Structure of the Vacuum in Light-Front Dynamics and Emergent Holographic QCD

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Vacuum and Symmetry Breaking:

From the Quantum to the Cosmos

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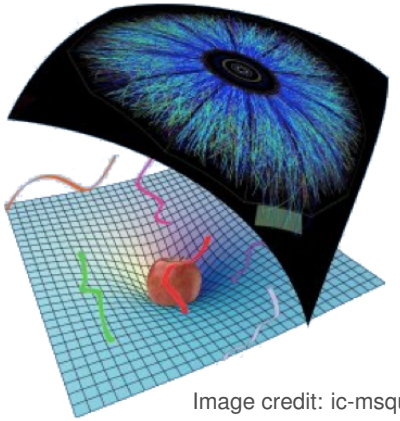

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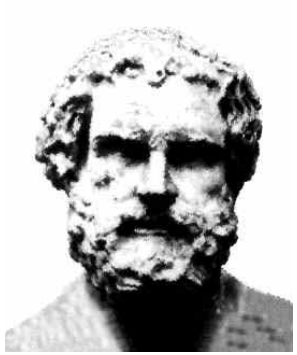
In collaboration with Stan Brodsky (SLAC) and Hans G. Dosch (Heidelberg)



$\Delta\eta\mu o\kappa\rho\iota\tau o\sigma$
460 - 370 BC

$\nu o\mu\omega\ \psi\upsilon\chi\rho o\nu,\ \nu o\mu\omega\ \theta\epsilon\rho\mu o\nu,\ \epsilon\tau\epsilon\eta\ \delta\epsilon\ \alpha\tau o\mu\alpha\ \kappa\alpha\iota\ \kappa\epsilon\nu o\nu$

(By convention cold, by convention hot, in reality atoms and the void)

Democritus 460 - 370 BC

Contents

1 Introduction

- SLAC and DESY 70's: elementary interactions of quarks and gluons at short distances remarkably well described by QCD
- No understanding of large distance strong dynamics of QCD: how quarks and gluons are confined and how hadrons emerge as asymptotic states
- Euclidean lattice important first-principles numerical simulation of nonperturbative QCD: excitation spectrum of hadrons requires enormous computational complexity beyond ground-state configurations
- Only known analytically tractable treatment in relativistic QFTh is perturbation theory: description of confinement from perturbation theory is not possible (Kinoshita-Lee-Nauenberg Theorem)
- Important theoretical goal: find initial analytic approximation to strongly coupled QCD, like Schrödinger or Dirac Eqs in atomic physics, corrected for quantum fluctuations
- Convenient frame-independent Hamiltonian framework for treating bound-states in relativistic theories is light-front (LF) quantization

- Definition of invariant LF vacuum within causal horizon $P^2|0\rangle = 0$
- Simple structure of vacuum allows definition of partonic content of hadron in terms of wavefunctions: quantum-mechanical probabilistic interpretation of hadronic states
- Calculation of matrix elements $\langle P+q|J|P\rangle$ requires boosting the hadronic bound state from $|P\rangle$ to $|P+q\rangle$: no change in particle number from boost in LF (kinematical problem)
- To a first semiclassical approximation one can reduce the strongly correlated multi-parton LF dynamics to an effective one-dim QFTh, which encodes the conformal symmetry of the classical QCD Lagrangian
- Uniqueness of the form of confining interaction from one dim conformal QFTh: extension of conformal quantum mechanics to LF bound-state dynamics
- Invariant Hamiltonian equation for bound states similar structure of AdS equations of motion: direct connection of QCD and AdS/CFT possible
- Description of strongly coupled gauge theory in physical space-time using a dual gravity description in a higher dimensional space (holographic)

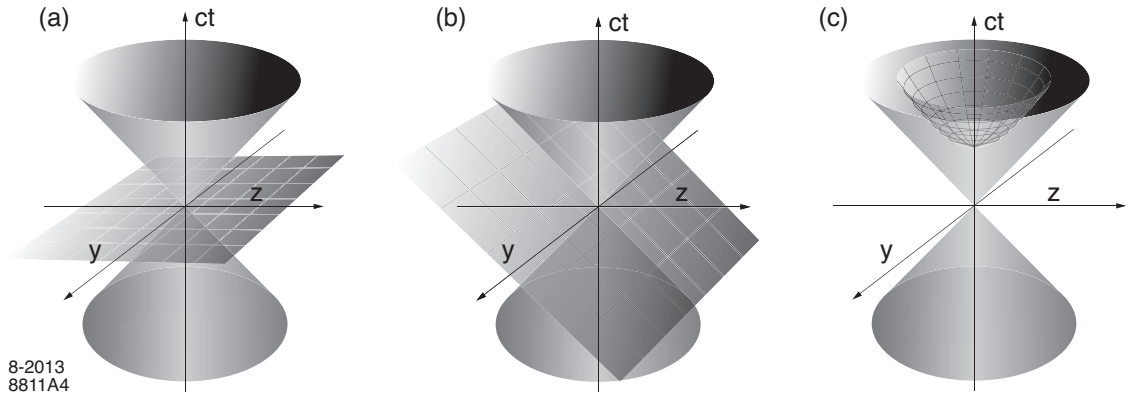
2 Dirac Forms of Relativistic Dynamics

[Dirac (1949)]

- The Poincaré group is the full symmetry group of any form of relativistic dynamics

$$
\begin{aligned}
[P^\mu, P^\nu] &= 0, \\
[M^{\mu\nu}, P^\rho] &= i\left(g^{\mu\rho}P^\nu - g^{\nu\rho}P^\mu\right), \\
[M^{\mu\nu}, M^{\rho\sigma}] &= i\left(g^{\mu\rho}M^{\nu\sigma} - g^{\mu\sigma}M^{\nu\rho} + g^{\nu\sigma}M^{\mu\rho} - g^{\nu\rho}M^{\mu\sigma}\right),
\end{aligned}
$$

- Poincaré generators separated into kinematical and dynamical
- Kinematical generators act along the initial hypersurface where initial conditions are imposed and contain no interactions (leave invariant initial surface)
- Dynamical generators are responsible for evolution of the system and depend on the interactions (map initial surface into another surface)
- Each form has its Hamiltonian and evolve with different time, but results computed in any form should be identical (different parameterizations of space-time)



