ELECTROWEAK VACUUM

Lecture 1: General picture

V Ferrara International School Niccolo' Cabeo 2014

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General Plan

The outline of these lectures is

- Lecture 1: General Picture of the Standard Model of EW interactions
- Lecture 2: Experimental Precision Data and Higgs Data
- Lecture 3: Theoretical Constraints from the Higgs discovery
- Lecture 4: Beyond the Standard Model

Outline

The outline of Lecture 1 is

General Picture

- Standard Model overview
- Electroweak breaking
- Higgs and Goldstone bosons
- Fermion gauge interactions
- Yukawa interactions
- Neutral currents
- Charged currents and CKM mixing
- GIM mechanism

Standard Model overview

- The Standard Model (SM) is a gauge theory based on the group

Gauge group

$$SU(3) \otimes SU(2) \otimes U(1)_Y$$

- $SU(3)$ describes the strong interactions (QCD) $\Rightarrow$ S. Scherer's lectures
- Since the gauge interactions conserve chirality we can decompose fermions as

$$f = f_L + f_R, \quad f_{L,R} = P_{L,R} f, \quad P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$$

- The SM choice was to place f_L in $SU(2)$ doublets and f_R in $SU(2)$ singlets
- One can instead replace f_R by

$f_R \to f_L^c = C\bar{f}^T$, where C=charge conjugation matrix

- They appear in (at least) three generations

SM fermions

$$\left[\begin{array}{c}\begin{pmatrix}\nu_i\\ \ell_i^-\end{pmatrix}_L \\ \\ \ell_{iR}^-\ [\ell_{iL}^+]\end{array}\right]\ \left[\begin{array}{c}\begin{pmatrix}u_i^\alpha\\ d_i^\alpha\end{pmatrix}_L \\ \\ u_{iR}^\alpha\ [u_{iL}^{c\ \alpha}]\quad d_{iR}^\alpha\ [d_{iL}^{c\ \alpha}]\end{array}\right]\qquad \begin{array}{c}\alpha = \mathit{colors}\\ i = \mathit{generations}\\ Q = T_3 + Y\end{array}$$

$$f_L\ \mathit{doublets}:\quad (1,2)_{-1/2} + (3,2)_{1/6}$$

$$f_L^c\ \mathit{singlets}:\quad (1,1)_1 + (\bar{3},1)_{-2/3} + (\bar{3},1)_{1/3}$$

- The pure gauge boson part Lagrangian is

Electroweak gauge bosons Lagrangian

$$\mathcal{L}_{gauge} = -\frac{1}{4}G_{\mu\nu a}G^{\mu\nu a} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mathcal{L}_{GF} + \mathcal{L}_{FP}$$

$$G_{\mu\nu a} \equiv \partial_\mu W_{\nu a} - \partial_\nu W_{\mu a} + g\epsilon_{abc}W_{\mu b}W_{\nu c}$$

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

- To properly quantize the theory we need the Faddeev-Popov gauge fixing

Faddeev-Popov Lagrangian (symmetric phase)

$$\mathcal{L}_{GF+FP} = \frac{1}{2\xi}(\partial^\mu W_\mu^a)^2 + \frac{1}{2\xi'}(\partial^\mu B_\mu)^2 + \bar{c}^a(-\partial^\mu D_\mu^{ab})c^b$$

$$D_\mu^{ab} = \partial_\mu \delta^{ab} + g\epsilon^{acb} W_\mu^c$$

- The interaction of gauge bosons with fermions is achieved in the gauge invariant Lagrangian

Fermion Lagrangian

$$\mathcal{L}_{fer} = i\sum_{f_L} \bar{f}_L \gamma^\mu (\partial_\mu - ig\frac{\sigma_a}{2} W_{\mu a} - ig' Y_{f_L} B_\mu) f_L$$

$$+ i\sum_{f_R} \bar{f}_R \gamma^\mu (\partial_\mu - ig' Y_{f_R} B_\mu) f_R$$

Electroweak breaking

- In the Standard Model the electroweak symmetry $SU(2) \otimes U(1)$ is spontaneously broken by the Higgs mechanism where an $SU(2)_L$ doublet Higgs boson is needed

Higgs mechanism

$$H = \begin{pmatrix} \chi^+ \\ H^0 \end{pmatrix}_{1/2}$$

$$\widetilde{H} = i\sigma_2 H^* = \begin{pmatrix} \bar{H}^0 \\ -\chi^- \end{pmatrix}_{-1/2}$$

$$\mathcal{L}_{Higgs} = \left|(\partial_\mu - ig\frac{\sigma_a}{2}W_{\mu a} - ig'\frac{1}{2}B_\mu)H\right|^2 - V(H)$$

$$V(H) = -m^2|H|^2 + \lambda|H|^4$$

- By minimization of the Higgs potential one obtains the VEV

$$\langle H\rangle = \frac{v}{\sqrt{2}}\left(\begin{array}{c}0\\1\end{array}\right),\quad v=\sqrt{\frac{m^2}{\lambda}},\quad m_h^2=2\lambda v^2$$

