
- We first decompose the scattering amplitude A into partial waves a_ℓ of orbital angular momentum ℓ

$$A = 16\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta)\, a_\ell$$

P_ℓ=Legendre polynomials and θ=scattering angle.
- For a $2 \to 2$ process, the cross section is given by

$$\mathrm{d}\sigma/\mathrm{d}\Omega = |A|^2/(64\pi^2 s), \quad d\Omega = 2\pi d(\cos\theta)$$

$$\sigma = \frac{16\pi}{s} \sum_{\ell=0}^{\infty} (2\ell+1)|a_\ell|^2$$

- Unitarity implies the

Optical theorem

$$\sigma = \frac{1}{s}\,\mathrm{Im}\,[\,A(\theta = 0)\,] \;=\; \frac{16\pi}{s}\sum_{\ell=0}^{\infty}(2\ell+1)|a_\ell|^2$$

- This leads to the

Unitarity condition

$$|a_\ell|^2 = \mathrm{Im}(a_\ell) \Rightarrow [\mathrm{Re}(a_\ell)]^2 + [\mathrm{Im}(a_\ell)]^2 = \mathrm{Im}(a_\ell)$$

$$[\mathrm{Re}(a_\ell)]^2 + \left[\mathrm{Im}(a_\ell) - \frac{1}{2}\right]^2 = \frac{1}{4}$$

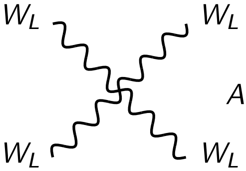
$$\Updownarrow$$

$$|\mathrm{Re}(a_\ell)| < \frac{1}{2}$$

- In particular for the $J = 0$ partial wave

$$|\mathrm{Re}(a_0)| < \frac{1}{2}$$

- The unitarity condition is badly violated by the quartic W_L interactions



$$A \propto g^2 \frac{s^2}{M_W^4} \Rightarrow s \leq M_W^2$$

- This problem can be partly cured by adding the other SM gauge interactions

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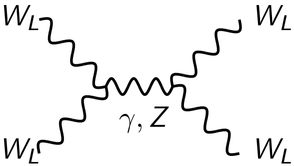

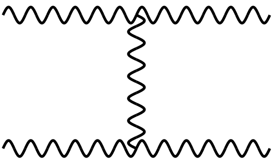

$$a_0 = \frac{g^2 s}{16\pi M_W^2} \Rightarrow \sqrt{s} \leq 1.7\,TeV$$

Outline

Unitarity constraints

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Post LHC phase diagram

- The problem is fully solved by introducing the Higgs interactions

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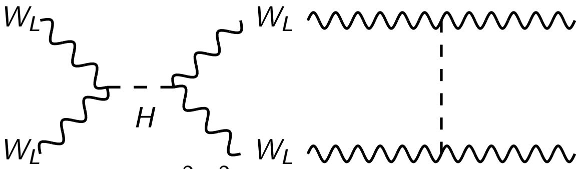


$$a_0 = \frac{g^2 m_H^2}{32\pi M_W^2} \Longrightarrow m_H \leq 870 \ GeV$$

- Channel $W_L^+ W_L^-$ considered above can be coupled with other neutral $Z_L Z_L$, HH and $Z_L H$ and charged $W_L^+ H$ and $W_L^+ Z_L$ channels. The scattering amplitude and a_0 is then given by a 6×6 matrix. The requirement that the largest eigenvalues of a_0, respects the unitarity constraint yields

$$M_H \lesssim 710 \text{ GeV}$$

- Goldstone bosons are useful tools to enforce unitarity because of the

Electroweak Equivalence Theorem

At very high energies, the longitudinal massive vector bosons can be replaced by the Goldstone bosons.

$$A(V^1 \cdots V^n \to V^1 \cdots V^{n'}) \sim A(V_L^1 \cdots V_L^n \to V_L^1 \cdots V_L^{n'})$$

$$\sim A(\chi^1 \cdots \chi^n \to \chi^1 \cdots \chi^{n'})$$

- Thus, in this limit, one can simply replace in the SM scalar potential, the W and Z bosons by their corresponding Goldstone bosons $\chi^\pm, \chi_0$, leading to