- *Instant-form*: initial surface defined by $x^0 = 0$

$$P^0,\ \mathbf{K}\ \text{dynamical}, \quad \mathbf{P},\ \mathbf{J}\ \text{kinematical}$$

- *Front-form*: initial surface tangent to the light cone $x^+ = x^0 + x^3 = 0 \quad (\ P^\pm = P^0 \pm P^3)$

$$P^-,\ J^x,\ J^y\ \text{dynamical}, \quad P^+,\ \mathbf{P}_\perp,\ J^3,\ \mathbf{K}\ \text{kinematical}$$

- *Point-form*: initial surface is the hyperboloid $x^2 = \kappa^2 > 0,\ x^0 > 0$

$$P^\mu\ \text{dynamical}, \quad M^{\mu\nu}\ \text{kinematical}$$

3 Light-Front Dynamics and Light-Front Vacuum

- Hadron with 4-momentum $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, mass-shell relation $P^2 = M^2$ leads to dispersion relation for LF Hamiltonian P^-

$$P^- = \frac{\mathbf{P}_\perp^2 + M^2}{P^+}, \qquad P^+ > 0$$

- n-particle bound state with $p_i^2 = m_i^2$, $p_i = \left(p_i^+, p_i^-, \mathbf{p}_{\perp i}\right)$, for each constituent i

$$p_i^- = \frac{\mathbf{p}_{\perp i}^2 + m_i^2}{p_i^+}, \qquad p_i^+ > 0$$

- Longitudinal momentum P^+ is kinematical: sum of single particle constituents p_i^+ of bound state

$$P^+ = \sum_i p_i^+, \qquad p_i^+ > 0$$

- LF Hamiltonian P^- is dynamical: bound-state is arbitrarly off the LF energy shell

$$P^- - \sum_i^n p_i^- < 0$$

- LF evolution given by relativistic Schrödinger-like equation ($x^+ = x^0 + x^3$ LF time)

$$i\frac{\partial}{\partial x^+}|\psi(P)\rangle = P^-|\psi(P)\rangle$$

where

$$P^-|\psi(P)\rangle = \frac{\mathbf{P}_\perp^2 + M^2}{P^+}|\psi(P)\rangle$$

- Construct LF invariant Hamiltonian $P^2 = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$

$$\boxed{P^2|\psi(P)\rangle = M^2|\psi(P)\rangle}$$

- Since P^+ and $\mathbf{P}_\perp$ are kinematical, linear QM evolution in LF Hamiltonian P^- is maintained
- State $|\psi(P)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n|n\rangle, \qquad |n\rangle = \{\,|uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle,\ \cdots\}$$

with $p_i^2 = m_i^2$, $p_i = (p_i^+, p_i^-, \mathbf{p}_{\perp i})$, for each constituent i in state n

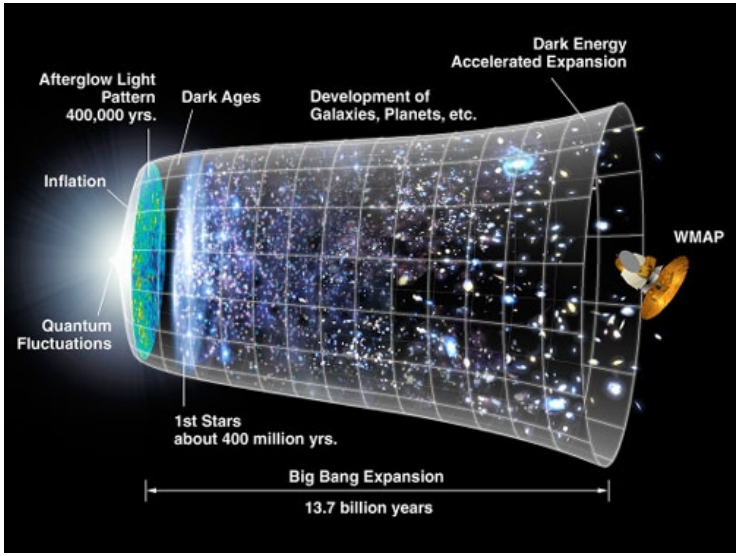
- <u>Fock component</u> ψ_n is the <u>light-front wave function</u> (LFWF)

Vaccum in the Front-Form of Dynamics

- $P^+ = \sum_i p_i^+,\ p_i^+ > 0$: LF vacuum is the state with $P^+ = 0$ and contains no particles: all other states have $P^+ > 0$ (usual vacuum bubbles are kinematically forbidden in the front form !)
- Frame independent definition of the vacuum within the causal horizon

$$\boxed{P^2|0\rangle = 0}$$

  (LF vacuum also has zero quantum numbers and $P^+ = 0$)
- LF vacuum is defined at fixed LF time $x^+ = x^0 + x^3$ over all $x^- = x^0 - x^3$ and $\mathbf{x}_\perp$, the expanse of space that can be observed within the speed of light
- Causality is maintained since LF vacuum only requires information within the causal horizon
- The front form is a natural basis for cosmology: universe observed along the front of a light wave



Vacuum in the Instant-Form of Dynamics

- Unsolved problems concerning the instant vacuum structure of QCD, confinement and chiral symmetry breaking
- In Lattice QCD the structure of the instant-form vacuum is sampled in the Euclidean region
- If monopoles or vortices are removed from the vacuum confinement and chiral symmetry breaking disappear
- Instantons provide a mechanism for symmetry breaking through the Banks-Casher relation

$$\langle 0|\overline{\psi}\psi|0\rangle = -\pi\rho(0),$$

  where $\rho(0)$ is the density or Dirac-zero modes $\not{D}\psi_\lambda = \lambda\psi_\lambda$, but do not lead to confinement
- No convincing proof that continuum QCD is confining in the infrared
- Confinement in continuum QCD is a fundamental unsolved problem in theoretical physics

4 Light-Front Quantization of QCD

- Start with $SU(3)_C$ QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu}$$

- Express the hadron four-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields $\psi_+ = \Lambda_\pm\psi$ and $\mathbf{A}_\perp$ $\left(\Lambda_\pm = \gamma^0\gamma^\pm\right)$ quantized in null plane $x^+ = x^0 + x^3 = 0$

$$\begin{aligned}
P^- &= \tfrac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+\,\gamma^+\frac{(i\nabla_\perp)^2 + m^2}{i\partial^+}\psi_+ + \text{interactions}\\
P^+ &= \int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+\gamma^+ i\partial^+\psi_+\\
\mathbf{P}_\perp &= \tfrac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+\gamma^+ i\nabla_\perp\psi_+
\end{aligned}$$

where the integrals are over initial surface $x^+ = 0$, where commutation relation for fields are defined