- By replacing $H=\langle H\rangle+\hat{H}$ in $\mathcal{L}_{Higgs}$ one obtains

$$\mathcal{L}_m = \frac{v^2}{8}(-g^2 W_{\mu a}W^{\mu a}+2gg' B_\mu W^{3\mu}-g'^2 B_\mu B^\mu)$$

$$= -\frac{1}{4}g^2 v^2 W_\mu^+ W_\mu^-$$

$$-\frac{1}{4}v^2\left(\begin{array}{cc}W_3^\mu & B^\mu\end{array}\right)\left(\begin{array}{cc}g^2 & -gg'\\ -gg' & g'^2\end{array}\right)\left(\begin{array}{c}W_\mu^3\\ B_\mu\end{array}\right)$$

$$W_\mu^\pm = \frac{W_\mu^1 \pm iW_\mu^2}{\sqrt{2}}$$

- The gauge boson mass spectrum is then

Gauge boson masses and relations

$$m_{W^\pm} = \frac{1}{2} g v; \quad m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v; \quad m_A = 0$$

$$Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu; \quad A_\mu = \cos\theta_W W_\mu^3 + \sin\theta_W B_\mu$$

$$\tan\theta_W = \frac{g'}{g}$$

- The mixing angle can be put in relation with gauge boson masses as

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}$$

- The muon decay lifetime determines the relation

$$v^2 = \frac{1}{\sqrt{2} G_\mu} = (246.22\ \mathrm{GeV})^2$$

Higgs and Goldstone bosons

- We can parametric the Higgs field as

$$H(x) = e^{i\chi_a(x)\sigma^a/v} \left(\begin{array}{c} 0 \\ \frac{1}{\sqrt{2}}(v+h(x)) \end{array} \right)$$

- The unitary gauge is defined as $(\chi^a \to 0)$

$$H(x) \to e^{-i\chi_a(x)\sigma^a/v}\, H(x) = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ v+h(x) \end{array} \right)$$

- In the unitary gauge the Goldston bosons decouple
- In the unitary gauge the gauge boson propagators are

$$\Delta^{\mu\nu}_{VV}(q) = \frac{-i}{q^2 - m_V^2 + i\epsilon}\left[g^{\mu\nu} - \frac{q^\mu q^\nu}{m_V^2}\right]$$

- It is more convenient to work in R_ξ gauge characterized by the GF lagrangian

$$\mathcal{L}_{\rm GF} = \frac{-1}{2\xi}\left[2(\partial^\mu W_\mu^+ - \xi m_W \chi^+)(\partial^\mu W_\mu^- - \xi m_W \chi^-) \right.$$

$$\left. + (\partial^\mu Z_\mu - \xi m_Z \chi^0)^2 + (\partial^\mu A_\mu)^2\right]$$

- The propagators in R_ξ gauge

R_ξ gauge

$$\Delta_{VV}^{\mu\nu}(q) = \frac{-i}{q^2 - m_V^2 + i\epsilon}\left[g^{\mu\nu} + (\xi - 1)\frac{q^\mu q^\nu}{q^2 - \xi m_V^2}\right]$$

$$\Delta_{\chi^0\chi^0}(q^2) = \frac{i}{q^2 - \xi m_Z^2 + i\epsilon}$$

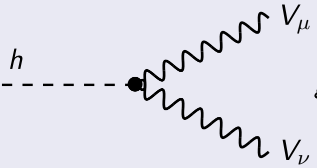
$$\Delta_{\chi^\pm\chi^\mp}(q^2) = \frac{i}{q^2 - \xi m_W^2 + i\epsilon}$$

- $\xi = 0$ is the Landau gauge
- $\xi = 1$ is the 't Hooft-Feynman gauge (the $q^\mu q^\nu$ term is absent
- $\xi \to \infty$ is the Unitary gauge.
- In gauge boson propagators the last term $(-q^\mu q^\nu / m_V^2)$ leads to very complicated cancellations in the invariant amplitudes involving the exchange of V bosons at high energies and, even worse, make the renormalization program very difficult to carry out, as the latter usually makes use of four–momentum power counting analyses of the loop diagrams.
- The Goldstone boson propagators vanish in the unitary gauge
- The Higgs propagator

