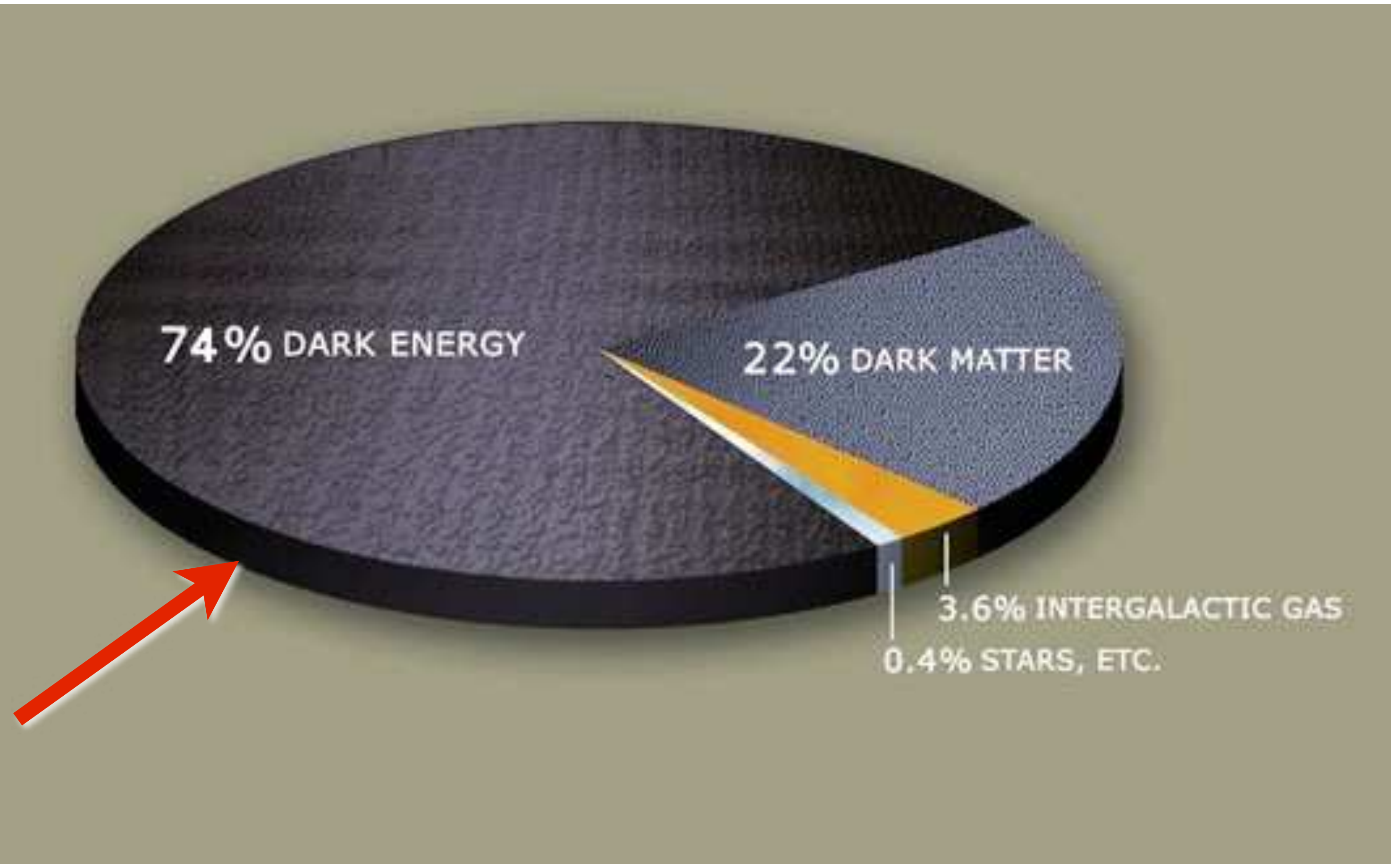
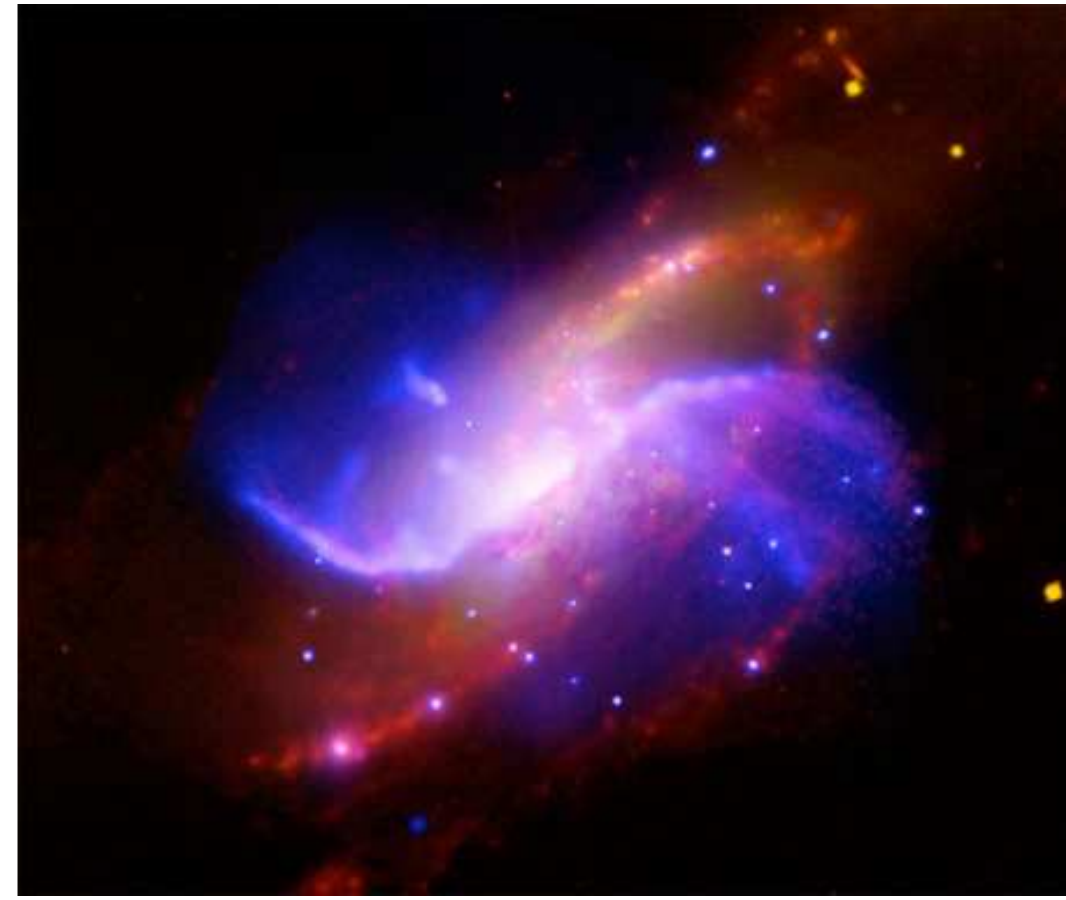
AdS/QCD, Light-Front Holography, and the Light-Front Vacuum

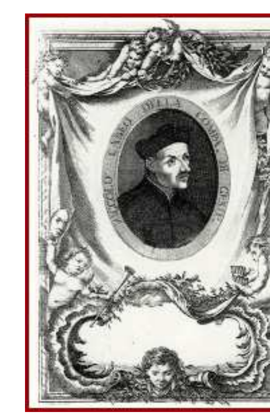
collaborations with Craig Roberts, Robert Shrock, Prem Srivastava, Peter Tandy,
Guy de Téramond, and Hans Günter Dosch





Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

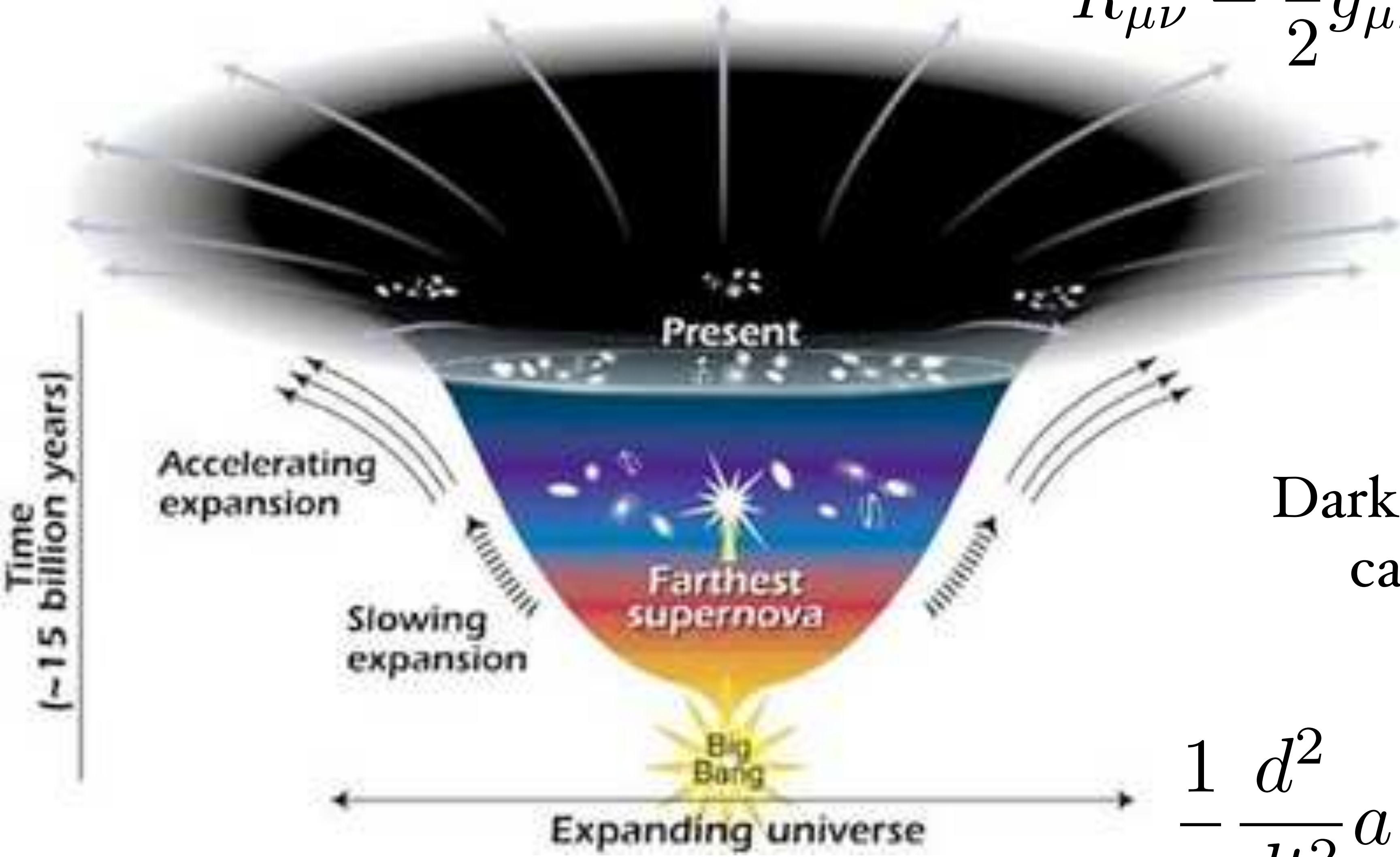


Ferrara I` ter` atio` al School Niccolò Cabeo

May 19-23, 2014

Vacuum and broken symmetries: from the quantum to the cosmos

IUSS - Ferrara 1391



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = (8\pi G_N) T_{\mu\nu}$$

Dark energy/cosmological constant causes accelerating expansion

$$\frac{1}{a} \frac{d^2}{dt^2} a = \Lambda/3 = (8\pi) G_N \rho_\Lambda / 3$$

If the vacuum energy is due to QCD condensates:

$$\rho_\Lambda^{\rm QCD} \simeq M_{\rm QCD}^4 \simeq 10^{46} \rho_\Lambda^{\rm obs}$$

$$\Omega_\Lambda = \frac{\rho_\Lambda^{\rm obs}}{\rho_c} \simeq 0.76 \qquad \rho_c = \frac{3H_0^2}{8\pi G_N}$$

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

Conventional Wisdom:

- Wikipedia: (http://en.wikipedia.org/wiki/QCD_vacuum)

  "The QCD vacuum is the vacuum state of quantum chromodynamics (QCD). It is an example of a non-perturbative vacuum state, characterized by many non-vanishing condensates such as the gluon condensate or the quark condensate. These condensates characterize the normal phase or the confined phase of quark matter."

$$(\Omega_\Lambda)_{QCD} \propto < 0|q\bar{q}|0 >^4$$

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

“ Most embarrassing observation in physics – that’s the only quick thing I can say about dark energy that’s also true.” **-- Edward Witten**

Two general problems:

- Why is the cosmological constant so small, $\Lambda < 10^{-120}$ in Planck density units ?
- Why $\Lambda \sim \rho_{matter}$?
  Coincidence problem.

addressed by anthropic principle, Weinberg 1987

Renata Kallosh

String Theory Landscape

Renata Kallosh

Metaphysics of the Vacuum

“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

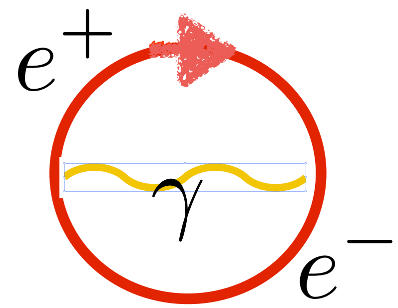
Elements of the solution:

(A) Light-Front Quantization: causal, frame-independent vacuum

(B) New understanding of QCD “Condensates”

(C) Higgs Light-Front Zero Mode

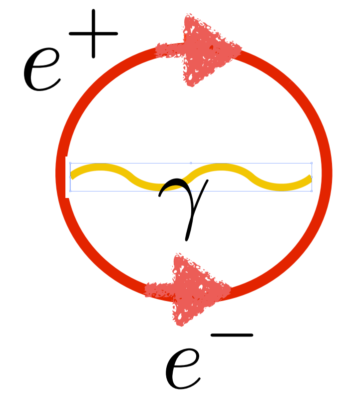
Instant-Form Vacuum in QED



- Loop diagrams of all orders contribute
- Huge vacuum energy: $\rho_\Lambda^{QED} \simeq 10^{120} \rho_\Lambda^{Observed}$
- $\frac{E}{V} = \int \frac{d^3k}{2(2\pi)^3}\sqrt{\vec{k}^2 + m^2}$ Cut off the quadratic divergence at M_{Planck}
- Frame-Dependence, Causality issues.
- Divide S-matrix by disconnected vacuum diagrams?
- In Contrast: Light-Front Vacuum trivial since plus momenta are positive and conserved: $k^+ = k^0 + k^3 > 0$

Front-Form Vacuum in QED

$$P^+ = 0$$



$$k_i^+ > 0 \qquad \sum_i k_i^+ \neq P^+ = 0$$

- Light-Front Vacuum is trivial since all plus momenta are positive and conserved. $k_i^+ > 0$
- All QED vacuum graphs vanish!

PHYSICAL REVIEW VOLUME 150, NUMBER 4 28 OCTOBER 1966

Dynamics at Infinite Momentum*

STEVEN WEINBERG
Department of Physics, University of California, Berkeley, California†

(Received 6 June 1966)

Old-fashioned perturbation theory is applied to a relativistic theory in a reference frame with infinite total momentum. It is found that many undesirable diagrams disappear. The contribution of the remaining diagrams is described by a new set of rules with properties intermediate between those of Feynman diagrams and old-fashioned diagrams, e.g., energy denominators become covariant, and Feynman parameters appear naturally. The new rules are used to derive some integral equations.

$$P^z \to \infty$$

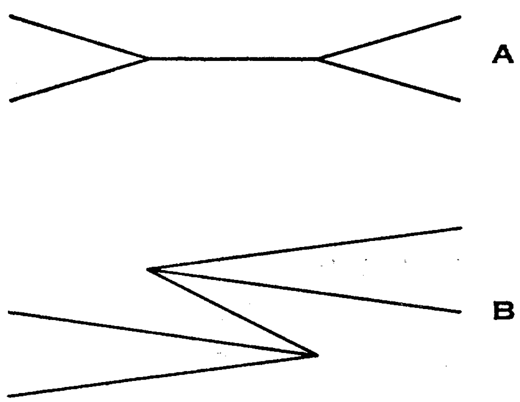


FIG. 1. Two old-fashioned diagrams for scattering in a theory with $\mathcal{H}=g\phi^3$. Under the new rules only A contributes.

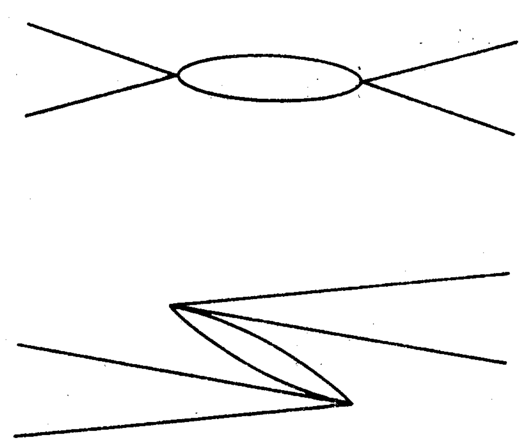


FIG. 2. Two old-fashioned diagrams for scattering in a theory with $\mathcal{H}=g\phi^4$. Under the new rules only A contributes.

The most important distinction between these new rules, and the old rules listed in Sec. II, is that the factors $\theta(\eta_n)$ under rule (b) eliminate some diagrams. This happens whenever a vertex has a number of lines coming in from the right but has no lines going out to the left, or vice versa, because η conservation would require that the sum of the η's of these lines would have to vanish, and this is forbidden by the requirement that all η's be positive. Therefore under rule (a) *we need not draw diagrams in which particles are created or destroyed out of the vacuum*. For instance, diagrams B of both Fig. 1 and Fig. 2 do not contribute to the matrix element. Also, there can be no vacuum fluctuation diagrams.

All $k^+ = \eta > 0$

No Vacuum Fluctuations at Infinite Momentum

Equivalent to Front Form

Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.[1] Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.[2] A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron's wave function and not to the vacuum.[3]

$P^z \to \infty$

Equivalent to Light-Front Formalism

Revised Gell Mann-Oakes-Renner Formula in QCD

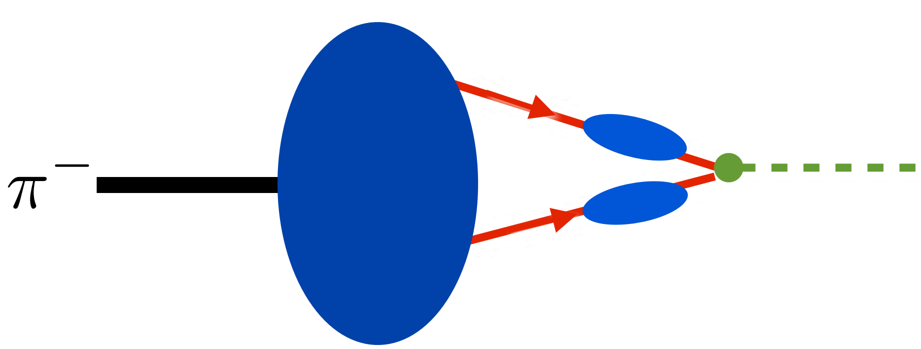
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is an "in-hadron condensate"



$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

PHYSICAL REVIEW C **82**, 022201(R) (2010)

New perspectives on the quark condensate

Stanley J. Brodsky,[1,2] Craig D. Roberts,[3,4] Robert Shrock,[5] and Peter C. Tandy[6]

[1] *SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*

[2] *Centre for Particle Physics Phenomenology: CP^3-Origins, University of Southern Denmark, Odense 5230 M, Denmark*

[3] *Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

[4] *Department of Physics, Peking University, Beijing 100871, China*

[5] *C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*

[6] *Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

(Received 25 May 2010; published 18 August 2010)

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

Quark and Gluon condensates reside within hadrons, not vacuum

Casher and Susskind   Maris, Roberts, Tandy   Shrock and sjb

- Bound-State Dyson Schwinger Equations
- AdS/QCD
- Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

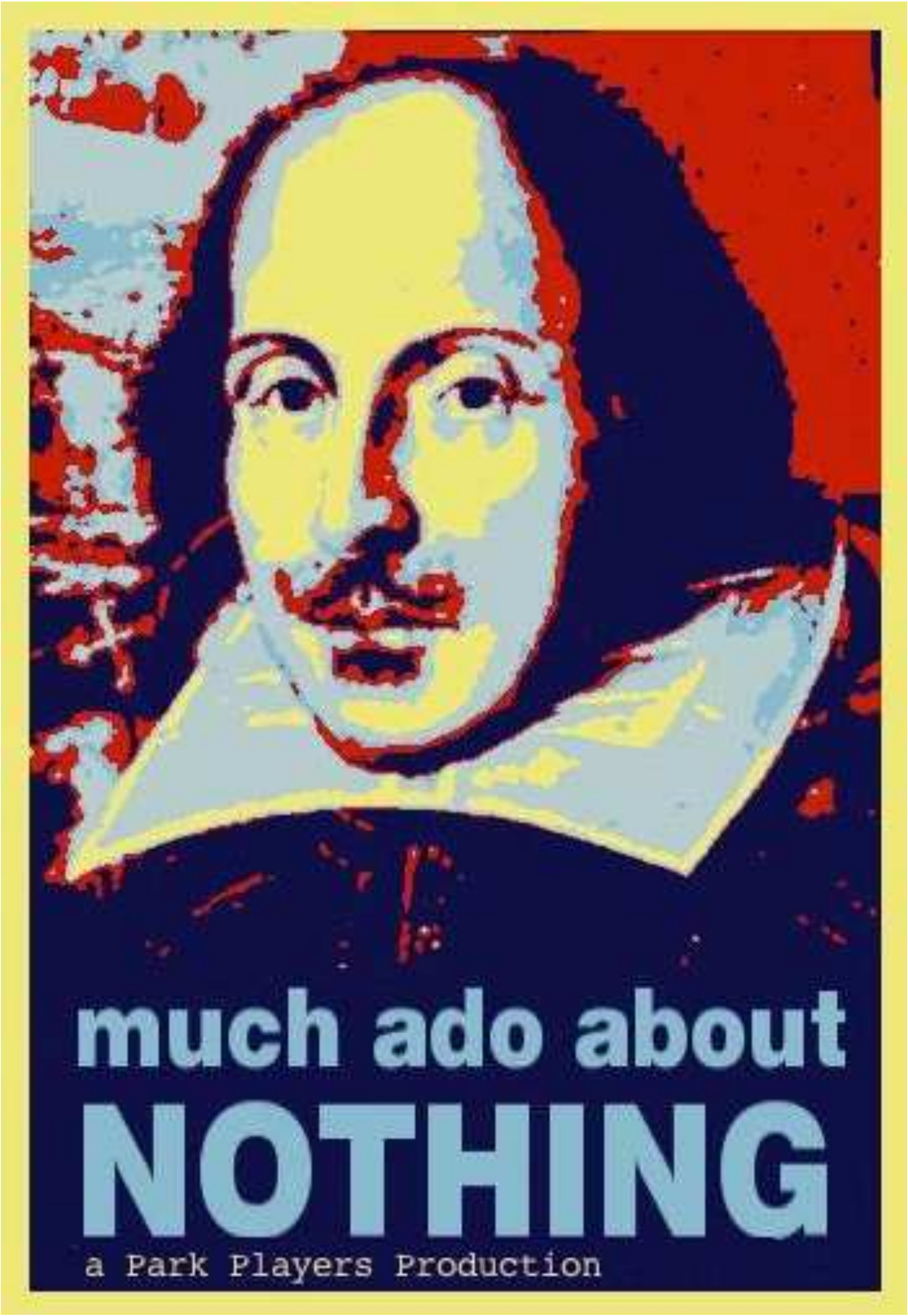
$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{QCD} \propto < 0|q\bar{q}|0 >^4$$

QCD Problem Solved if quark and gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb Proc.Nat.Acad.Sci. 108 (2011) 45-50 "Condensates in Quantum Chromodynamics and the Cosmological Constant"

C. Roberts, R. Shrock, P. Tandy, sjb Phys.Rev. C82 (2010) 022201 "New Perspectives on the Quark Condensate"

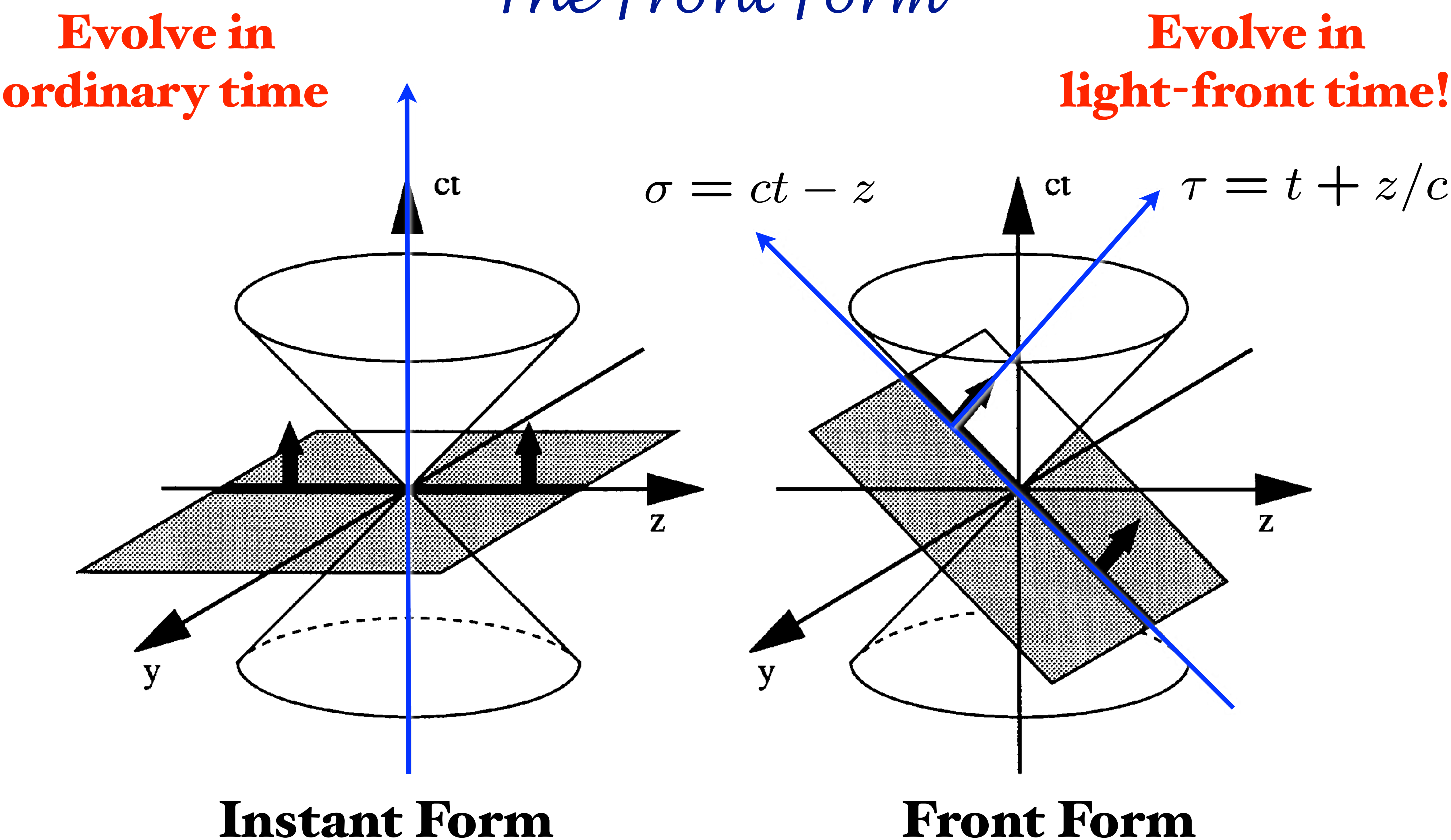

much ado about
NOTHING
a Park Players Production

Outline

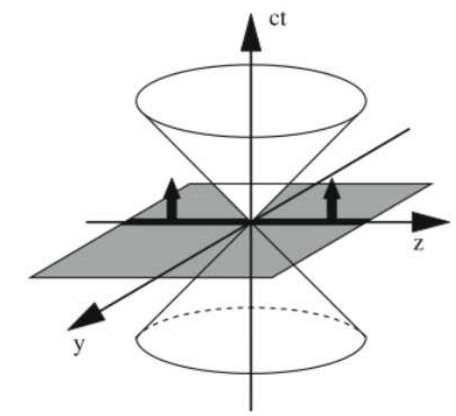
- Light Front Quantization
- The LF Vacuum and the Physical Universe
- QCD Condensates and the Cosmological Constant
- Higgs Model on The LF and the Cosmological Constant
- Light-Front Holography and AdS/QCD

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Dirac's Amazing Idea: The Front Form



- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different "times" and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



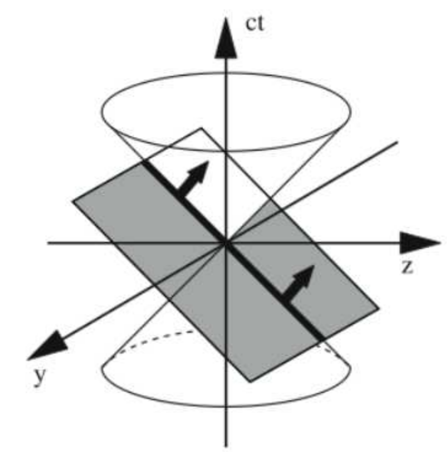
- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

τ= $x^+ = x^0 + x^3$ light-front time

σ= $x^- = x^0 - x^3$ longitudinal space variable

$k^+ = k^0 + k^3$ longitudinal momentum $(k^+ \geq 0)$

$k^- = k^0 - k^3$ light-front energy



$$k \cdot x = \tfrac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

Quantum chromodynamics and other field theories on the light cone.
Stanley J. Brodsky (SLAC), Hans-Christian Pauli (Heidelberg, Max Planck Inst.),
Stephen S. Pinsky (Ohio State U.). SLAC-PUB-7484, MPIH-V1-1997. Apr 1997. 203 pp.
Published in **Phys.Rept. 301 (1998) 299-486**
e-Print: **hep-ph/9705477**

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Images in a photograph show object at a single light-front time

HELEN BRADLEY - PHOTOGRAPHY

Each element of flash photograph illuminated along the light front ***at a fixed***

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i \frac{d}{d\tau}$$

Eigenvalue

$$P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

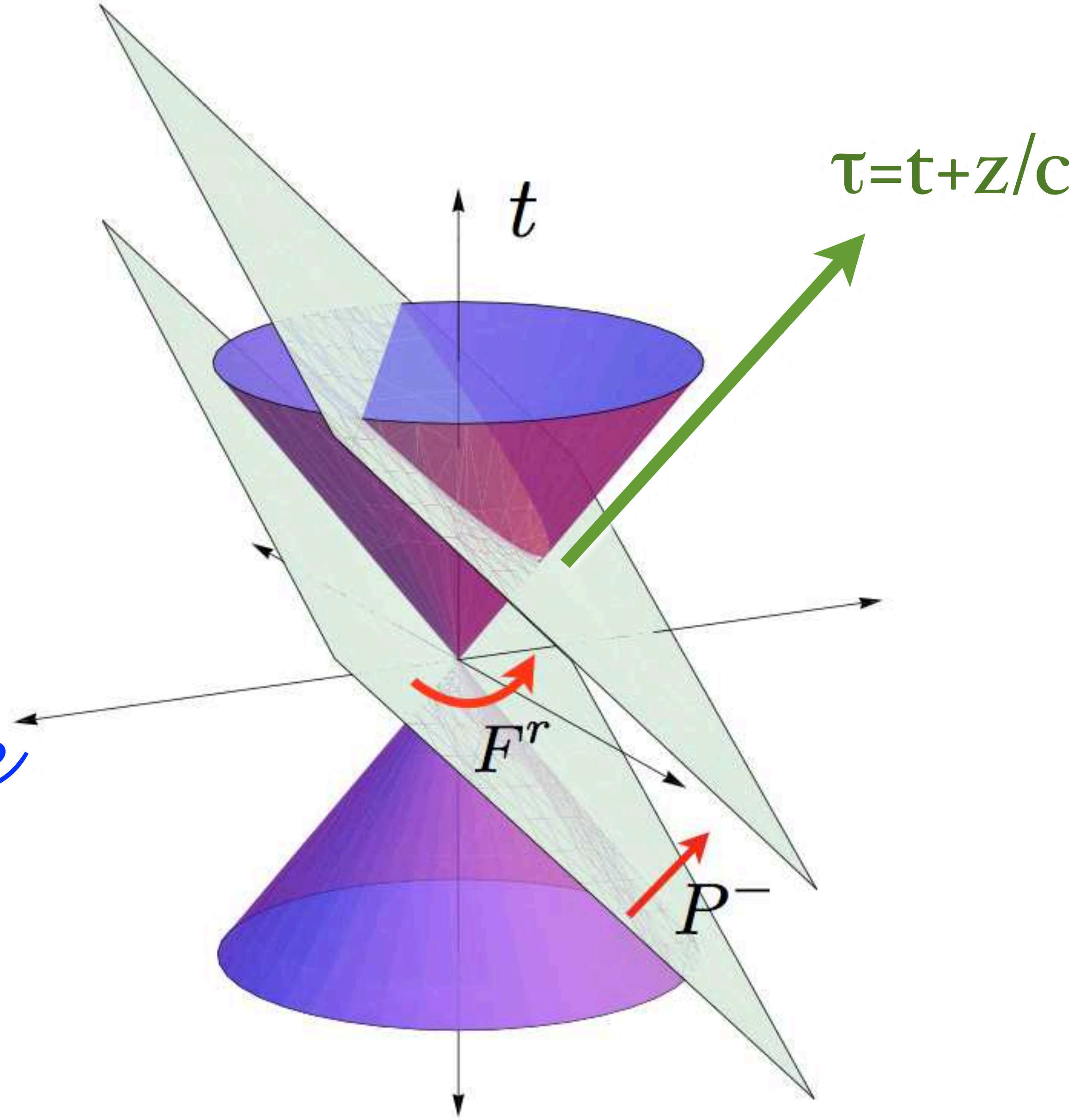
$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$

$$\tau = t + z/c$$

$$P^{-} = P^{0} - P^{z}$$

Evolve in LF time

$$P^{-} = i\frac{d}{d\tau}$$

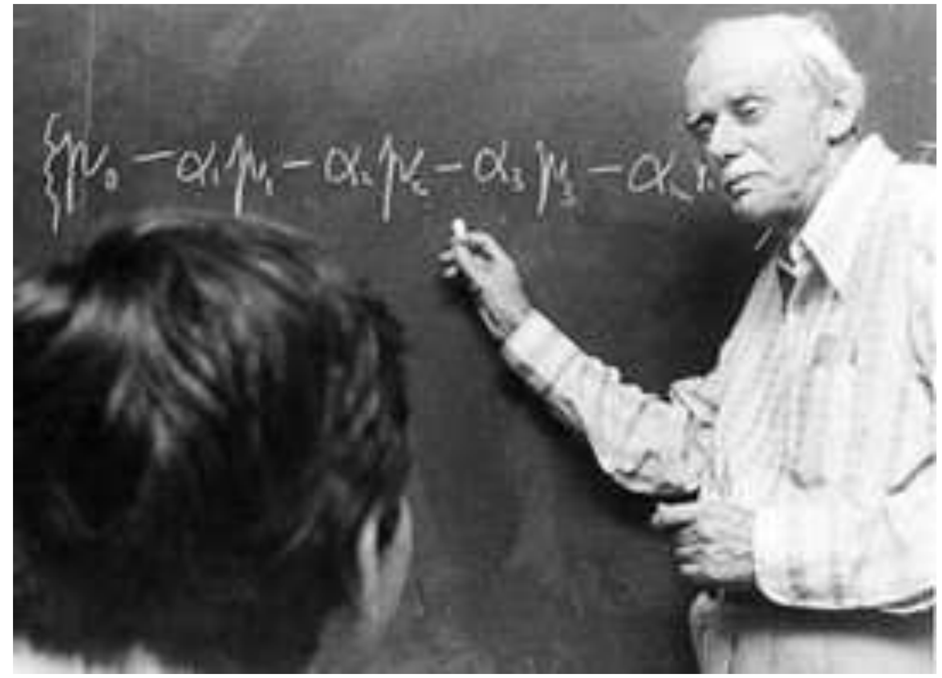


Null plane: a surface tangent to the light cone.

The null-plane Light-Front Hamiltonian maps the initial light-like surface onto some other surface, and therefore describe the dynamical evolution of the system.

The energy P- translates the system in the null-plane time coordinate τ, whereas the spin Hamiltonians Fr rotate the initial surface about the surface of the light cone.

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)



"Working with a front is a process that is unfamiliar to physicists.

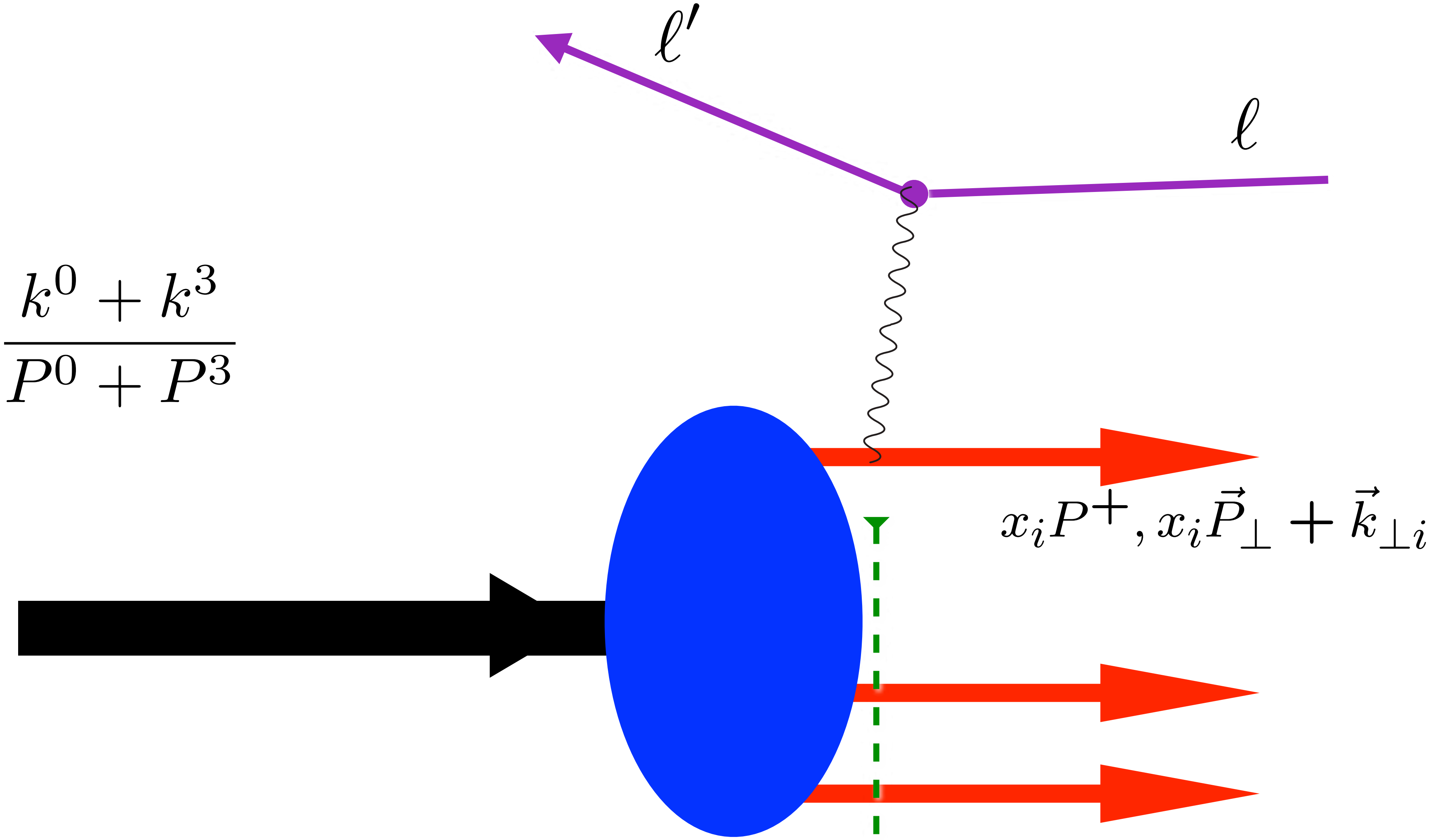
But still I feel that the mathematical simplification that it introduces is all-important.