Higgs-Goldstone interactions

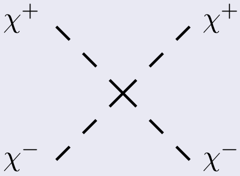
$$V = \frac{m_h^2}{2v}(h^2 + \chi_0^2 + 2\chi^+\chi^-)h + \frac{m_h^2}{8v^2}(h^2 + \chi_0^2 + 2\chi^+\chi^-)^2$$

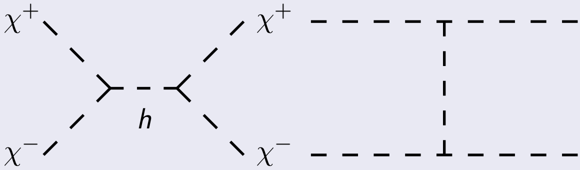
and use this potential to calculate the amplitudes

Exercise: compute a_0 as

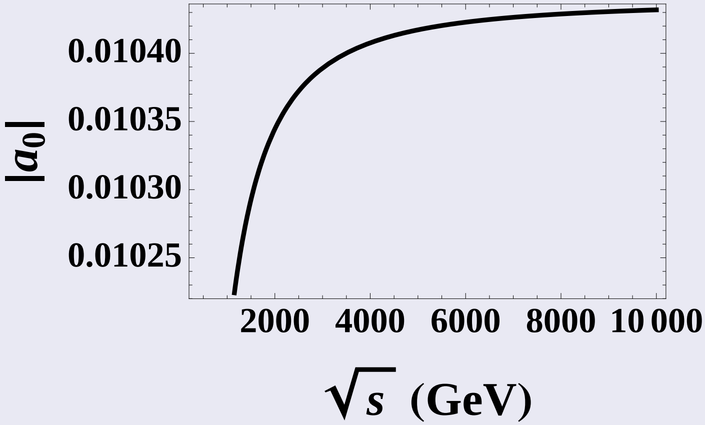
$$a_0 = -\frac{M_H^2}{16\pi v^2}\left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s}\log\left(1 + \frac{s}{M_H^2}\right)\right]$$

from the set of diagrams





For $m_H = 126$ GeV



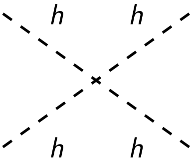
Triviality constraints

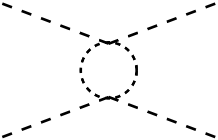
- The running of the Higgs quartic coupling with the energy scale Q is described by the Renormalization Group Equation (RGE)

RGE

$$\frac{d\lambda}{d\log Q^2} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda h_t^2 - 3h_t^4 \right.$$

$$\left. -\frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$





- For large values of the Higgs mass (λ) the quartic coupling dominates the RGE and its solution can be written analytically

$$\lambda(Q^2) \simeq \lambda(v^2)\left[1-\frac{3}{4\pi^2}\,\lambda(v^2)\log\frac{Q^2}{v^2}\right]^{-1}$$

- When the energy is much higher than the weak scale, $Q^2 \gg v^2$, the quartic coupling grows and eventually becomes infinite. This point is called Landau pole

$$\Lambda \simeq v\,\exp\left(\frac{4\pi^2}{3\lambda}\right) = v\,\exp\left(\frac{4\pi^2 v^2}{m_h^2}\right)$$

The general triviality argument

The scalar sector of the SM is a ϕ^4–theory, and for these theories to remain perturbative at all scales one needs to have a coupling $\lambda = 0$ [which in the SM, means that the Higgs boson is massless], thus rendering the theory trivial, i.e. non–interacting

- One can turn the argument around: fixing the value of m_h one can use the RGE for the quartic Higgs self–coupling to establish the energy domain in which the SM is valid, i.e. the energy cut–off Λ below which the self–coupling λ remains finite
- Alternatively, fixing Λ one can determine an upper bound on the Higgs mass for the theory to remain perturbative i.e. for the self–coupling λ to remain finite
- As the experimental value of the Higgs mass ($m_H \simeq 126$ GeV) is not too large, the previous approximation is not a good one and there is no problem with the triviality bound for any value of the cutoff below M_P

No triviality bound at least for

$$\Lambda \lesssim M_P$$

- Next we will study the stability bound. It is a lower bounds and applies for light Higgs masses.