- Dirac field ψ_+ expanded in terms of ladder operators $\quad b(q)|0\rangle = d(q)|0\rangle = 0$

$$\psi_+(x^-, \mathbf{x}_\perp)_\alpha = \sum_\lambda \int_{q^+>0} \frac{dq^+}{\sqrt{2q^+}} \frac{d^2\mathbf{q}_\perp}{(2\pi)^3} \left[b_\lambda(q) u_\alpha(q,\lambda) e^{-iq\cdot x} + d_\lambda(q)^\dagger v_\alpha(q,\lambda) e^{iq\cdot x} \right]$$

with

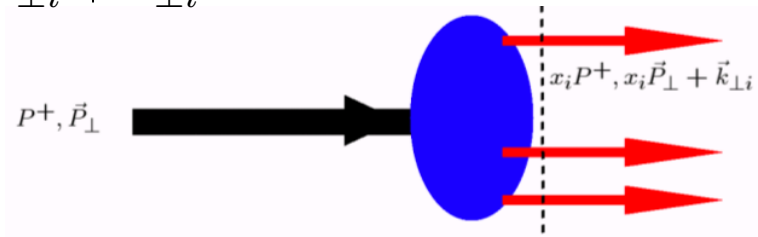
$$\left\{ b(q), b^\dagger(q') \right\} = \left\{ d(q), d^\dagger(q') \right\} = (2\pi)^3\, \delta(q^+ - q'^+)\, \delta^{(2)}\big(\mathbf{q}_\perp - \mathbf{q}'_\perp\big)$$

$$\begin{aligned}
P^- &= \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3} \left(\frac{\mathbf{q}_\perp^2 + m^2}{q^+} \right) b_\lambda^\dagger(q) b_\lambda(q) + \text{interactions} \\
P^+ &= \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3}\, q^+\, b_\lambda^\dagger(q) b_\lambda(q) \\
\mathbf{P}_\perp &= \sum_\lambda \int \frac{dq^+ d^2\mathbf{q}_\perp}{(2\pi)^3}\, \mathbf{q}_\perp\, b_\lambda^\dagger(q) b_\lambda(q)
\end{aligned}$$

where $q^- = \frac{\mathbf{q}_\perp^2 + m^2}{q^+}$

- Relative partonic coordinates: $k_i^+ = x_i P^+, \quad \mathbf{p}_{\perp i} = x_i \mathbf{P}_{\perp i} + \mathbf{k}_{\perp i}$

$$\sum_{i=1}^{n} x_i = 1, \quad \sum_{i=1}^{n} \mathbf{k}_{\perp i} = 0$$



- Fock component $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ depends only on relative coordinates: longitudinal momentum fraction $x_i = k_i^+ / P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin λ_i^z

- LFWF ψ_n is <u>frame independent</u>

- Momentum conservation: $P^+ = \sum_{i=1}^n k_i^+, \;\; k_i^+ > 0$, but LFWF represents a bound-state which is off the LF energy shell, $P^- - \sum_i^n k_i^- < 0$

- Express off-shell dependence of bound-state in terms of invariant variable (invariant mass of constituents)

$$M_n^2 = (k_1 + k_2 + \cdots + k_n)^2 = \Big(\sum_{i=1}^{n} k_i^\mu \Big)^2 = \sum_i \frac{\mathbf{k}_{\perp i}^2 + m_i^2}{x_i}$$

- $M^2 - M_n^2$ is the measure of the off-energy shell: key variable which controls the bound state

5 Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Effective reduction of strongly coupled LF multiparticle dynamics to a 1-dim QFT with no particle creation and absorption: LF quantum mechanics !
- Central problem is derivation of effective interaction which acts only on the valence sector: express higher Fock states as functionals of the lower ones
- Advantage: Fock space not truncated and symmetries of the Lagrangian preserved
  [H. C. Pauli, EPJ, C7, 289 (1999)
- Compute M^2 from hadronic matrix element $\langle\psi(P')|P_\mu P^\mu|\psi(P)\rangle = M^2\langle\psi(P')|\psi(P)\rangle$
- Find

$$M^2 = \sum_n \int [dx_i]\,[d^2\mathbf{k}_{\perp i}] \sum_q \left(\frac{\mathbf{k}_{\perp q}^2 + m_q^2}{x_q}\right) |\psi_n(x_i, \mathbf{k}_{\perp i})|^2 + \text{interactions}$$

with phase space normalization

$$\sum_n \int [dx_i]\,[d^2\mathbf{k}_{\perp i}]\,|\psi_n(x_i, \mathbf{k}_{\perp i})|^2 = 1$$

- In terms of $n-1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$,

$$M^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi_n^*(x_i, \mathbf{b}_{\perp i}) \sum_q \left(\frac{-\nabla^2_{\mathbf{b}_{\perp q}} + m_q^2}{x_q} \right) \psi_n(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

with normalization

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \left|\psi_n(x_j, \mathbf{b}_{\perp j})\right|^2 = 1.$$

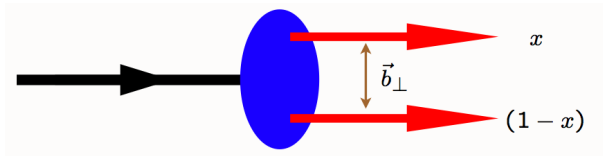
- Semiclassical approximation

$$\psi_n(k_1, k_2, \ldots, k_n) \to \phi_n\Big(\underbrace{(k_1 + k_2 + \cdots + k_n)^2}_{M_n^2 = \sum_i \frac{\mathbf{k}_{\perp i}^2 + m_i^2}{x_i}} \Big), \quad m_q \to 0$$

- For a two-parton system in the $m_q \to 0$ $\quad M^2_{q\bar{q}} = \frac{\mathbf{k}_\perp^2}{x(1-x)}$

- Conjugate invariant variable in transverse impact space is

$$\boxed{\zeta^2 = x(1-x)\mathbf{b}_\perp^2}$$



- To first approximation LF dynamics depend only on the invariant variable ζ, and dynamical properties are encoded in the hadronic mode $\phi(\zeta)$

$$\psi(x,\zeta,\varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}},$$

where we factor out the longitudinal $X(x)$ and orbital kinematical dependence from LFWF ψ