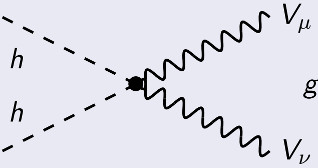
$$\Delta_{hh}(q^2) = \frac{i}{q^2 - m_h^2 + i\epsilon}$$

- The couplings of the Higgs bosons to gauge bosons

Higgs-gauge bosons



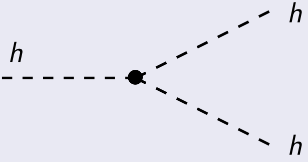
$$g_{hVV} = -i g_{\mu\nu}\, 2m_V^2/v$$



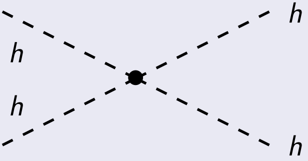
$$g_{hhVV} = -i g_{\mu\nu}\, 2m_V^2/v^2$$

- The self-couplings of the Higgs bosons

Higgs-Higgs bosons



$$g_{hhh} = i\ 3m_h^2/v$$



$$g_{hhhh} = i\ 3m_h^2/v^2$$

Fermion gauge interactions

Using the lagrangian $\mathcal{L}_{fer}$ one obtains the interaction of fermions with gauge boson eigenvectors in the broken phase

- The weak isospin current of $SU(2)$ is

$$J_a^\mu = \sum_{f_L} \bar{f}_L \gamma^\mu \frac{\sigma_a}{2} f_L$$

- The hypercharge current is

$$J_Y^\mu = \sum_{f_L} \bar{f}_L \gamma^\mu Y_{f_L} f_L + \sum_{f_R} \bar{f}_R \gamma^\mu Y_{f_R} f_R$$

- They are coupled to gauge bosons (W, Z, A) as

$$g J_a^\mu W_a^\mu + g' J_Y^\mu B_\mu$$

with the decomposition

$$W_\mu^3 = \cos\theta_W Z_\mu + \sin\theta_W A_\mu;$$

$$B_\mu = -\sin\theta_W Z_\mu + \cos\theta_W A_\mu$$

- $W_\mu^\pm$ couple to the weak charged currents

Charged currents lagrangian

$$\mathcal{L}_{int}^{CC} = \frac{g}{\sqrt{2}}(W_\mu^+ J_-^\mu + W_\mu^- J_+^\mu)$$

$$J_\pm^\mu = \frac{1}{2}(J_1^\mu \mp i J_2^\mu)$$

- The electromagnetic interactions are

Electromagnetic lagrangian

$$\mathcal{L}_{int}^{EM} = e J_\mu^{EM} A^\mu$$

$$J_\mu^{EM} = \sum_f [\bar{f}_L \gamma_\mu Q f_L + \bar{f}_R \gamma_\mu Q f_R]$$

$$Q = T_3 + Y; \quad e = \frac{g g'}{\sqrt{g^2 + g'^2}}$$

- Z_μ couples to neutral current

Neutral current lagrangian

$$\mathcal{L}_{int}^{NC} = \sqrt{g^2 + g'^2} J_\mu^0 Z^\mu$$

$$J_\mu^0 = J_\mu^3 - \sin^2\theta_W J_\mu^{EM}$$

- Notice that the neutral currents

Neutral currents

$$\propto \bar{f}_{L,R}\gamma^\mu f_{L,R}$$

and charged currents

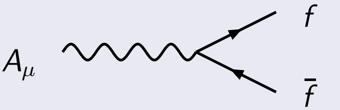
Charged currents

$$\propto \bar{u}_{L,R}\gamma^\mu d_{L,R}$$

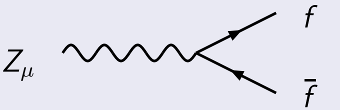
are all flavor-diagonal in the interaction basis

Diagrammatically the Feynman rules are

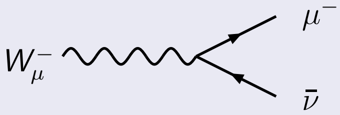
Fermion gauge interactions



$$ieQ_f\gamma_\mu$$



$$\frac{ie}{sc}\gamma_\mu\left[(T_f^3 - Q_f s^2)P_L - Q_f s^2 P_R\right]$$



$$\frac{ie}{s\sqrt{2}}\gamma_\mu P_L$$

Yukawa interactions

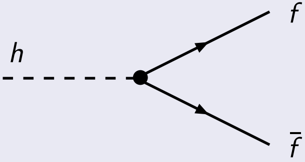
- Fermion masses and mixing appear from the Yukawa interactions

Quarks Yukawa lagrangian

$$\mathcal{L}_Y = -Y_{ij}^U (\bar{u}_L, \bar{d}_L)_i \begin{pmatrix} \bar{H}^0 \\ -\chi^- \end{pmatrix} u_{Rj}$$