Stability constraints

- In the region of light Higgs there is another effect of the RGE on the quartic coupling

RGE (reminder)

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\log Q^2} \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda h_t^2 - 3h_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}\left(2g_2^4 + (g_2^2 + g_1^2)^2\right)\right]$$

- For small values of λ the RGE is dominated by the h_t^4 coupling

$$8\pi^2 \frac{d\lambda}{d\log\Lambda} \simeq -3h_t^4$$

and λ decreases with Λ

$$\lambda(\Lambda) \simeq \lambda(v) - \frac{3}{8\pi^2} h_t^4 \log\frac{\Lambda}{v}$$

- When $\lambda(\Lambda) < 0$ the potential is unbounded from below
- For fixed Λ there is a lower bound on the Higgs mass

$$m_h^2 \gtrsim \frac{3h_t^2 m_t^2}{2\pi^2} \log \frac{\Lambda}{v}$$

- For fixed m_h there is an upper bound on Λ

$$\Lambda \lesssim v \exp(2\pi^2 m_h^2 / 3h_t^2 m_t^2)$$

- A more precise bound of course requires the numerical solution to the system of coupled differential RGE to find out the scale where $\lambda(\Lambda) = 0$
- Going beyond the one-loop result can be achieved by using RGE techniques to resum the effective potential as we will show next

- The SM effective potential can be written in the 't Hooft-Landau gauge and the $\overline{MS}$ renormalization scheme as $V_{\rm eff} = V_0 + V_1$

Tool 1: SM effective potential

$$V_0 = -\frac{1}{2}m^2(t)\phi^2(t) + \frac{1}{8}\lambda(t)\phi^4(t)$$

$$V_1 = \sum_{i=W,Z,t} \frac{n_i}{64\pi^2} m_i^4(\phi)\left[\log\frac{m_i^2(\phi)}{\mu^2(t)} - C_i\right] + \Omega(t)$$

$$C_W = C_Z = \frac{5}{6}, \quad C_t = \frac{3}{2}, n_W = 6, \quad n_Z = 3, \quad n_t = -12,$$

$$m_i^2 = \kappa_i \phi^2(t), \quad \phi(t) = \xi(t)\phi_c$$

$$\xi(t) = \exp\left\{-\int_0^t \gamma(t')dt'\right\}, \quad \mu(t) = m_Z e^t$$

$$\kappa_W = \frac{1}{4}g^2(t), \quad \kappa_Z = \frac{1}{4}[g^2(t) + g'^2(t)], \quad \kappa_t = \frac{1}{2}h^2(t).$$

Tool 2: The pole masses M_h and M_t

$$M_h^2 = m_h^2[M_h] + \mathrm{Re}\left[\Pi_{HH}(p^2 = M_h^2) - \Pi_{HH}(p^2 = 0)\right]$$

$$M_t = \left[1 + \frac{4}{3}\frac{\alpha_s(M_t)}{\pi}\right] m_t[M_t].$$

- The tree-level potential improved by the RGE is highly scale dependent
- The one-loop effective potential improved by RGE is highly scale independent. This allows fixing the renormalization scale as

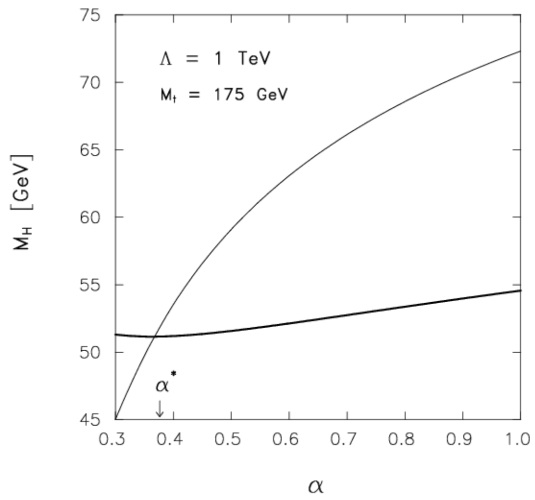
$$\mu(t) \sim \alpha\phi(t), \quad \alpha \simeq \mathcal{O}(1)$$

in order to tame potentially dangerous logarithms at large values of the field (where the instability is expected to appear)

- And determine when α is "more" scale invariant

The scale dependence (tree-level RGE-improved Vs. one-loop RGE-improved) in the appropriate region is shown in the figure