I consider the method to be promising and have recently been making an extensive study of it.

It offers new opportunities, while the familiar instant form seems to be played out " -
P.A.M. Dirac (1977)

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$P^+, \vec{P}_\perp$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

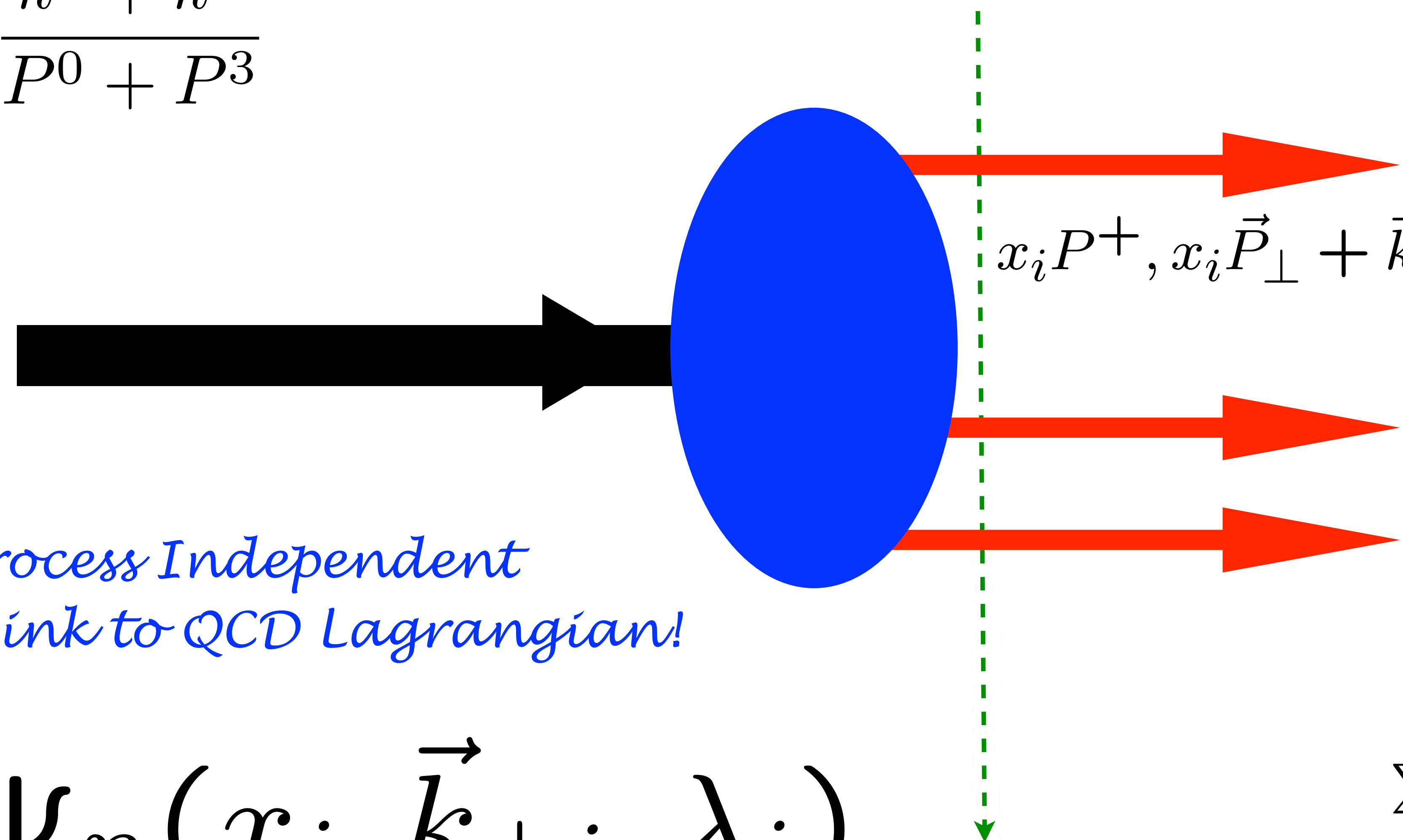
Like a flash photograph

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$P^+, \vec{P}_\perp$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

Process Independent
Direct Link to QCD Lagrangian!

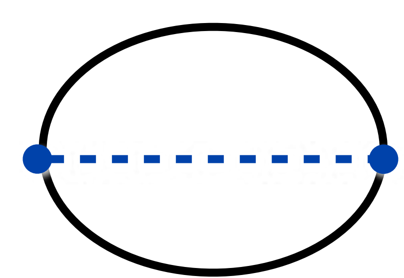
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Plus momenta conserved; all $k^+ \geq 0$



zero !!

Causal Commutators, Trivial Vacuum

Angular Momentum on the Light-Front

LC gauge $A^+=0$

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock state by Fock State

Gluon orbital angular momentum defined in physical lc gauge

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Orbital Angular Momentum is a property of LFWFS

Nonzero Anomalous Moment -->
Nonzero quark orbital angular momentum!

The Light-Front Vacuum

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

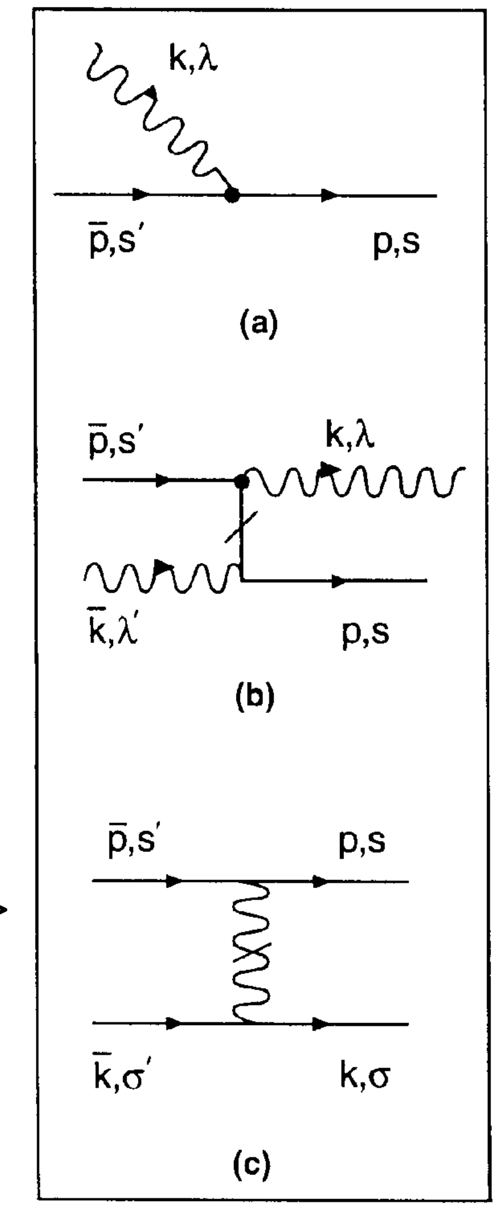
$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i \left[\frac{m^2 + k^2_\perp}{x}\right]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

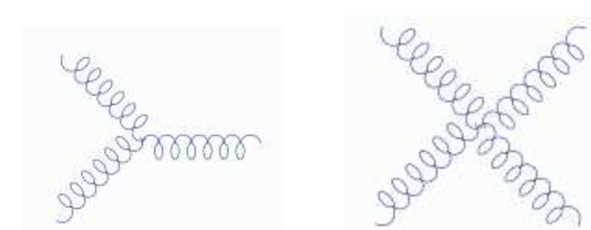
$$H^{QCD}_{LF} |\Psi_h > = \mathcal{M}^2_h |\Psi_h >$$

$$|p, J_z > = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i >$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

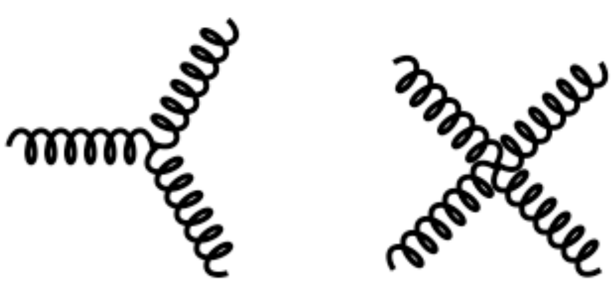
LFWFs: Off-shell in P- and invariant mass

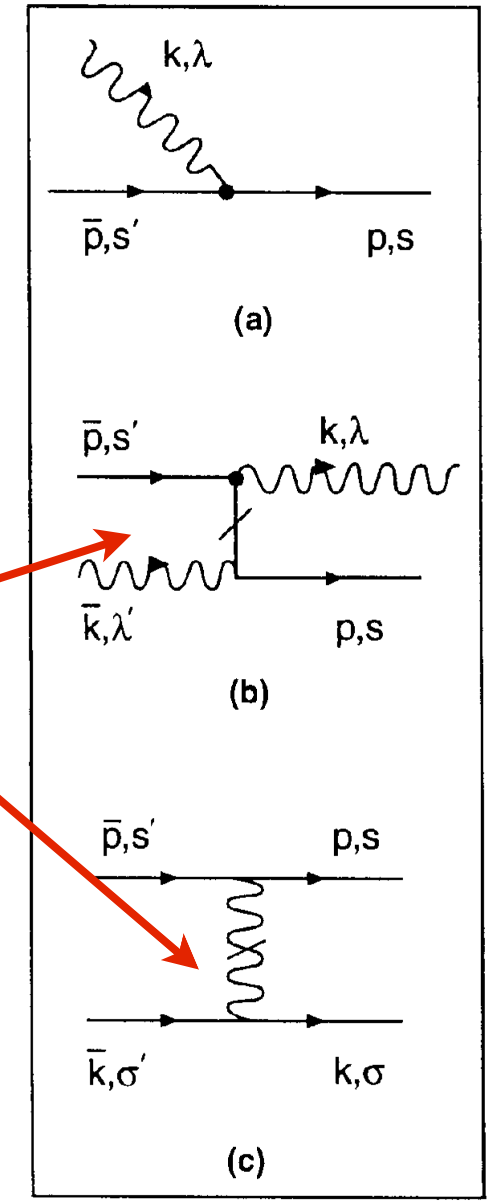


H^{int}_{LF}

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$\begin{aligned} H^{LF}_{QCD} = {} & \frac{1}{2}\int d^3x \bar{\tilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\tilde{\psi} - A^i_a(\mathrm{i}\partial^\perp)^2 A_{ia} \\ & - \frac{1}{2}g^2 \int d^3x \mathbf{Tr}\left[\tilde{A}^\mu, \tilde{A}^\nu\right]\left[\tilde{A}_\mu, \tilde{A}_\nu\right] \\ & + \frac{1}{2}g^2 \int d^3x \bar{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \frac{1}{(\mathrm{i}\partial^+)^2} \bar{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \\ & - g^2 \int d^3x \bar{\tilde{\psi}}\gamma^+ \left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right)\tilde{\psi} \\ & + g^2 \int d^3x \mathbf{Tr}\left(\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right) \\ & + \frac{1}{2}g^2 \int d^3x \bar{\tilde{\psi}}\tilde{\not{A}}\frac{\gamma^+}{\mathrm{i}\partial^+}\tilde{\not{A}}\tilde{\psi} \\ & + g \int d^3x \bar{\tilde{\psi}}\tilde{\not{A}}\tilde{\psi} \\ & + 2g \int d^3x \mathbf{Tr}\left(\mathrm{i}\partial^\mu \tilde{A}^\nu \left[\tilde{A}_\mu, \tilde{A}_\nu\right]\right) \end{aligned}$$





Physical gauge: $A^+ = 0$

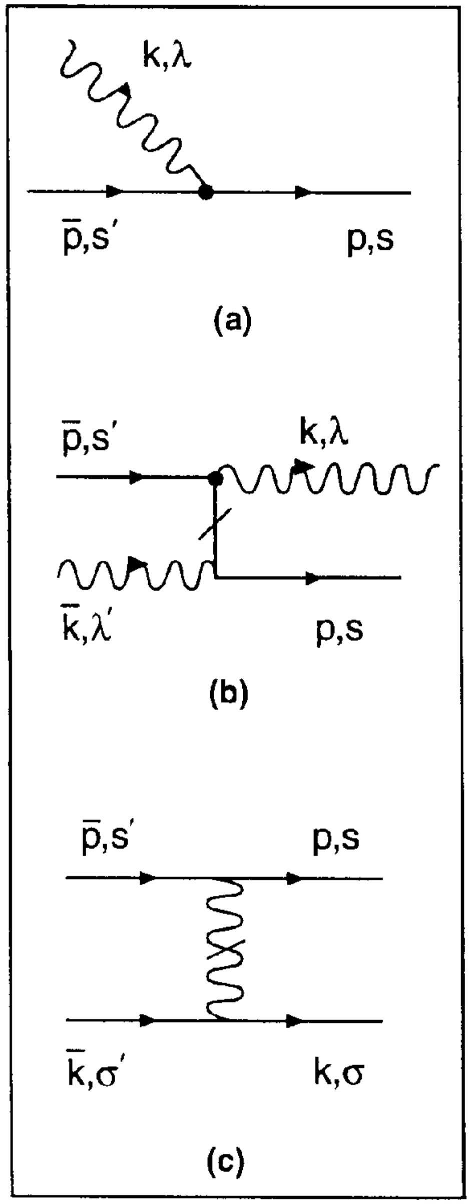
Rigorous First-Principle Formulation of Non-Perturbative QCD

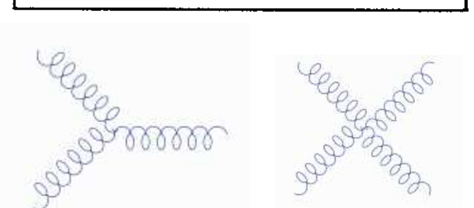
Light-Front QCD Heisenberg Equation

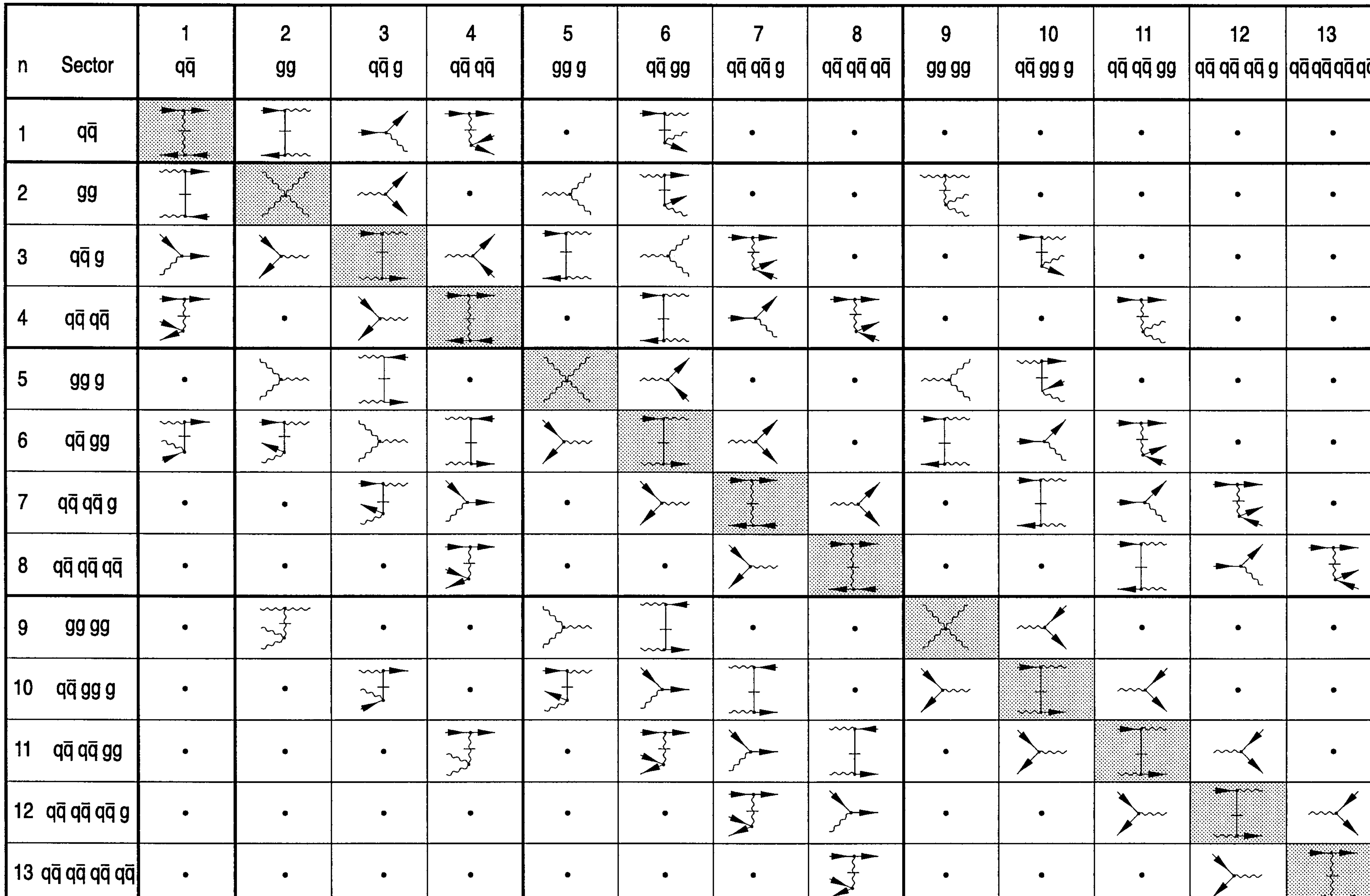
$$H^{QCD}_{LC}|\Psi_h\rangle = \mathcal{M}^2_h\, |\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb







Minkowski space; frame-independent; no fermion doubling; no ghosts trivial vacuum

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

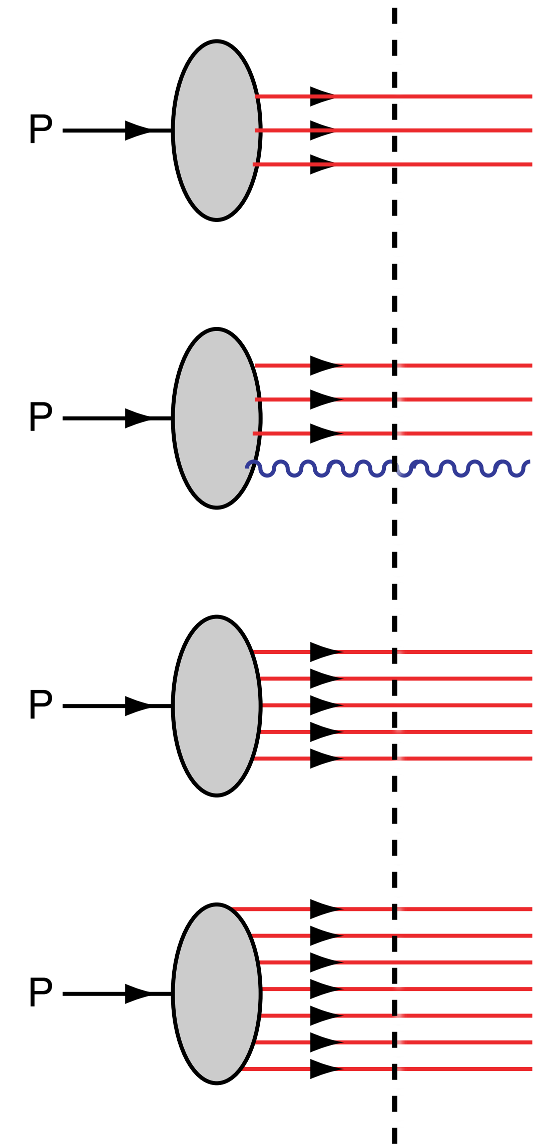
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks
c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Mueller: gluon Fock states **BFKL Pomeron**

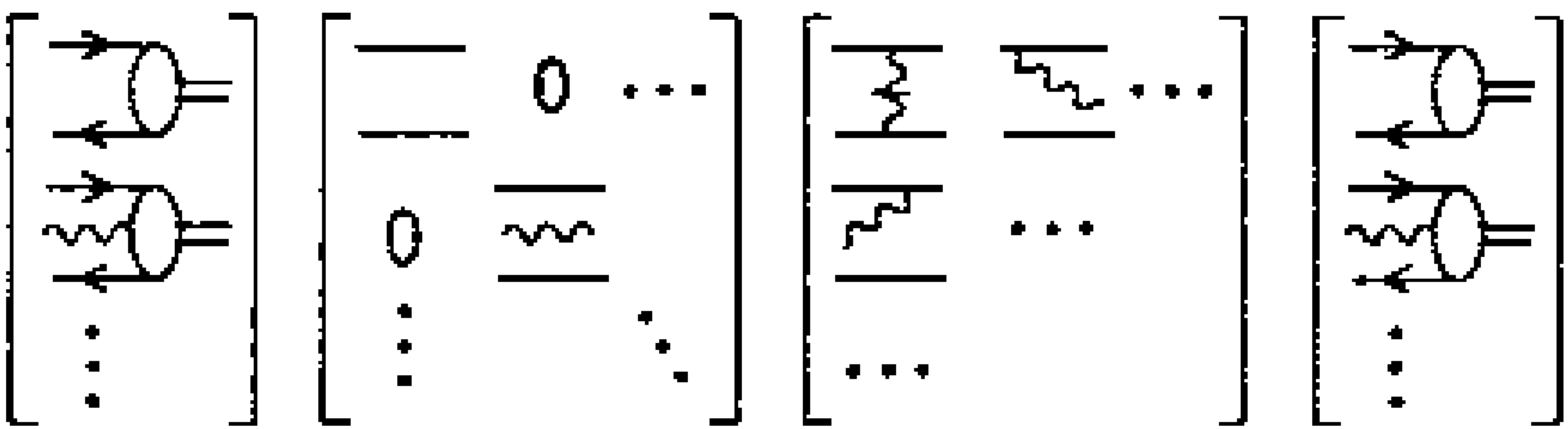
Hidden Color

LIGHT-FRONT MATRIX EQUATION

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right)\begin{bmatrix}\psi_{q\bar{q}/\pi}\\ \psi_{q\bar{q}g/\pi}\\ \vdots\end{bmatrix} = \begin{bmatrix}\langle q\bar{q}|\,V\,|q\bar{q}\rangle & \langle q\bar{q}|\,V\,|q\bar{q}g\rangle & \cdots\\ \langle q\bar{q}g|\,V\,|q\bar{q}\rangle & \langle q\bar{q}g|\,V\,|q\bar{q}g\rangle & \cdots\\ \vdots & \vdots & \ddots\end{bmatrix}\begin{bmatrix}\psi_{q\bar{q}/\pi}\\ \psi_{q\bar{q}g/\pi}\\ \vdots\end{bmatrix}$$

$$A^+ = 0$$

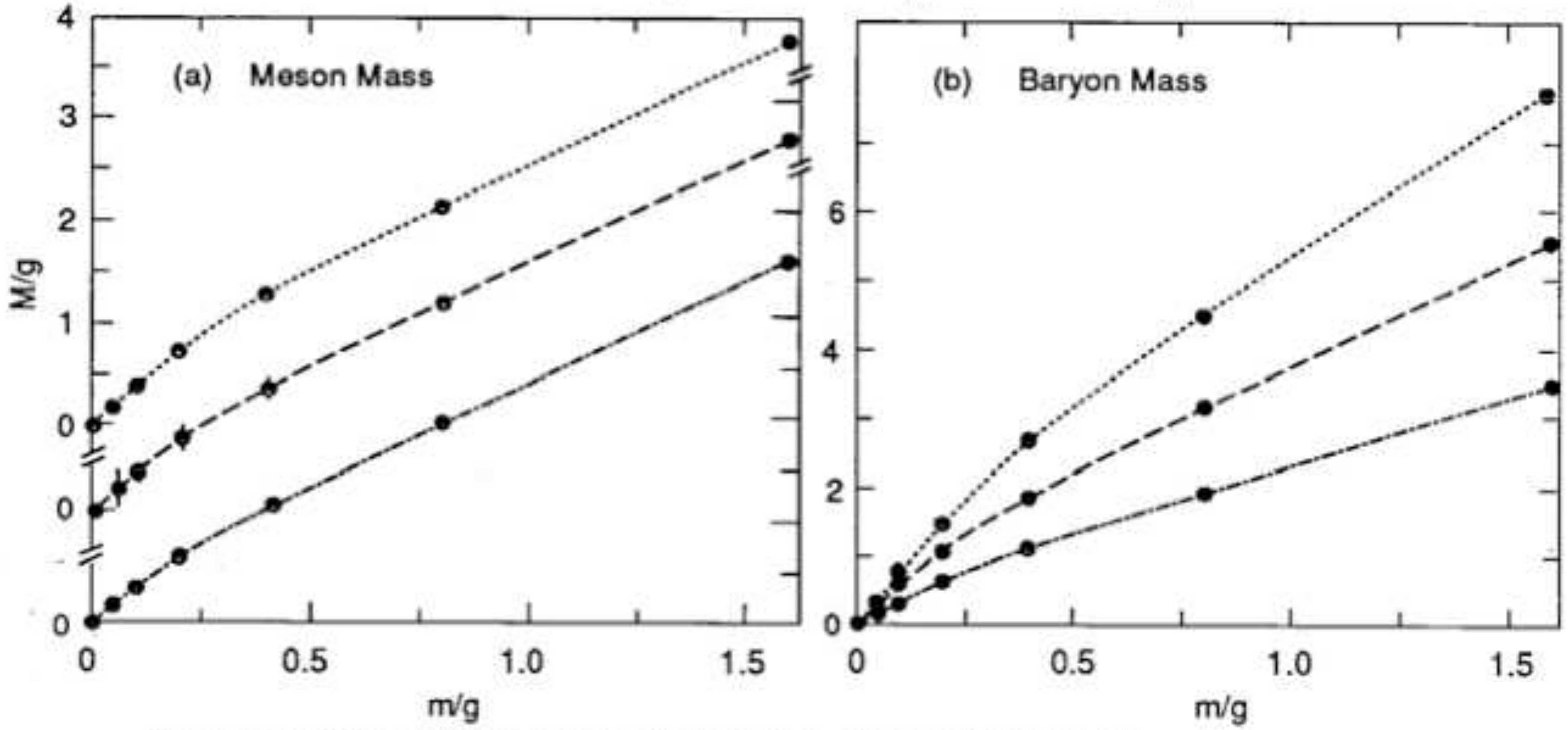


Minkowski space; frame-independent; no fermion doubling; no ghosts

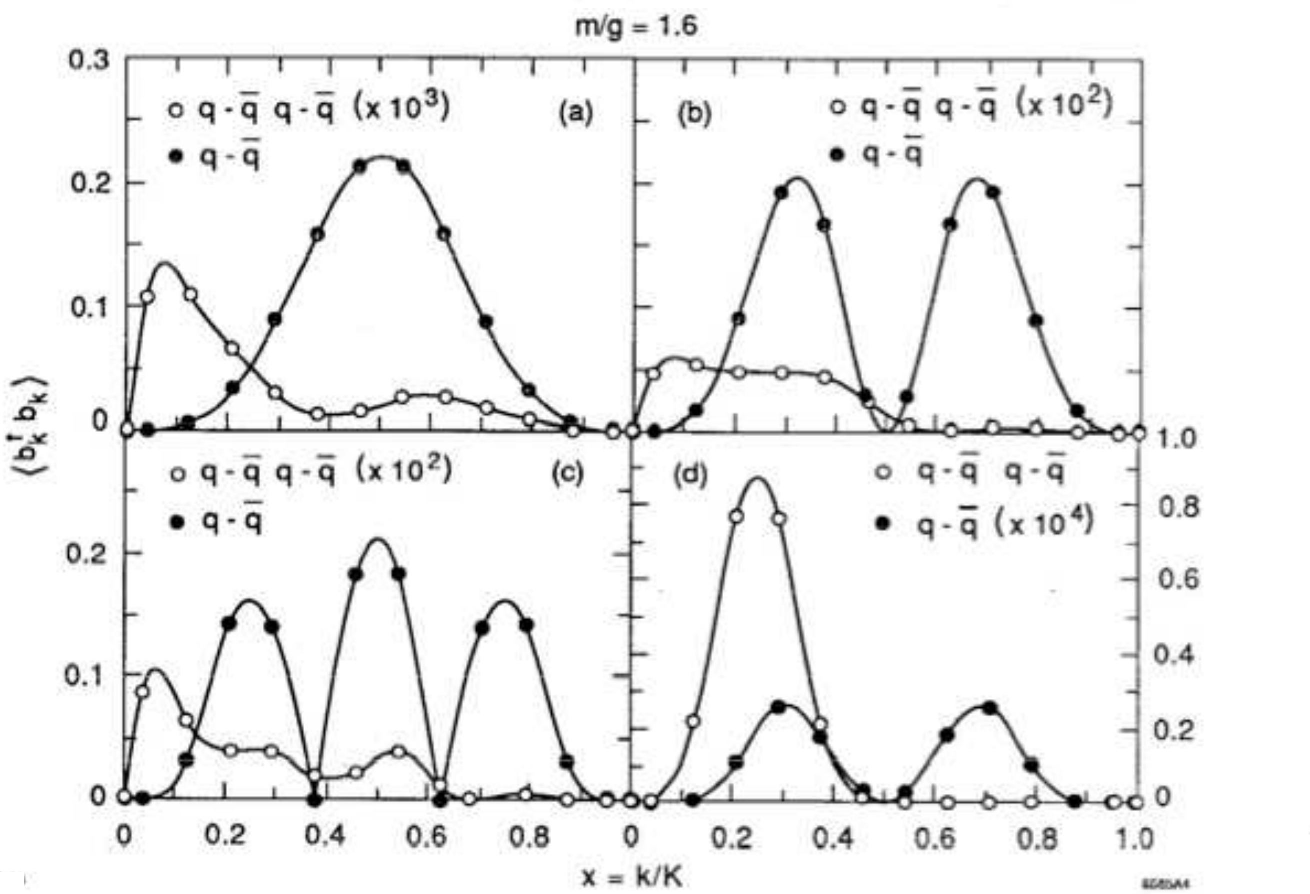
- Light-Front Vacuum = vacuum of free Hamiltonian!

The Light-Front Vacuum

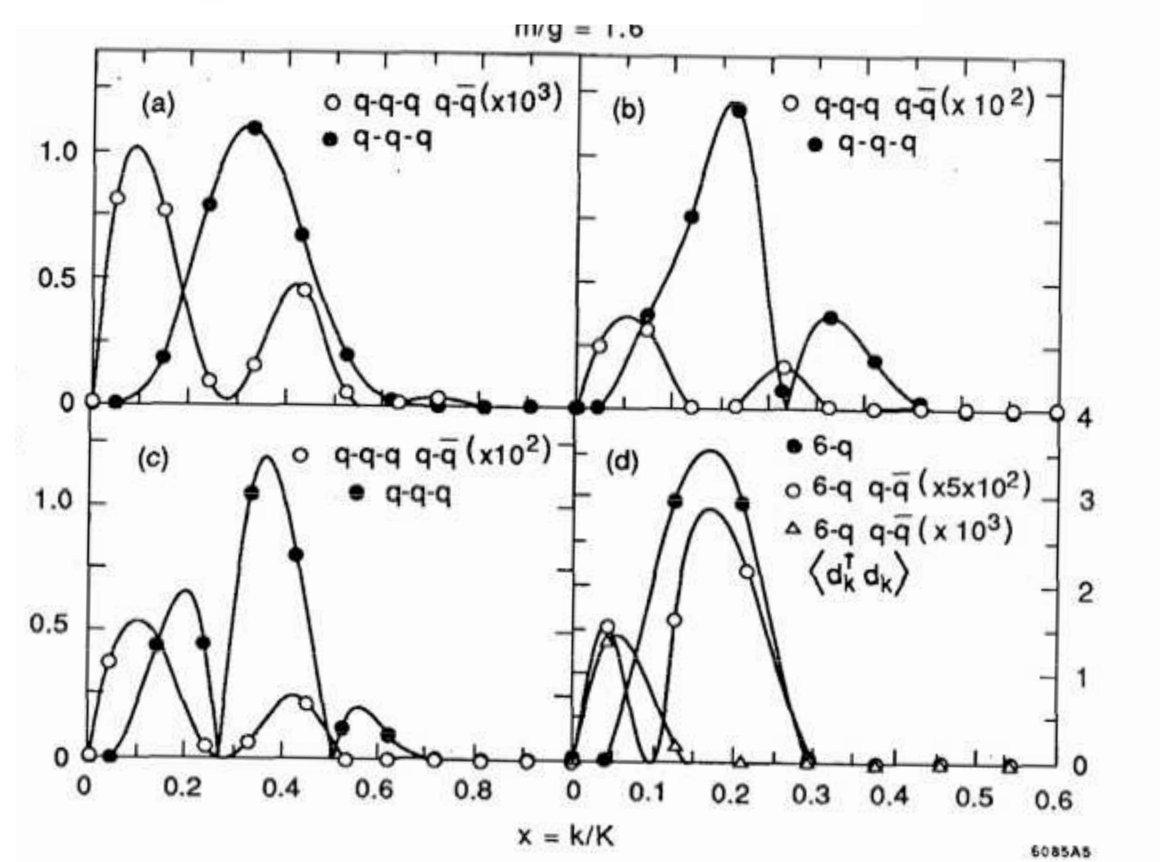
DLCQ: Solve QCD(1+1) for any quark mass and flavors



Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, 2K=24. d) Eleventh state.



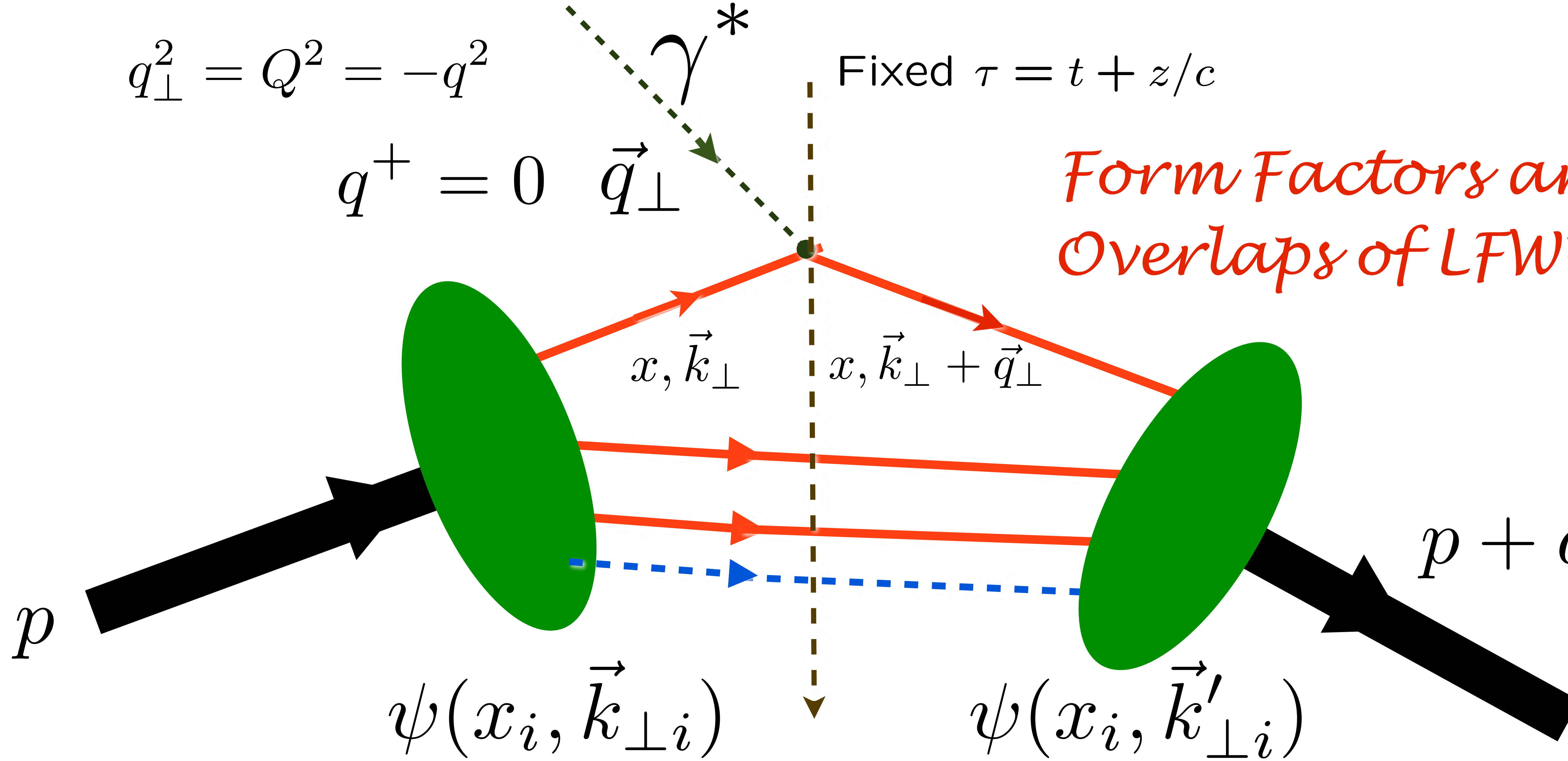
a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Hornbostel, Pauli, sjb

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Exact LF Formula for Pauli Form Factor

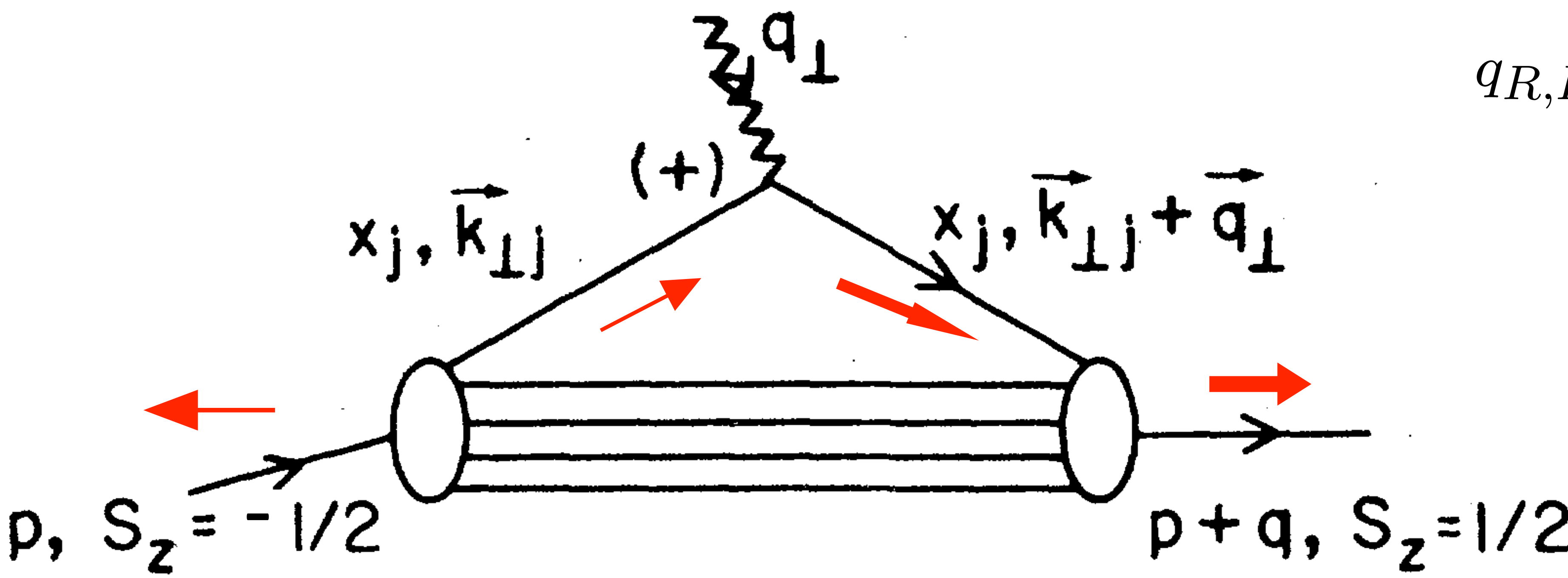
Drell, sjb

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

The Light-Front Vacuum

Gravitational Form Factors

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \overline{u}(P')\Big[A(q^2)\gamma^{(\mu}\overline{P}^{\nu)} + B(q^2)\frac{i}{2M}\overline{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]u(P)\ ,$$

where $q^\mu = (P' - P)^\mu,\ \overline{P}^\mu = \frac{1}{2}(P' + P)^\mu,\ a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$