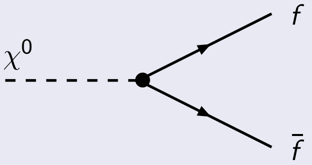
$$-Y_{ij}^D (\bar{u}_L, \bar{d}_L)_i \begin{pmatrix} \chi^+ \\ H^0 \end{pmatrix} d_{Rj} + h.c.$$

Higgs fermion interactions

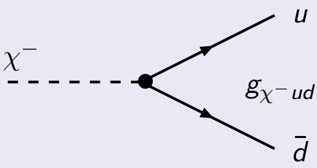


$$g_{Hff} = i\ m_f / v$$

Goldstone bosons fermion interactions



$$g_{\chi^0 ff} = -2T_f^3 m_f/v$$



$$g_{\chi^- ud} = -\frac{i\sqrt{2}}{v} V_{ud}[m_d P_L - m_u P_R]$$

- After electroweak breaking it gives rise to the mass terms

Mass lagrangian

$$\mathcal{L}_{mass} = -\frac{v}{\sqrt{2}}\bar{u}_L^i Y_{ij}^U u_R^j + h.c.$$

$$-\frac{v}{\sqrt{2}}\bar{d}_L^i Y_{ij}^D d_R^j + h.c.$$

- We can diagonalize the bilinear mass terms by unitary transformations

$$u_{L,R} \to V_{L,R}^u u_{L,R}; \quad d_{L,R} \to V_{L,R}^d d_{L,R}$$

interaction $\to$ mass eigenstates basis

- The mass Lagrangian becomes

Mass Lagrangian

$$\mathcal{L}_{mass} \to -\frac{v}{\sqrt{2}}\bar{u}_L V_L^{u\dagger} Y^U V_R^u u_R + h.c.$$
$$-\frac{v}{\sqrt{2}}\bar{d}_L V_L^{d\dagger} Y^D V_R^d d_R + h.c.$$

- With

$$V_L^{u\dagger} Y^U V_R^u \propto diag(m_u, m_c, m_t)$$

$$V_L^{d\dagger} Y^D V_R^d \propto diag(m_d, m_s, m_b)$$

- Where now the states $u_{L,R}, d_{L,R}$ are mass eigenstates

Neutral currents in mass eigenbasis

- Neutral currents which were flavor-diagonal in the interaction basis remain flavor-diagonal in the mass eigenstate basis

Neutral currents in mass eigenstates

$$\bar{f}_{L,R}\gamma^\mu f_{L,R} \to \bar{f}_{L,R} V^{f\dagger}_{L,R}\gamma^\mu V^f_{L,R} f_{L,R} = \bar{f}_{L,R}\gamma^\mu f_{L,R}$$

- This ensures that

FCNC will not be generated at tree level

- In agreement with experimental data

Charged currents in mass eigenbasis: CKM mixing

- Charged currents which were flavor-diagonal in the interaction basis do not remain flavor diagonal in the mass eigenstate basis

Charged currents in mass eigenstates

$$W_\mu^+ \bar{u}_L \gamma^\mu d_L \to W_\mu^+ \bar{u}_L \gamma^\mu V_L^{u\dagger} V_L^d d_L = W_\mu^+ \bar{u}_L \gamma^\mu V_{CKM} d_L$$

$$V_{CKM} = V_L^{u\dagger} V_L^d$$

- V_{CKM} is the Cabbibo-Kobayashi-Maskawa matrix defined as

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- A standard parametrization for the CKM matrix is

$$V_{CKM} =$$

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

- A good approximation is

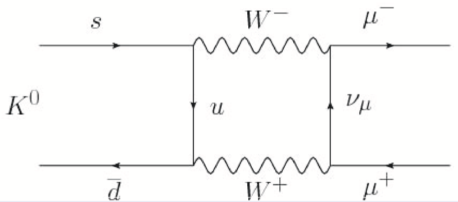
$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Where $\lambda = s_{12}$, $s_{23} = A\lambda^2$, $s_{13}e^{i\delta} = A\lambda^3(\rho + i\eta)$
- $\lambda \simeq \sin\theta_C = 0.23$
- The experimental values for the V_{CKM} entries can be found in RPP

The GIM mechanism

- The GIM mechanism explains the smallness of processes as $K_L \to \mu^+\mu^-$ as given by the diagrams in the figure

GIM mechanism



- CKM mixing ($V_{ud}^* V_{us}$) leads to the three diagrams where the vertical line is (u, c, t).
- In the limit of exact flavor symmetry the three diagrams cancel by virtue of unitarity

$$\sum_{i=u,c,t} V_{is} V_{id}^* = 0$$

- Exercise: Estimate the suppression of previous process