Scale (in)dependence



- We can write the potential as

$$V_{\rm eff}=-\frac{1}{2}m^2(t)\phi^2(t)+\frac{1}{8}\lambda_{\rm eff}(t)\phi^4(t)+\Omega(t)$$

from where

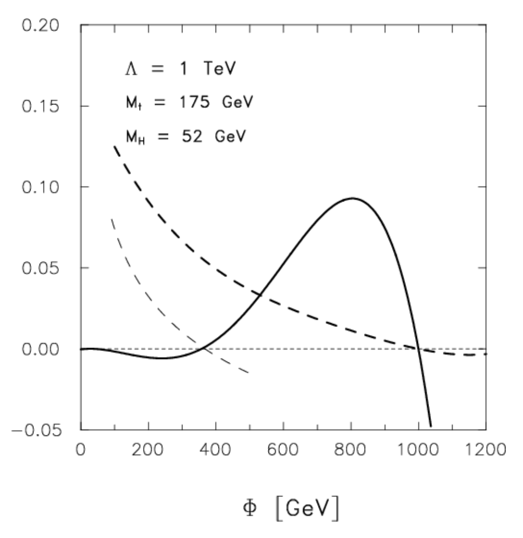
$$\lambda_{\rm eff}(t)=\lambda(t)+\sum_i\frac{n_i}{8\pi^2}\kappa_i^2\left[\log\frac{\kappa_i}{\alpha^2}-C_i\right].$$

- The value of the scale Λ where new physics has to stabilize the SM potential is given by the value of the field ϕ where the depth of the potential equals the depth of the potential at the standard electroweak minimum
- Due to the steepness of the potential around that point, we can identify Λ with the value of the field where the potential vanishes, i.e.

$$V_{\rm eff}(\phi)|_{\phi=\Lambda}=0\quad,$$

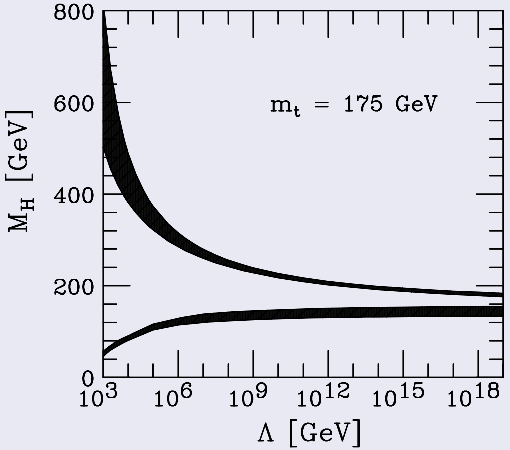
The effective potential is destabilized at a given value of the field [dashed line is $\lambda_{\rm eff}(\phi)$]

Effective potential



- The summary of pre-LHC triviality and stability bounds

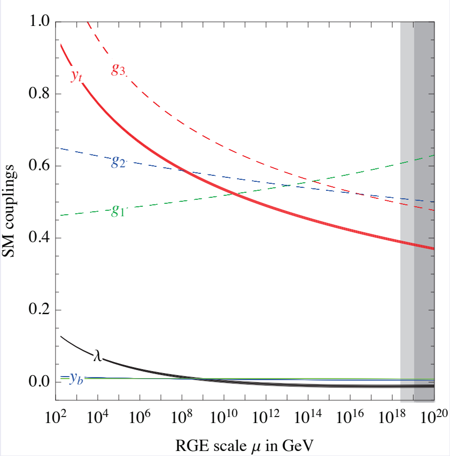
The pre-LHC Standard Model Window



Stability bounds post-LHC

Stability bounds have been re-considered after the Higgs discovery at LHC [1]