$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\uparrow\right\rangle = A(q^2)\ ,$$

$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\downarrow\right\rangle = -(q^1 - \mathrm{i}q^2)\frac{B(q^2)}{2M}\ .$$

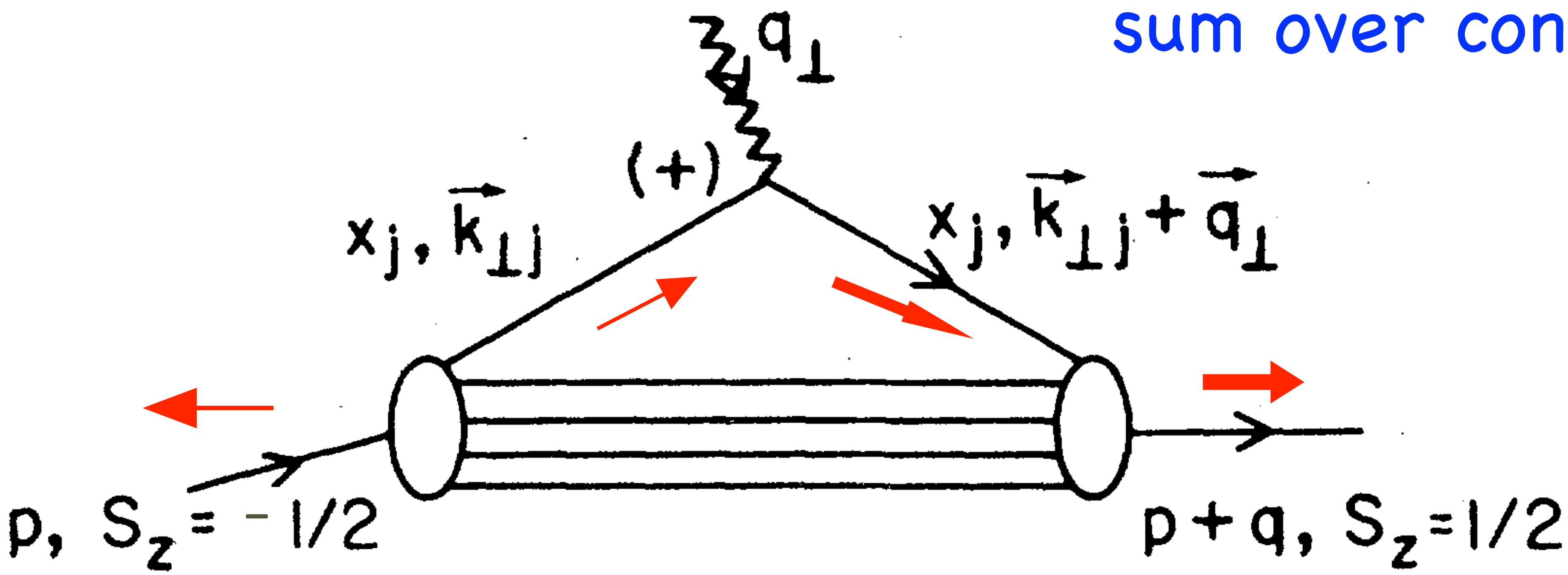
Vanishing Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*

Crucial Test of Consistency with Gravity!

graviton

sum over constituents



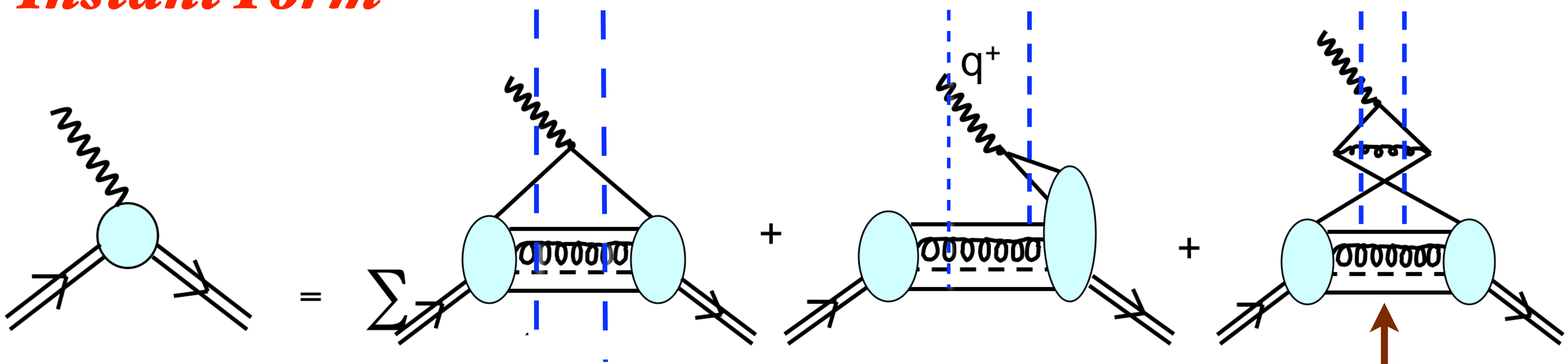
Hwang, Schmidt, sjb; Holstein et al

$$B(0) = 0$$

Each Fock State

Calculation of Form Factors in Equal-Time Theory

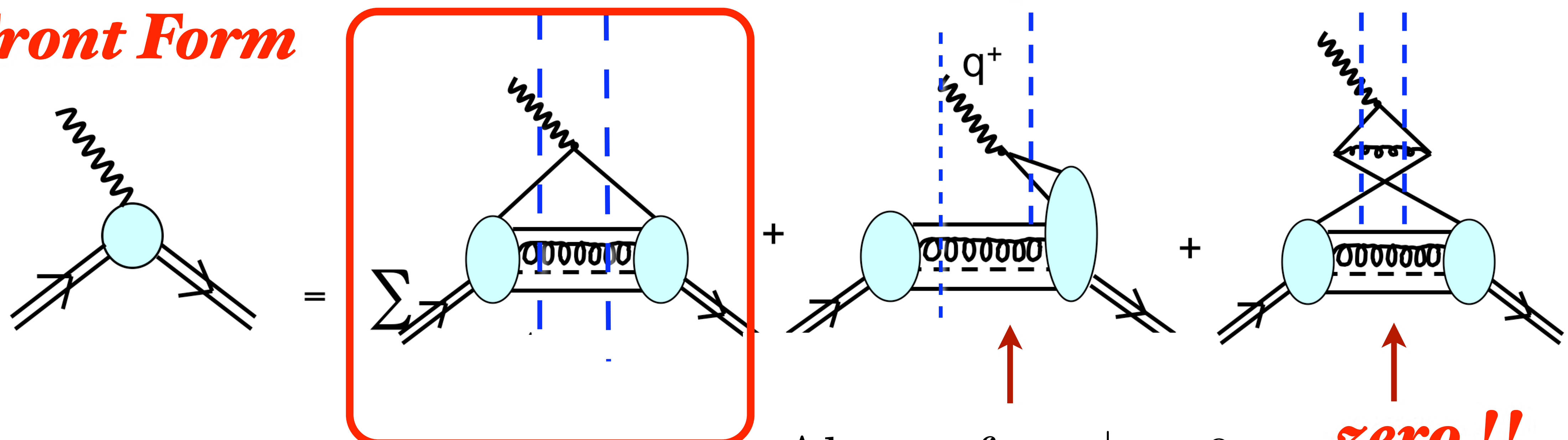
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form

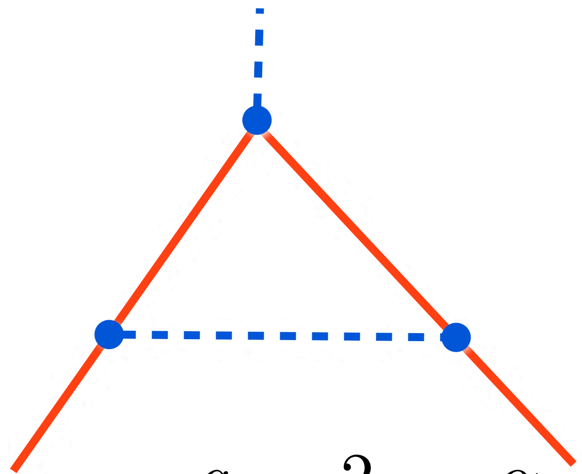


Complete Answer

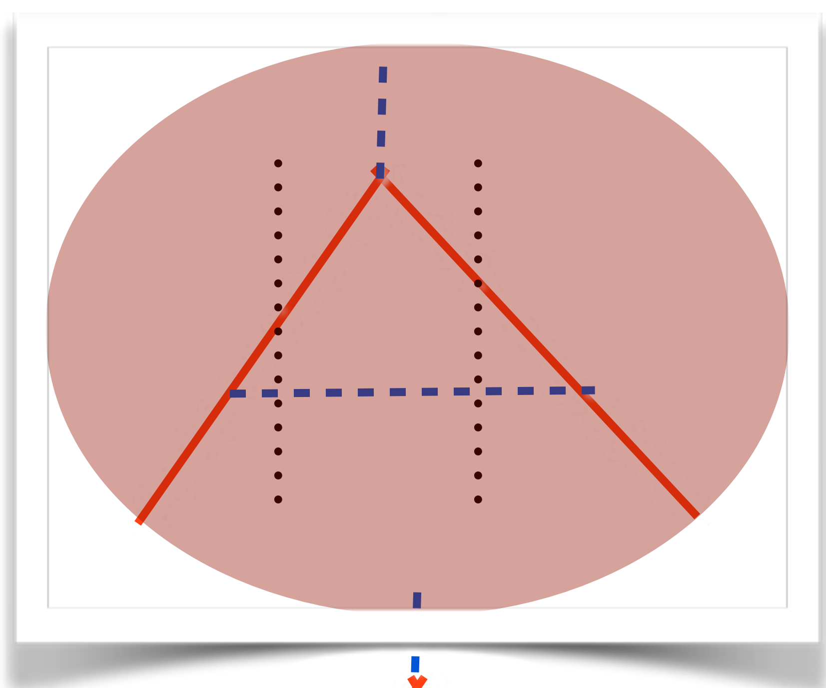
Absent for $q^+ = 0$

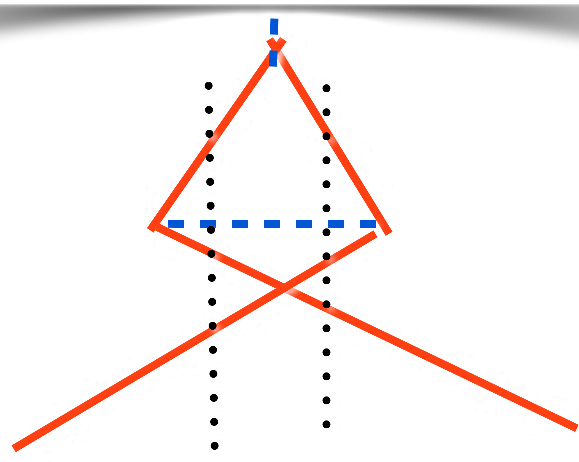
zero !!

No vacuum graphs



$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$

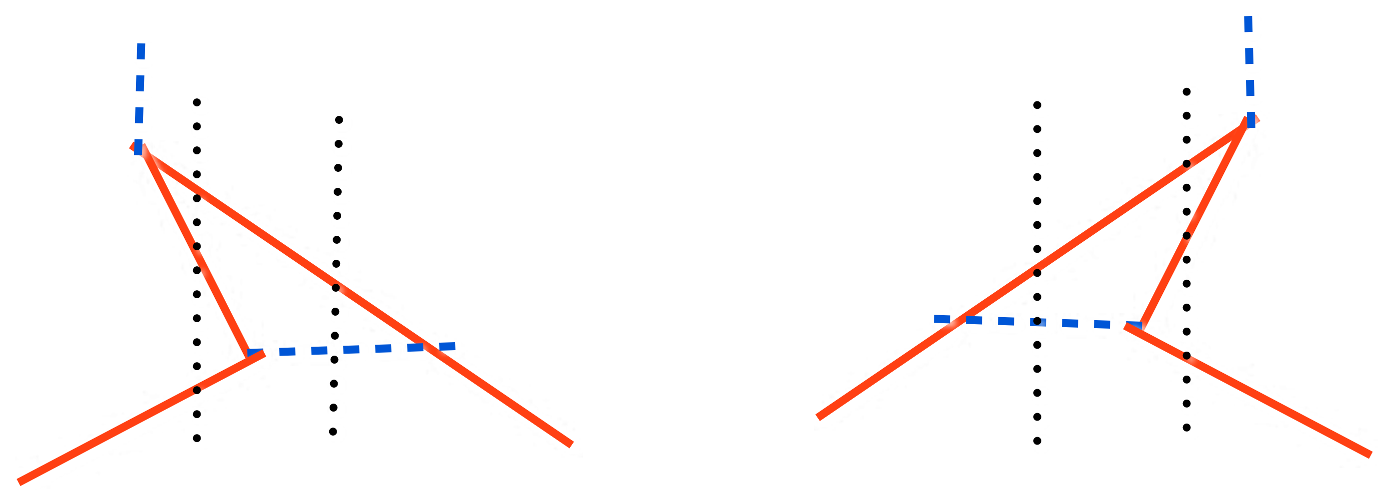




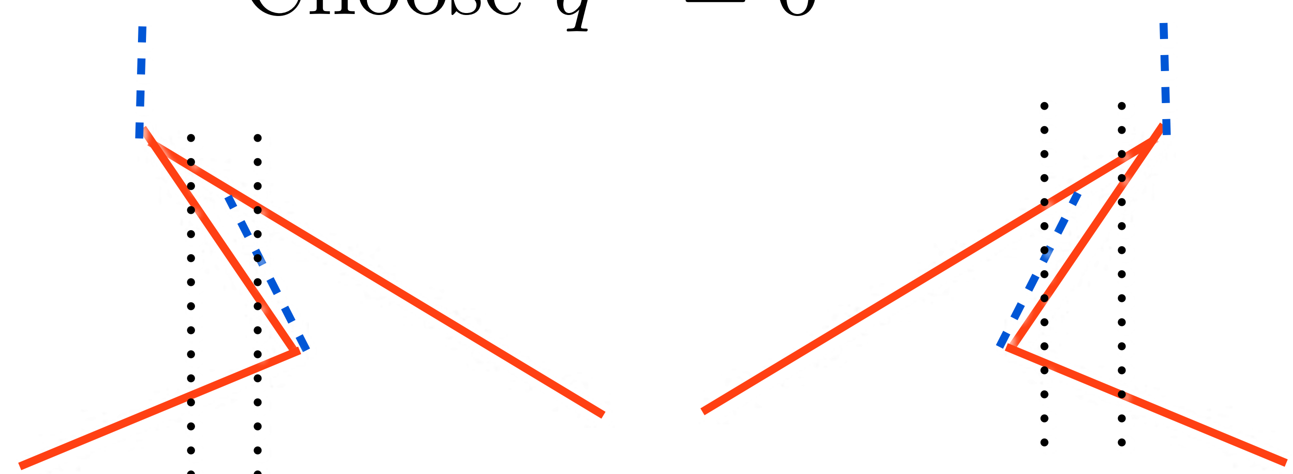
Wick Theorem

Feynman diagram =
single front-form time-ordered diagram!

Also $P \to \infty$ observer frame (Weinberg)

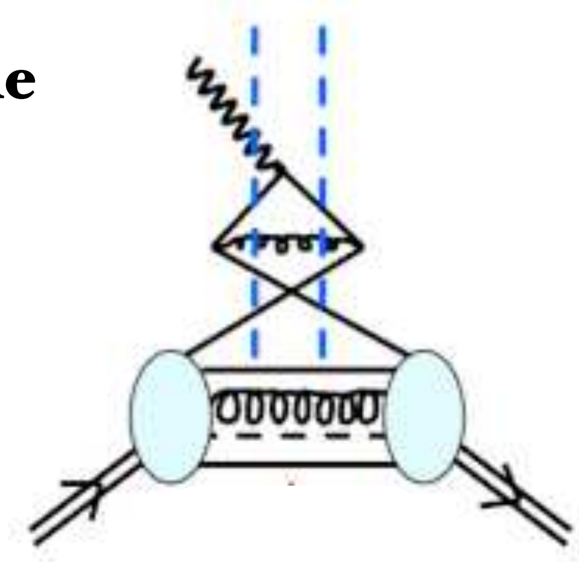


Choose $q^+ = 0$



Disadvantages of the Instant Form

- Boosts are dynamical, change particle number: not Melosh!
- Famous wrong proof showing violation of LET and DHG sum rule
- Each Amplitude is Frame-Dependent
- States defined at one instant of time over all space - acausal!
- Current matrix elements involve connected vacuum currents -- eigensolutions insufficient!
- N! time-ordered graphs, each frame-dependent
- Vacuum is complex: apparently gives huge vacuum energy density
- Normal-ordering required to compute observables
- Cluster decomposition theorem fails in relativistic systems
- Virtually no valid calculations of dynamics of relativistic composite systems use the instant form
- Why Feynman invented Feynman diagrams!



PHYSICAL REVIEW VOLUME 174, NUMBER 5 25 OCTOBER 1968

Electromagnetic Interactions of Loosely-Bound Composite Systems*

STANLEY J. BRODSKY AND JOEL R. PRIMACK
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 13 June 1968)

Contrary to popular assumption, the interaction of a composite system with an external electromagnetic field is not equal to the sum of the individual Foldy-Wouthyusen interactions of the constituents if the constituents have spin. We give the correct interaction, and note that it is consistent with the Drell-Hearn-Gerasimov sum rule and the low-energy theorem for Compton scattering. We also discuss the validity of additivity of the individual Dirac interactions, and the corrections to this approximation, with particular reference to the atomic Zeeman effect, which is of importance in the fine-structure and Lamb-shift measurements.

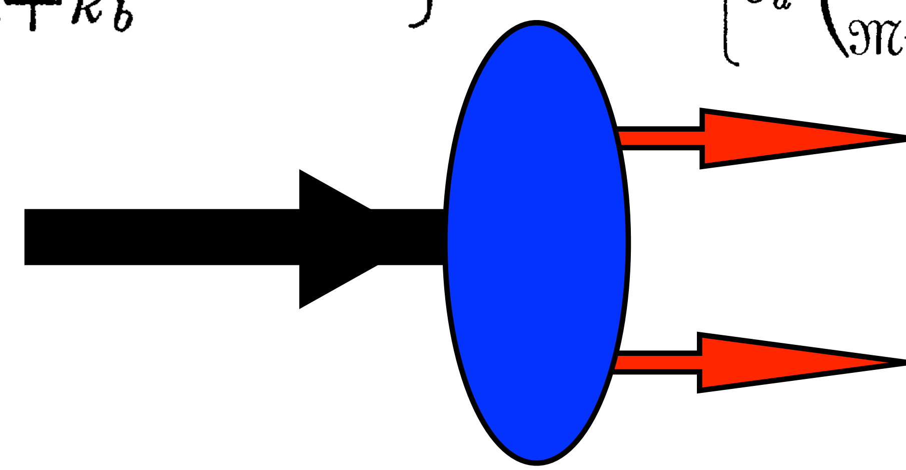
Dynamical boost contribution

$$\left\{\begin{matrix} 1 \\ \dfrac{1}{2m_a+k_a}\boldsymbol{\sigma}_a\cdot\mathbf{p}\end{matrix}\right\}\otimes\left\{\begin{matrix} 1 \\ \dfrac{1}{2m_b+k_b}\boldsymbol{\sigma}_b\cdot(-\mathbf{p})\end{matrix}\right\}$$

$\vec{P} \neq 0$

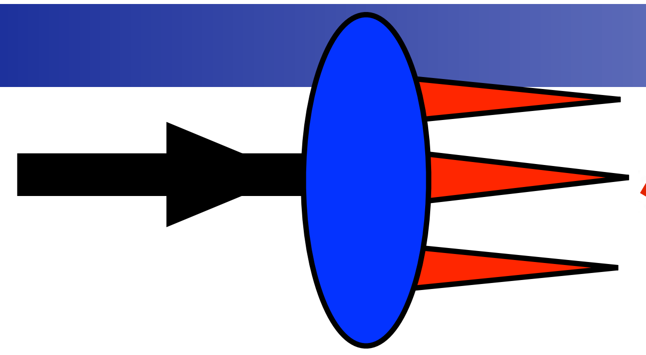
$$\left\{\begin{matrix} 1+\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{P}}{\mathfrak{M}+E}\,\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{p}}{2m_a+k_a} \\ \boldsymbol{\sigma}_a\cdot\left(\dfrac{\mathbf{P}}{\mathfrak{M}+E}+\dfrac{\mathbf{p}}{2m_a+k_a}\right)\end{matrix}\right\}\otimes\left\{\begin{matrix} 1-\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{P}}{\mathfrak{M}+E}\,\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{p}}{2m_b+k_b} \\ \boldsymbol{\sigma}_b\cdot\left(\dfrac{\mathbf{P}}{\mathfrak{M}+E}-\dfrac{\mathbf{p}}{2m_b+k_b}\right)\end{matrix}\right\}$$

Instant Form WF



Also: Hugh Osborne

• Light Front Wavefunctions:



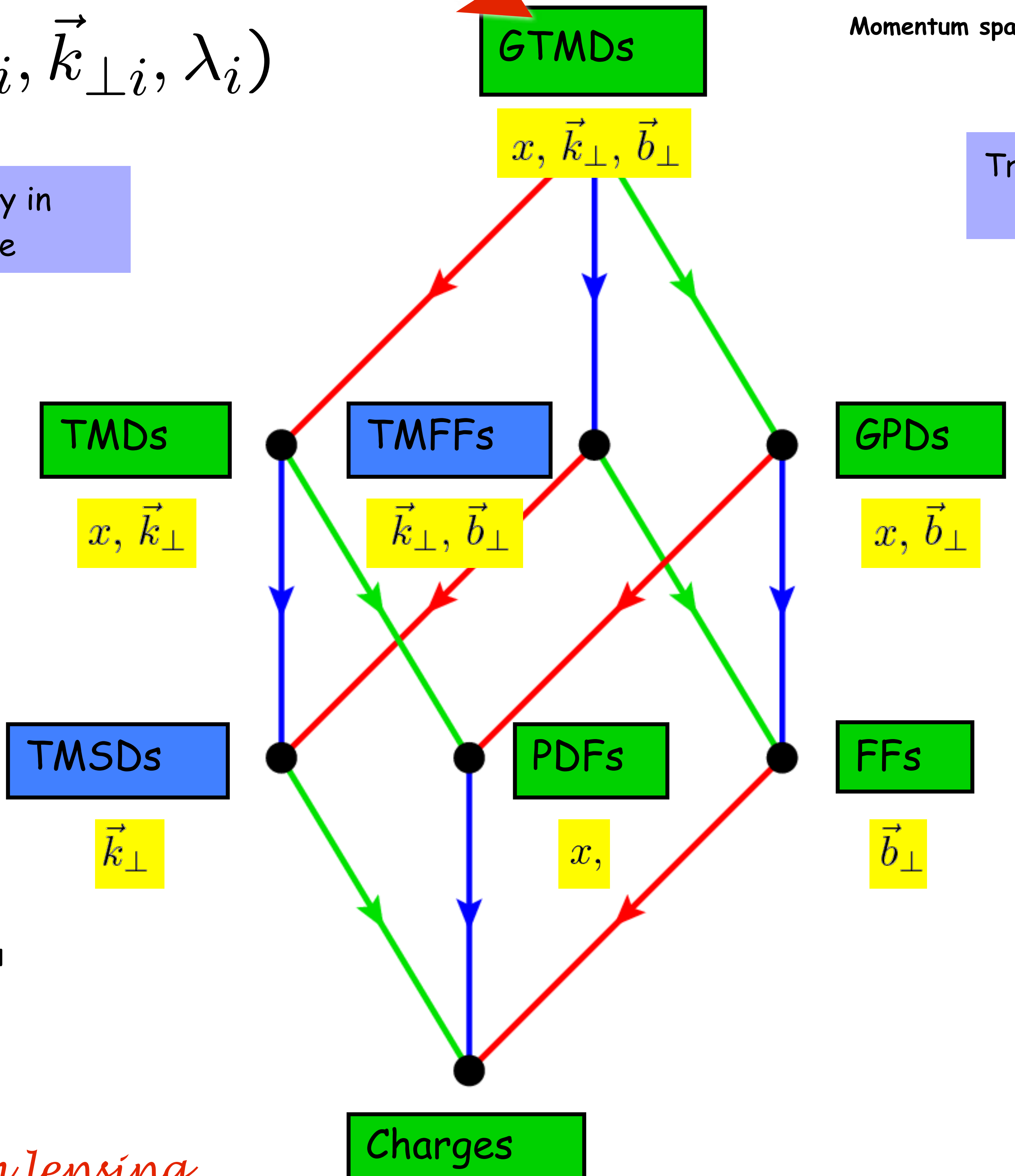
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

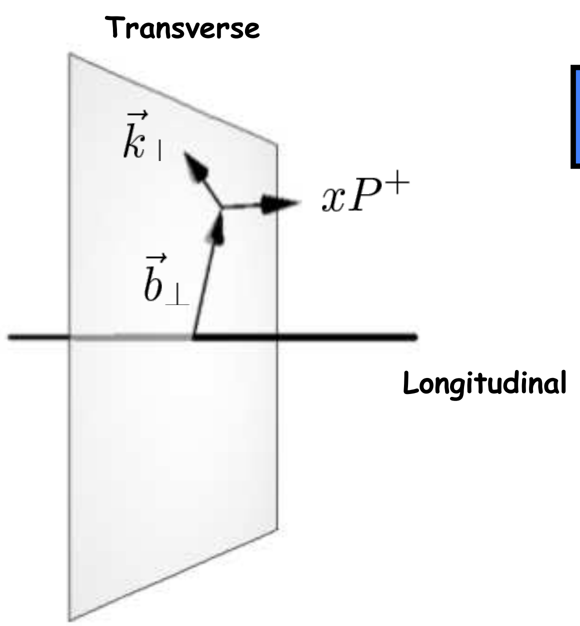
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

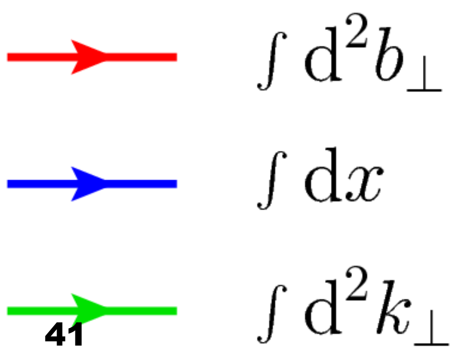
Transverse density in momentum space

Transverse density in position space



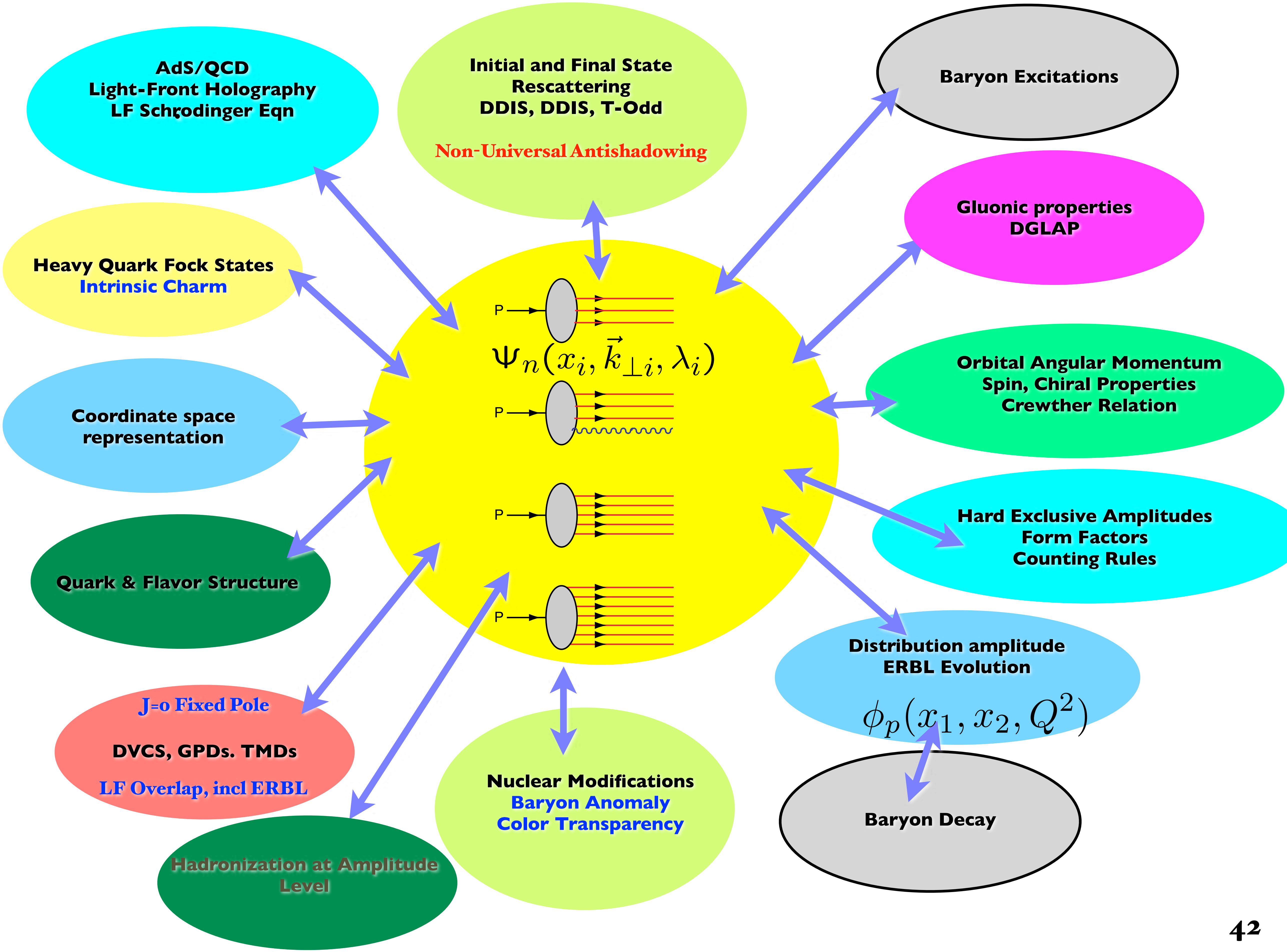
Lorce, Pasquini





Sivers, T-odd from lensing

QCD and the LF Hadron Wavefunctions



Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

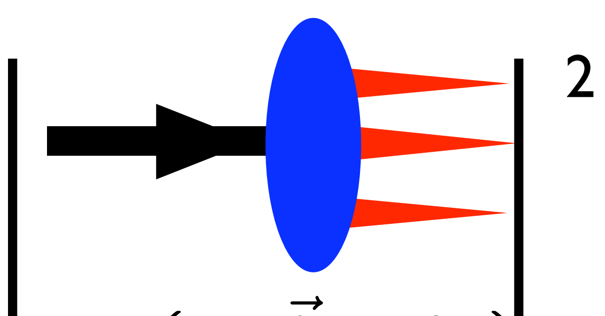
Hwang, Schmidt, sjb,

Mulders, Boer

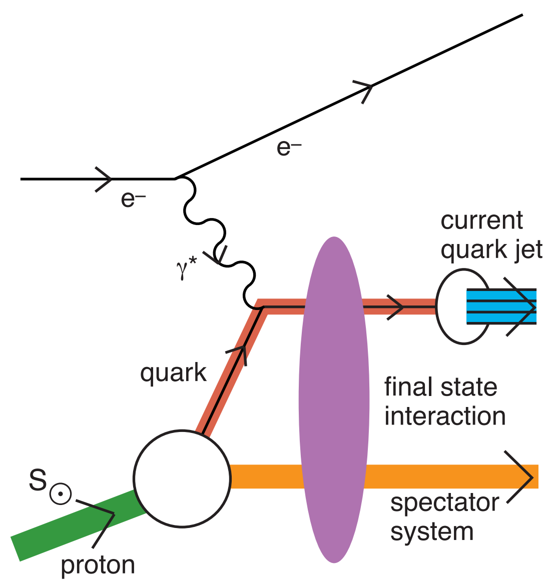
Qiu, Sterman

Collins, Qiu

Pasquini, Xiao, Yuan, sjb



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



Single-spin asymmetries

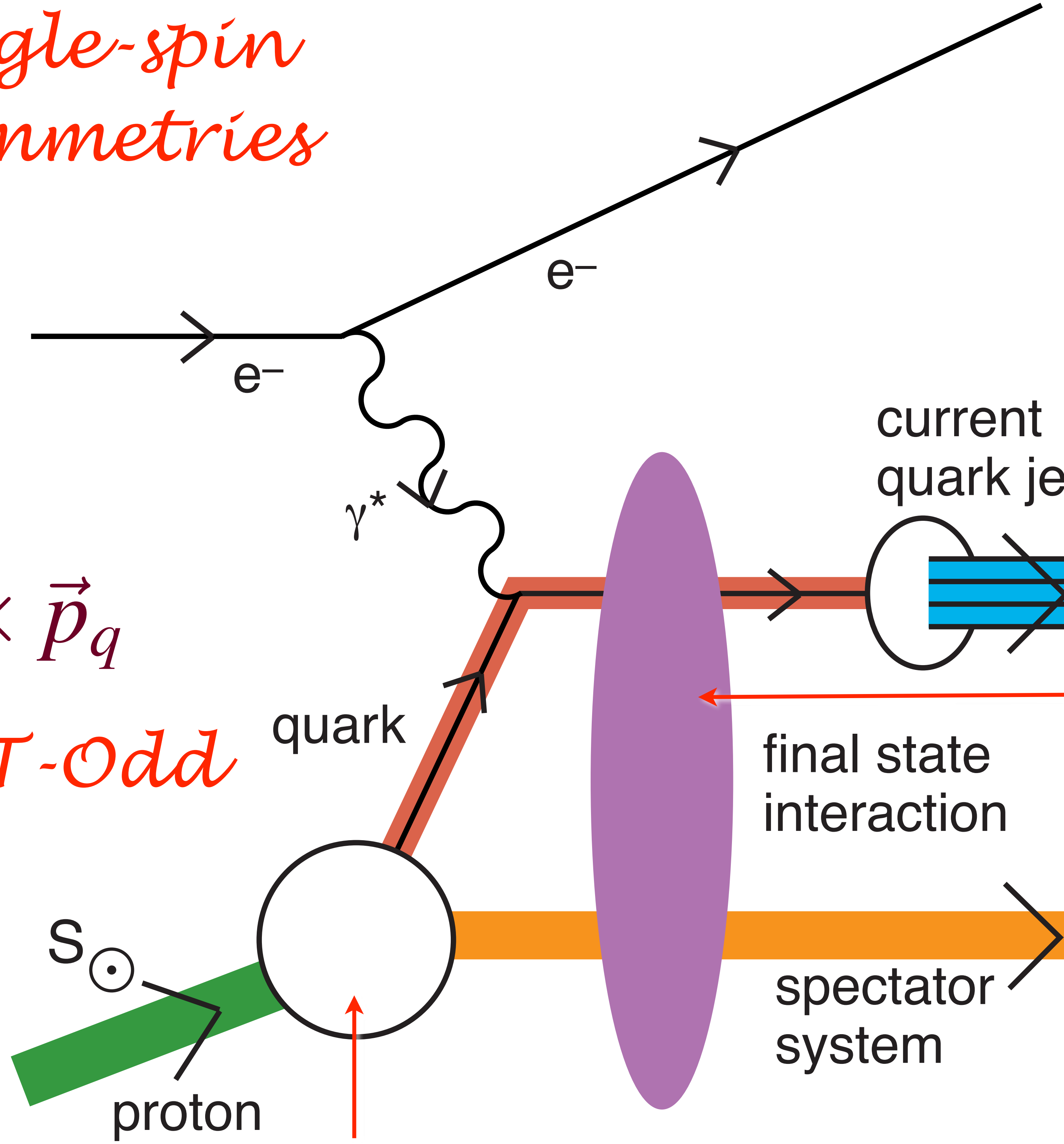
Leading-Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd



QCD S- and P- Coulomb Phases --Wilson Line

Analog of QED FSIs

Light-Front Wavefunction S and P- Waves

Remarkable Advantages of the Front Form

- **Light-Front Time-Ordered Perturbation Theory: Elegant, Physical**
- **Frame-Independent**
- **Few LF Time-Ordered Diagrams (not n!) -- all k^+ must be positive**
- **J^z conserved at each vertex**
- **LF Vacuum trivial up to zero modes**
- **Renormalization: Alternate Denominator Subtractions: Tested to three loops in QED**
- **Reproduces Parke-Taylor Rules and Amplitudes (Stasto)**
- **Hadronization at the Amplitude Level with Confinement**

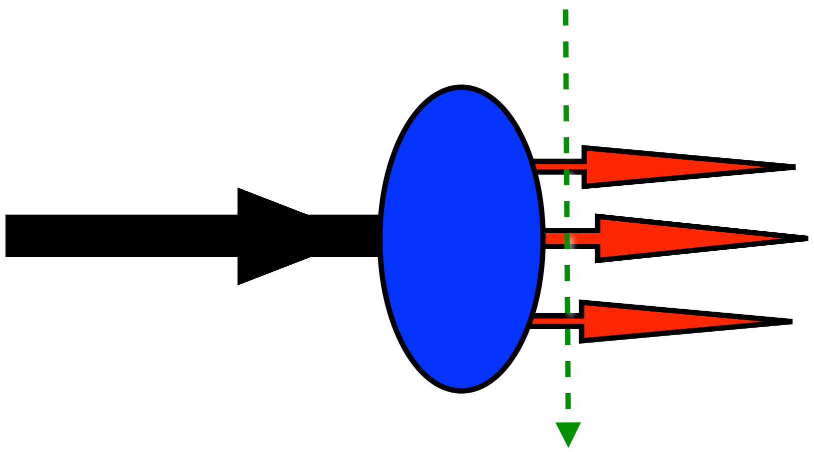
Advantages of the Dirac's Front Form for Hadron Physics

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent -- no boosts
- No dependence on observer's frame
- Dual to AdS/QCD
- LF Vacuum trivial -- no vacuum condensates
- Implications for Cosmological Constant

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

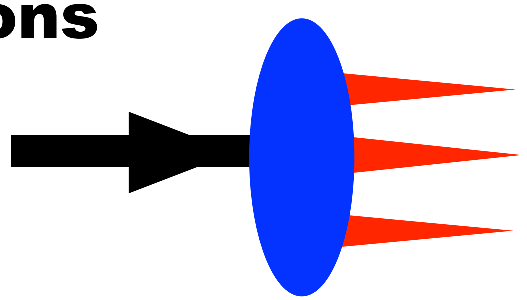
Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

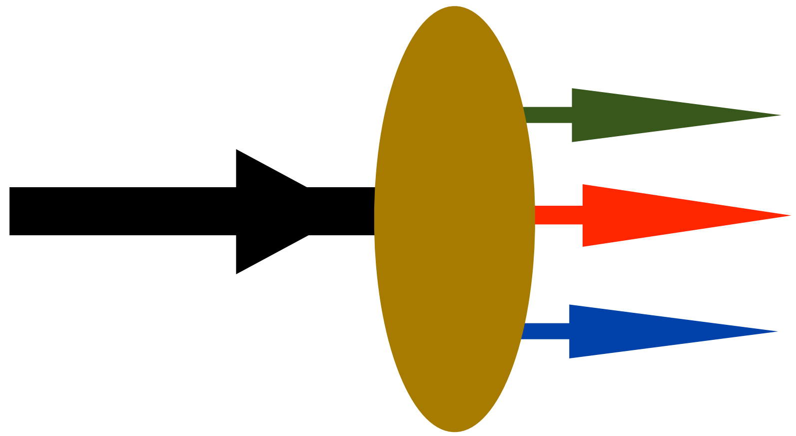
Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**
- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**
- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs**
- **Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!**
- **Hadron Physics without LFWFs is like Biology without DNA!**



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- *Hadron Physics without LFWFs is like Biology without DNA!*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Definition of the Vacuum in Quantum Field Theory

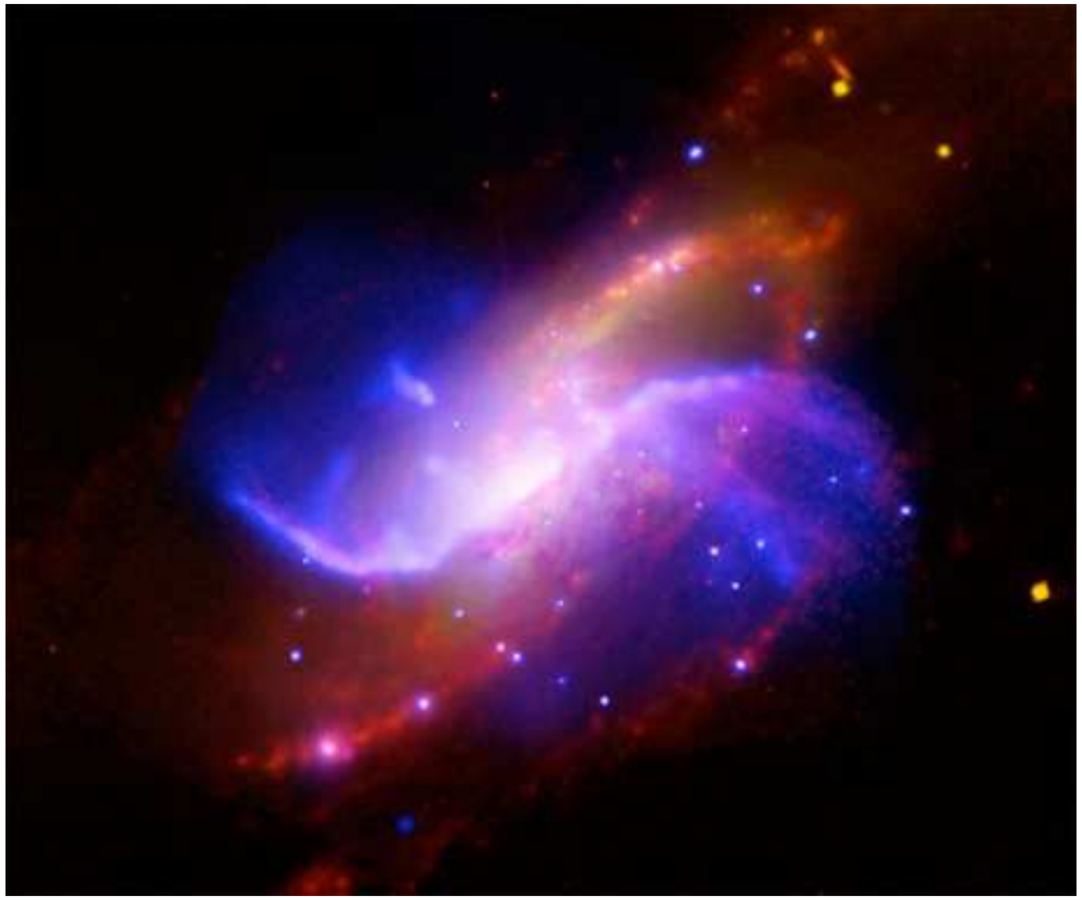
- Lowest Energy Ground State of the Hamiltonian

- The **ground state** of a quantum mechanical system is its lowest-energy state; the energy of the ground state is known as the zero-point energy of the system. An excited state is any state with energy greater than the ground state. The ground state of a quantum field theory is usually called the vacuum state or the vacuum.