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple $\quad (L = L^z)$

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta)\, U(\zeta)\, \phi(\zeta),$$

where effective potential U includes all interaction terms upon integration of the higher Fock states

- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = M^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\boxed{\Big(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \Big) \phi(\zeta) = M^2 \phi(\zeta)}$$

- Relativistic and frame-independent LF Schrödinger equation: U is instantaneous in LF time

- Critical value $L = 0$ corresponds to lowest possible stable solution, the ground state of the LF invariant Hamiltonian P^2

- Compare invariant mass in the instant-form in the hadron center-of-mass system $\mathbf{P} = 0$,

$$M_{q\overline{q}}^2 = 4\, m_q^2 + 4\mathbf{p}^2$$

with the invariant mass in the front-form in the constituent rest frame, $\mathbf{k}_q + \mathbf{k}_{\overline{q}} = 0$

$$M_{q\overline{q}}^2 = \frac{\mathbf{k}_\perp^2 + m_q^2}{x(1-x)}$$

obtain

$$U = V^2 + 2\sqrt{\mathbf{p}^2 + m_q^2}\, V + 2\, V \sqrt{\mathbf{p}^2 + m_q^2}$$

where $\mathbf{p}_\perp^2 = \frac{\mathbf{k}_\perp^2}{4x(1-x)}$, $p_3 = \frac{m_q(x-1/2)}{\sqrt{x(1-x)}}$, and V is the effective potential in the instant-form

- For small quark masses a linear instant-form potential V implies a harmonic front-form potential U and thus linear Regge trajectories

  [A. P. Trawiński, S. D. Glazek, S. J. Brodsky, GdT, H. G. Dosch, arXiv: 1403.5651]

6 Effective Confinement from Underlying Conformal Invariance

[S. J. Brodsky, GdT and H.G. Dosch, PLB **729**, 3 (2014)]

- Incorporate in a 1-dim QFT – as an effective theory, the fundamental conformal symmetry of the 4-dim classical QCD Lagrangian in the limit of massless quarks

- Invariance properties of 1-dim field theory under the full conformal group from dAFF action [V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A **34**, 569 (1976)]

$$S = \frac{1}{2}\int dt\Big(\dot{Q}^2 - \frac{g}{Q^2}\Big)$$

where g is a dimensionless number (Casimir operator which depends on the representation)

- The equation of motion

$$\ddot{Q} - \frac{g}{Q^3} = 0$$

and the generator of evolution in t, the Hamiltonian

$$H = \frac{1}{2}\Big(\dot{Q}^2 + \frac{g}{Q^2}\Big)$$

follow from the dAFF action

- Absence of dimensional constants implies that the action

$$S = \tfrac{1}{2}\int dt\Big(\dot{Q}^2 - \frac{g}{Q^2}\Big)$$

is invariant under a larger group of transformations, the general conformal group

$$t' = \frac{\alpha t + \beta}{\gamma t + \delta}, \qquad Q'(t') = \frac{Q(t)}{\gamma t + \delta}$$

with $\alpha\delta - \beta\gamma = 1$

- Applying Noether's theorem one obtains 3 conserved operators

I. Translations in t: $H = \frac{1}{2}\left(\dot{Q}^2 + \frac{g}{Q^2}\right)$,

II. Dilatations: $D = \frac{1}{2}\left(\dot{Q}^2 + \frac{g}{Q^2}\right)t - \frac{1}{4}\left(\dot{Q}Q + Q\dot{Q}\right)$,

III. Special conformal transformations: $K = \frac{1}{2}\left(\dot{Q}^2 + \frac{g}{Q^2}\right)t^2 - \frac{1}{2}\left(\dot{Q}Q + Q\dot{Q}\right)t + \frac{1}{2}Q^2$,

- Using canonical commutation relations $[Q(t), \dot{Q}(t)] = i$ find

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK,$$

the algebra of the generators of the conformal group $Conf(R^1)$

- Introduce the linear combinations

$$J^{12} = \frac{1}{2}\Big(\frac{1}{a}K + aH\Big), \;\; J^{01} = \frac{1}{2}\Big(\frac{1}{a}K - aH\Big), \;\; J^{02} = D,$$

where a has dimension t since H and K have different dimensions

- Generators J have commutation relations

$$[J^{12}, J^{01}] = iJ^{02}, \quad [J^{12}, J^{02}] = -iJ^{01}, \quad [J^{01}, J^{02}] = -iJ^{12},$$

the algebra of $SO(2,1)$

- $J^{0i}, \; i = 1, 2$, boost in space direction i and J^{12} rotation in the (1,2) plane
- J^{12} is compact and has thus discrete spectrum with normalizable eigenfunctions and a ground state
- The relation between the generators of conformal group and generators of $SO(2,1)$ suggests that the scale a may play a fundamental role

- dAFF construct a new generator as a superposition of the 3 constants of motion

$$G = uH + vD + wK$$

and introduce new time variable τ and field operator $q(\tau)$

$$d\tau = \frac{dt}{u + vt + wt^2}, \qquad q(\tau) = \frac{Q(t)}{[u + vt + wt^2]^{\frac{1}{2}}}$$

- Find usual quantum mechanical evolution for time τ

$$G|\psi(\tau)\rangle = i\frac{d}{d\tau}|\psi(\tau)\rangle$$

$$i\,[G, q(\tau)] = \frac{dq(\tau)}{d\tau}$$

and usual equal-time quantization $[q(t), \dot{q}(t)] = i$

- In terms of τ and $q(\tau)$

$$\begin{aligned} S &= \tfrac{1}{2}\int dt\Big(\dot{Q}^2 - \frac{g}{Q^2}\Big) \\ &= \tfrac{1}{2}\int d\tau\Big(\dot{q}^2 - \frac{g}{q^2} - \frac{4u\omega - v^2}{4}q^2\Big) + \text{surface term} \end{aligned}$$

  Action is conformal invariant invariant up to a surface term !

- The corresponding Hamiltonian

$$G = \tfrac{1}{2}\Big(\dot{q}^2 + \frac{g}{q^2} + \frac{4u\omega - v^2}{4}q^2\Big)$$

  is a compact operator for

$$\frac{4u\omega - v^2}{4} > 0$$

- Scale appears in the Hamiltonian without affecting the conformal invariance of the action !