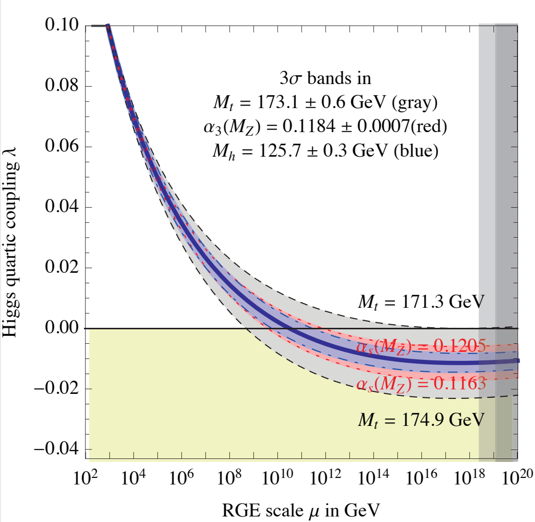
The SM RGE for $m_H = 125.5$ GeV



[1] G. Degrassi, et al. arXiv:1205.6497 [hep-ph]

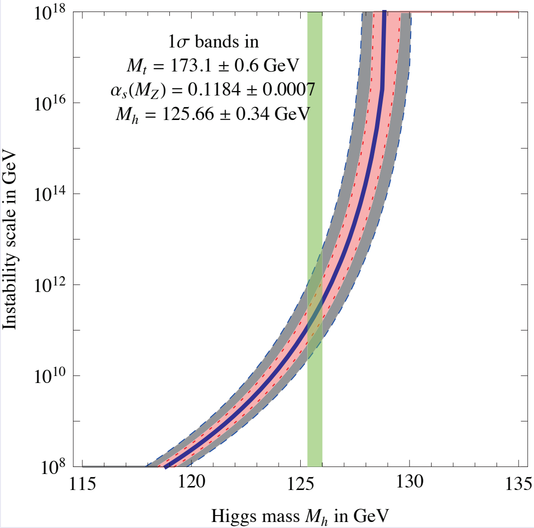
Detail of the running of the Higgs quartic coupling

At NNLO



Some detail about the instability region

The instability region



Some detail about the instability region

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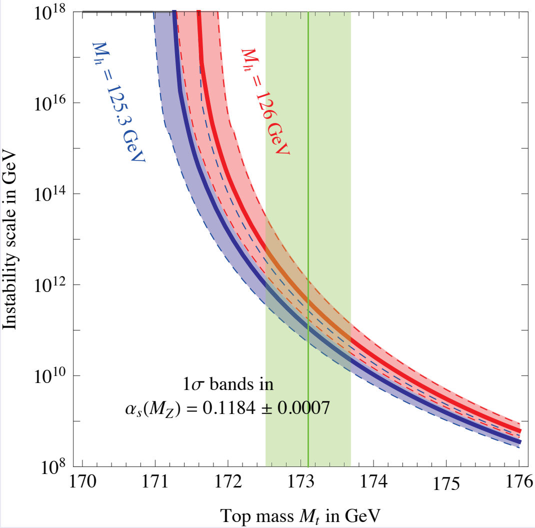
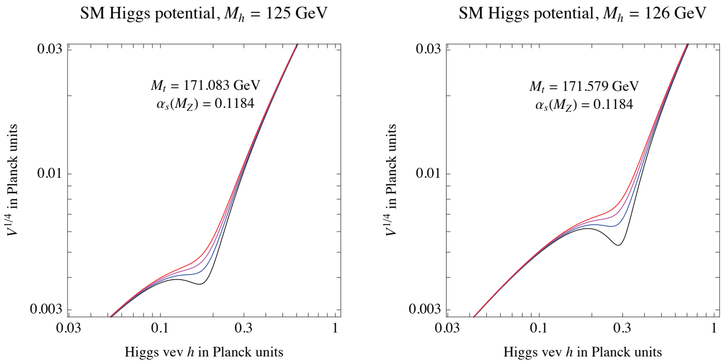


Identifying the Higgs with the inflaton field [2]

Inflationary potential: curves are $\Delta M_t = 0.1$ MeV



[2] I. Masina and A. Notari, arXiv:1112.5430 [hep-ph].