But which Hamiltonian to use? Front Form or Instant Form?

We view the universe as light reaches us along the light-front at fixed

$$\tau = t + z/c$$



Front Form Vacuum Describes the Empty, Causal Universe

Two Definitions of Vacuum State

Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian

$$H|\psi_0 >= E_0|\psi_0 >, E_0 = min\{E_i\}$$

Eigenstate defined at one time t over all space; Acausal! Frame-Dependent

Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian

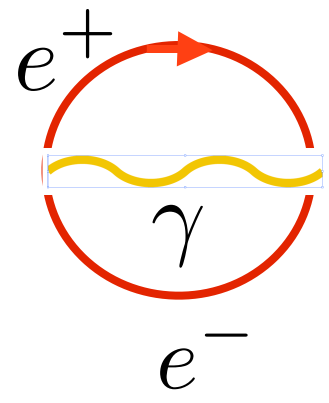
$$H_{LF}|\psi_0 >_{LF}= M_0^2|\psi_0 >_{LF}, M_0^2 = 0.$$

Frame-independent eigenstate at fixed LF time τ = t+z/c within causal horizon

Front Form Vacuum Describes the Empty, Causal Universe

Front-Form Vacuum in QED

$P^+ = 0$



$\sum_i k_i^+ \neq P^+ = 0$

$k_i^+ > 0$

- **All Light-Front Vacuum Graphs Vanish!**
- **Light-Front Vacuum is trivial since all plus momenta are positive and conserved. Zero energy density in LF Vacuum.**
- **Zero modes ($k^+=0$) in vacuum allowed in some theories with massless fermions.**
- **Zero contribution to Λ from QED LF Vacuum**

Calphysics Home | Research | Scientific Articles | Popular Articles | Recommended Books

Nature of Mass | Origin of Inertia | Gravitation | **Zero-Point Energy** | Questions and Answers

The Casimir force is widely cited as evidence that underlying the universe there must be a sea of real zero-point energy. This argument follows from Casimir's analysis and prediction. It is not necessarily true, however. It is perfectly possible to explain the Casimir effect by taking into account the quantum-induced motions of atoms in each plate and examining the retarded potential interactions of atoms in one plate with those in the other.

Phys. Rev. D 72, 021301(R) (2005)

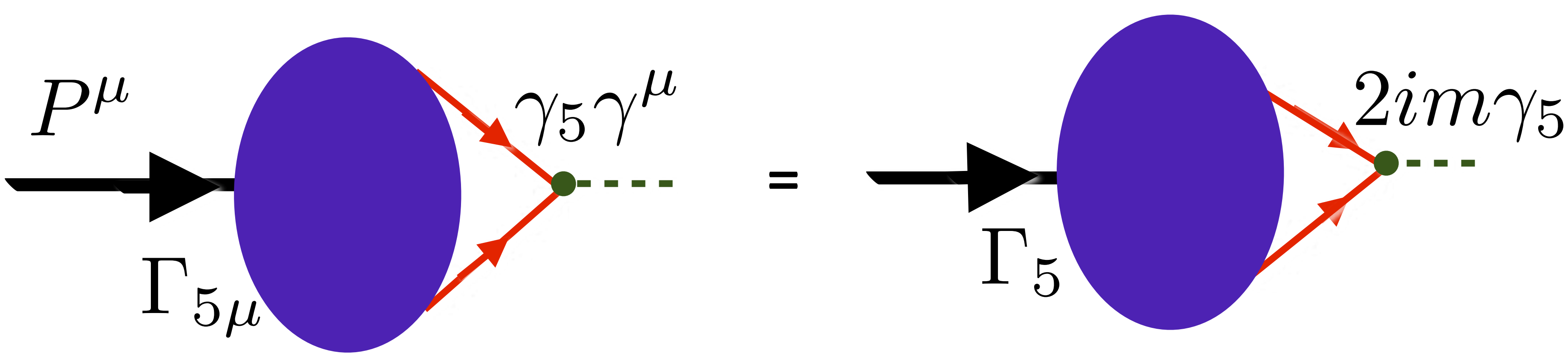
Casimir effect and the quantum vacuum

R.L. Jaffe

Ward-Takahashi Identity for axial current

$$P^\mu \Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k+P/2)i\gamma_5 + i\gamma_5 S^{-1}(k-P/2)$$

$$S^{-1}(\ell) = i\gamma\cdot\ell A(\ell^2) + B(\ell^2) \qquad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$



plus non-pole

Identify pion pole at $P^2 = m_\pi^2$

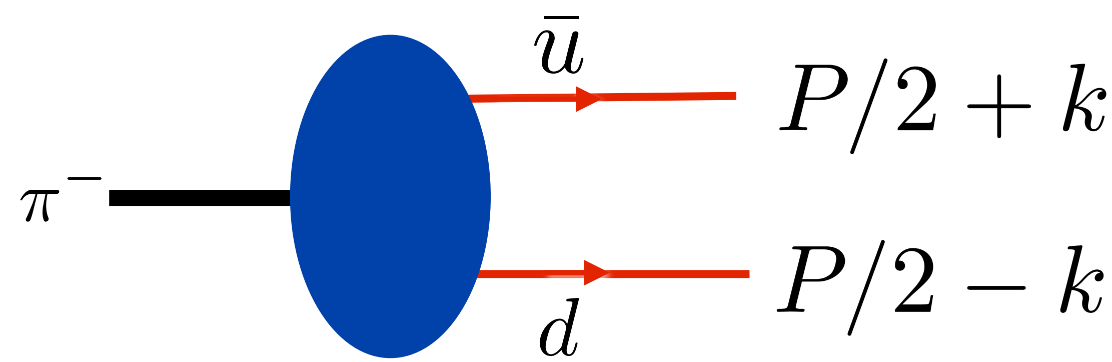
$$P^\mu < 0|\bar{q}\gamma_5\gamma^\mu q|\pi >= 2m < 0|\bar{q}i\gamma_5 q|\pi >$$

$$f_\pi m_\pi^2 = -(m_u + m_d)\rho_\pi$$

General Form of Bethe-Salpeter Wavefunction

$$\Gamma_\pi(k;P) = i\gamma_5 E_\pi(k,P) + \gamma_5\gamma\cdot P F_\pi(k;P)$$
$$+\gamma_5\gamma\cdot k G_\pi(k;P) - \gamma_5\sigma_{\mu\nu}k^\mu P^\nu H_\pi(k;P)$$

$\Gamma_\pi(k;P)$

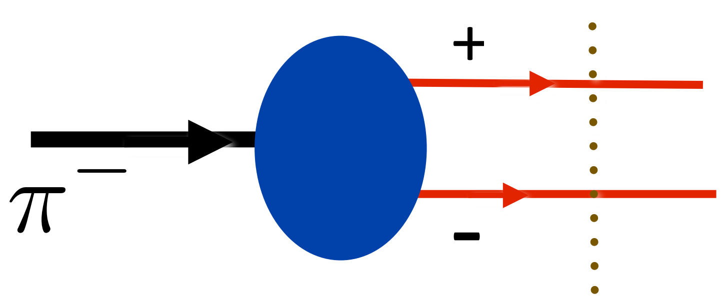


Imaging dynamical chiral symmetry breaking: pion wave function on the light front

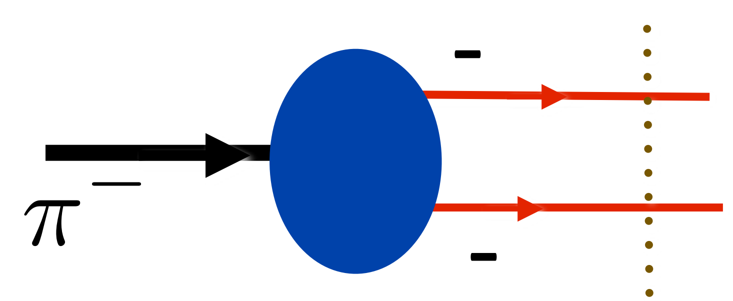
Lei Chang, I.C. Cloet, J.J. Cobos-Martinez, C.D. Roberts, S.M. Schmidt, P.C. Tandy

Allows both $< 0|\bar{q}\gamma_5\gamma_\mu q|\pi >$ and $< 0|\bar{q}\gamma_5 q|\pi >$ LFWFs

$S^z = 0, L^z = 0$



$S^z = -1, L^z = +1$



Revised Gell Mann-Oakes-Renner Formula in QCD

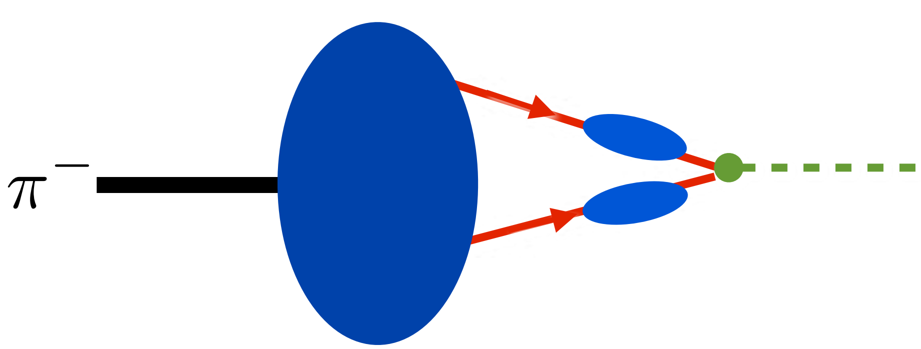
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is an "in-hadron condensate"

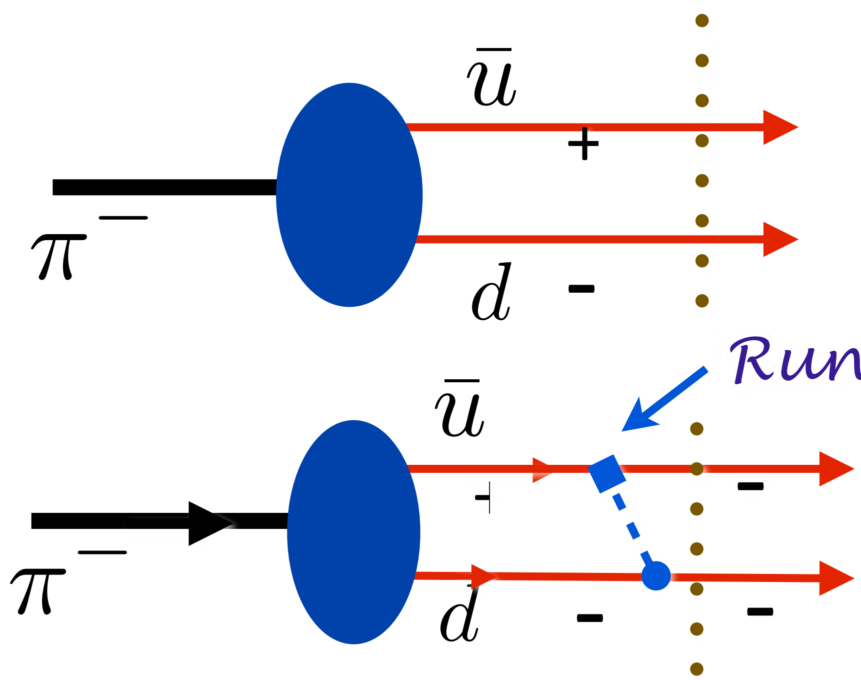


$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

Light-Front Pion Valence Wavefunctions

$S^z_{\bar{u}} + S^z_d = +1/2 - 1/2 = 0$



$S^z_{\bar{u}} + S^z_d = -1/2 - 1/2 = -1$

Couples to

$$L^z = 0, S^z = 0 \quad < \pi|\bar{\gamma}^\mu q \gamma_5 q|0 > \sim f_\pi$$

Running constituent mass at vertex

Couples to

$$L^z = +1, S^z = -1 \quad < \pi|\bar{q} \gamma_5 q|0 > \ \sim \rho_\pi$$

Angular Momentum Conservation

$$J^z = \sum_i^n S^z_i + \sum_i^{n-1} L^z_i$$

PHYSICAL REVIEW C **82**, 022201(R) (2010)

New perspectives on the quark condensate

Stanley J. Brodsky,[1,2] Craig D. Roberts,[3,4] Robert Shrock,[5] and Peter C. Tandy[6]

[1] *SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA*

[2] *Centre for Particle Physics Phenomenology: CP^3-Origins, University of Southern Denmark, Odense 5230 M, Denmark*

[3] *Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

[4] *Department of Physics, Peking University, Beijing 100871, China*

[5] *C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*

[6] *Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*

(Received 25 May 2010; published 18 August 2010)

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

Summary on QCD `Condensates'

- Condensates do not exist as space-time-independent phenomena -- consistent with LF Theory

- Property of hadron wavefunctions: Bethe-Salpeter or Light-Front: "In-Hadron Condensates"

- Find: $$\frac{<0|\bar{q}q|0>}{f_\pi} \rightarrow -<0|i\bar{q}\gamma_5 q|\pi>=\rho_\pi$$
  $$<0|\bar{q}i\gamma_5 q|\pi> \text{ similar to } <0|\bar{q}\gamma^\mu\gamma_5 q|\pi>$$

- Zero contribution to cosmological constant! Included in hadron mass

- ρ_π survives for small m_q -- enhanced running mass from gluon loops / multiparton Fock states

Quark and Gluon condensates reside within hadrons, not vacuum

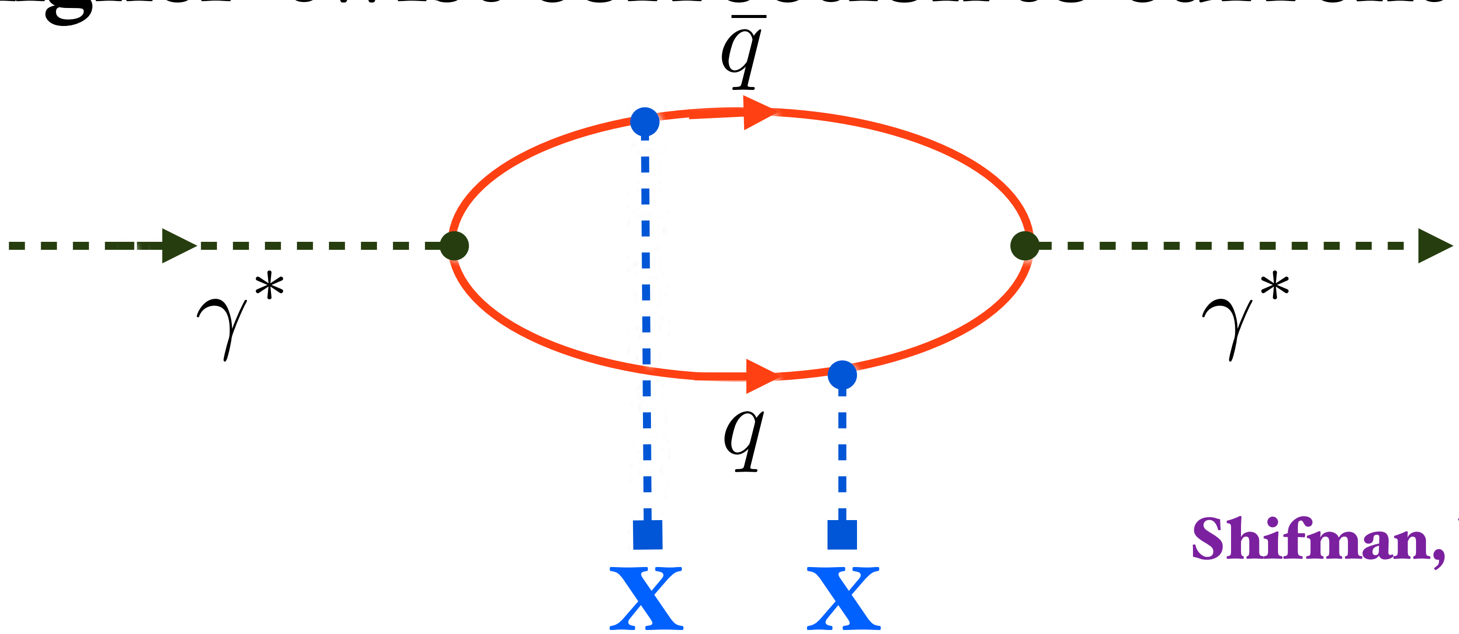
Casher and Susskind Maris, Roberts, Tandy Shrock and sjb

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

Is there empirical evidence for a gluon vacuum condensate?

$$< 0|\frac{\alpha_s}{\pi}G^{\mu\nu}(0)G_{\mu\nu}(0)|0 >$$

Look for higher-twist correction to current propagator



Shifman, Vainshtein, Zakharov

$$e^+e^- \to X,\ \tau \text{ decay},\ Q\bar{Q} \text{ phenomenology}$$

$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \frac{\alpha_s}{\pi}\frac{\Lambda^4_{\rm QCD}}{s^2} + \cdots)$$

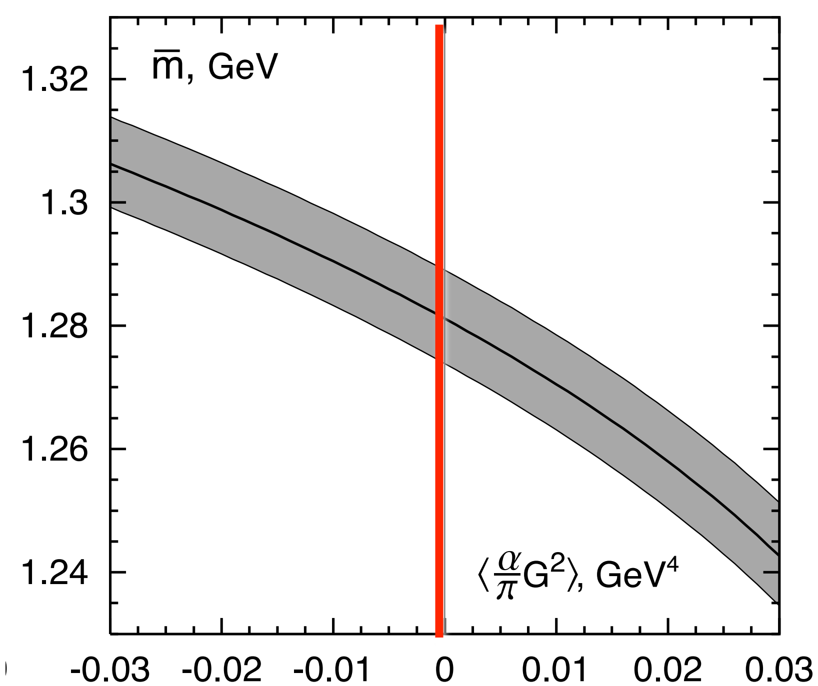
Determinations of the vacuum Gluon Condensate

$$< 0|\frac{\alpha_s}{\pi} G^2|0 > [\mathrm{GeV}^4]$$

$$-0.005 \pm 0.003 \text{ from } \tau \text{ decay.}$$ Davier et al.

$$+0.006 \pm 0.012 \text{ from } \tau \text{ decay.}$$ Geshkenbein, Ioffe, Zyablyuk

$$+0.009 \pm 0.007 \text{ from charmonium sum rules}$$ Ioffe, Zyablyuk

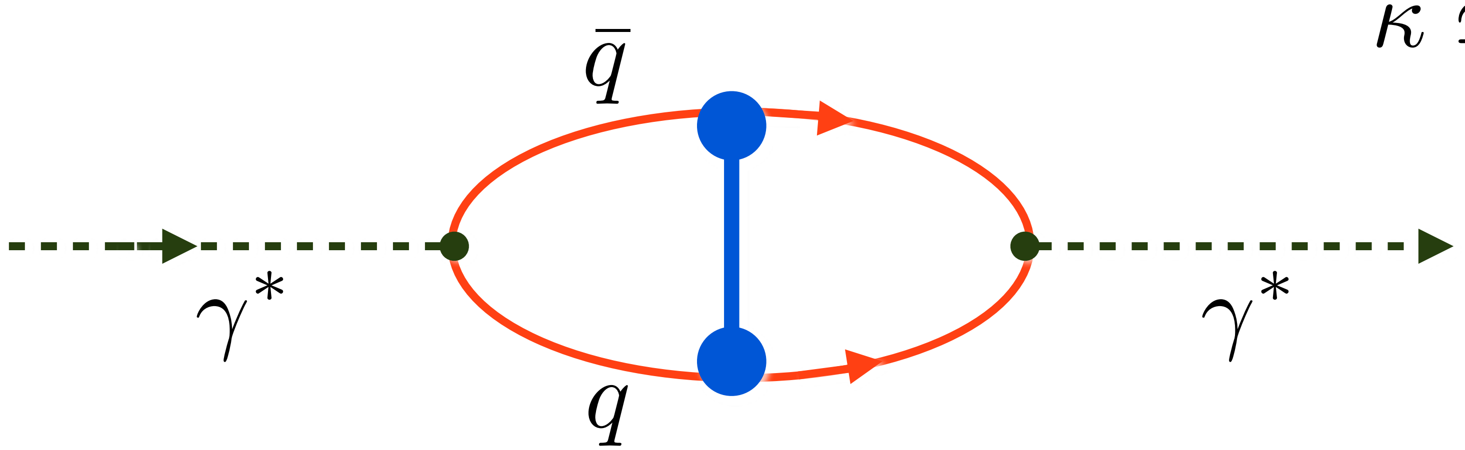


Consistent with zero vacuum condensate

Effective Confinement potential from soft-wall AdS/QCD gives Regge Spectroscopy plus higher-twist correction to current propagator

$$M^2 = 4\kappa^2(n + L + S/2)$$ light-quark meson spectra

$$\kappa \simeq 0.5 \text{ GeV}$$



$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \mathcal{O}\frac{\kappa^4}{s^2} + \cdots)$$

mimics dimension-four gluon condensate $< 0|\frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0)|0 >$ *in*

$e^+e^- \to X$, τ decay, $Q\bar{Q}$ phenomenology

- de Teramond, Dosch, sjb

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z) \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

Derived from variation of Action Dilaton-Modified AdS_5

PHYSICAL REVIEW D **66**, 045019 (2002)

Light-front formulation of the standard model

Prem P. Srivastava*
Instituto de Física, Universidade do Estado de Rio de Janeiro, RJ 20550, Brazil,
Theoretical Physics Department, Fermilab, Batavia, Illinois 60510,
and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

Stanley J. Brodsky†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309
(Received 20 February 2002; published 20 August 2002)

Light-front (LF) quantization in the light-cone (LC) gauge is used to construct a renormalizable theory of the standard model. The framework derived earlier for QCD is extended to the Glashow-Weinberg-Salam (GWS) model of electroweak interaction theory. The Lorentz condition is automatically satisfied in LF-quantized QCD in the LC gauge for the free massless gauge field. In the GWS model, with the spontaneous symmetry breaking present, we find that the 't Hooft condition accompanies the LC gauge condition corresponding to the massive vector boson. The two transverse polarization vectors for the massive vector boson may be chosen to be the same as found in QCD. The nontransverse and linearly independent third polarization vector is found to be parallel to the gauge direction. The corresponding sum over polarizations in the standard model, indicated by $K_{\mu\nu}(k)$, has several simplifying properties similar to the polarization sum $D_{\mu\nu}(k)$ in QCD. The framework is unitary and ghost free (except for the ghosts at $k^+=0$ associated with the light-cone gauge prescription). The massive gauge field propagator has well-behaved asymptotic behavior. The interaction Hamiltonian of electroweak theory can be expressed in a form resembling that of covariant theory, plus additional instantaneous interactions which can be treated systematically. The LF formulation also provides a transparent discussion of the Goldstone boson (or electroweak) equivalence theorem, as the illustrations show.

P. Srivastava, sjb

Abelian U(1) LF Model with Spontaneous Symmetry Breaking

$$\mathcal{L} = \partial_+\phi^\dagger\partial_-\phi + \partial_-\phi^\dagger\partial_+\phi - \partial_\perp\phi^\dagger\partial_\perp\phi - \mathcal{V}(\phi^\dagger\phi)$$

where $V(\phi^\dagger\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$ with $\lambda > 0,\ \mu^2 < 0$

Constraint equation: $\int d^2x_\perp dx^- \left[\partial_\perp\partial_\perp\phi - \frac{\delta V}{\delta\phi^\dagger}\right] = 0$

$$\phi(\tau, x^-, x_\perp) = \omega(\tau, x_\perp) + \varphi(\tau, x^-, x_\perp)$$

$\omega(\tau, x_\perp)$ is a $k^+ = 0$ zero mode

$\omega = v/\sqrt{2}$ where $v = \sqrt{-\mu^2/\lambda}$

Thus a c-number in LF replaces conventional Higgs VEV

No coupling to gravity!

Possibility: $\partial_\perp\omega \neq 0$

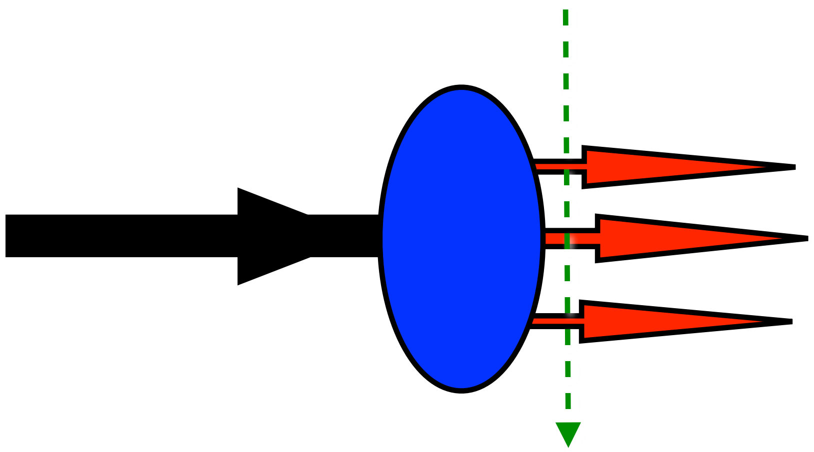
P. Srivastava, sjb

Standard Model on the Light-Front

- **Same phenomenological predictions**
- **Higgs field has three components**
- **Real part creates Higgs particle**
- **Imaginary part (Goldstone) become longitudinal components of W, Z**
- ***Higgs VEV of instant form becomes $k^+=0$ LF zero mode!***
- **Analogous to a background static classical Zeeman or Stark Fields**
- **Zero contribution to $T^\mu{}_\mu$; zero coupling to gravity**

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD} |\psi> = M^2 |\psi>$$

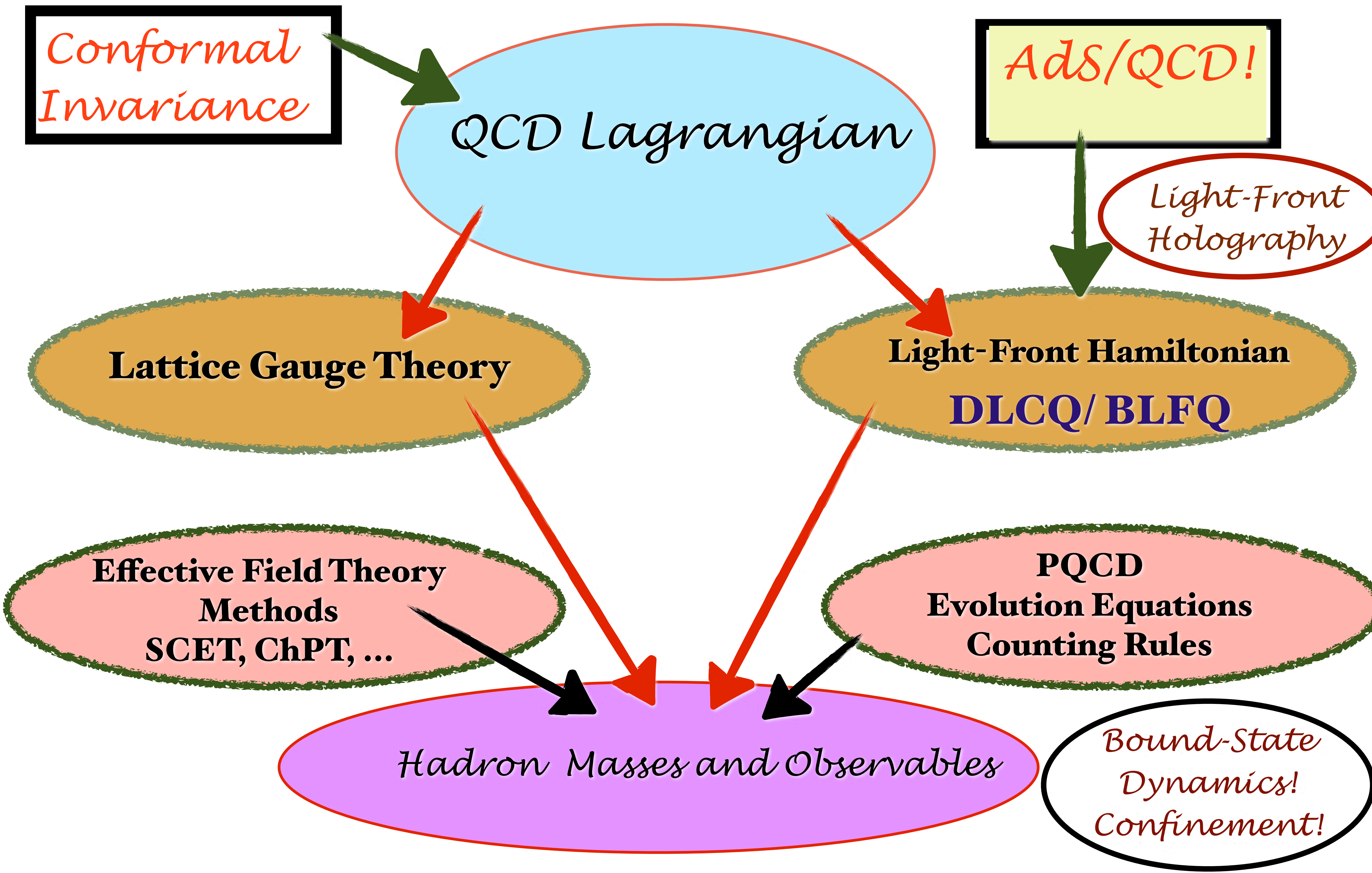
Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- Confinement in QCD -- What sets the QCD mass scale?
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Structure Functions,Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Insights into QCD Condensates
- Chiral Symmetry
- Systematically improvable

Predict Hadron Properties from First Principles!



QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics

quark kinetic energy + quark-gluon dynamics

quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement
Classically Conformal if m_q=0

QCD Mass Scale from Confinement not Explicit

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi> = E\ |\Psi>$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

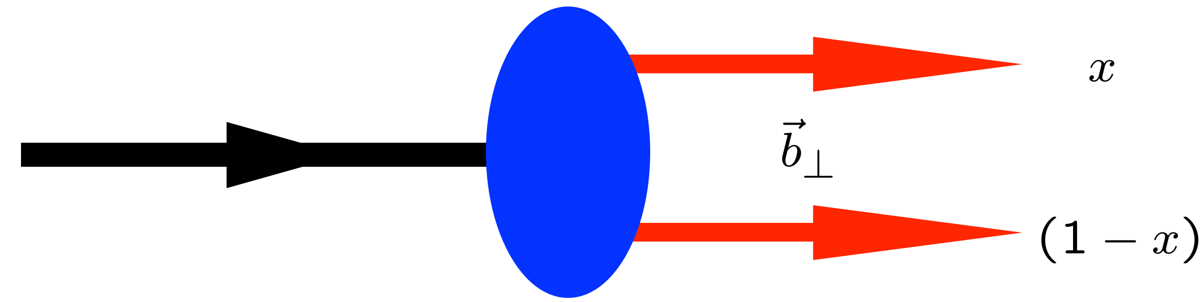
Coulomb potential

Semiclassical first approximation to QED --> **Bohr Spectrum**

G. de Teramond, sjb

Light-Front QCD

$$H^{LF}_{QCD}$$



$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}] \; \psi_{LF}(x, \vec{k}_\perp) = M^2 \; \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)] \; \psi_{LF}(\zeta) = M^2 \; \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)b^2_\perp$$

Azimuthal Basis ζ, ϕ

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

Semiclassical first approximation to QCD

Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

$$\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left|\psi(x,\vec{k}_\perp)\right|^2 + \text{interactions}$$

$$= \int_0^1 \frac{dx}{x(1-x)} \int d^2\vec{b}_\perp \, \psi^*(x,\vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_{\perp\ell}}\right) \psi(x,\vec{b}_\perp) + \text{interactions}.$$

Change variables $(\vec{\zeta},\varphi),\ \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta}\frac{d}{d\zeta}\left(\zeta\frac{d}{d\zeta}\right) + \frac{1}{\zeta^2}\frac{\partial^2}{\partial\varphi^2}$

$$\mathcal{M}^2 = \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta) U(\zeta)\phi(\zeta)$$

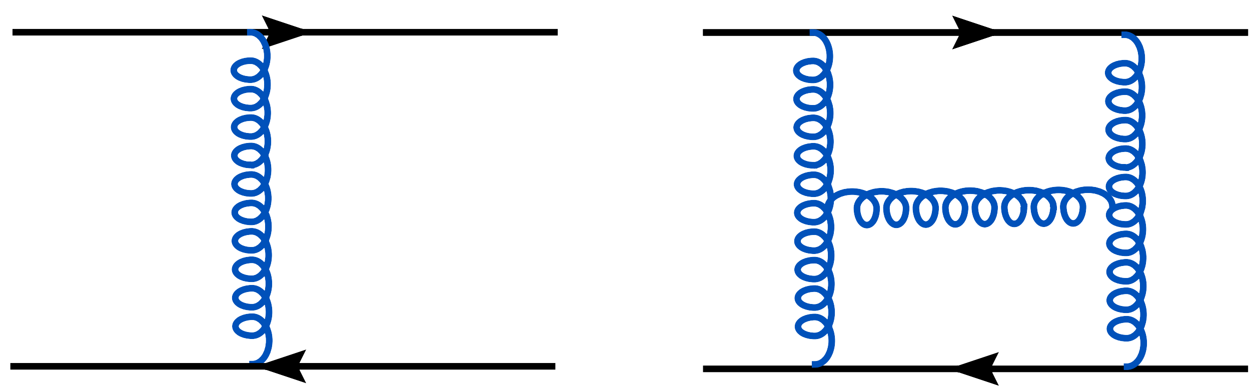
$$= \int d\zeta\, \phi^*(\zeta)\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta)$$

Heavy Quark Potential is IR Divergent in QCD

$$V(Q^2) = -\frac{(4\pi)^2 C_F}{Q^2} a(Q^2) \Big[1 + (c_{2,0} + c_{2,1} N_f) a(Q^2) + (c_{3,0} + c_{3,1} N_f + c_{3,2} N_f^2) a(Q^2)^2$$

$$+ (c_{4,0} + c_{4,1} N_f + c_{4,2} N_f^2 + c_{4,3} N_f^3) a(Q^2)^3 + 8\pi^2 C_A^3 \ln \frac{\mu_{IR}^2}{Q^2} a(Q^2)^3 \Big]$$

Smirnov, Smirnov, Steinhauser, 2010



$$\log \kappa^2 \zeta^2$$

Summation of H graphs: confining potential

Confinement eliminates IR divergences

Self-consistent mass scale κ

Light-Front Schrödinger Equation

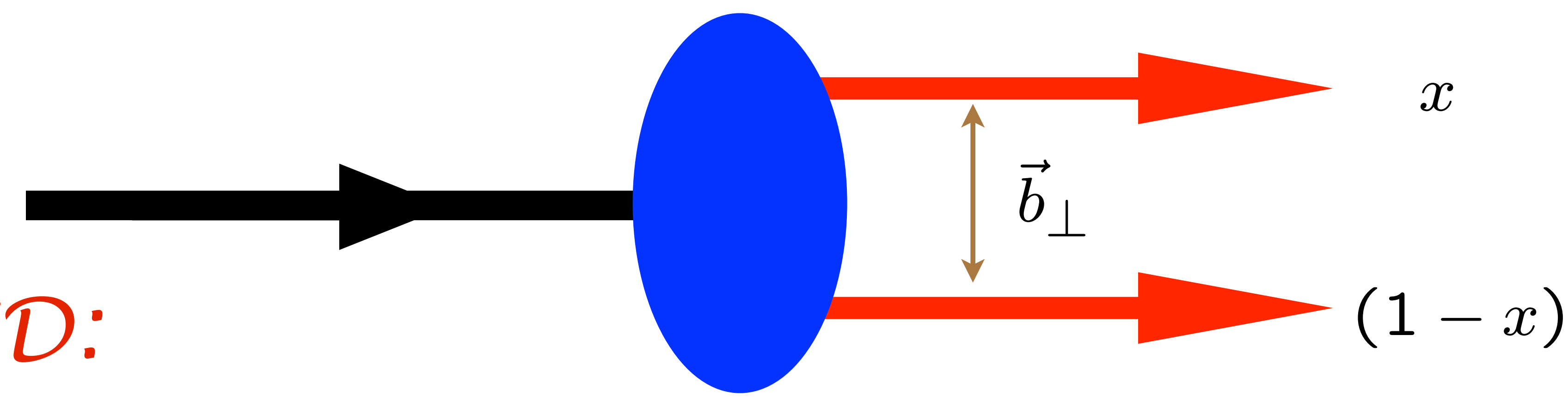
G. de Teramond, G. Dosch, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

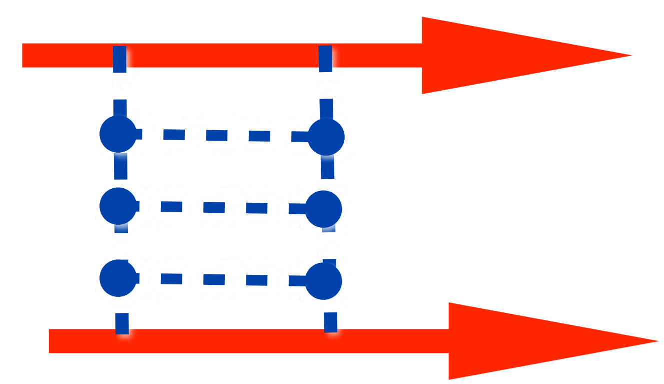
$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$