- Evolution in τ in terms of original field operator $Q(t)$ at $t = 0$

$$\begin{aligned} G(Q, \dot{Q}) &= \frac{1}{2} u \left(\dot{Q}^2 + \frac{g}{Q^2} \right) - \frac{1}{4} v \left(Q\dot{Q} + \dot{Q}Q \right) + \frac{1}{2} w Q^2 \\ &= uH + vD + wK, \end{aligned}$$

- The Schrödinger picture follows from the representation of Q and $P = \dot{Q}$

$$Q \to x, \;\; \dot{Q} \to -i \frac{d}{dx}$$

- dAFF Conformal Quantum Mechanics !

- Schrödinger wave equation determines evolution of bound states in terms of the variable τ

$$i \frac{\partial}{\partial \tau} \psi(x, \tau) = H_\tau \left(x, -i \frac{d}{dx} \right) \psi(x, \tau)$$

- dAFF Hamiltonian G

$$G = \frac{1}{2} u \left(-\frac{d^2}{dx^2} + \frac{g}{x^2} \right) + \frac{i}{4} v \left(x \frac{d}{dx} + \frac{d}{dx} x \right) + \frac{1}{2} w x^2$$

- Compare dAFF Hamiltonian

$$G = \frac{1}{2}u\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right) + \frac{i}{4}v\left(x\,\frac{d}{dx} + \frac{d}{dx}\,x\right) + \frac{1}{2}wx^2$$

with LF Hamiltonian for $u = 2, \quad v = 0,$

$$P^2 = -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)$$

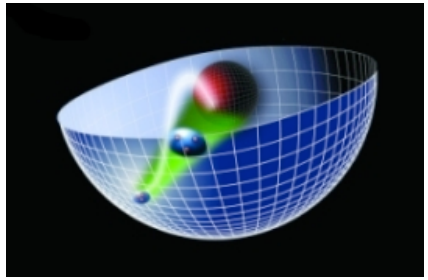
- x is identified with the LF variable ζ: $\quad x = \zeta$
- Casimir g with the LF orbital angular momentum L: $g = L^2 - \frac{1}{4}$
- $w = 2\lambda^2 \sim \frac{1}{a^2}$ fixes effective LF confining interaction to quadratic $\lambda^2\,\zeta^2$ dependence

$$U \sim \lambda^2\zeta^2$$

- Also, dAFF evolution variable τ is identified with LF time $\tau = x^+/P^+$

7 Gravity in AdS and Light Front Holographic Mapping

$$\mathcal{R}_{NKLM} = -\frac{1}{R^2}\left(g_{NL}g_{KM} - g_{NM}g_{KL}\right)$$



- Why is AdS space important?

  AdS$_5$ is space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$ with the group of isometries of AdS$_5$ Dim isometry group of AdS$_{d+1}$: $\frac{(d+1)(d+2)}{2}$

- AdS$_5$ metric $x^M = (x^\mu, z)$:

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$$

- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable z acts like a scaling variable in Minkowski space

- Short distances $x_\mu x^\mu \to 0$ map to UV conformal AdS$_5$ boundary $z \to 0$

- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ map to IR region of AdS$_5$, $z \sim 1/\Lambda_{\rm QCD}$, thus AdS geometry has to be modified at large z to include the scale of strong interactions

8 Higher Integer-Spin Wave Equations in AdS Space

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)
- Integer spin-J fields in AdS conveniently described by tensor field $\Phi_{N_1 \ldots N_J}$ with effective action

$$S_{\mathit{eff}} = \int d^d x \, dz \, \sqrt{|g|} \, e^{\varphi(z)} \, g^{N_1 N_1'} \cdots g^{N_J N_J'} \Big(g^{MM'} D_M \Phi^*_{N_1 \ldots N_J} \, D_{M'} \Phi_{N_1' \ldots N_J'} - \mu^2_{\mathit{eff}}(z) \, \Phi^*_{N_1 \ldots N_J} \, \Phi_{N_1' \ldots N_J'} \Big)$$

D_M is the covariant derivative which includes affine connection and dilaton $\varphi(z)$ breaks conformality

- Effective mass $\mu_{\mathit{eff}}(z)$ is determined by precise mapping to light-front physics
- Non-trivial geometry of pure AdS encodes the kinematics and the additional deformations of AdS encode the dynamics, including confinement

- Physical hadron has plane-wave and polarization indices along $3{+}1$ physical coordinates and a profile wavefunction $\Phi(z)$ along holographic variable z

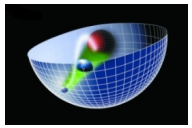
$$\Phi_P(x,z)_{\mu_1\cdots\mu_J} = e^{iP\cdot x}\Phi(z)_{\mu_1\cdots\mu_J}, \quad \Phi_{z\mu_2\cdots\mu_J} = \cdots = \Phi_{\mu_1\mu_2\cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu = M^2$

- Further simplification by using a local Lorentz frame with tangent indices

- Variation of the action gives AdS wave equation for spin-J field $\Phi(z)_{\nu_1\cdots\nu_J} = \Phi_J(z)\epsilon_{\nu_1\cdots\nu_J}$

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right) + \left(\frac{mR}{z}\right)^2\right]\Phi_J = M^2\Phi_J}$$



with

$$(m\,R)^2 = (\mu_{\mathit{eff}}(z)R)^2 - Jz\,\varphi'(z) + J(d-J+1)$$

<u>and</u> the kinematical constraints

$$\eta^{\mu\nu}P_\mu\,\epsilon_{\nu\nu_2\cdots\nu_J} = 0, \quad \eta^{\mu\nu}\,\epsilon_{\mu\nu\nu_3\cdots\nu_J} = 0.$$

- Kinematical constrains in the LF imply that m must be a constant

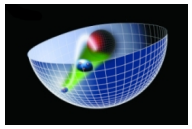
[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