Metastability constraint

- Even if the stability requirements are a valuable indication, they cannot be considered as absolute lower bounds in the SM since we cannot logically exclude the possibility of the physical electroweak minimum being a metastable one, provided the probability, normalized with respect to the expansion rate of the Universe, for decay to the unphysical (true) minimum, be negligibly small
- In view of the recent discovery of the Higgs boson it is then extremely important to consider this latter possibility and study cases where the electroweak vacuum is not stable, but metastable
- The main tools for that should be
  - Thermal corrections to the effective potential including plasma effects by one-loop resummation of Debye masses
  - Numerical calculation of the bounce solution and the energy of the critical bubble

Thermal corrections

- The thermal correction to the effective potential can be computed using the rules of field theory at finite temperature. Including plasma effects by one-loop ring resummation of Debye masses
- It can be written as

$$\Delta V_{\rm eff}(\phi, T) = V_1(\phi, T) + V_{\rm ring}(\phi, T)$$

- The one-loop thermal correction

One-loop correction

$$V_1(\phi, T) = \frac{T^4}{2\pi^2}\left[\sum_{i=W,Z} n_i J_B\left(\frac{m_i^2(\phi)}{T^2}\right) + n_t J_F\left(\frac{m_t^2(\phi)}{T^2}\right)\right]$$

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- The thermal functions are given by

Thermal integrals

$$J_B(y) = \int_0^\infty dx\ x^2 \log\left[1 - e^{-\sqrt{x^2+y^2}}\right]$$

$$J_F(y) = \int_0^\infty dx\ x^2 \log\left[1 + e^{-\sqrt{x^2+y^2}}\right]$$

- Plasma effects in the leading approximation can be accounted for by the one-loop effective potential improved by the daisy diagrams

Hard thermal loops

$$V_{\rm ring}(\phi, T) = \sum_{i=W_L,Z_L,\gamma_L} n_i \left\{ \frac{m_i^3(\phi)T}{12\pi} - \frac{\mathcal{M}_i^3(\phi)T}{12\pi} \right\}$$

- Only the longitudinal degrees of freedom of gauge bosons, $\frac{1}{2}n_{W_L} = n_{Z_L} = n_{\gamma_L} = 1$, are accounted for
- The thermal masses are

Debye corrected masses

$$\mathcal{M}^2_{W_L} = m^2_W(\phi) + \frac{11}{6} g^2 T^2$$

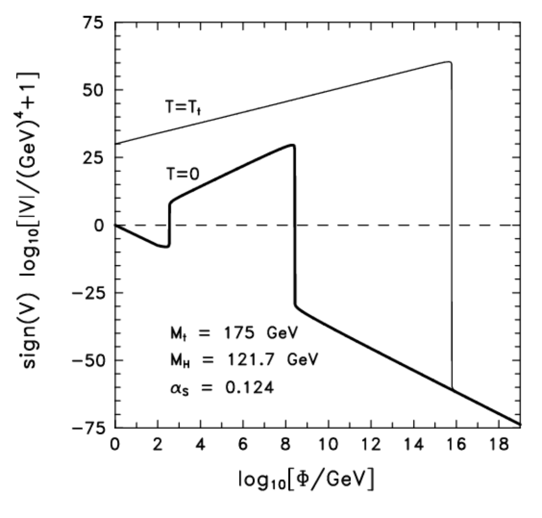
$$\mathcal{M}^2_{Z_L} = \frac{1}{2}\left[m^2_Z(\phi) + \frac{11}{6}\frac{g^2}{\cos^2\theta_W} T^2 + \Delta(\phi, T)\right]$$

$$\mathcal{M}^2_{\gamma_L} = \frac{1}{2}\left[m^2_Z(\phi) + \frac{11}{6}\frac{g^2}{\cos^2\theta_W} T^2 - \Delta(\phi, T)\right]$$

- The discriminant is responsible for the rotation at finite temperature from the basis (W_{3L}, B_L) to the mass eigenstate basis (Z_L, γ_L)

$$\Delta^2 = m^4_Z(\phi) + \frac{11}{3}\frac{\cos^2 2\theta_W}{\cos^2\theta_W}\left[m^2_Z(\phi) + \frac{11}{12}\frac{g^2}{\cos^2\theta_W} T^2\right] T^2$$

Effective potential at $T = T_t = 2.5 \times 10^{15}$ GeV (thin solid line), for $M_t = 175$ GeV and $M_H = 122$ GeV



Thermal tunneling

- In a first-order phase transition the tunneling probability rate per unit time per unit volume is given by

$$\frac{\Gamma}{\nu} \sim \omega T^4 e^{-E_b/T} \ ,$$

- E_b (the energy of a bubble of critical size) is given by the three-dimensional euclidean action S_3 evaluated at the *bounce* solution

$$E_b = S_3[\phi_B(r)]$$

- At high temperature the bounce has $O(3)$ symmetry

Euclidean action

$$S_3 = 4\pi \int_0^\infty r^2 dr \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V_{\text{eff}}(\phi(r), T) \right]$$

$$r^2 = \vec{x}^2$$

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- The bounce ϕ_B satisfies the Euclidean equation of motion and boundary conditions