AdS/QCD:

U is the exact QCD potential
Conjecture: 'H'-diagrams generate

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$



The Light-Front Vacuum

Remarkable Features of Light-Front Schrödinger Equation

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Meson Spectrum in Soft Wall Model

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2 \left(n + \frac{J+L}{2}\right)$$

G. de Teramond, H. G. Dosch, sjb

Quark separation increases with L

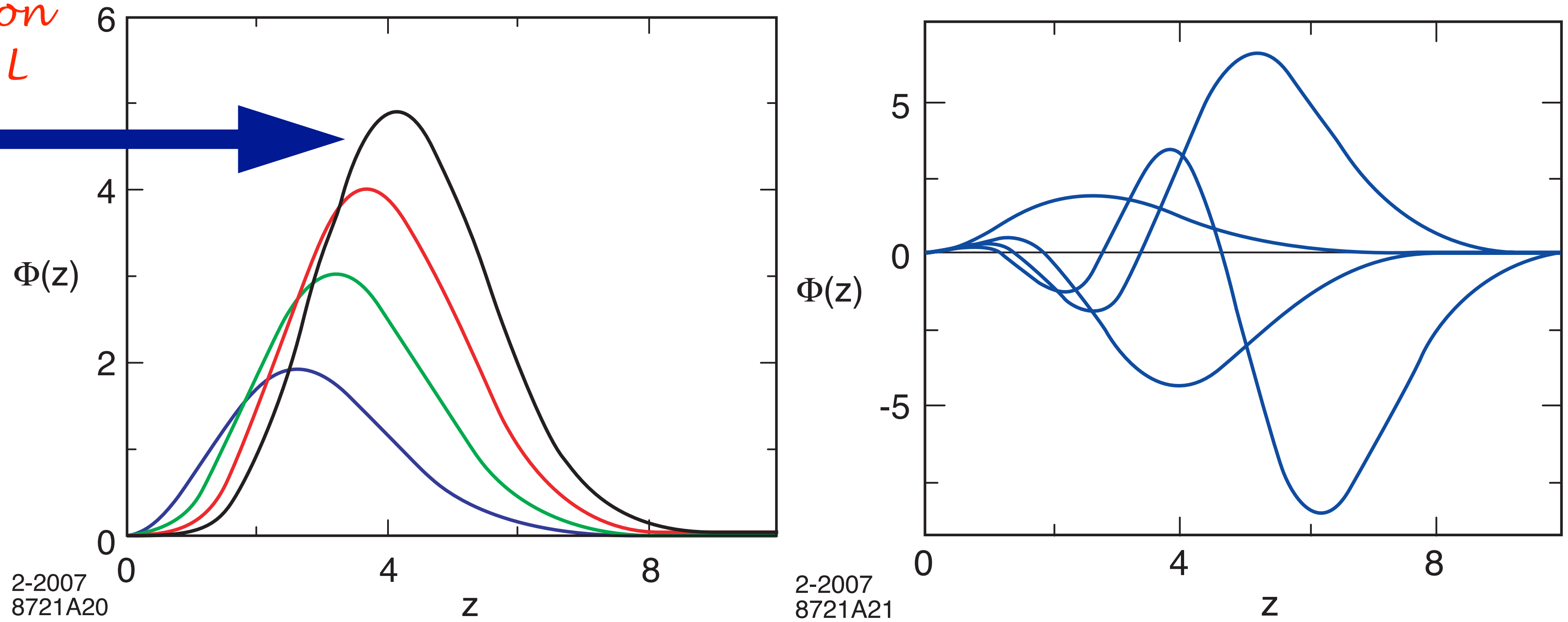
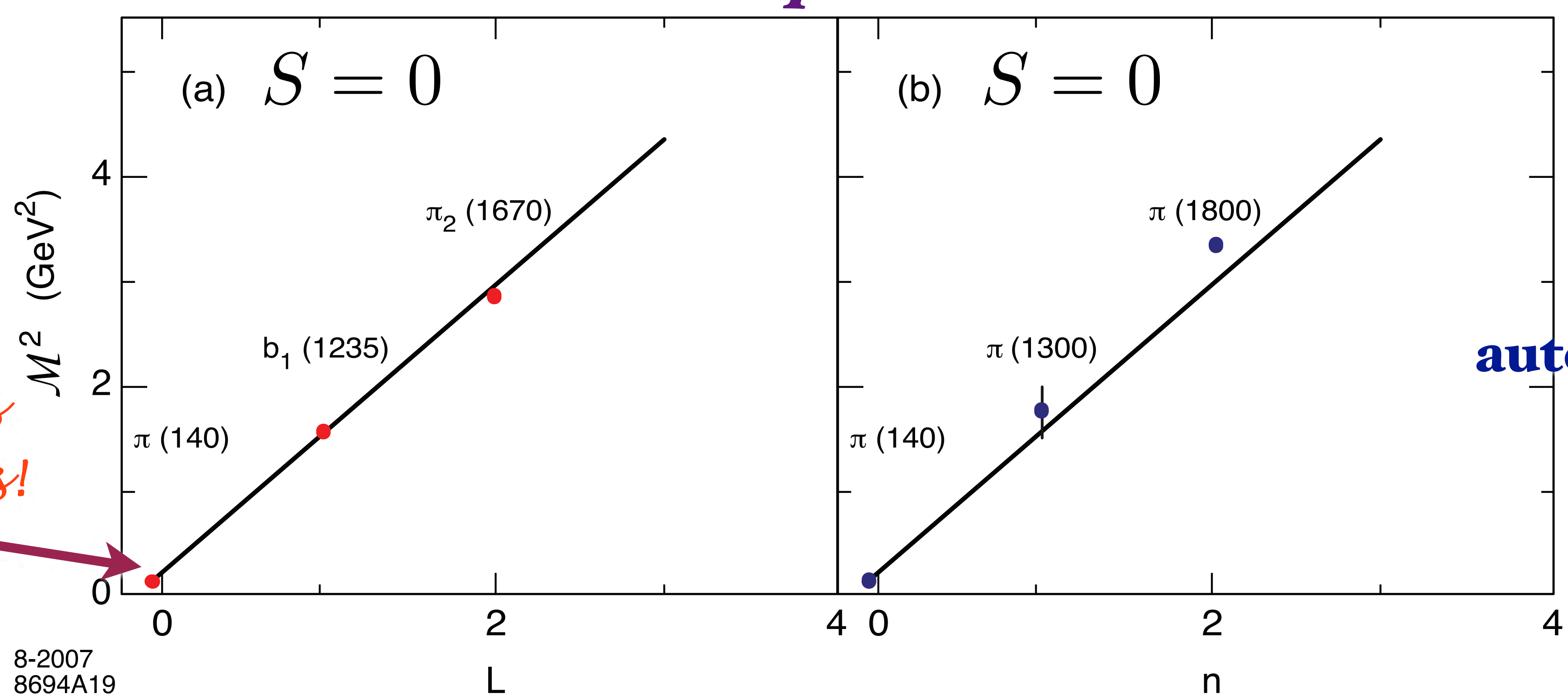


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Soft Wall Model

Same slope in n and L!



Pion has zero mass!

Pion mass automatically zero!

$$m_q = 0$$

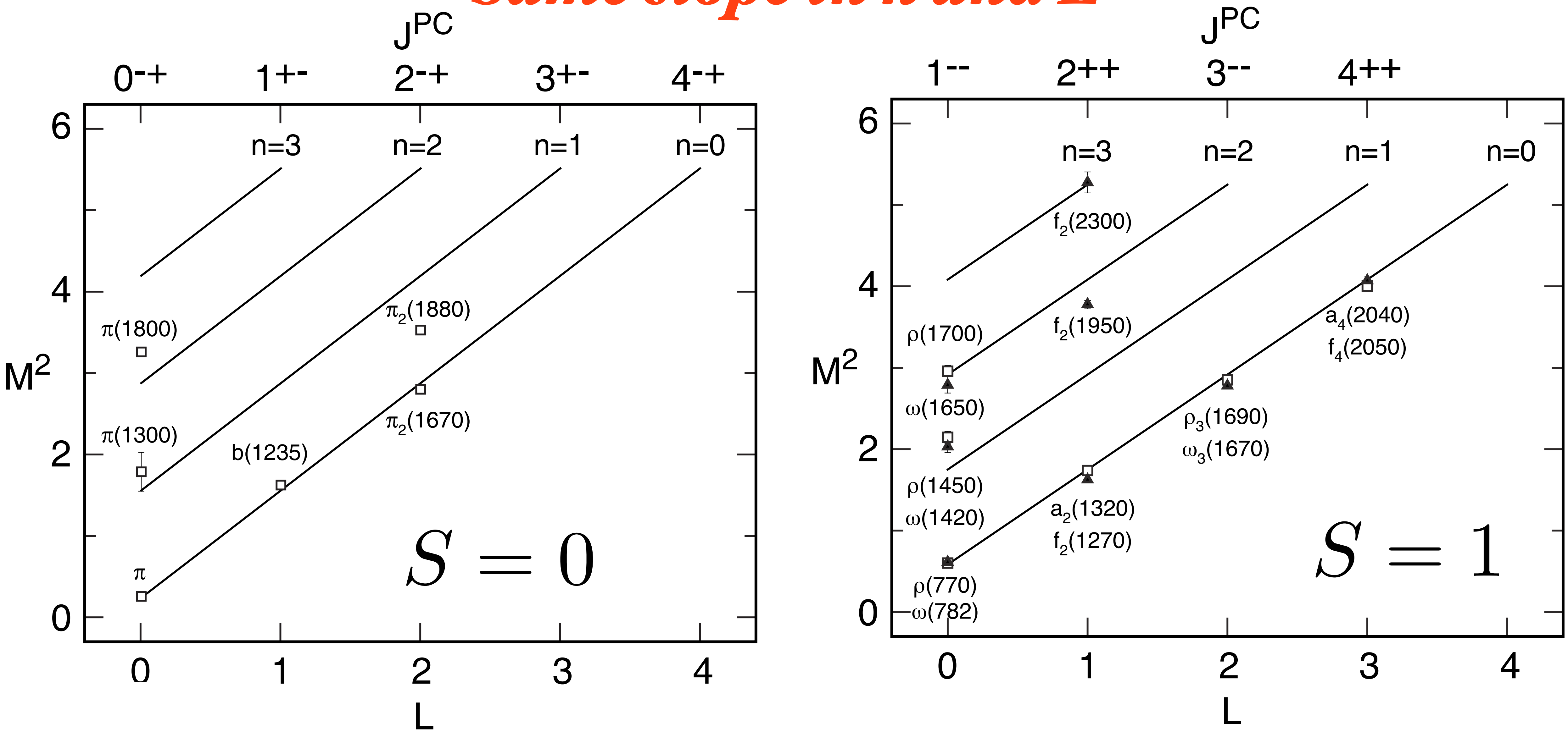
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Bosonic Modes and Meson Spectrum

$$\mathcal{M}^2 = 4\kappa^2(n + J/2 + L/2) \to 4\kappa^2(n + L + S/2)$$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

Same slope in n and L



Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I = 1$ ρ-meson and $I = 0$ ω-meson families ($\kappa = 0.54$ GeV)

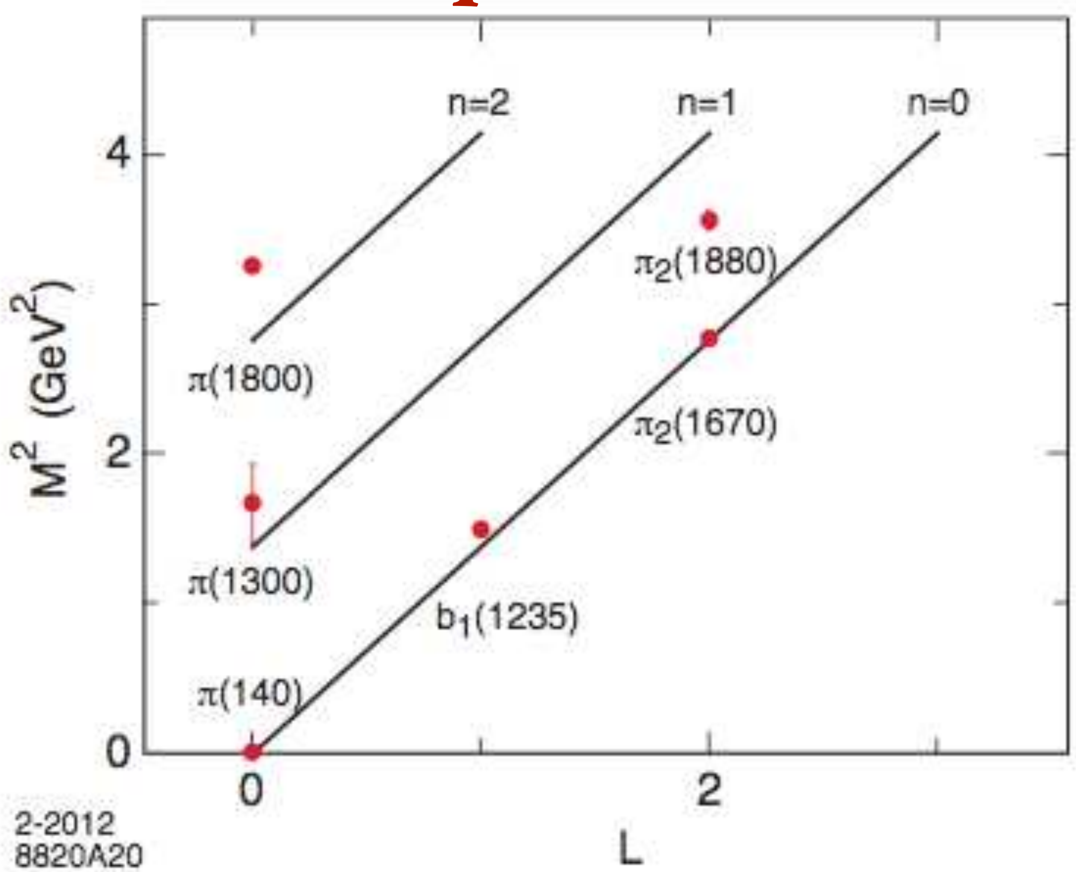
Balmer series of QCD

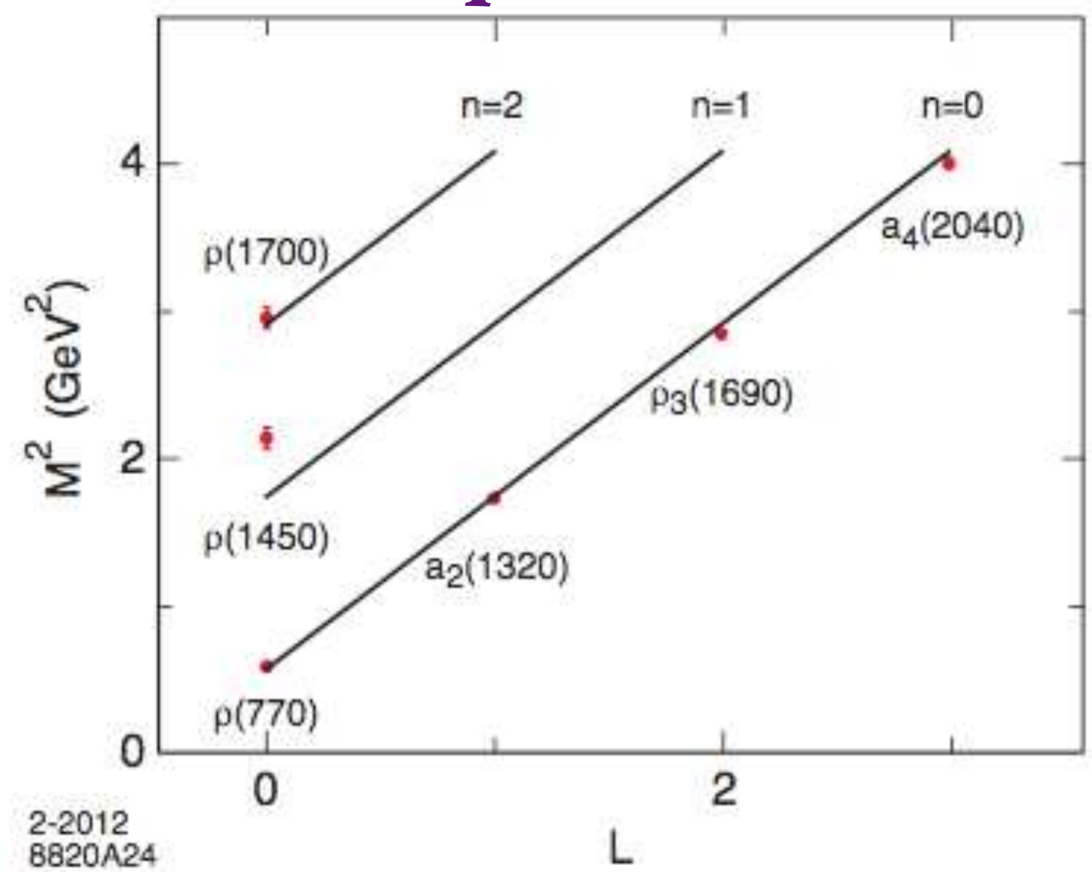
- $J = L + S, I = 1$ meson families $\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + L + S/2\right)$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$m_q = 0$

Massless pion in Chiral Limit! ***Same slope in n and L!***





I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

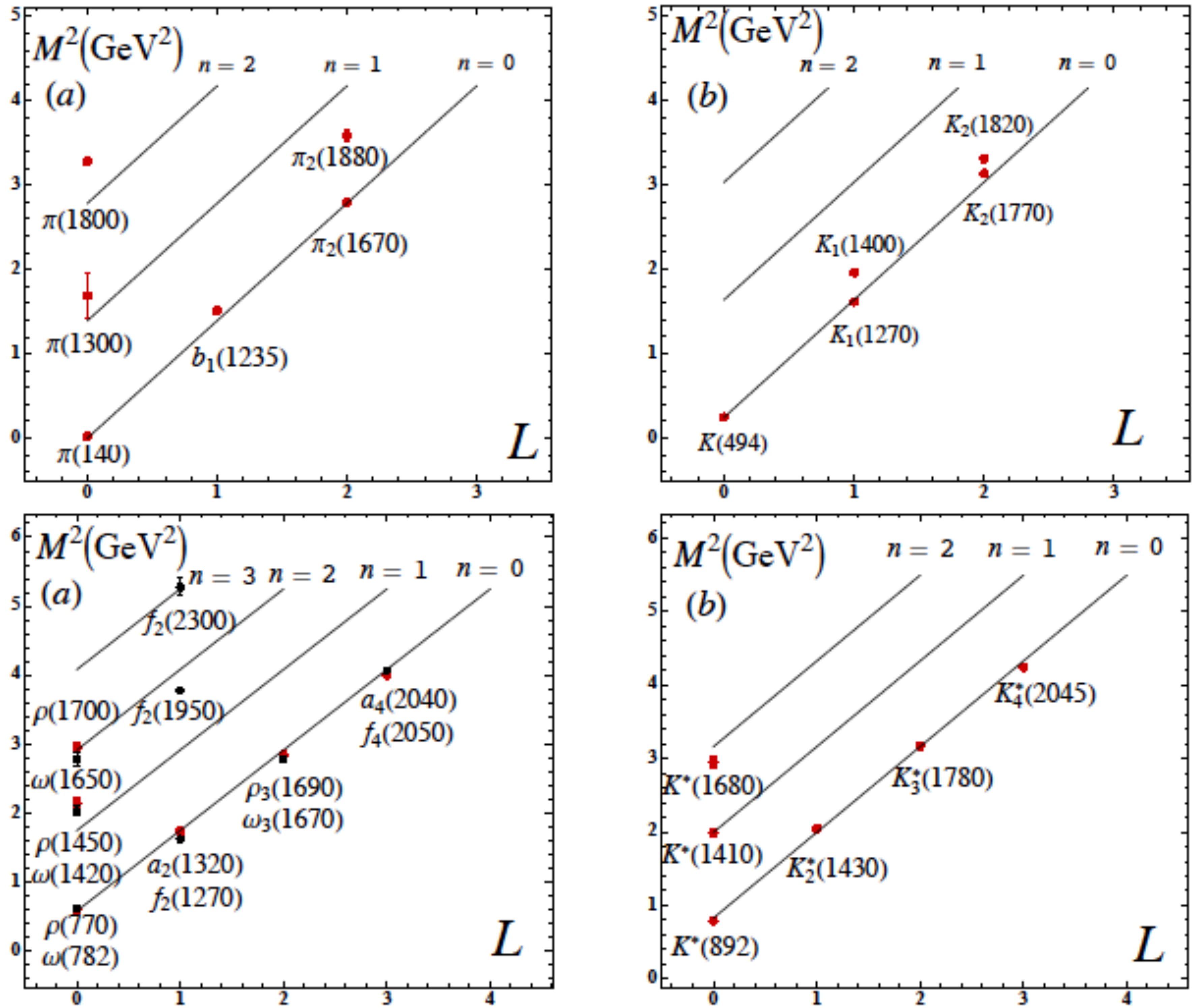
- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a-states

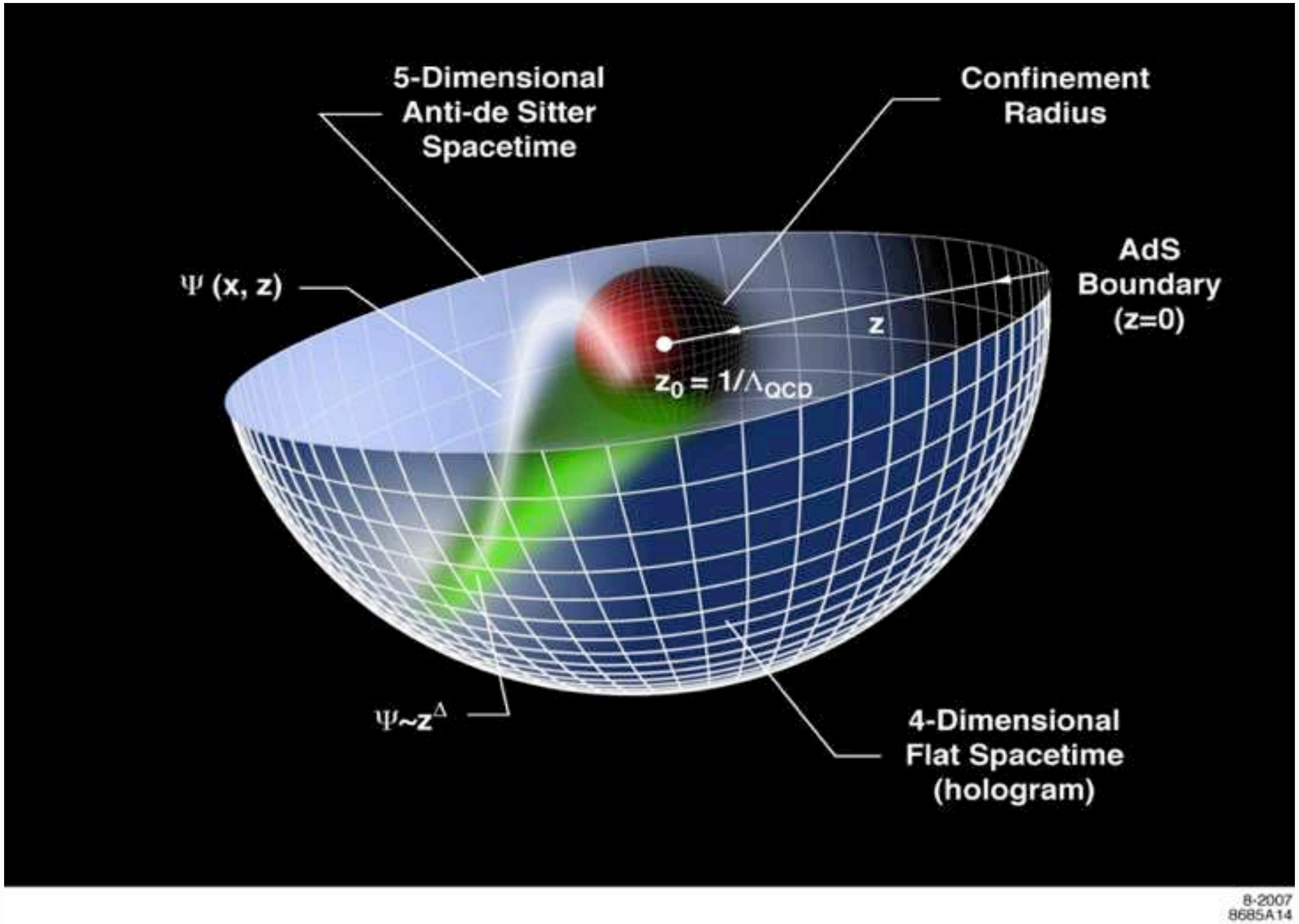
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

Application to Strange Hadrons

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$





Changes in physical length scale mapped to evolution in the 5th dimension z

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$ ← *invariant measure*

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$

- Soft-wall dilaton profile breaks conformal invariance $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- Color Confinement
- Introduces confinement scale κ

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

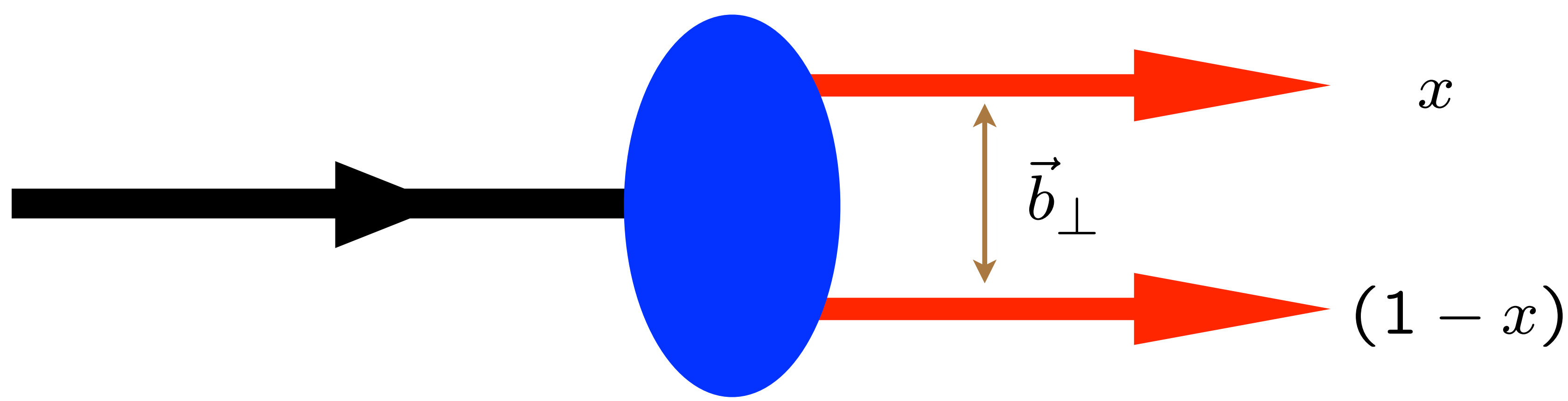
$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

de Teramond,
Dosch, sjb

LF(3+1) $\longleftrightarrow$ AdS_5

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}} \longleftrightarrow z$$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formulae for EM and gravitational current matrix elements and identical equations of motion*

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- Substituting in the AdS scalar wave equation for Φ

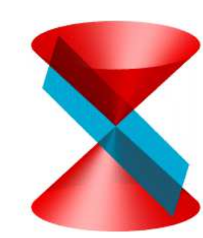
$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2} \, \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}$$



with $(\mu R)^2 = -(2 - J)^2 + L^2$

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

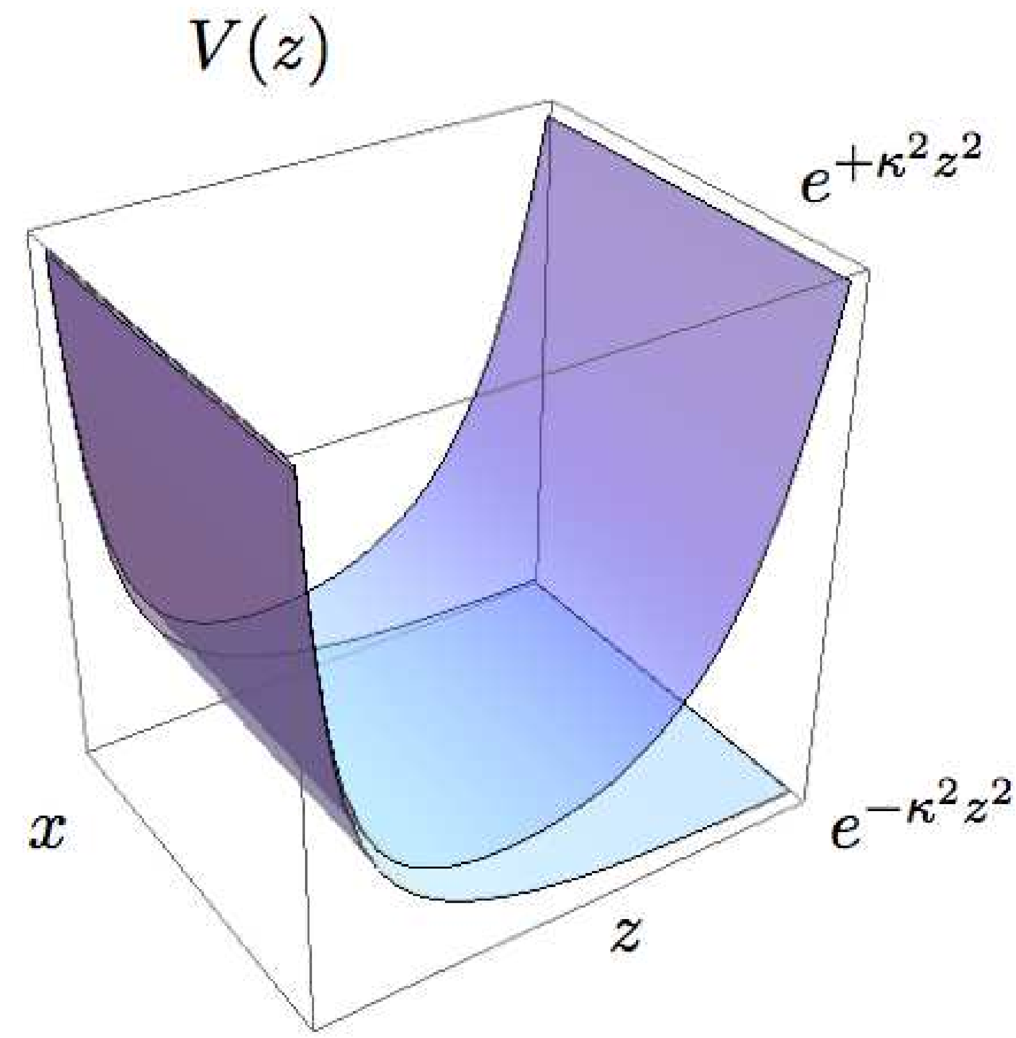
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

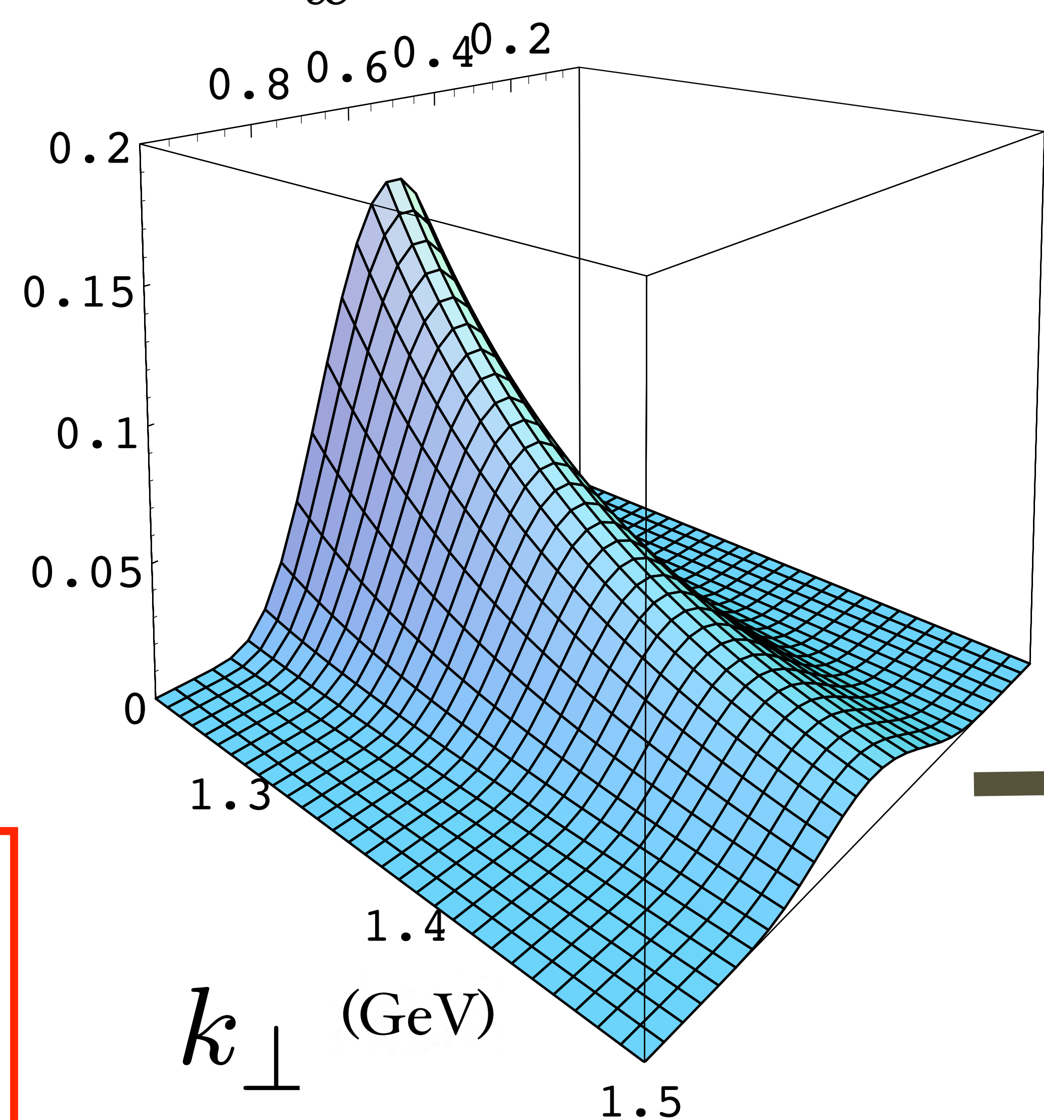
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

Positive-sign dilaton

- de Teramond, sjb

Prediction from AdS/QCD: Meson LFWF

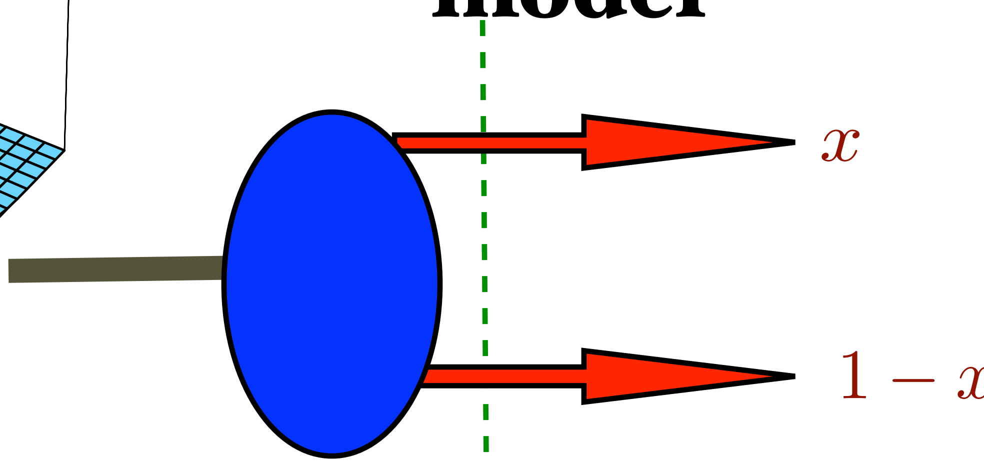
$\psi_M(x, k_\perp^2)$



$k_\perp$ (GeV)

de Teramond, Dosch, sjb

"Soft Wall" model



massless quarks

Note coupling

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Provides Connection of Confinement to TMDs

PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

J. R. Forshaw*

Consortium for Fundamental Physics, School of Physics and Astronomy, University of Manchester, Oxford Road, Manchester M13 9PL, United Kingdom

R. Sandapen†

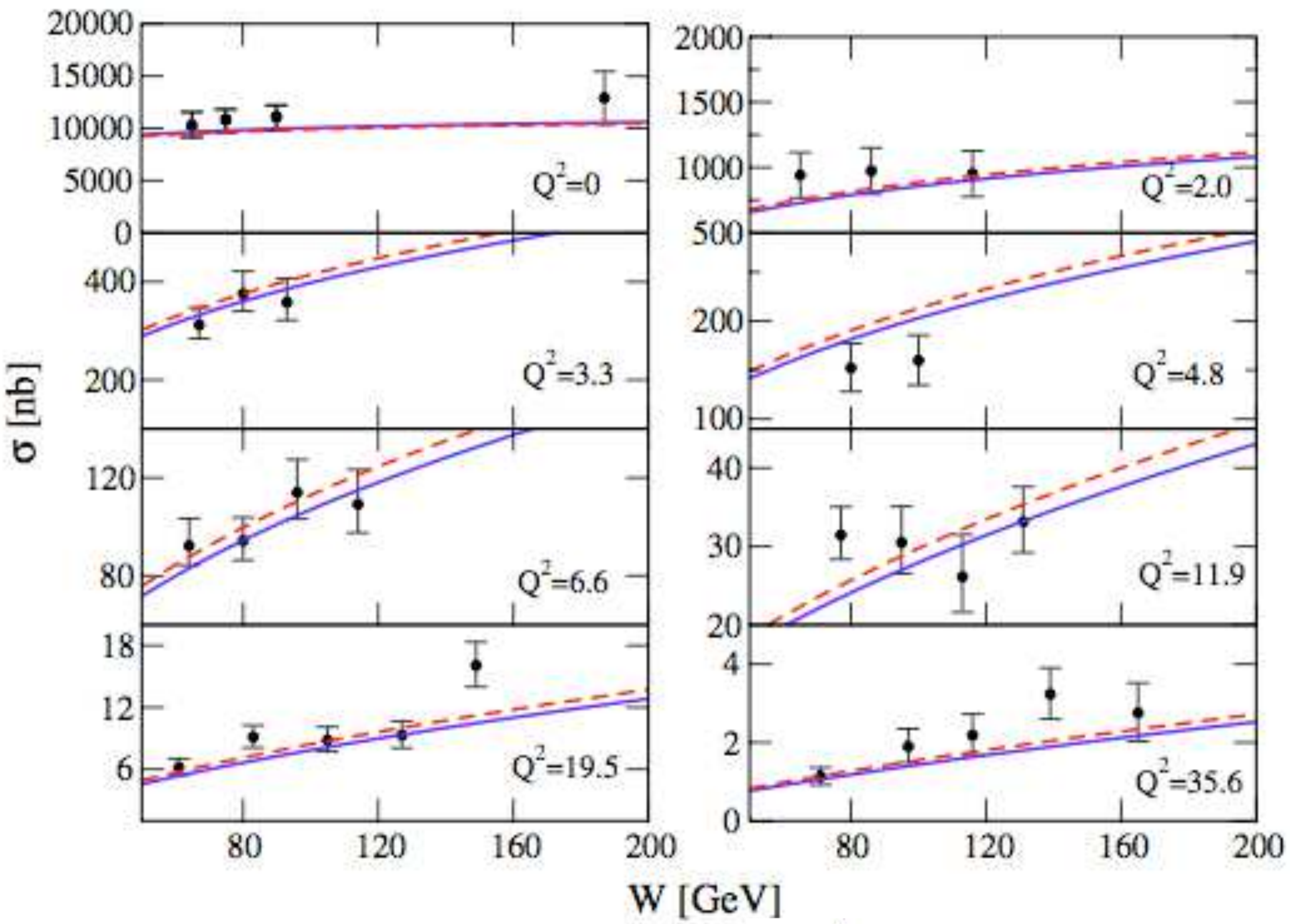
Département de Physique et d'Astronomie, Université de Moncton, Moncton, New Brunswick E1A3E9, Canada