Light-Front Mapping

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Upon substitution $\Phi_J(z) \sim z^{(d-1)/2-J} e^{-\varphi(z)/2}\, \phi_J(z)$ and $z \to \zeta = \sqrt{x(1-x)}b$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z^{d-1-2J}}\partial_z\right)+\left(\frac{mR}{z}\right)^2\right]\Phi_J(z) = M^2\Phi_J(z)$$



we find LFWE $(d=4)$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

with

$$U(\zeta) = \tfrac{1}{2}\varphi''(\zeta)+\frac{1}{4}\varphi'(\zeta)^2+\frac{2J-3}{2z}\varphi'(\zeta)$$

and $(mR)^2 = -(2-J)^2 + L^2$

- Unmodified AdS equations correspond to the kinetic energy terms of the partons inside a hadron
- Interaction terms in the QCD Lagrangian build the effective confining potential $U(\zeta)$ and correspond to the truncation of AdS space in an effective dual gravity approximation
- AdS Breitenlohner-Freedman bound $(mR)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

9 Meson Spectrum

- Dilaton profile in the dual gravity model determined from one-dim QFTh (dAFF)

$$\varphi(z) = \lambda z^2, \qquad \lambda^2 = w/2$$

- Effective potential: $U = \lambda^2\zeta^2 + 2\lambda(J-1)$
- LFWE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

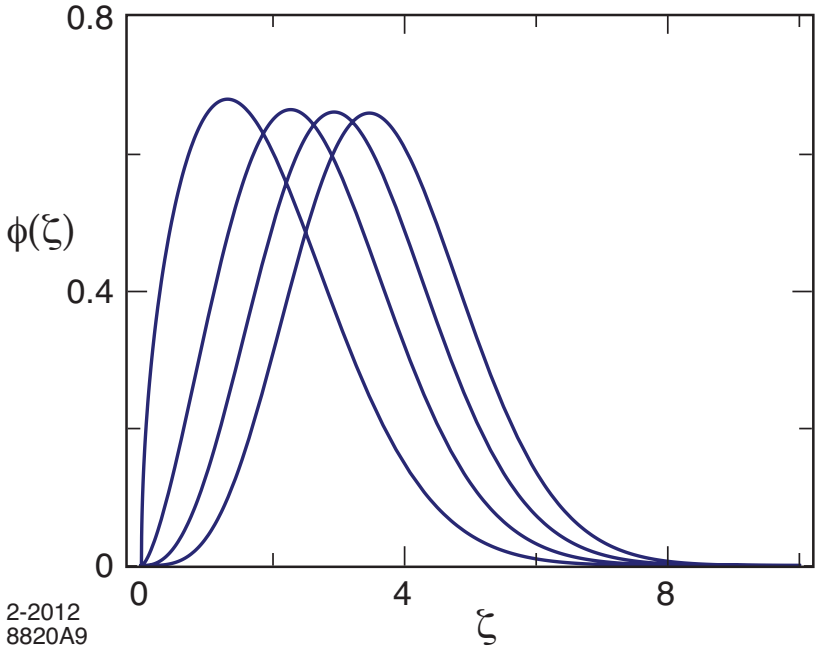
- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z) = 1$

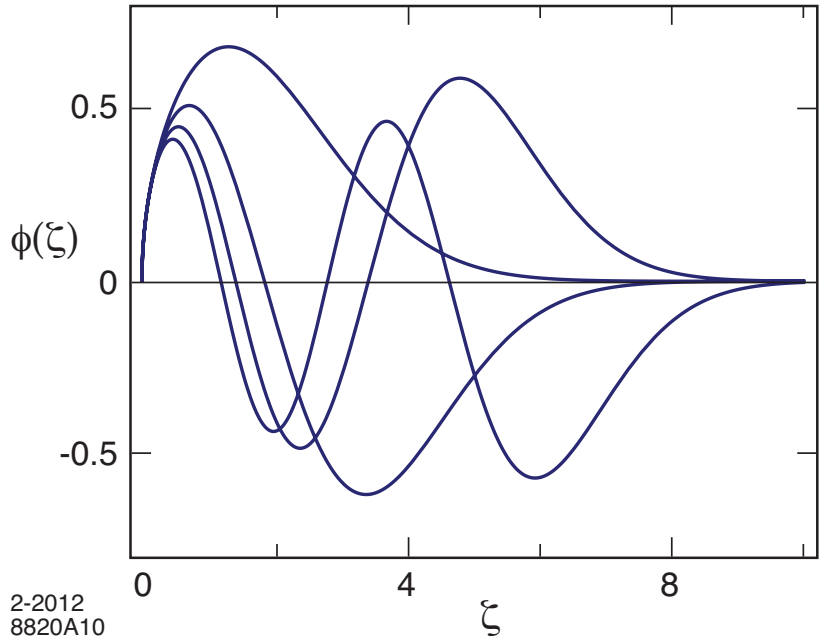
$$\phi_{n,L}(\zeta) = |\lambda|^{(1+L)/2}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-|\lambda|\zeta^2/2}L_n^L(|\lambda|\zeta^2)$$

- Eigenvalues for $\lambda > 0$

$$\mathcal{M}^2_{n,J,L} = 4\lambda\left(n + \frac{J+L}{2}\right)$$

- $\lambda < 0$ incompatible with LF constituent interpretation





LFWFs $\phi_{n,L}(\zeta)$ in physical space-time: (L) orbital modes and (R) radial modes

- Results easily extended to light quarks masses (Ex: K-mesons)

  [GdT, S. J. Brodsky and H. G.Dosch, arXiv:1405.2451 [hep-ph]]

- First order perturbation in the quark masses

$$\Delta M^2 = \langle\psi| \sum_a m_a^2/x_a |\psi\rangle$$

- Holographic LFWF with quark masses

  [S. J. Brodsky and GdT, arXiv:0802.0514 [hep-ph]

$$\psi(x,\zeta) \sim \sqrt{x(1-x)}\, e^{-\frac{1}{2\lambda}\left(\frac{m_q^2}{x}+\frac{m_{\bar{q}}^2}{1-x}\right)} e^{-\frac{1}{2}\lambda\,\zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA

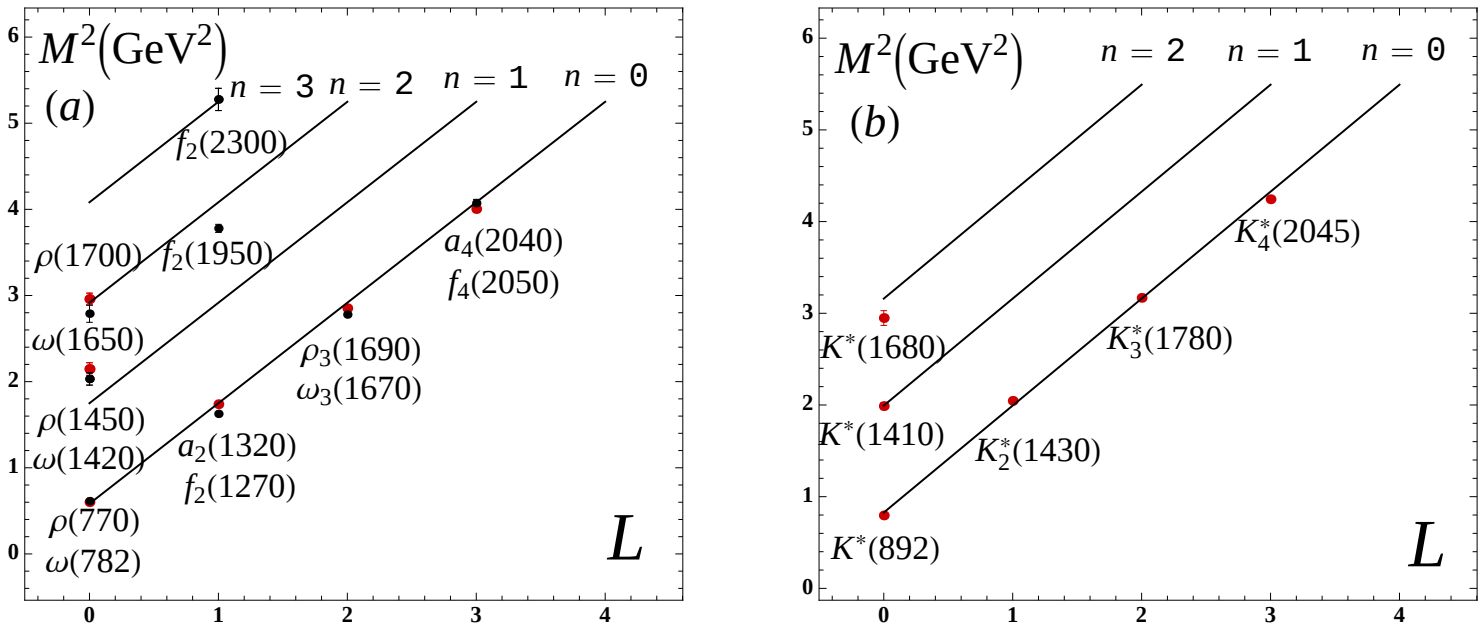
  [J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

- For the K^*

$$M^2_{n,L,S} = M^2_{K^\pm} + 4\lambda\left(n + \frac{J+L}{2}\right)$$

- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

$$m_u = m_d = 46\,\mathrm{MeV}, \quad m_s = 357\,\mathrm{MeV}$$



Orbital and radial excitations for vector mesons for $\sqrt{\lambda} = 0.54$ GeV

- Linear Regge trajectories, a massless pion and relation between the ρ and a_1 mass $M_{a_1}/M_\rho = \sqrt{2}$ usually obtained from Weinberg sum rules described by LF harmonic confinement model

10 Higher Half-Integer Spin Wave Equations in AdS Space

[J. Polchinski and M. J. Strassler, JHEP **0305**, 012 (2003)]

[GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]

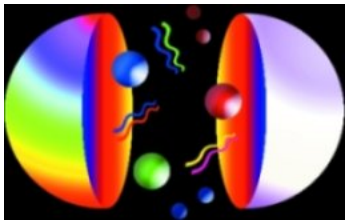


Image credit: N. Evans

- The gauge/gravity duality can give important insights into the strongly coupled dynamics of nucleons using simple analytical methods: analytical exploration of systematics of light-baryon resonances
- Extension of holographic ideas to spin-$\frac{1}{2}$ (and higher half-integral J) hadrons by considering wave equations for Rarita-Schwinger spinor fields in AdS space and their mapping to light-front physics
- LF clustering decomposition of invariant variable ζ: same multiplicity of states for mesons and baryons $\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$ (spectators vs active quark)
- But in contrast with mesons there is important degeneracy of states along a given Regge trajectory for a given L: no spin-orbit coupling

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

- Half-integer spin $J = T + \frac{1}{2}$ conveniently represented by RS spinor $\left[\Psi_{N_1 \cdots N_T}\right]_\alpha$ with effective AdS action

$$S_{eff} = \frac{1}{2} \int d^d x \, dz \, \sqrt{|g|} \, g^{N_1 \, N_1'} \cdots g^{N_T \, N_T'}$$
$$\left[\overline{\Psi}_{N_1 \cdots N_T} \Big(i \, \Gamma^A \, e_A^M \, D_M - \mu - U(z) \Big) \Psi_{N_1' \cdots N_T'} + h.c. \right]$$

where the covariant derivative D_M includes the affine connection and the spin connection

- e_M^A is the vielbein and Γ^A tangent space Dirac matrices $\left\{\Gamma^A, \Gamma^B\right\} = \eta^{AB}$

- LF mapping $z \longrightarrow \zeta$ find coupled LFWE

$$\begin{aligned} -\frac{d}{d\zeta}\psi_- - \frac{\nu + \frac{1}{2}}{\zeta}\psi_- - V(\zeta)\psi_- &= M\psi_+ \\ \frac{d}{d\zeta}\psi_+ - \frac{\nu + \frac{1}{2}}{\zeta}\psi_+ - V(\zeta)\psi_+ &= M\psi_- \end{aligned}$$

provided that $|\mu R| = \nu + \frac{1}{2}$ and

$$V(\zeta) = \frac{R}{\zeta} U(\zeta)$$

a J-independent potential – No spin-orbit coupling along a given trajectory !