Bounce equations: $\phi = \phi_B$

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{dV_{\rm eff}(\phi, T)}{d\phi}$$

$$\lim_{r\to\infty} \phi(r) = 0$$

$$\left.\frac{d\phi}{dr}\right|_{r=0} = 0$$

- The semiclassical picture is that unstable bubbles (either expanding or collapsing) are nucleated behind the barrier, at $\phi_B(0)$, with a probability rate given by Γ
- The actual probability P is obtained by multiplying the probability rate by the volume of our current horizon scaled back to the temperature T and by the time the Universe spent at temperature T

- The probability is then

$$\frac{dP}{d\log T} = \left(2\zeta\frac{M_{P\ell}}{T}\right)^4 e^{-E_b/T}$$

$$\zeta = \frac{1}{4\pi}\sqrt{\frac{45}{\pi g_*}} \sim 0.03$$

- The total integrated probability is defined as

$$P(T) = \int_T^{T_c} \frac{dP(T')}{dT'}\, dT',$$

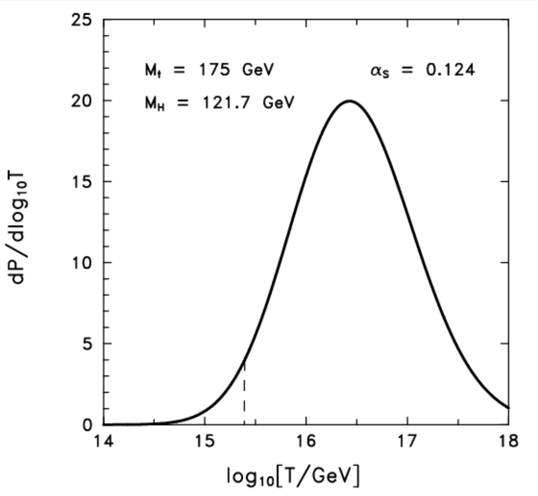
T_c $[E_b(T_c) \to \infty]$ is the temperature at which the two minima of the effective potential become degenerate

- The physical meaning of the integrated probability

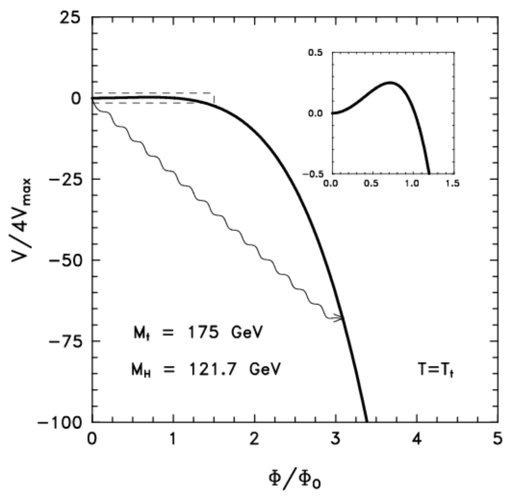
Fraction of space in the old metastable (new stable) phase

$$f_{\rm old} = e^{-P}, \quad f_{\rm new} = 1 - e^{-P}$$

Plot of $dP/d\log_{10} T$. Dashed line indicates temperature $T_t = 2.5 \times 10^{15}$ GeV at which the integrated probability is 1



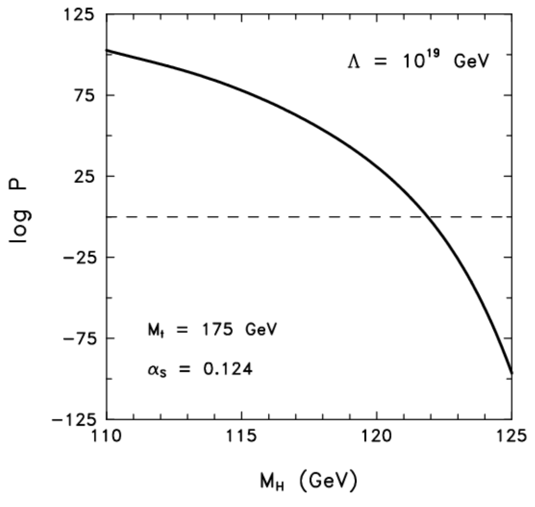
Plot of the effective potential at $T_t = 2.5 \times 10^{15}$ GeV



Constraints

- We have systematically analyzed cases with different values of M_h and different values of the cutoff Λ as e.g.