(Received 5 April 2012; published 20 August 2012)

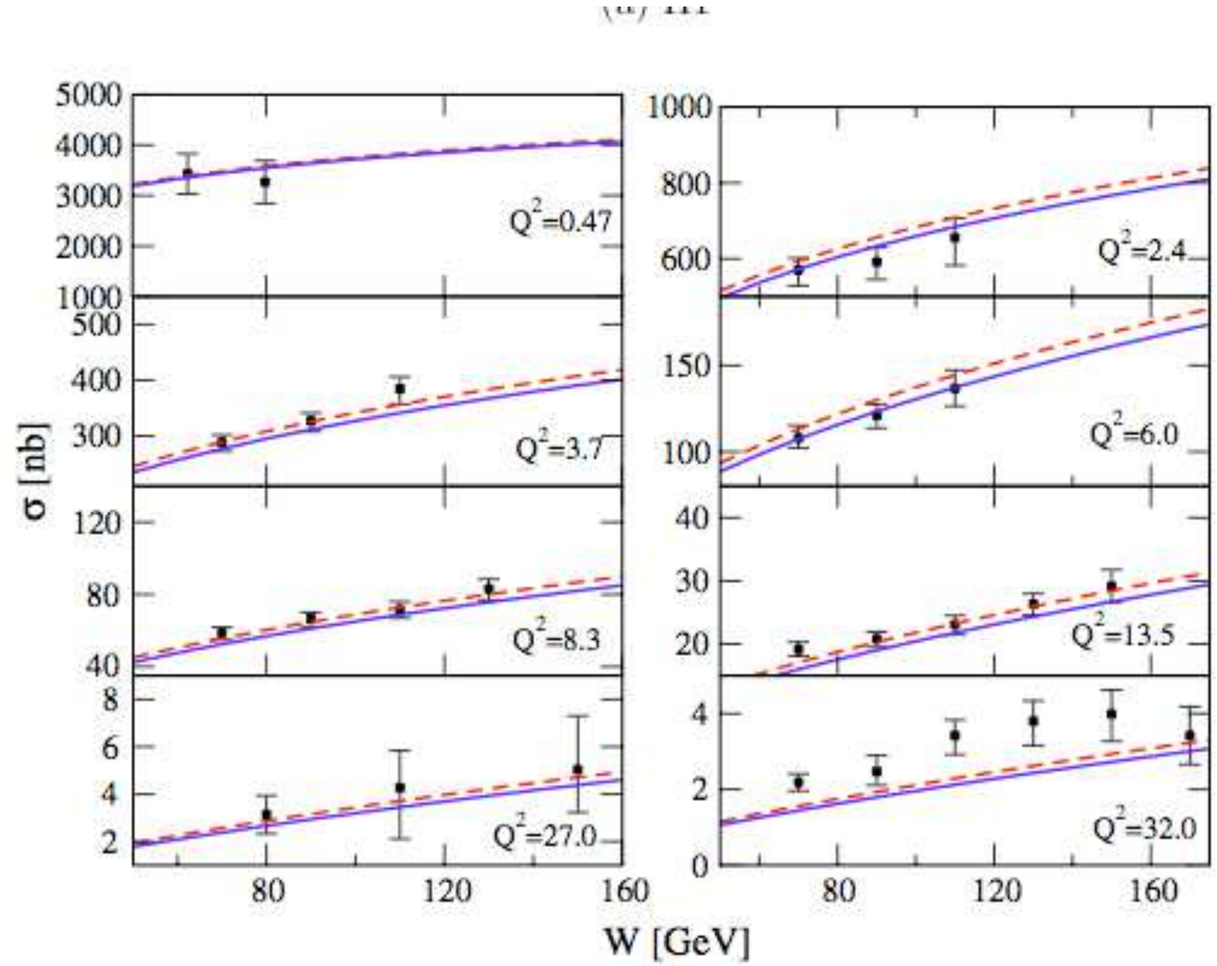
We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

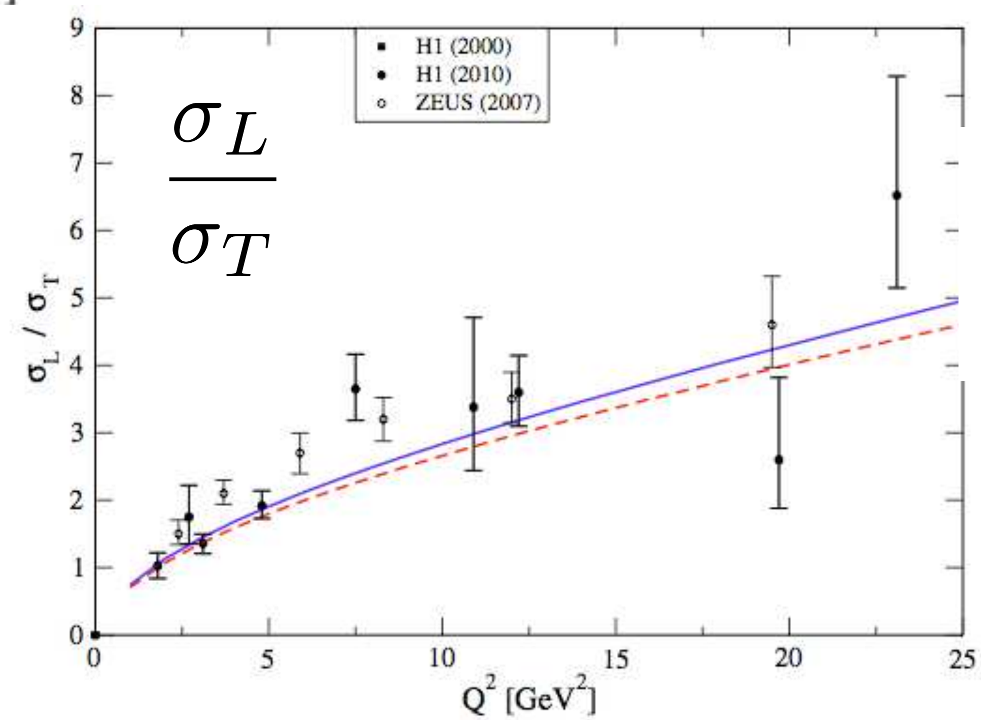
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H



(b) ZEUS



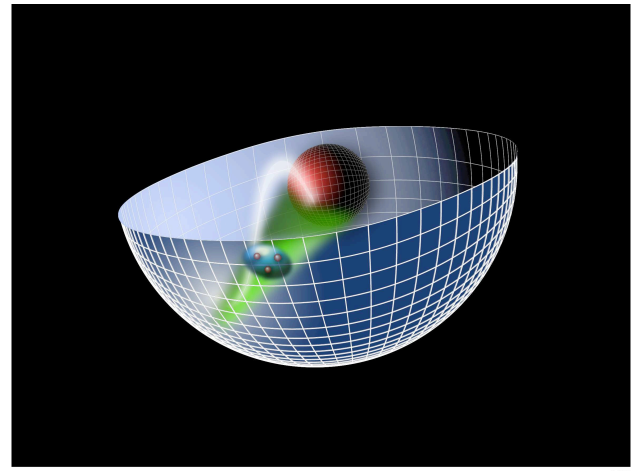
$$\frac{\sigma_L}{\sigma_T}$$

J. R. Forshaw,
R. Sandapen

$$\gamma^* p \to \rho^0 p'$$

$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

AdS/QCD Soft-Wall Model



Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

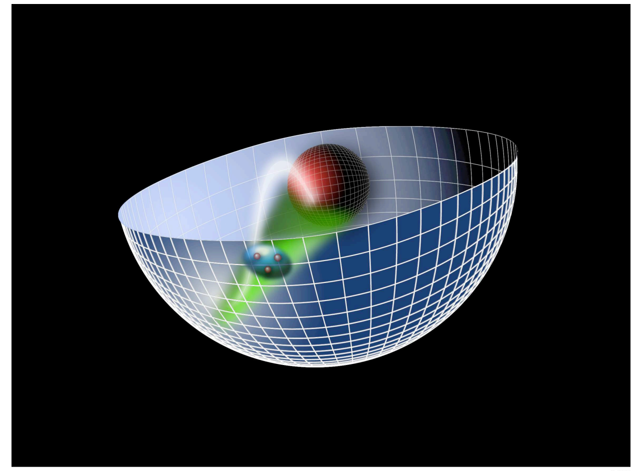
$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

$1/\kappa \simeq 0.4\ fm$

de Teramond, Dosch, sjb



AdS/QCD Soft-Wall Model

Light-Front Holography

Semi-Classical Approximation to QCD

Relativistic, frame-independent

Unique color-confining potential

Zero mass pion for massless quarks

Regge trajectories with equal slopes in n and L

Light-Front Wavefunctions

Conformal Symmetry

Light-Front Schrödinger Equation

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale Λ_{QCD} come from?

How does color confinement arise?

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique potential!

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

What determines the QCD mass scale Λ_{QCD}?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)
- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$
- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK$ $\quad 4uw - v^2 = \kappa^4 = [M]^4$
- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ
- Only dimensionless mass ratios (and M times R) predicted
- Mass and time units [GeV] and [sec] from physics external to QCD
- New feature: bounded frame-independent relative time between constituents

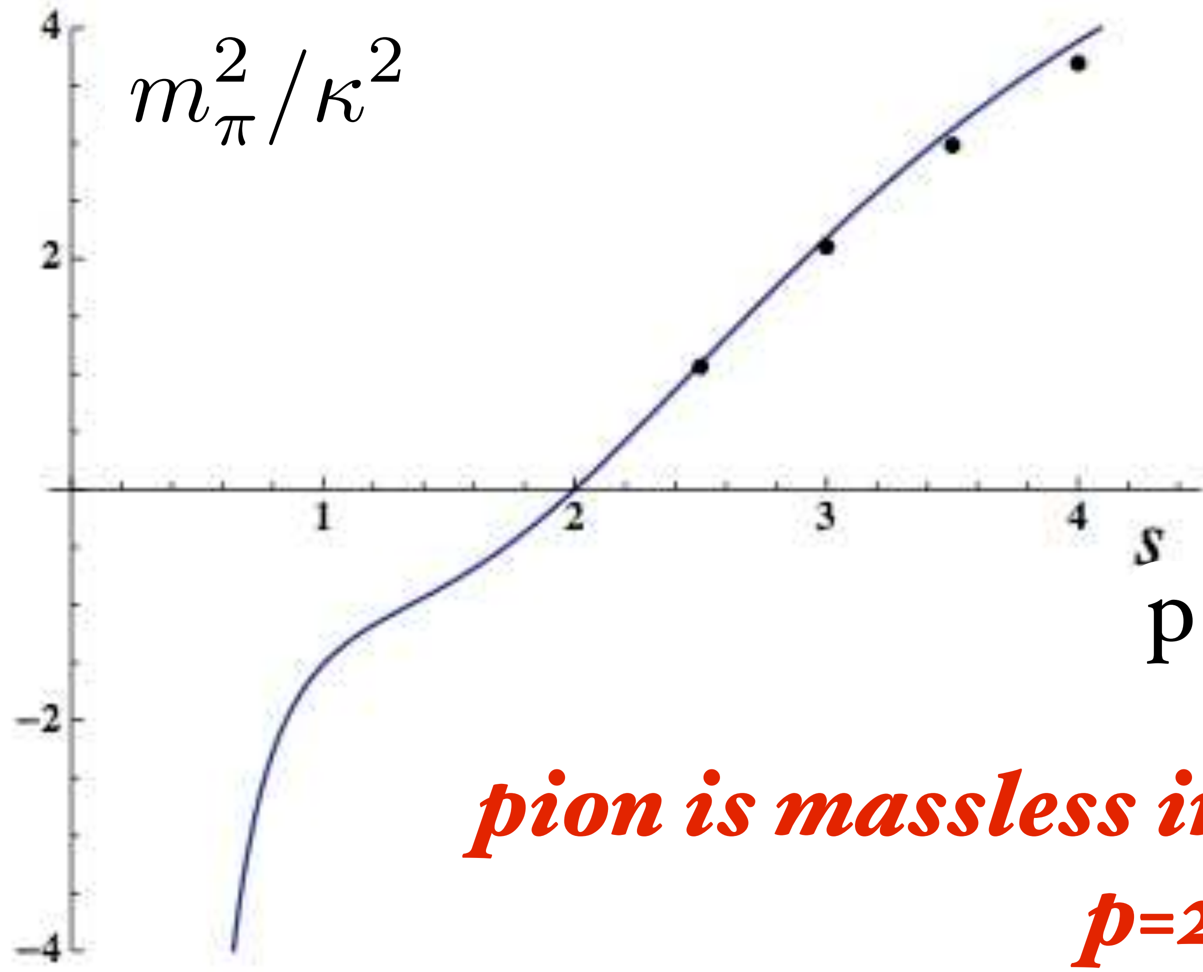
Uniqueness

de Teramond, Dosch, sjb

- ζ_2 confinement potential and dilaton profile unique!
- Linear Regge trajectories in n and L: same slope!
- Massless pion in chiral limit!   No vacuum condensate!
- Derive from conformal invariance: conformally invariant action for massless quarks despite mass scale
- Same principle, equation of motion as de Alfaro, Fubini, Furlan
- Conformal Invariance in Quantum Mechanics Nuovo Cim. A34 (1976) 569

Uniqueness

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Teramond, sjb**

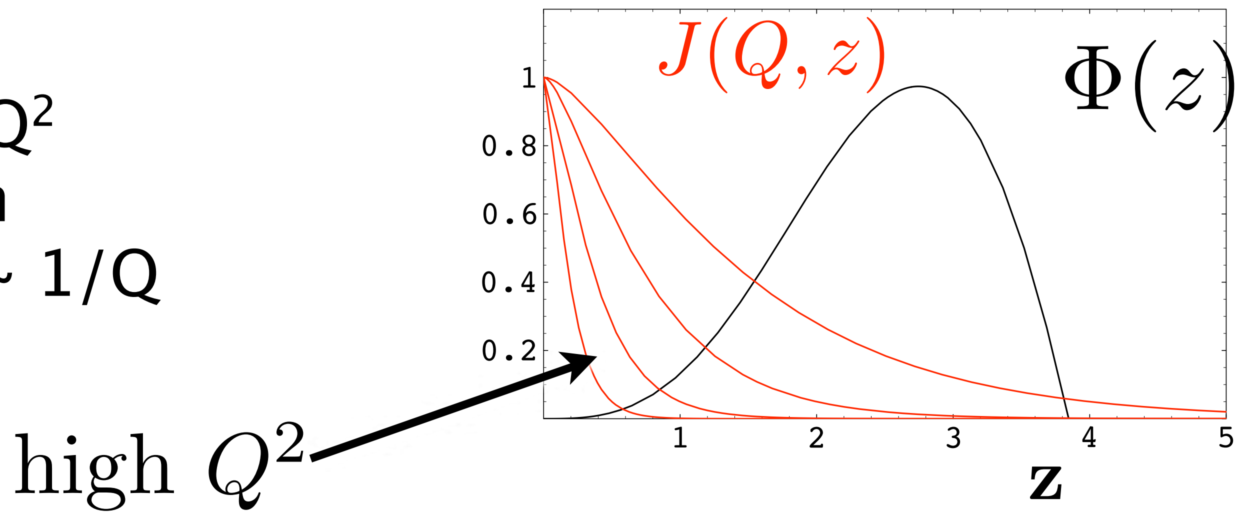
Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

Dressed Current in Soft-Wall Model

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

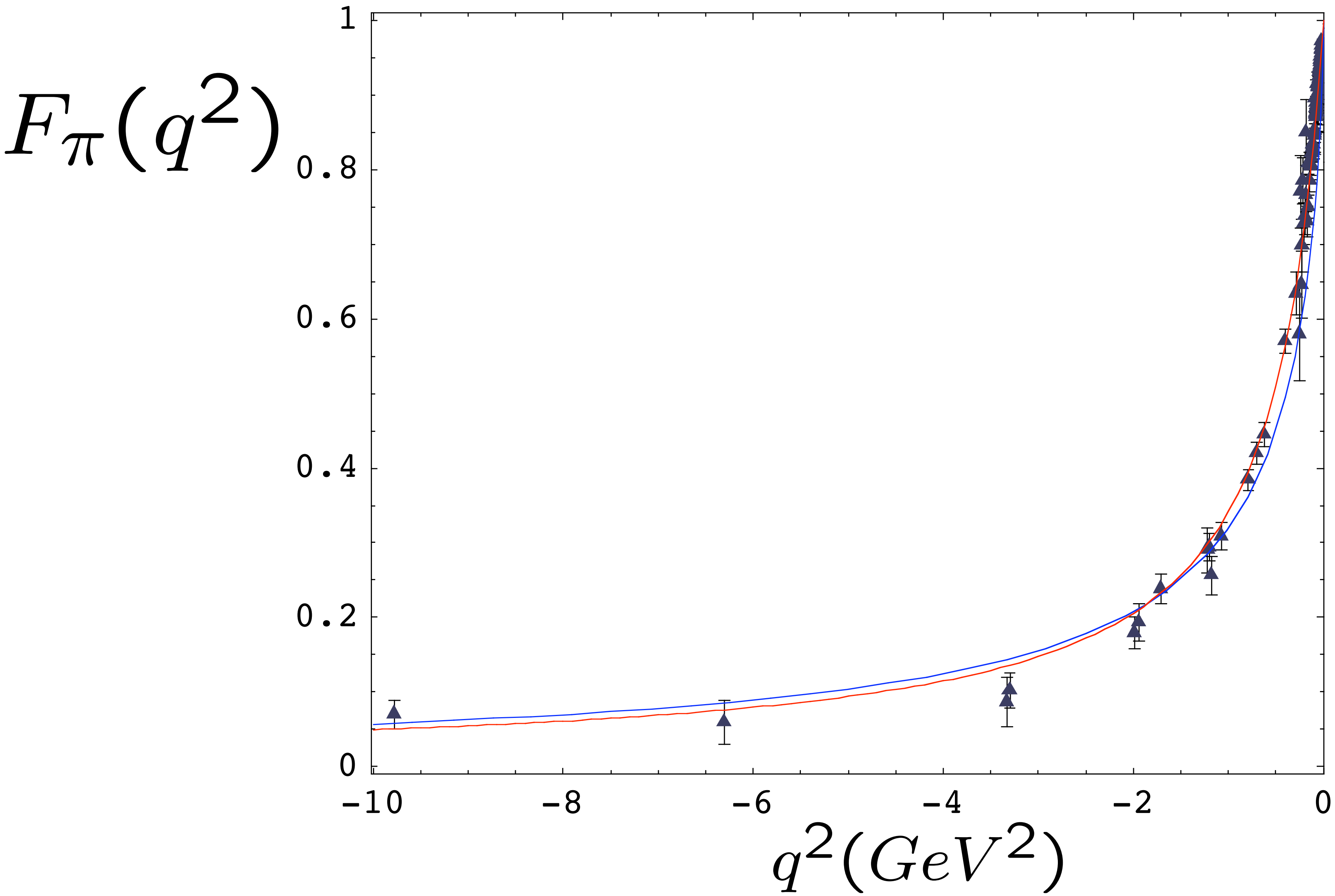
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2}\, \Phi(z) J_\kappa(Q, z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \to zQK_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

Spacelike pion form factor from AdS/CFT



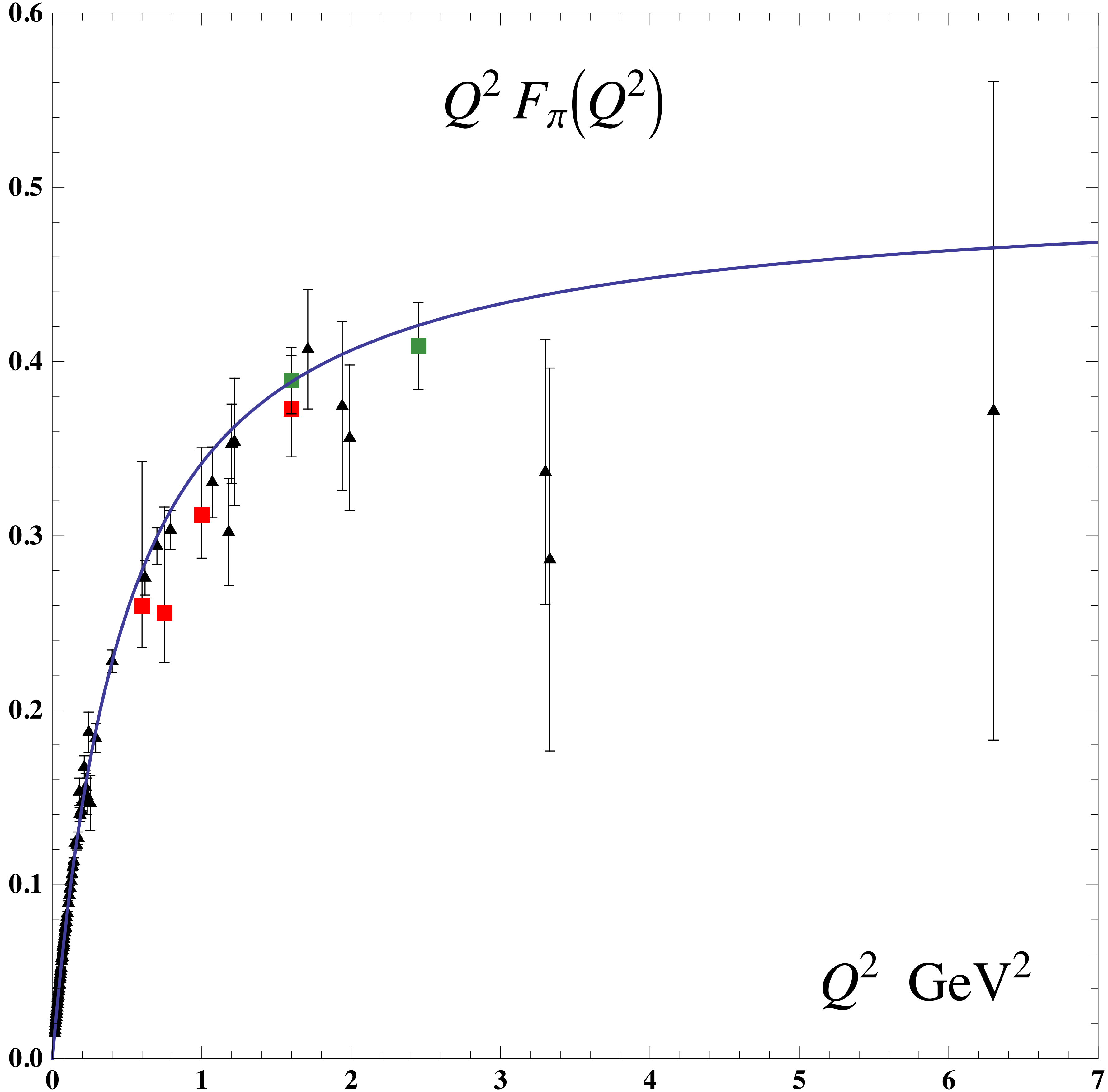
Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

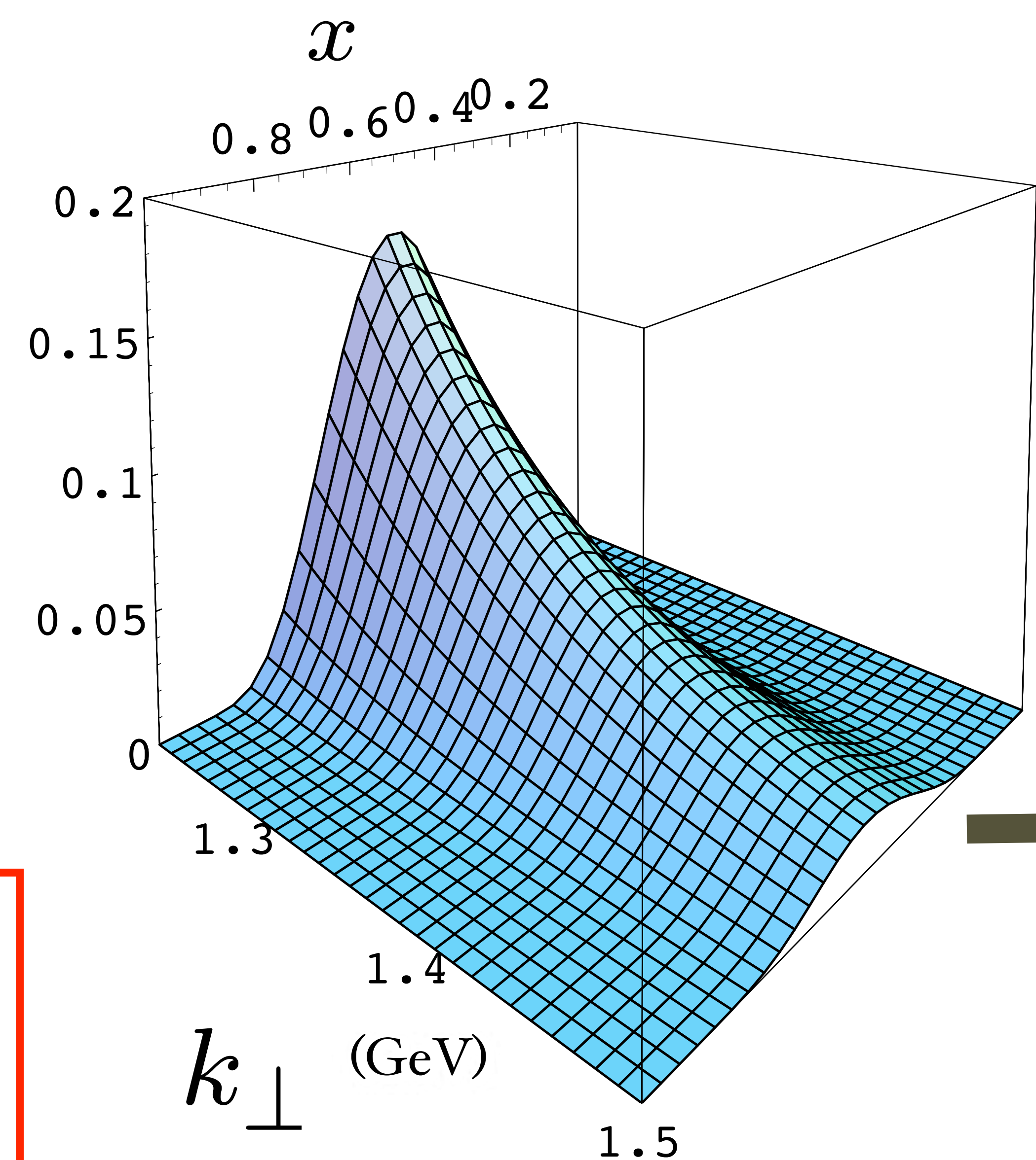


$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

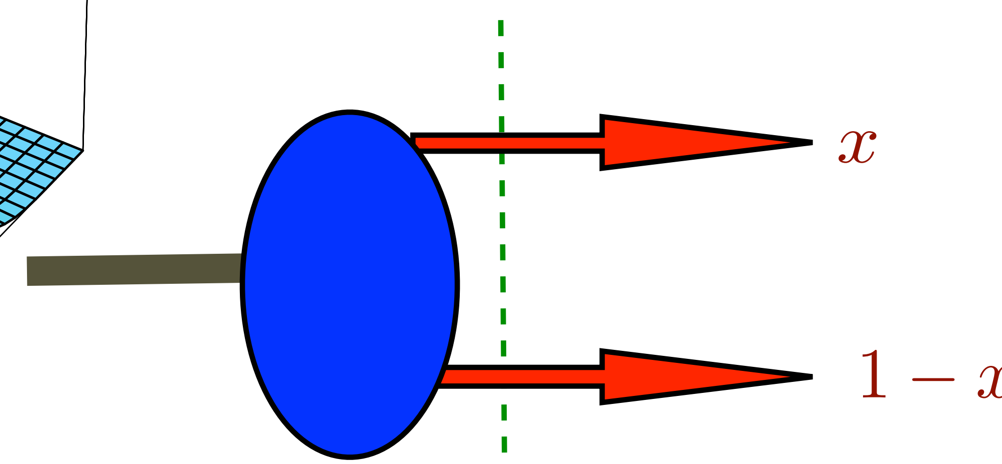
Prediction from AdS/QCD: Meson LFWF

de Teramond, Cao, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model



massless quarks

Note coupling

$k_\perp^2, x$

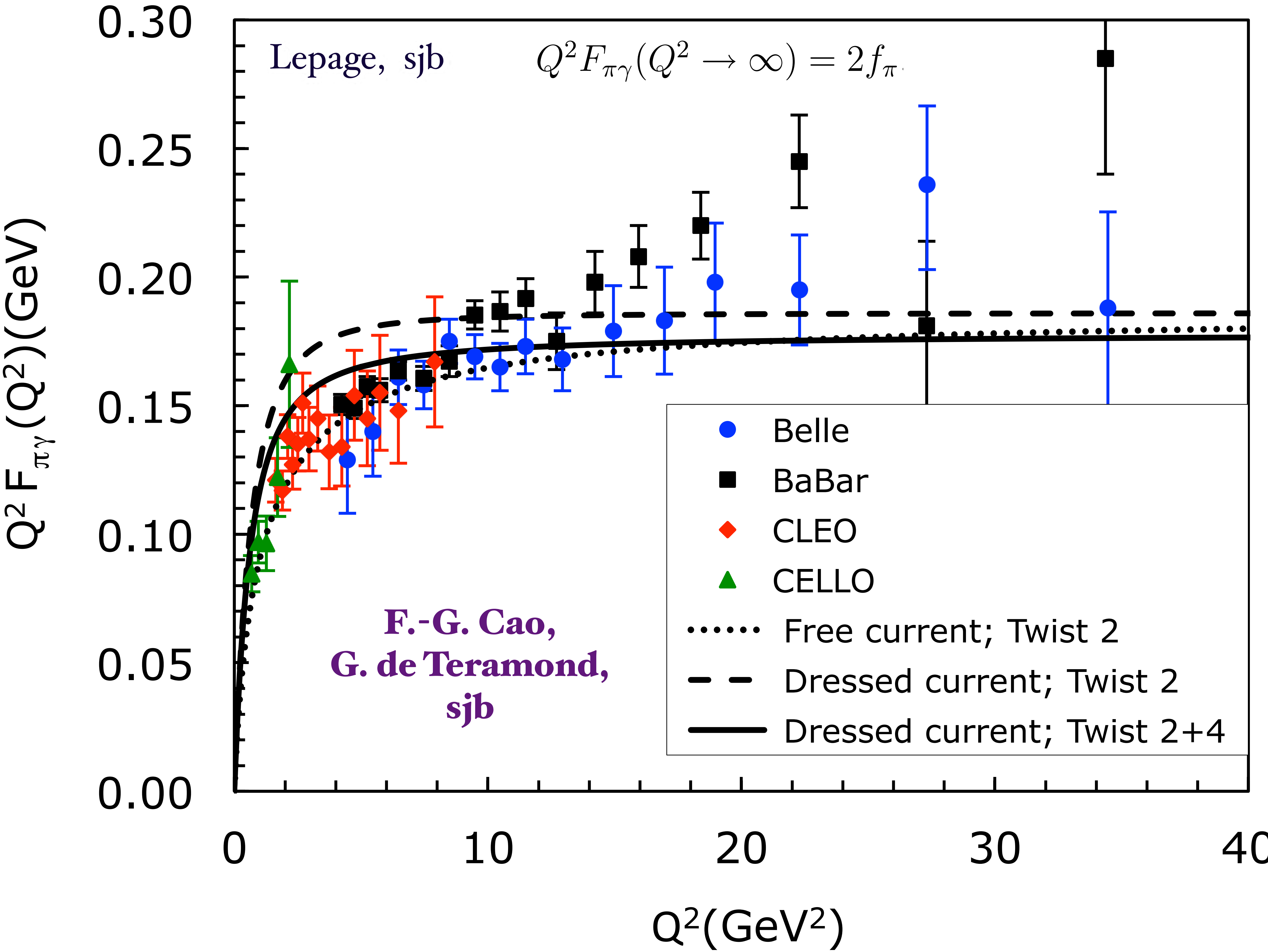
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Provides Connection of Confinement to Hadron Structure

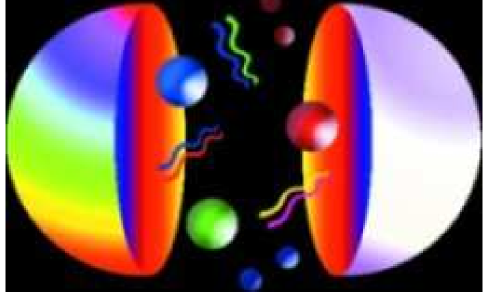
Photon-to-pion transition form factor



Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)

Yukawa interaction in 5 dimensions



From Nick Evans

- Action for Dirac field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1} \sqrt{g}\, e^{\varphi(z)} \left(i\overline{\Psi} e^M_A \Gamma^A D_M \Psi + h.c + \varphi(z)\overline{\Psi}\Psi - \mu \overline{\Psi}\Psi \right)$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP\cdot x}\Psi(z)$

$$\left[i \left(z\eta^{\ell m}\Gamma_\ell \partial_m + 2\Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^\ell) = 0.$$

- Solution $(\nu = \mu R - \frac{1}{2},\ \nu = L+1)$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1)$$

positive parity

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions
- Large N_C: $\mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C}\, \Lambda_{\rm QCD}$

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

and its adjoint $\Pi^\dagger$, with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).$$

$$\nu = L + 1$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

Baryon Spectrum in Soft-Wall Model

- Upon substitution $z \to \zeta$ and

$$\Psi_J(x,z) = e^{-iP\cdot x} z^2 \psi^J(z) u(P),$$

find LFWE for $d = 4$

$$\begin{aligned} \frac{d}{d\zeta}\psi_+^J + \frac{\nu+\frac{1}{2}}{\zeta}\psi_+^J + U(\zeta)\psi_+^J &= \mathcal{M}\psi_-^J, \\ -\frac{d}{d\zeta}\psi_-^J + \frac{\nu+\frac{1}{2}}{\zeta}\psi_-^J + U(\zeta)\psi_-^J &= \mathcal{M}\psi_+^J, \end{aligned}$$

where $U(\zeta) = \frac{R}{\zeta} V(\zeta)$

- Choose linear potential $U = \kappa^2 \zeta$

- Eigenfunctions

$$\psi_+^J(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2), \qquad \psi_-^J(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2)$$

- Eigenvalues

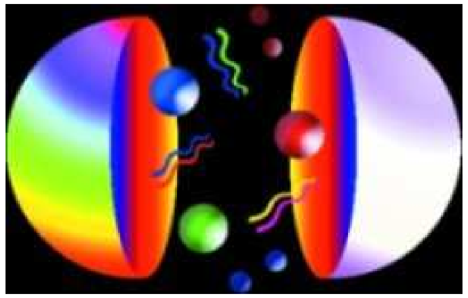
$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1), \quad \nu = L+1 \quad (\tau = 3)$$

- Full $J-L$ degeneracy (different J for same L) for baryons along given trajectory !

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

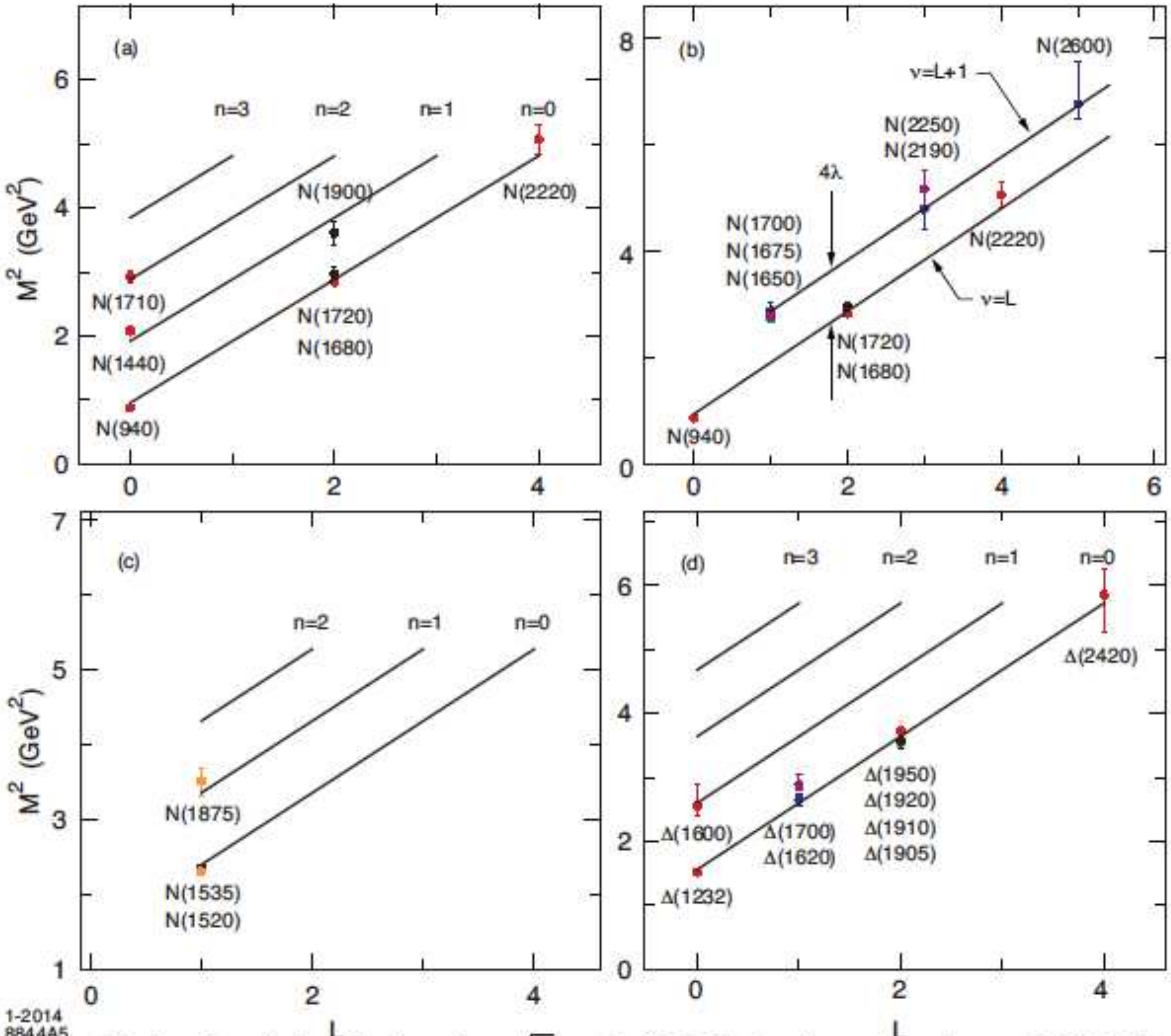
$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$



Baryon orbital and radial excitations for $\sqrt{\lambda} = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas)

$$\sqrt{\lambda} = \kappa$$

Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels S, L and n refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta\frac{5}{2}^-(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n=0$, $L=3$.