11 Baryon Spectrum

- Choose linear potential $V = \lambda\,\zeta, \quad \lambda > 0$ to satisfy dAFF
- Eigenfunctions

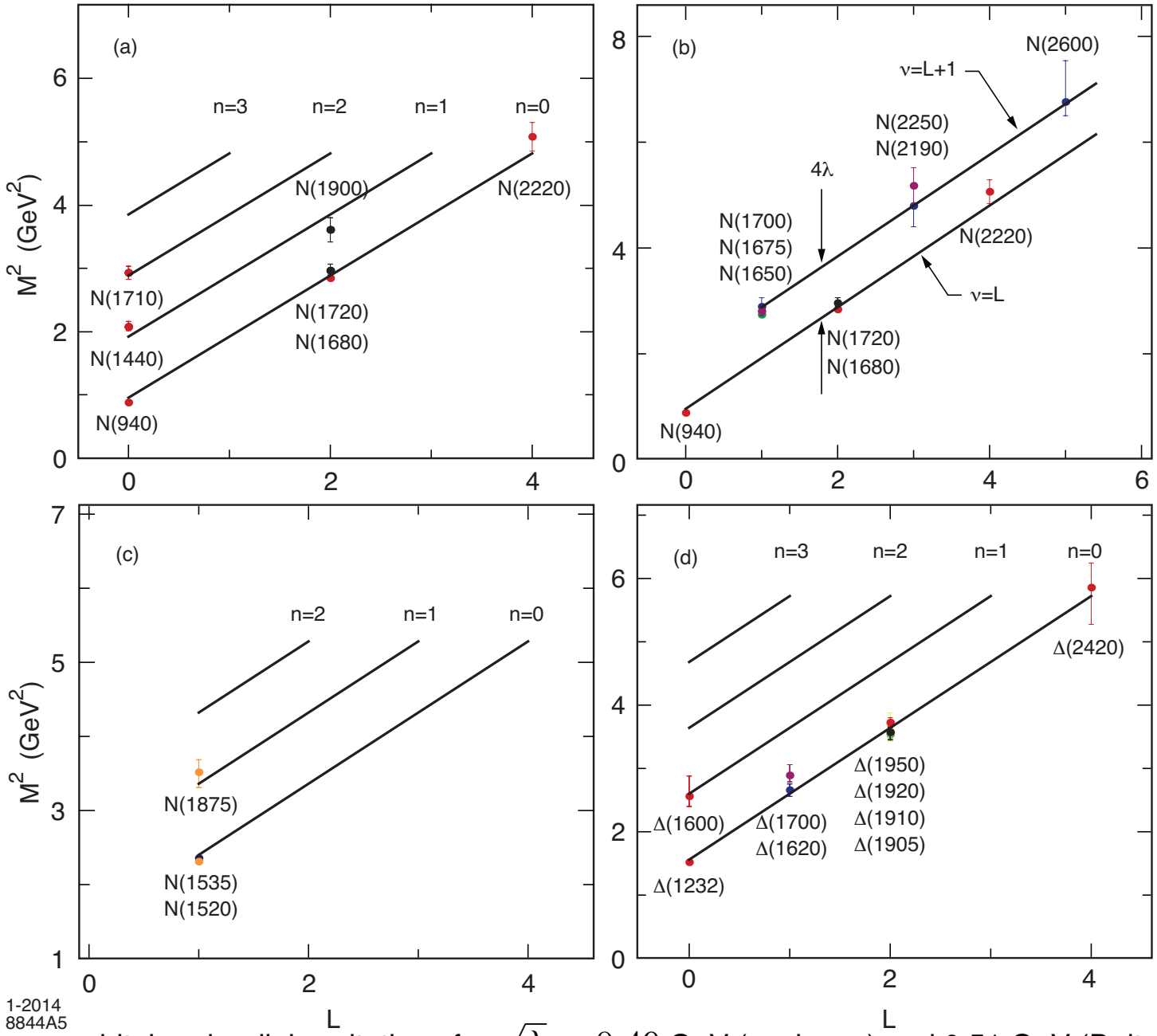
$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu}(\lambda\zeta^2), \qquad \psi_-(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu+1}(\lambda\zeta^2)$$

- Eigenvalues: $M^2 = 4\lambda(n+\nu+1)$
- Lowest possible state $n = 0$ and $\nu = 0$: orbital excitations $\nu = 0, 1, 2 \cdots = L$
- L is the relative LF angular momentum between the active quark and spectator cluster
- ν depends on internal spin and parity

  The assignment

	$S = \frac{1}{2}$	$S = \frac{3}{2}$
P = +	$\nu = L$	$\nu = L + \frac{1}{2}$
P = -	$\nu = L + \frac{1}{2}$	$\nu = L + 1$

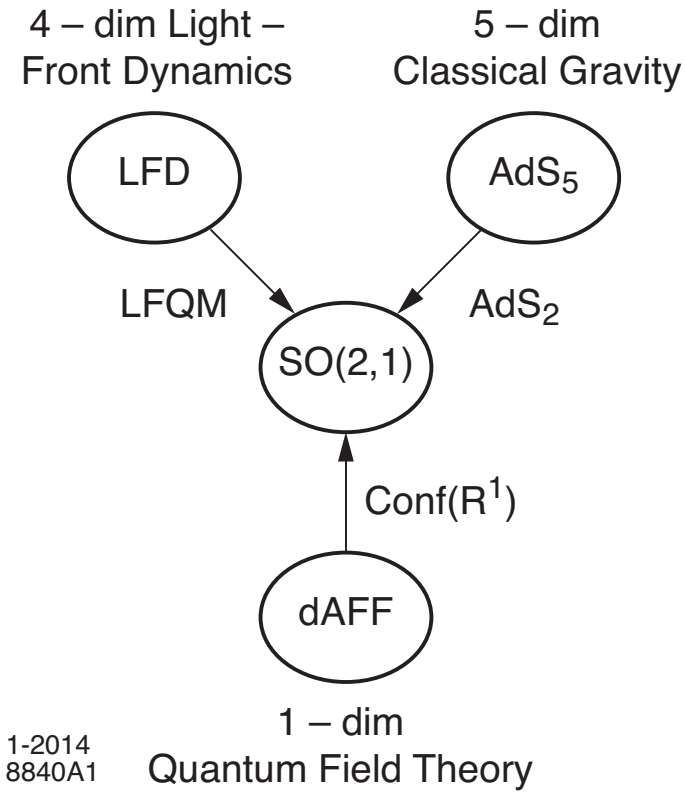
  describes the full light baryon orbital and radial excitation spectrum



Baryon orbital and radial excitations for $\sqrt{\lambda} = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas)

12 Conclusions

We have described a threefold connection between a one-dimensional QFT semiclassical approximation to light-front QCD with gravity in a higher dimensional AdS space, and the constraints imposed by the invariance properties under the full conformal group in one dimension. This provides a new insight into the physics underlying the QCD vacuum, confinement, chiral invariance and the QCD mass scale



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Thank you !