The case $\Lambda = 10^{19}$ GeV



A fit to the case $\Lambda = 10^{19}$ GeV

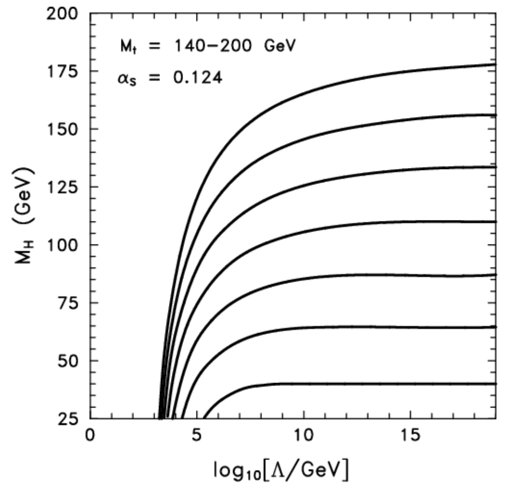
$$M_h/\mathrm{GeV} = [2.278 - 4.654\,(\alpha_S - 0.124)]\,(M_t/\mathrm{GeV}) - 277$$

A general fit

$$M_H/\mathrm{GeV} = A(\Lambda)(M_t/\mathrm{GeV}) - B(\Lambda)$$

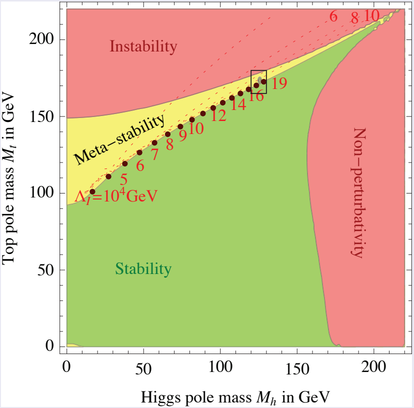
$\log_{10}(\Lambda/\mathrm{GeV})$	$A(\Lambda)$	$B(\Lambda)$
4	1.219	157
5	1.533	186
7	1.805	212
9	1.958	230
11	2.071	245
13	2.155	258
15	2.221	268
19	2.278	277

M_h as a function of Λ

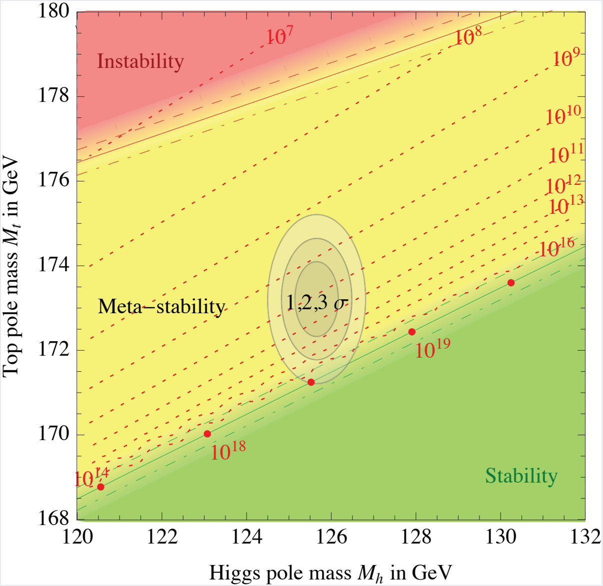


Post LHC phase diagram

Recently working at NNLO [3] the phase diagram in the plane (M_h, M_t) has been worked out



[3] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio and A. Strumia, arXiv:1307.3536



Instability
Meta−stability
Stability
1,2,3 σ
10^7
10^8
10^9
10^10
10^11
10^12
10^13
10^14
10^16
10^18
10^19
Top pole mass M_t in GeV
Higgs pole mass M_h in GeV