$SU(6)$	S	L	n	Baryon State
56	$\frac{1}{2}$	0	0	$N\frac{1}{2}^+(940)$
	$\frac{1}{2}$	0	1	$N\frac{1}{2}^+(1440)$
	$\frac{1}{2}$	0	2	$N\frac{1}{2}^+(1710)$
	$\frac{3}{2}$	0	0	$\Delta\frac{3}{2}^+(1232)$
	$\frac{3}{2}$	0	1	$\Delta\frac{3}{2}^+(1600)$
70	$\frac{1}{2}$	1	0	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	0	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{3}{2}$	1	1	$N\frac{1}{2}^-$ $N\frac{3}{2}^-(1875)$ $N\frac{5}{2}^-$
	$\frac{1}{2}$	1	0	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	0	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{1}{2}$	2	1	$N\frac{3}{2}^+(1900)$ $N\frac{5}{2}^+$
	$\frac{3}{2}$	2	0	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	0	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	0	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	0	$\Delta\frac{5}{2}^-$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	0	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	0	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	0	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	0	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

PDG 2012

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$\begin{aligned} F_+(Q^2) &= g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\ F_-(Q^2) &= g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2, \end{aligned}$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$\begin{aligned} F_1^p(Q^2) &= \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\ F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right], \end{aligned}$$

where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

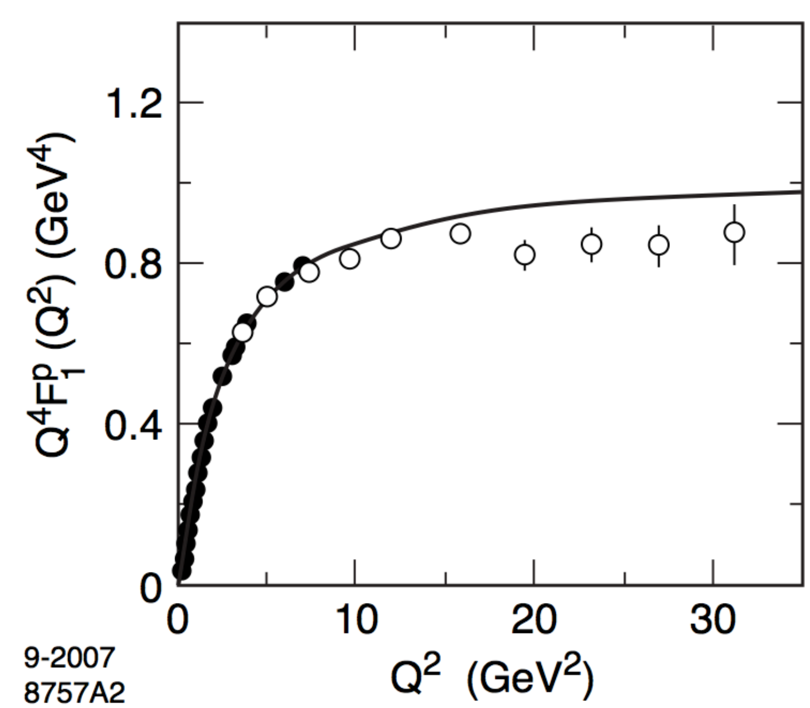
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} \, x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

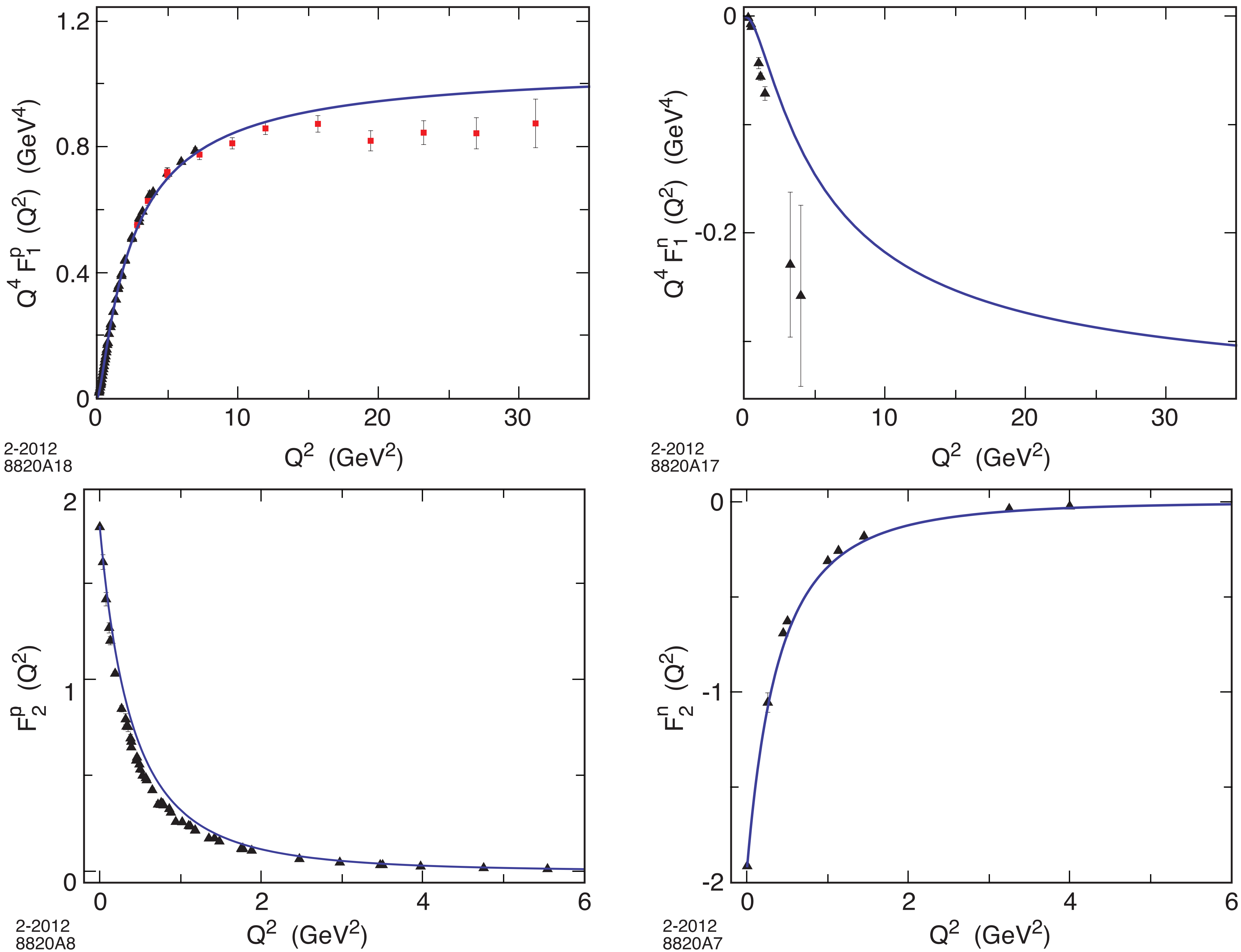
$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n+1/2)$



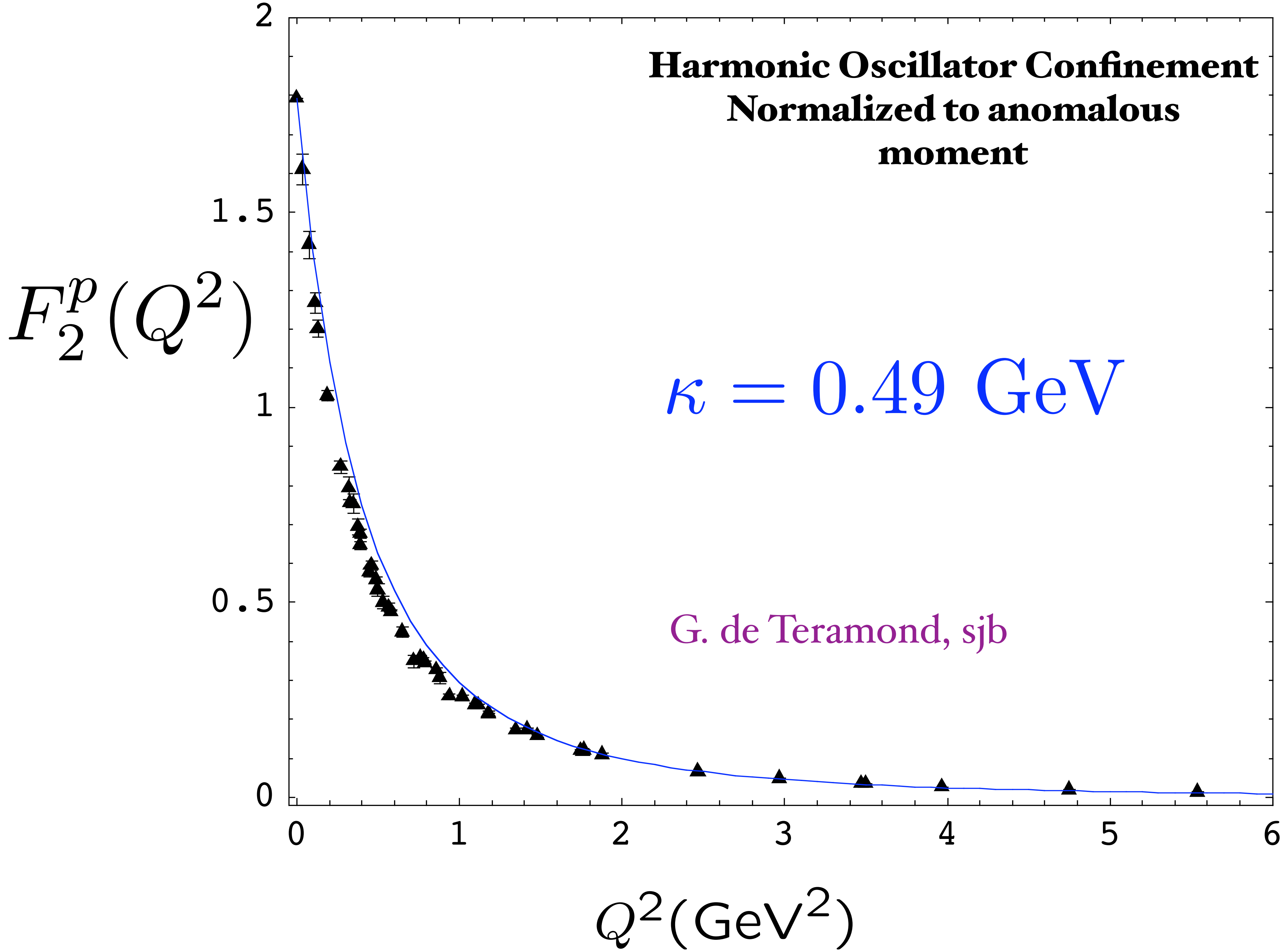
9-2007
8757A2

Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

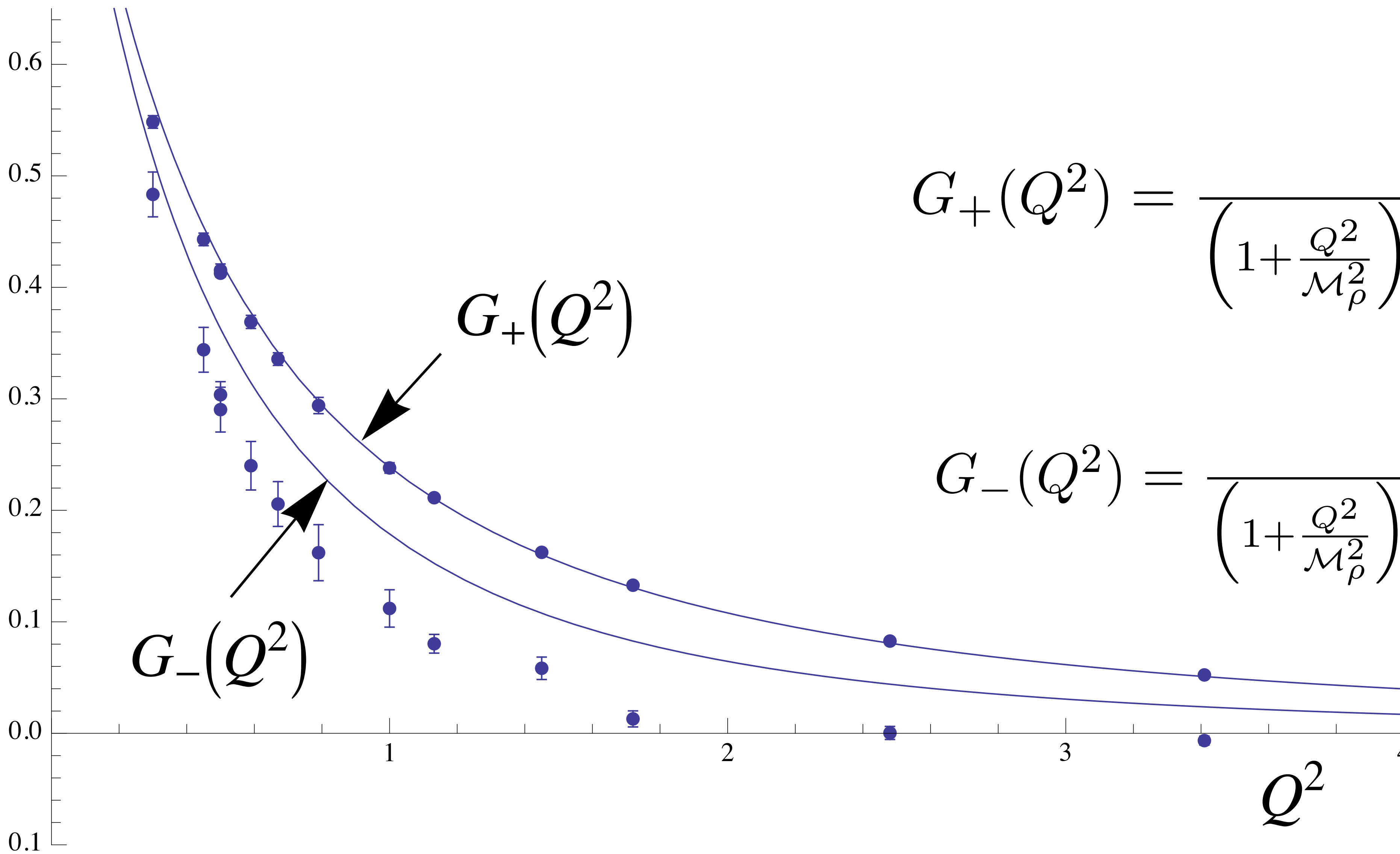
From overlap of $L = 1$ and $L = 0$ LFWFs



Flavor Decomposition of Elastic Nucleon Form Factors

G. D. Cates *et al.* Phys. Rev. Lett. **106**, 252003 (2011)

- Proton SU(6) WF: $F^p_{u,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-$, $F^p_{d,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-$
- Neutron SU(6) WF: $F^n_{u,1} = \frac{1}{3}G_+ + \frac{2}{3}G_-$, $F^n_{d,1} = \frac{5}{3}G_+ + \frac{1}{3}G_-$



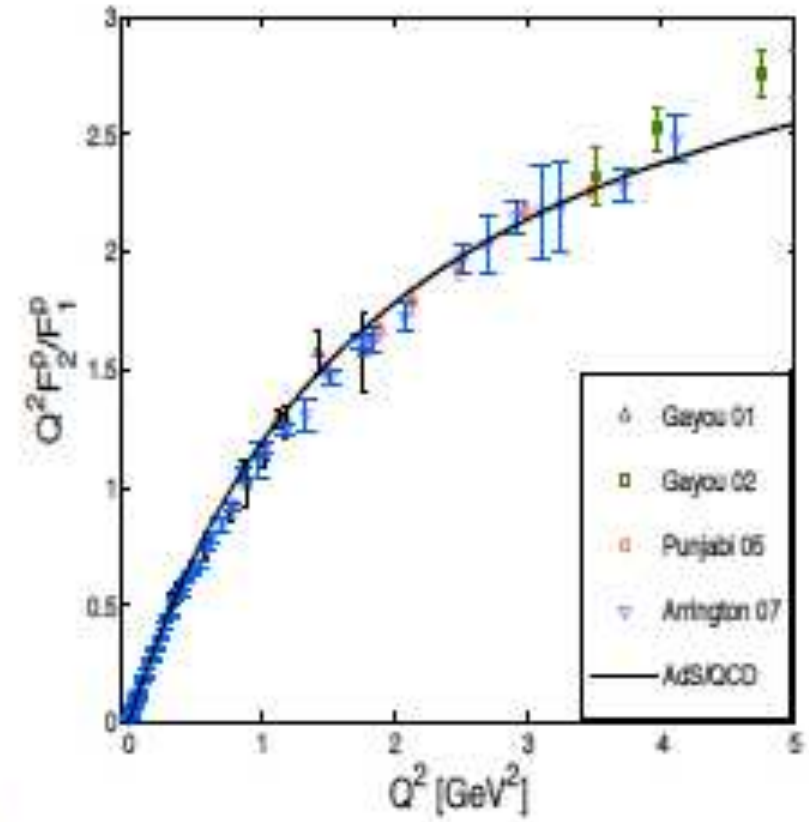
$$G_+(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)}$$

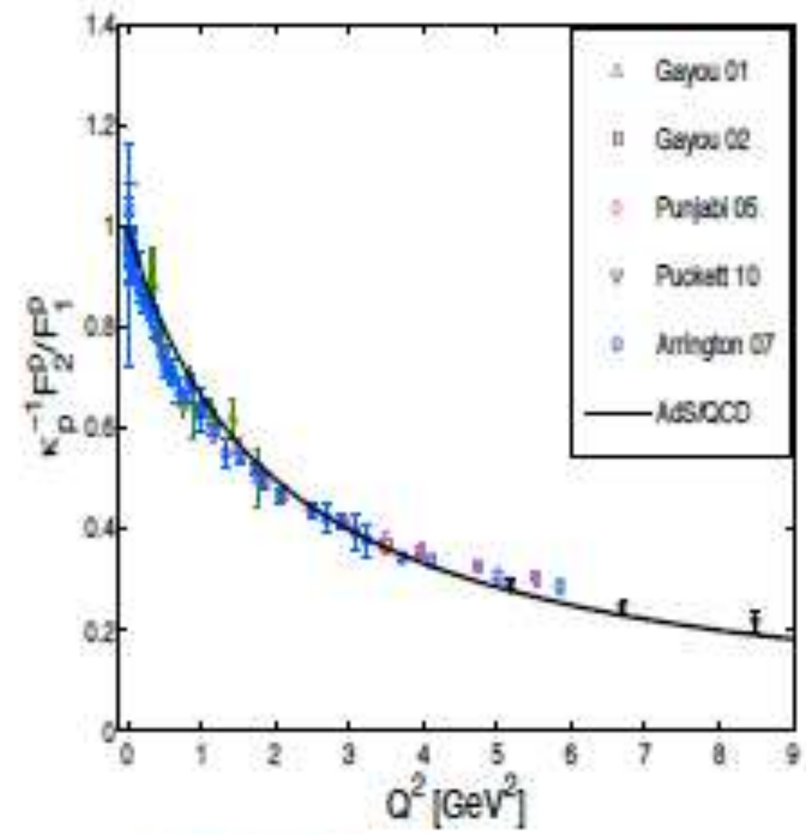
$$G_-(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho''}}\right)}$$

Nucleon and flavor form factors in a light front quark model in AdS/QCD

Dipankar Chakrabarti, Chandan Mondal

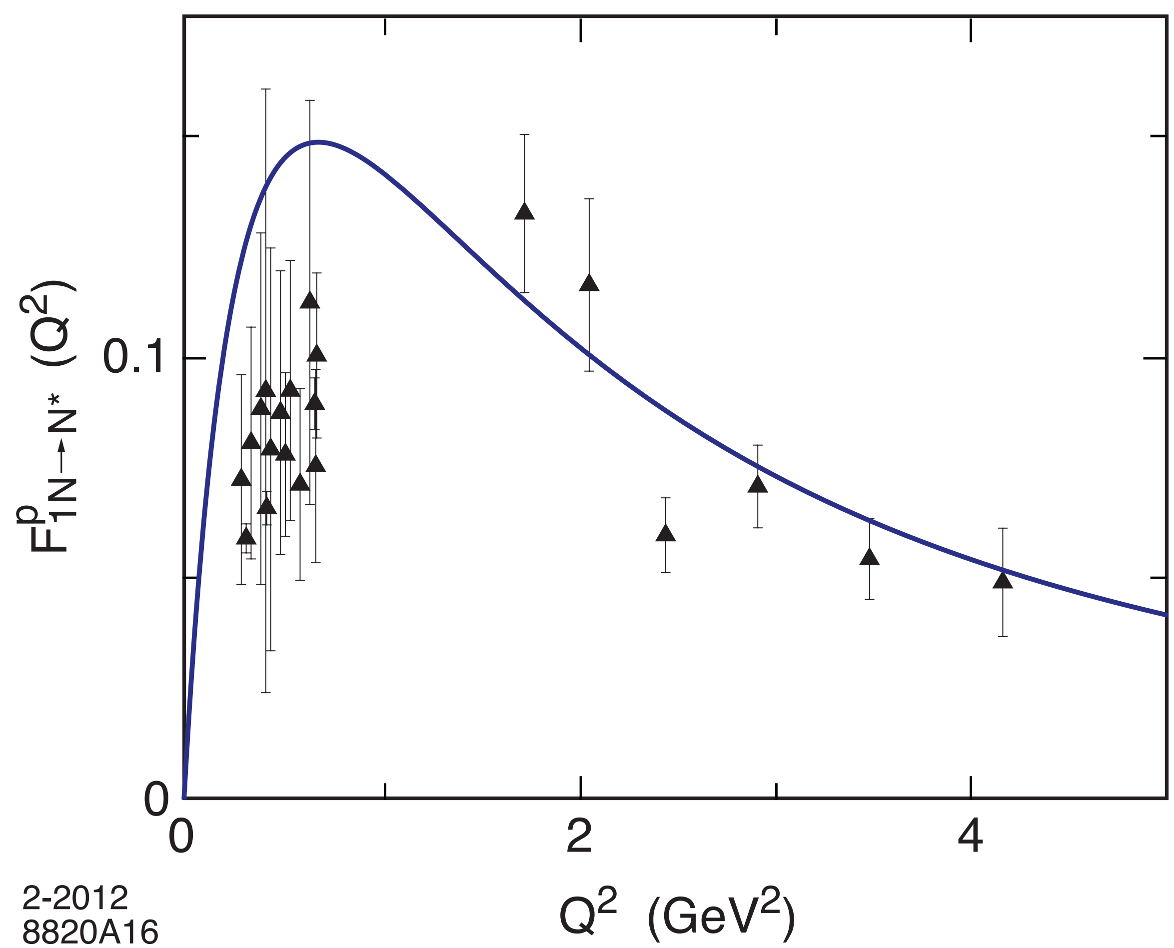
[1]Department of Physics, Indian Institute of Technology Kanpur, Kanpur-208016, India.





Nucleon Transition Form Factors

$$F^p_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab

Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1\,N\to N^*}^{\,p}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_{1\,N\to N^*}^{\,p}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1\,N\to N^*}^{\,p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

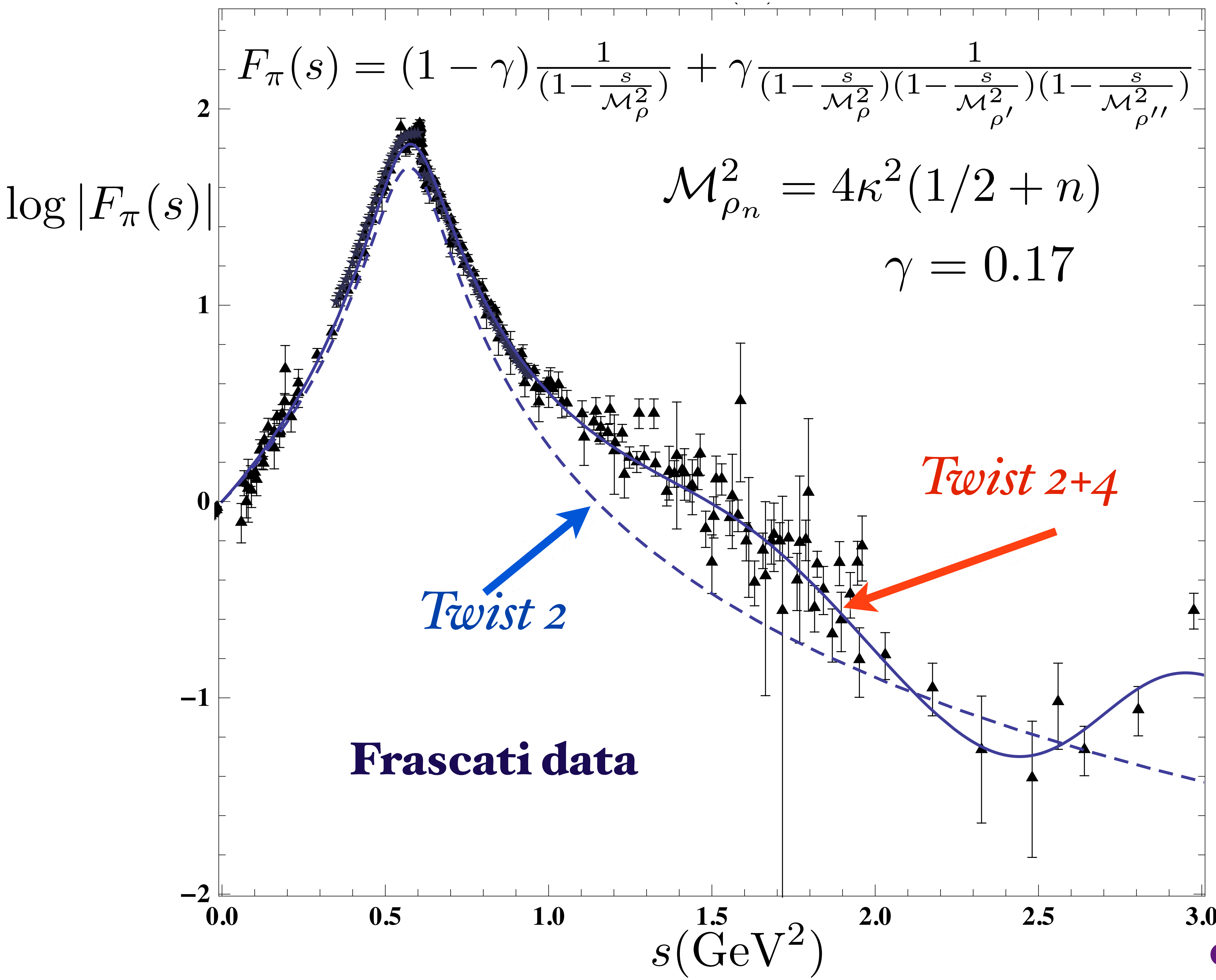
Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S_q^z = 0 >$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



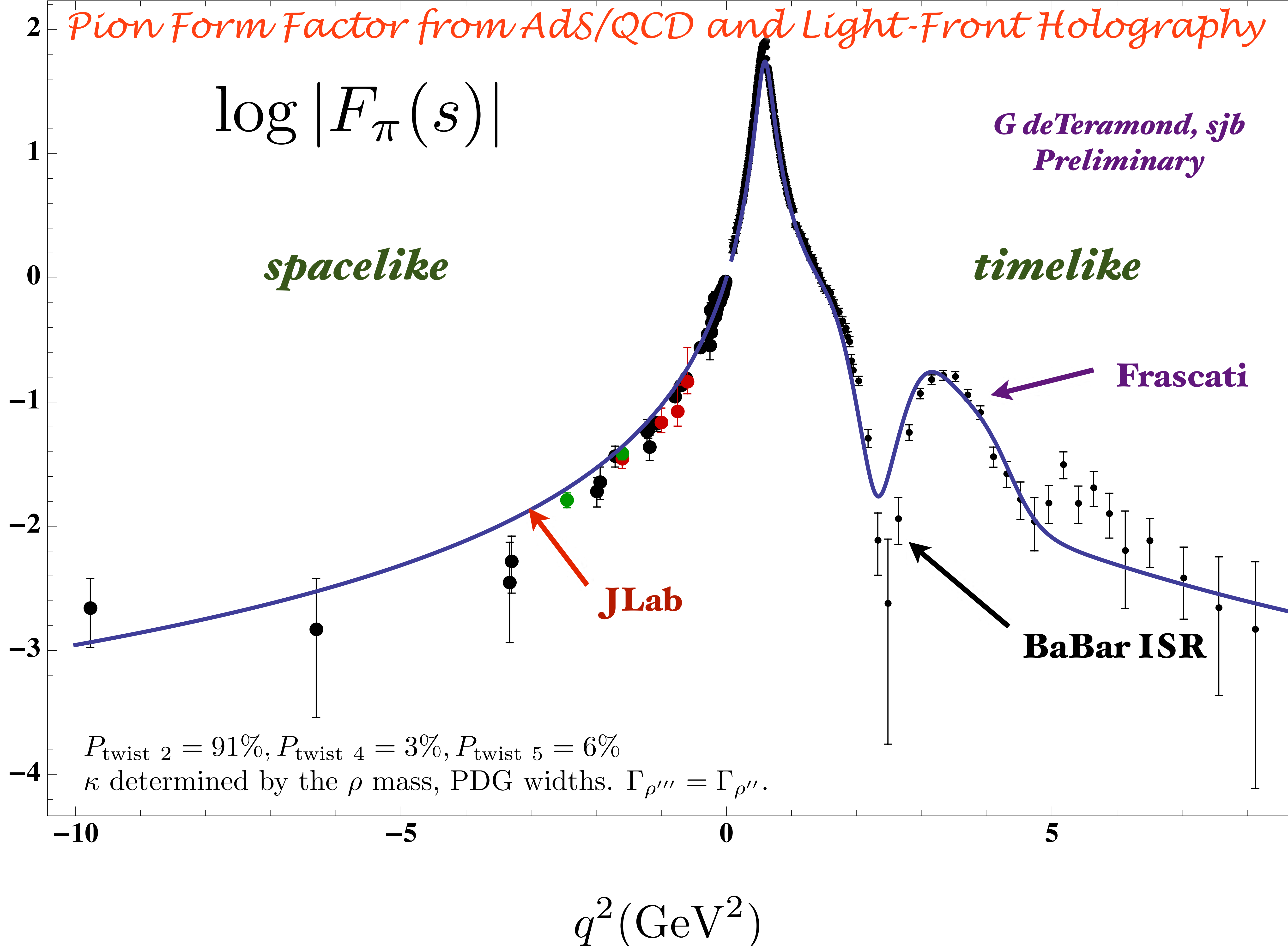
Prescription for Timelike poles :

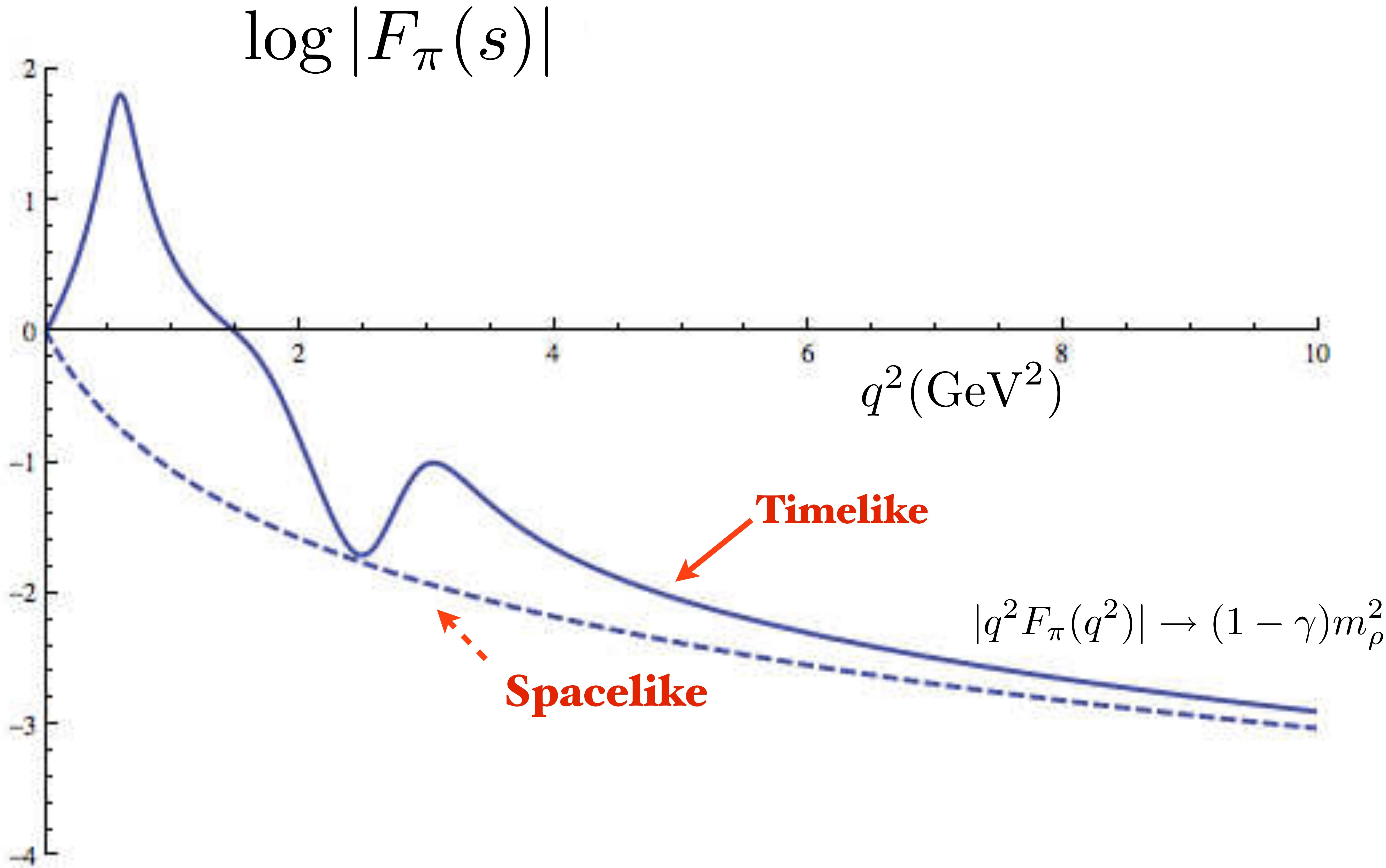
$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography





$\log|F_\pi(s)|$
2
1
0
-1
-2
-3
-4
2
4
6
8
10
$q^2(\mathrm{GeV}^2)$
Timelike
Spacelike
$|q^2F_\pi(q^2)| \to (1-\gamma)m_\rho^2$

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS_5 space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

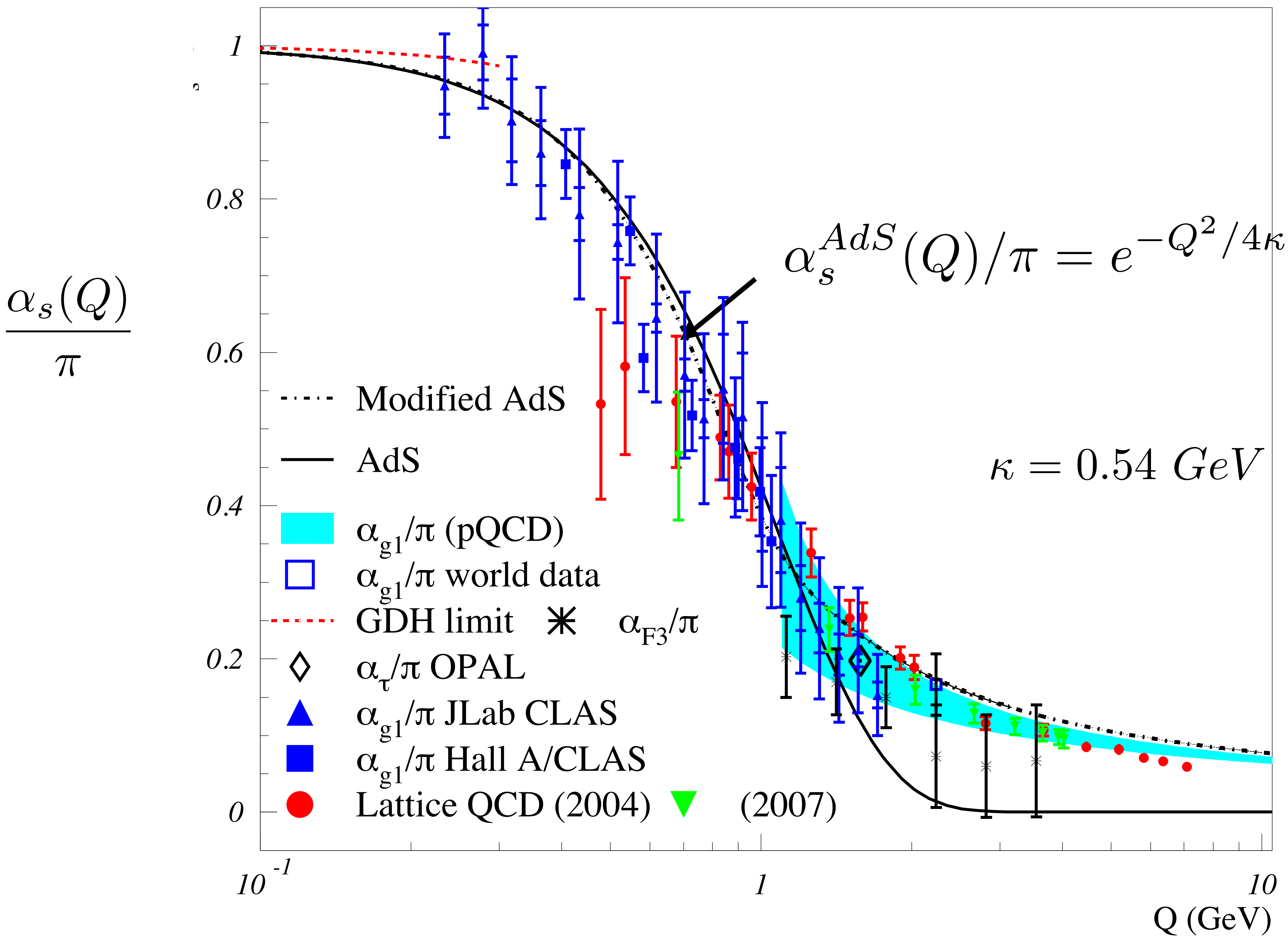
- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



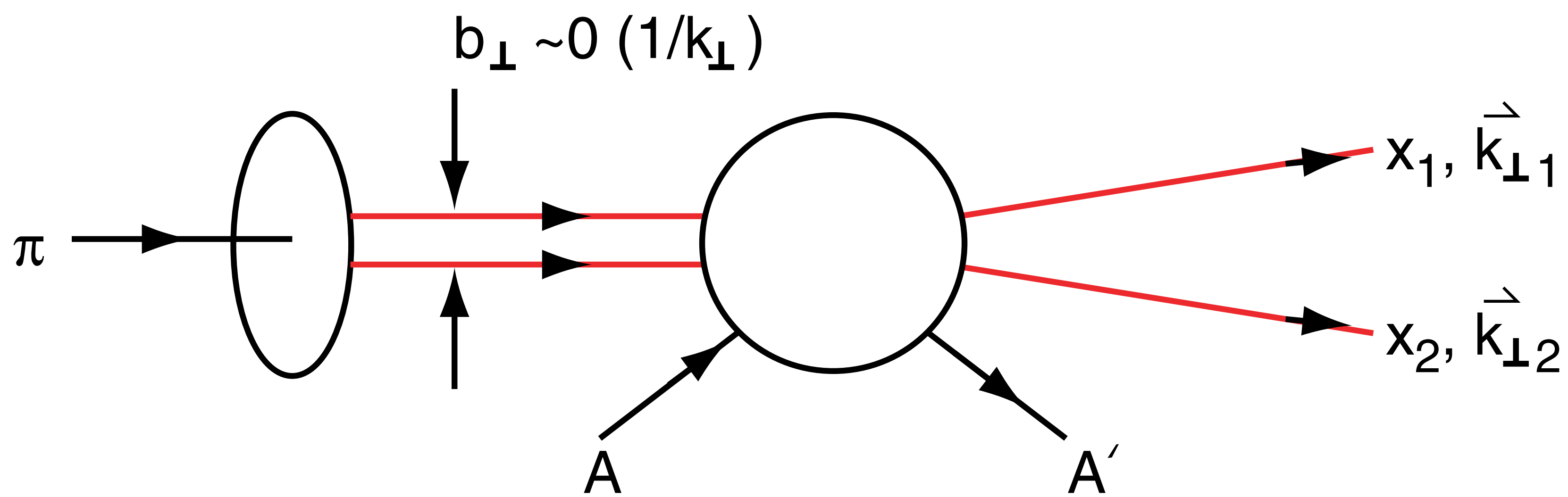
AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



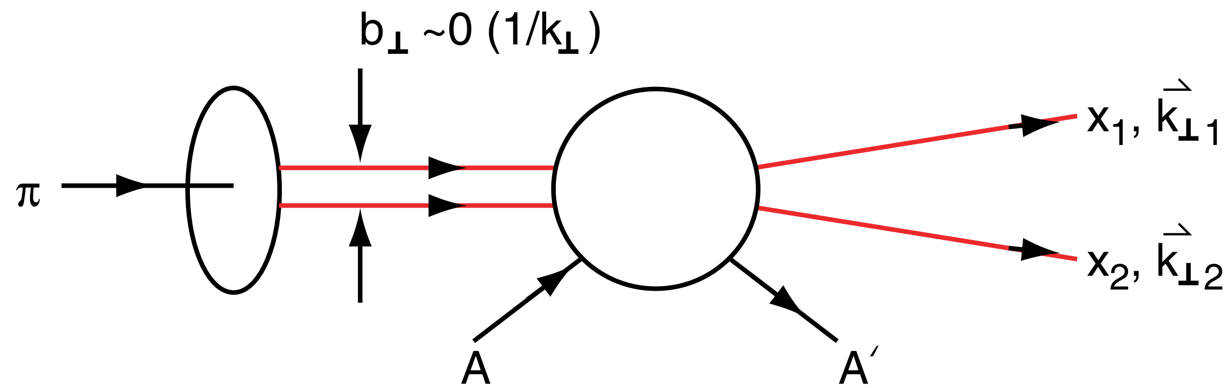
$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

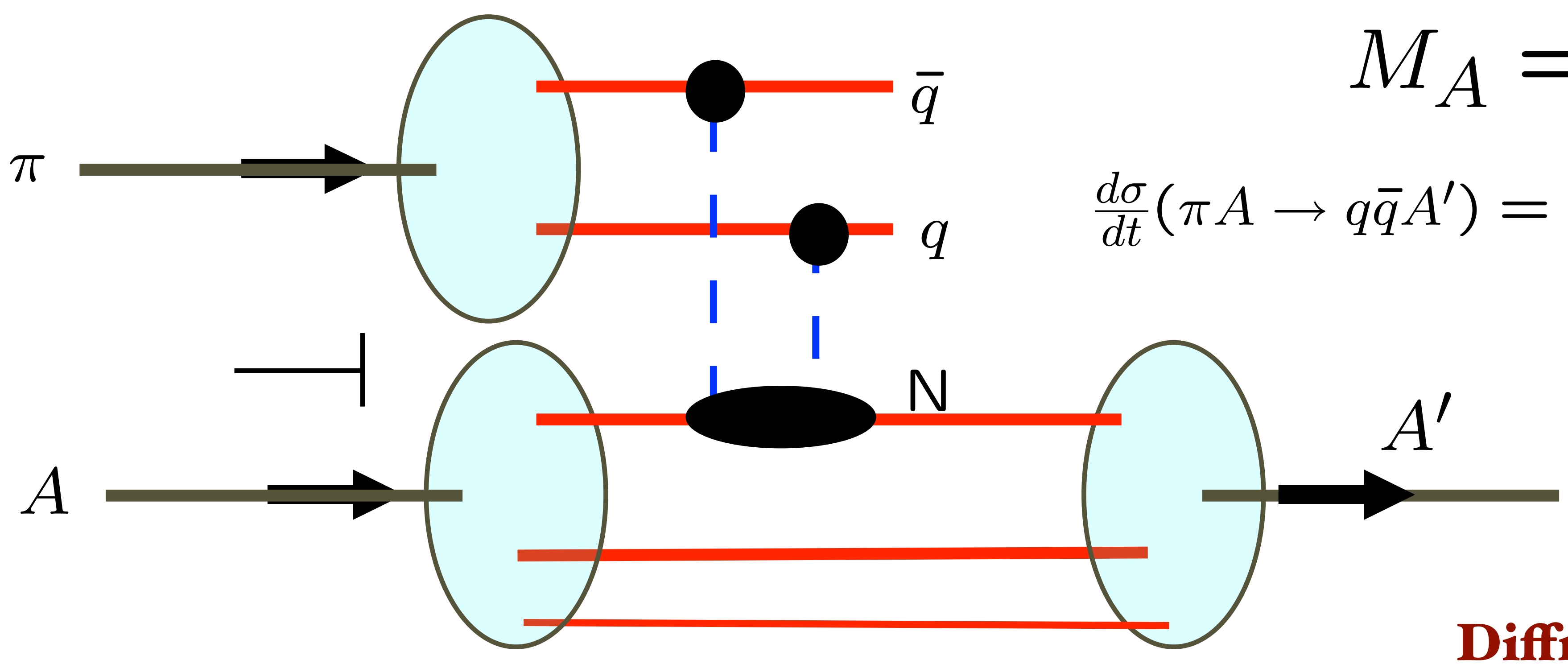
Nucleus left Intact!

Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed;
interacts with <u>each</u> nucleon coherently
<u>QCD COLOR Transparency</u>



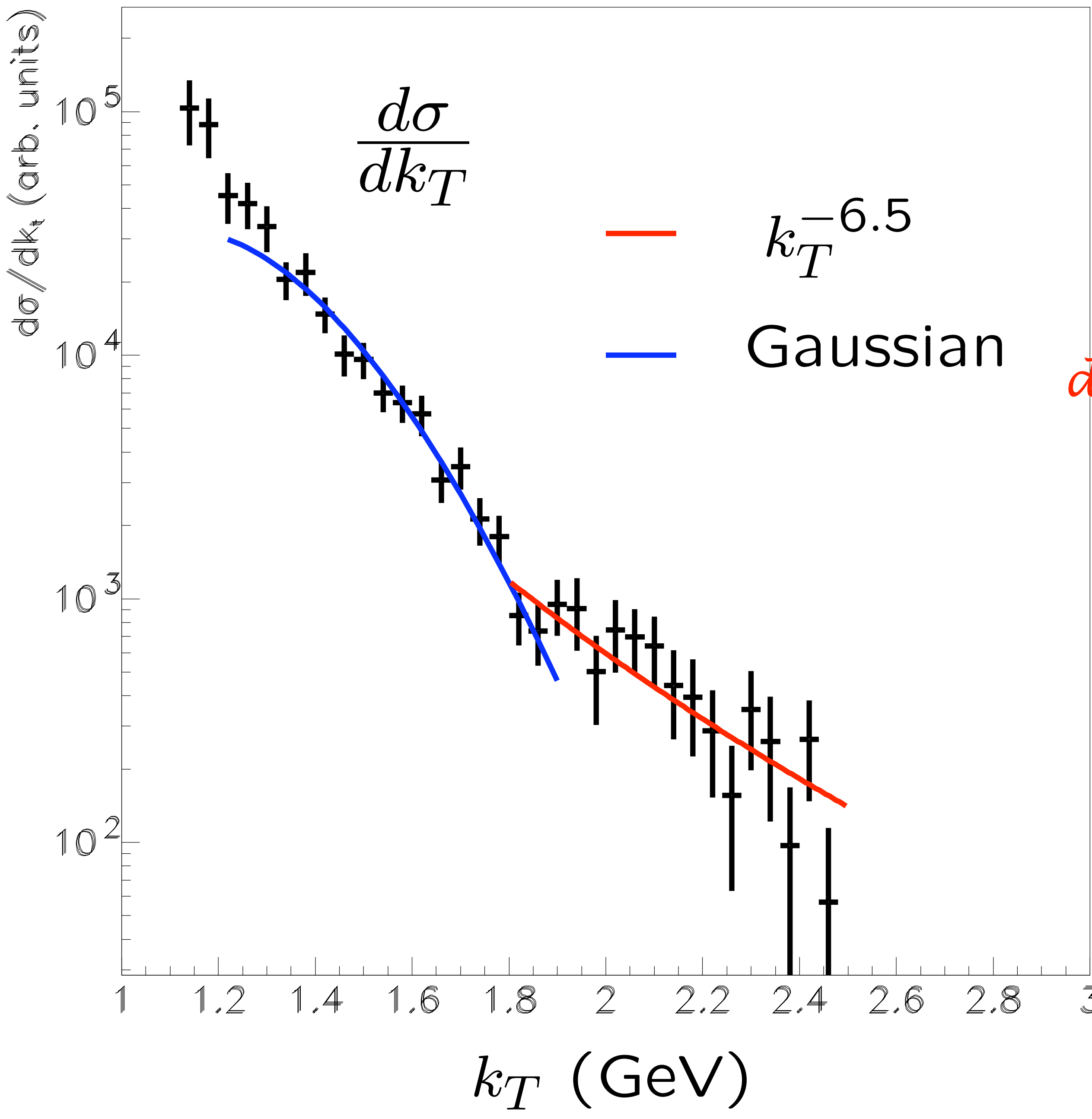
$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F_A^2(t)$$

Target left intact

Diffraction, Rapidity gap

E791 Diffractive Di-Jet transverse momentum distribution



Two Components

High Transverse momentum dependence consistent with PQCD, ERBL Evolution $k_T^{-6.5}$

Gaussian component similar to AdS/CFT HO LFWF

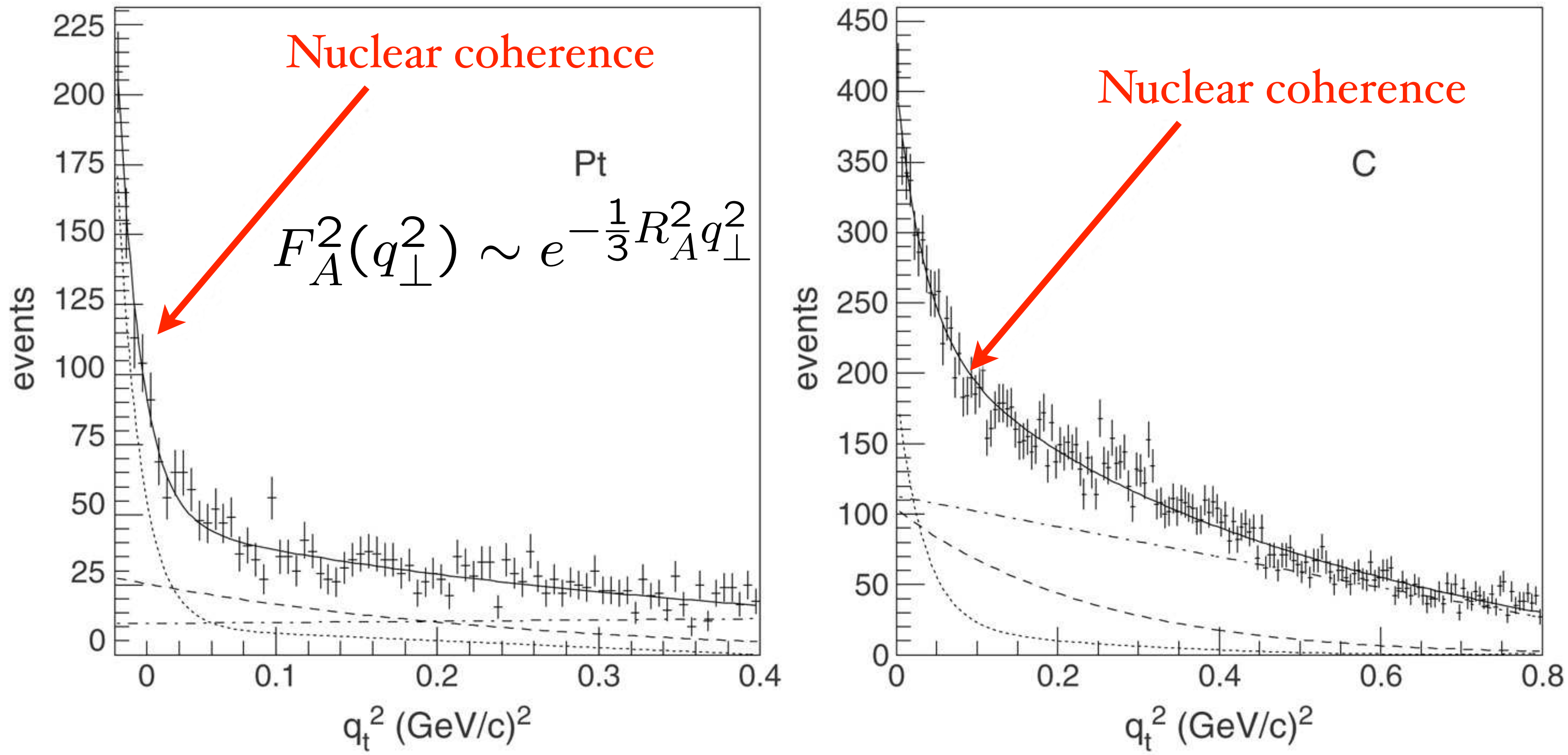
- **Fully coherent interactions between pion and nucleons.**

- **Emerging Di-Jets do not interact with nucleus.**

$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



Mueller, sjb; Bertsch et al;
Frankfurt, Miller, Strikman

Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: $\sigma \propto A^{\alpha}$

k_t range (GeV/c)	α	α (CT)
$1.25 < k_t < 1.5$	1.64 +0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

Ashery E791

α (Incoh.) $= 0.70 \pm 0.1$

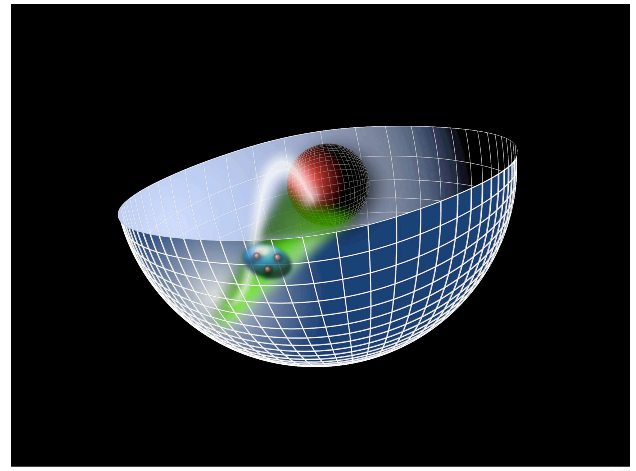
Conventional Glauber Theory Ruled Out !

Factor of 7

de Tèramond, Dosch, sjb

AdS/QCD
Soft-Wall Model

Single scheme-independent fundamental mass scale
κ



Light-Front Holography

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique
Confinement Potential!

Conformal Symmetry
of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

(m_q=0)

$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:** *Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!*

Conformal Template

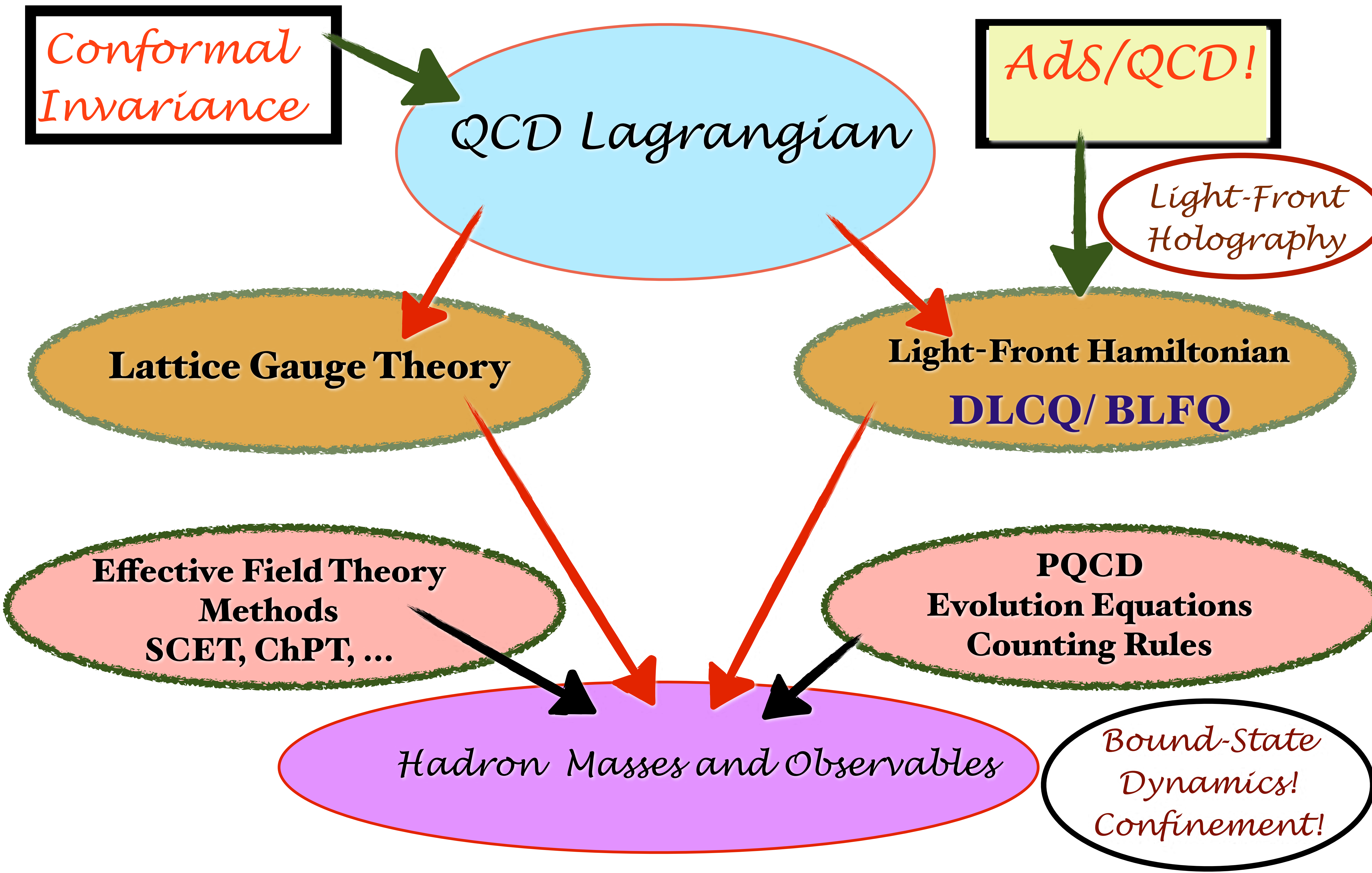
- Spontaneous breaking of scale invariance--Unique Confining Potential and Dilaton
- Non-Perturbative QCD Running Coupling
- Principle of Maximum Conformality -- sets renormalization scale in PQCD -- result is scheme independent!
- ERBL evolution and eigensolutions

Frishman, Sachrajda, Lepage, sjb; Braun

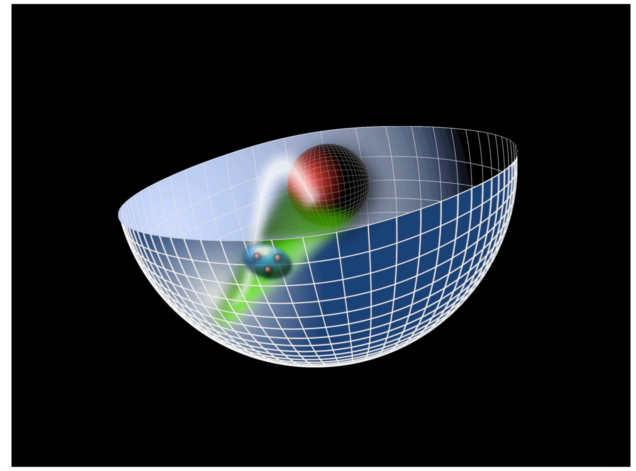
The Renormalization Scale Problem

- No renormalization scale ambiguity in QED
- Gell Mann-Low QED Coupling defined from physical observable
- Sums all Vacuum Polarization Contributions
- Recover conformal series
- Renormalization Scale in QED scheme: Identical to Photon Virtuality
- Analytic: Reproduces lepton-pair thresholds
- Examples: muonic atoms, g-2, Lamb Shift
- Time-like and Space-like QED Coupling related by analyticity
- Uses Dressed Skeleton Expansion
- Results are scheme independent
- High precision predictions

Predict Hadron Properties from First Principles!



de Teramond, Dosch, sjb



AdS/QCD Soft-Wall Model

Light-Front Holography

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

$$1/\kappa \simeq 0.4\ fm$$

Remarkable Features of Light-Front Schrödinger Equation

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**

New Directions

- **Hadronization at the Amplitude Level**
- **Direct Processes: Hadron production in subprocess**
- **Compute QCD Corrections at Soft-Scales -e.g. Sivers, Boer-Mulders, DDIS**
- **Double-Parton Processes**
- **Eliminate Factorization Scale: Fracture function determines off-shellness**
- **Sublimated Gluons: Gluons appear only at high virtuality**
- **Heavy Quark Fock States from Confinement Potential**
- **Hidden Color of Nuclear Wavefunctions**
- **Duality: Confinement effects absent at small x^2**

Gell-Mann Oakes Renner Formula in QCD

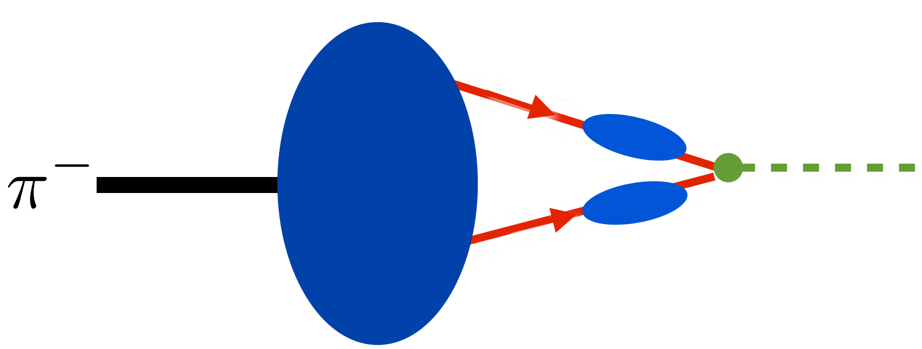
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is an "in-hadron condensate"



$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

THE COSMOLOGICAL CONSTANT

Sean M. Carroll

In Section (1.3) we discussed the large difference between the magnitude of the vacuum energy expected from zero-point fluctuations and scalar potentials, $^{theor} \sim 2 \times 10^{110}$ erg/cm^3, and the value we apparently observe, $^{(obs)} \sim 2 \times 10^{-10}$ erg/cm^3 (which may be thought of as an upper limit, if we wish to be careful). It is somewhat unfair to characterize this discrepancy as a factor of 10^{120}, since energy density can be expressed as a mass scale to the fourth power. Writing $= M_{vac}{}^4$, we find $M_{vac}{}^{(theory)} \sim M_{Pl} \sim 10^{18}$ GeV and $M_{vac}{}^{(obs)} \sim 10^{-3}$ eV, so a more fair characterization of the problem would be

$$\frac{M_{vac}^{(theory)}}{M_{vac}^{(observed)}} \sim 10^{30}$$

Of course, thirty orders of magnitude still constitutes a difference worthy of our attention.

Although the mechanism which suppresses the naive value of the vacuum energy is unknown, it seems easier to imagine a hypothetical scenario which makes it exactly zero than one which sets it to just the right value to be observable today.

(Keeping in mind that it is the zero-temperature, late-time vacuum energy which we want to be small; it is expected to change at phase transitions, and a large value in the early universe is a necessary component of inflationary universe scenarios.

If the recent observations pointing toward a cosmological constant of astrophysically relevant magnitude are confirmed, we will be faced with the challenge of explaining not only why the vacuum energy is smaller than expected, but also why it has the specific nonzero value it does.

"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE
Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

Elements of the solution:
(A) Light-Front Quantization: causal frame-independent vacuum
(B) New understanding of QCD "Condensates"
(C) Higgs Light-Front Zero Mode

“One of the gravest puzzles of theoretical physics”

**DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX**

A. ZEE
Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$
$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$
$$\Omega_\Lambda = 0.76(expt)$$

QCD gives Λ=zero if Quark and Gluon condensates reside within hadrons, not vacuum!

Electroweak contribution gives Λ=zero from Zero Mode solution to Higgs Potential

Electroweak Problem also could be solved in technicolor-- condensates within technihadrons

$$(\Omega_\Lambda)_{QCD} = 0 \qquad (\Omega_\Lambda)_{EW} = 0$$

Central Question: What is the source of Dark Energy?

$$\Omega_\Lambda = 0.76(expt)$$ *Higgs Zero-Mode Curvature?*

- **Light Front Quantization**
- **The LF Vacuum and the Physical Universe**
- **QCD Condensates and the Cosmological Constant**
- **Higgs Model on The LF and the Cosmological Constant**
- **Light-Front Holography and AdS/QCD**

String Theory

Goal: First Approximant to QCD

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+
space
to AdS5 space

AdS/CFT

Counting rules for Hard
Exclusive Scattering
Regge Trajectories

Conformal behavior at short
distances
+ Confinement at large distance

QCD at the Amplitude Level

AdS/QCD

Holography

Semi-Classical QCD / Wave Equations

Unique Potential,
Dilaton

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

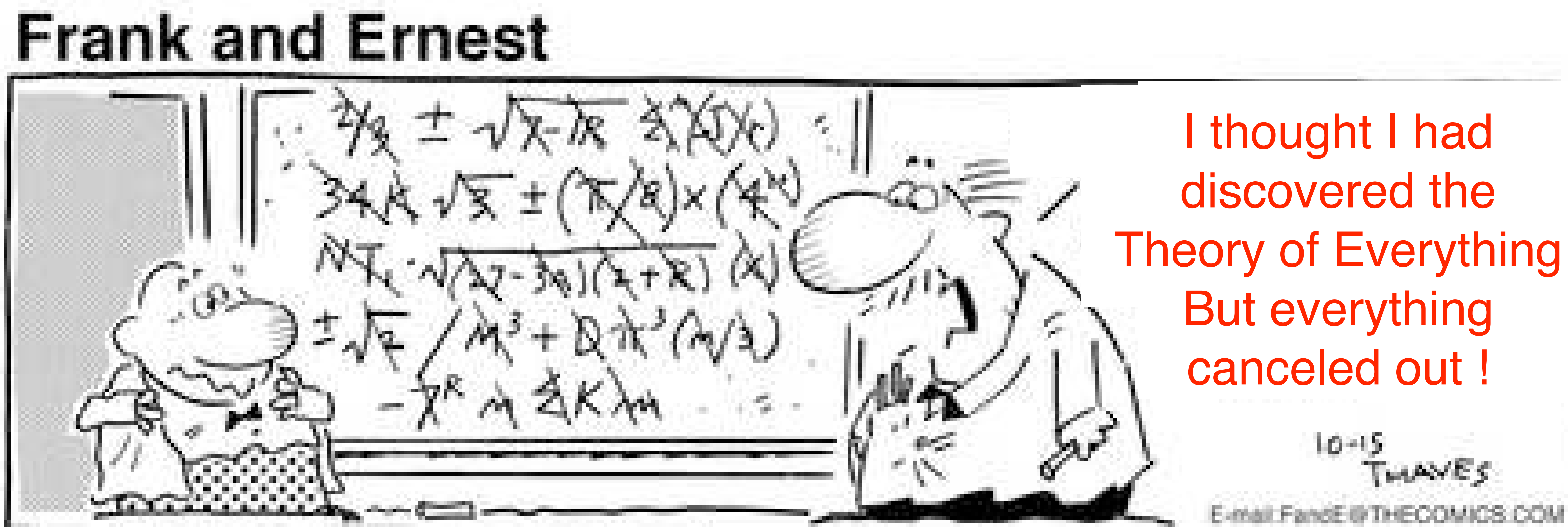
Integrable!

Hadron Spectra, Wavefunctions, Dynamics

SCIENCE VOL 265 15 SEPTEMBER 1995

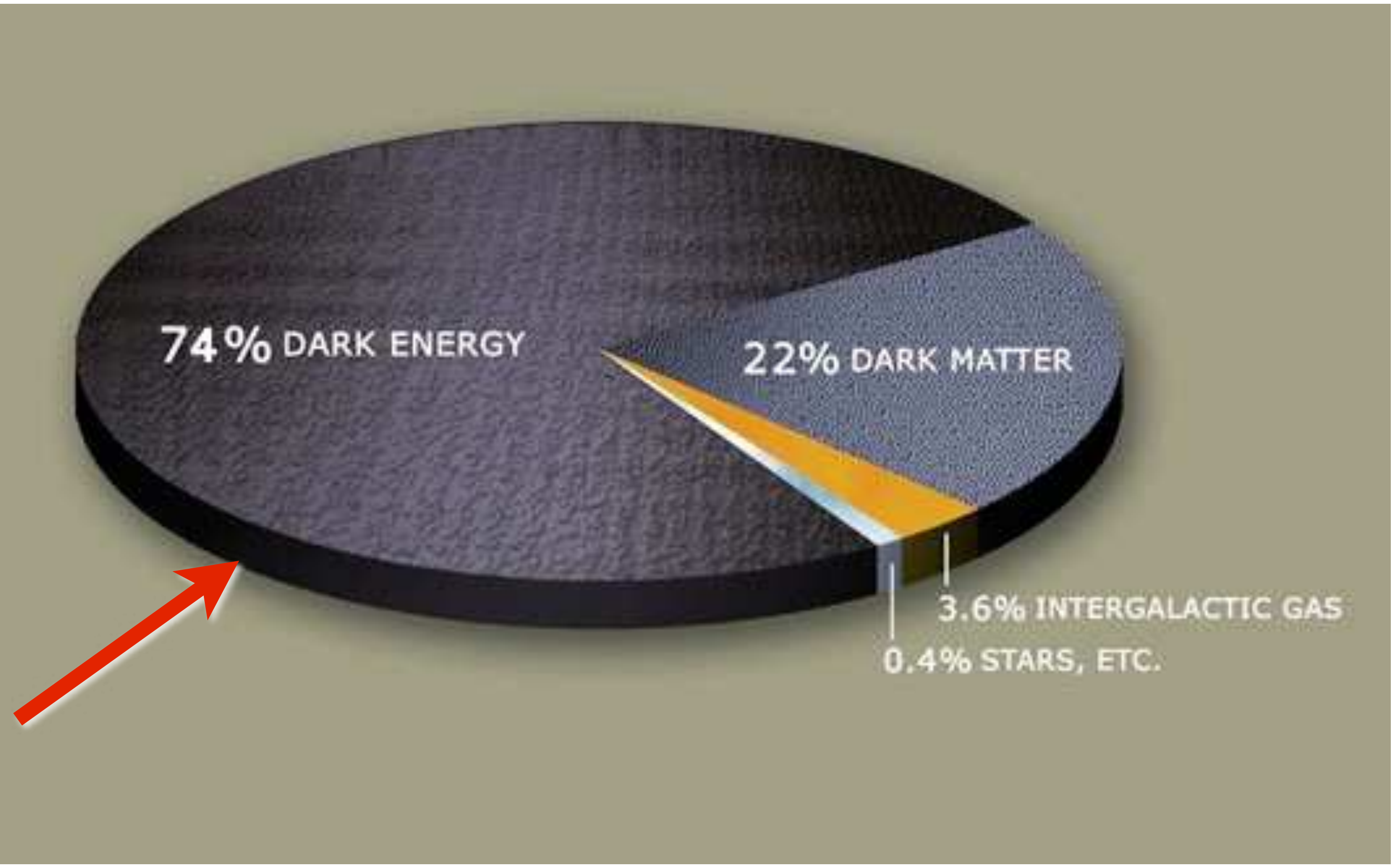
A Theory of Everything Takes Place

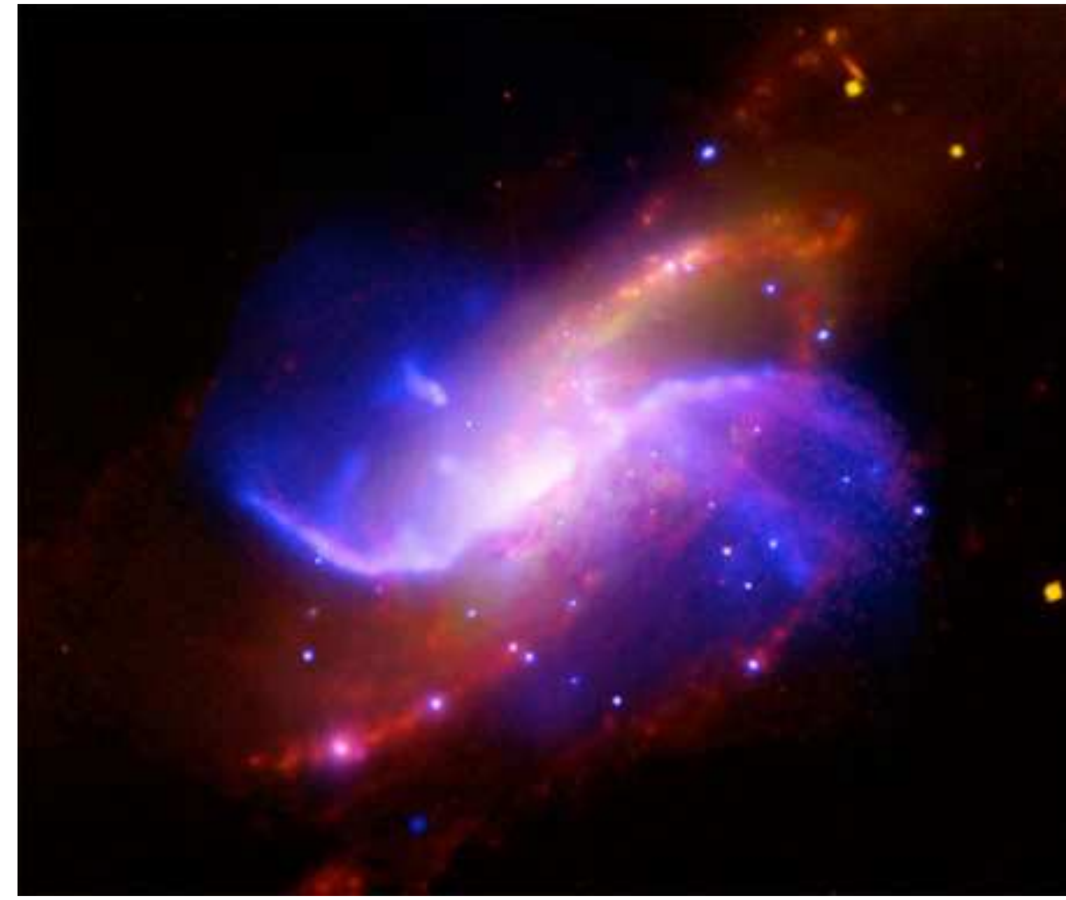
String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.



AdS/QCD, Light-Front Holography, and the Light-Front Vacuum

collaborations with Craig Roberts, Robert Shrock, Prem Srivastava, Peter Tandy, Guy de Téramond, and Hans Günter Dosch





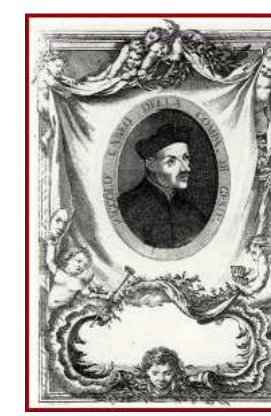
Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

Ferrara International School Niccolò Cabeo

May 19-23, 2014

Vacuum and broken symmetries: from the quantum to the cosmos



IUSS - FERRARA 1391

QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- heavy quarks only from gluon splitting
- renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- Infrared Slavery
- Nuclei are composites of nucleons only
- Real part of DVCS arbitrary

Goals

- **Test QCD to maximum precision**
- **High precision determination of $\alpha_s(Q^2)$ at all scales**
- **Relate observable to observable --no scheme or scale ambiguity**
- **Eliminate renormalization scale ambiguity in a scheme-independent manner**
- **Relate renormalization schemes without ambiguity**
- **Maximize sensitivity to new physics at the colliders**

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

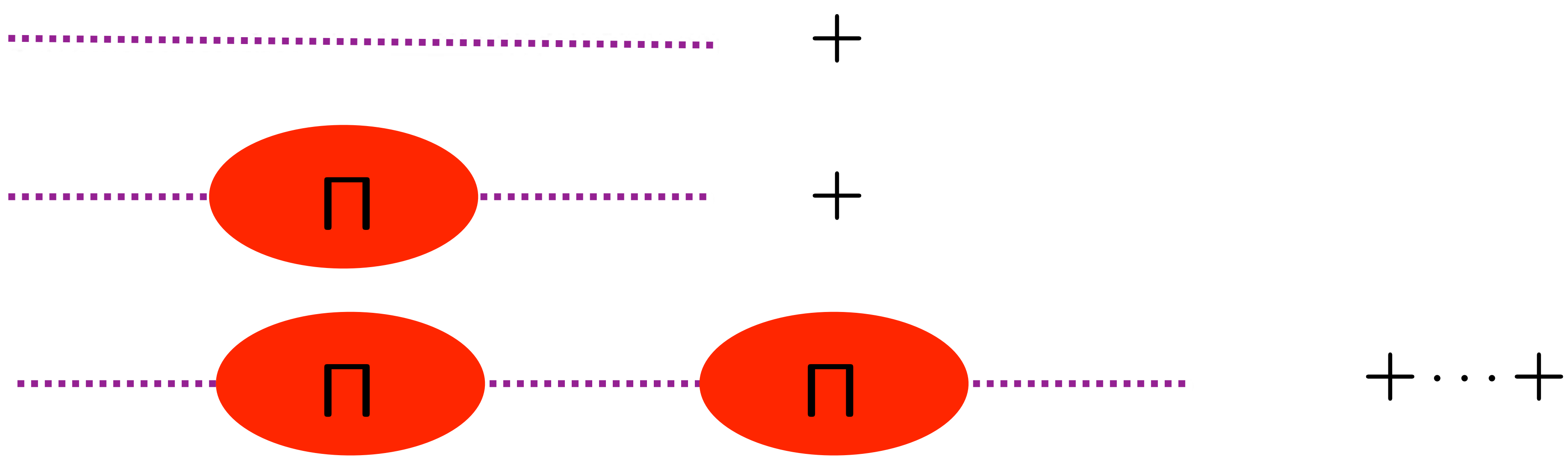
$$\alpha(t) = \frac{\alpha(0)}{1-\Pi(t)}$$

Gell-Mann--Low Effective Charge

QED Effective Charge

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

All-orders lepton loop corrections to dressed photon propagator

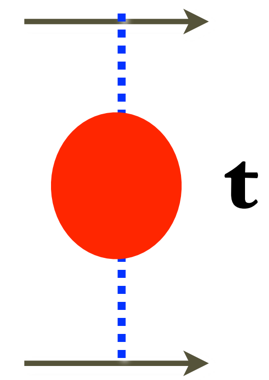


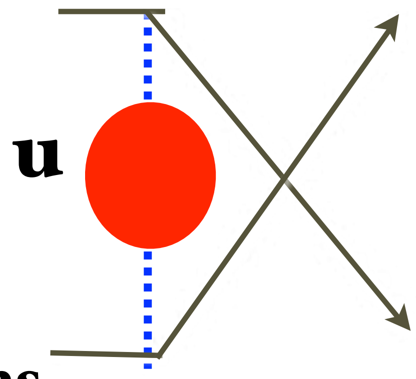
$$\alpha(t) = \frac{\alpha(t_0)}{1 - \Pi(t, t_0)} \qquad \Pi(t, t_0) = \frac{\Pi(t) - \Pi(t_0)}{1 - \Pi(t_0)}$$

Initial scale t_o is arbitrary -- Variation gives RGE Equations
Physical renormalization scale t not arbitrary

Electron-Electron Scattering in QED

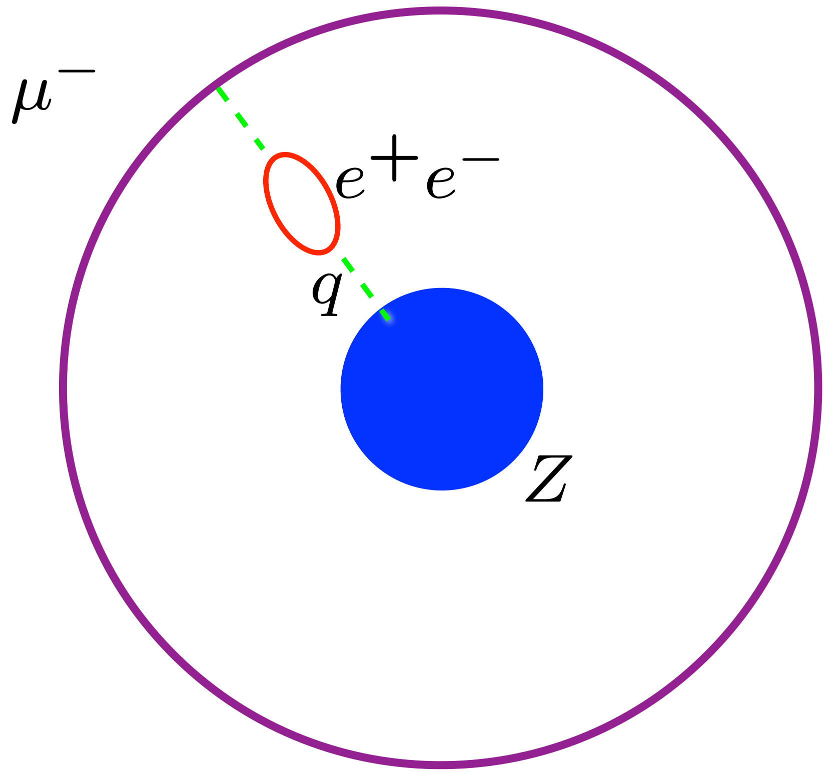
$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$





- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling.**
- **If one chooses a different scale, one can sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- *No renormalization scale ambiguity!*
- *Two separate gauge invariant physical scales.*

Scale Setting in QED: Muonic Atoms



$$V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2}$$

$$\mu_R^2 \equiv q^2$$

$$\alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1-\Pi(q^2)}$$

Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to 0.1% precision in μ Pb

Features of BLM/PMC Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb

Phys.Rev.D28:228,1983

- **"Principle of Maximum Conformality"**

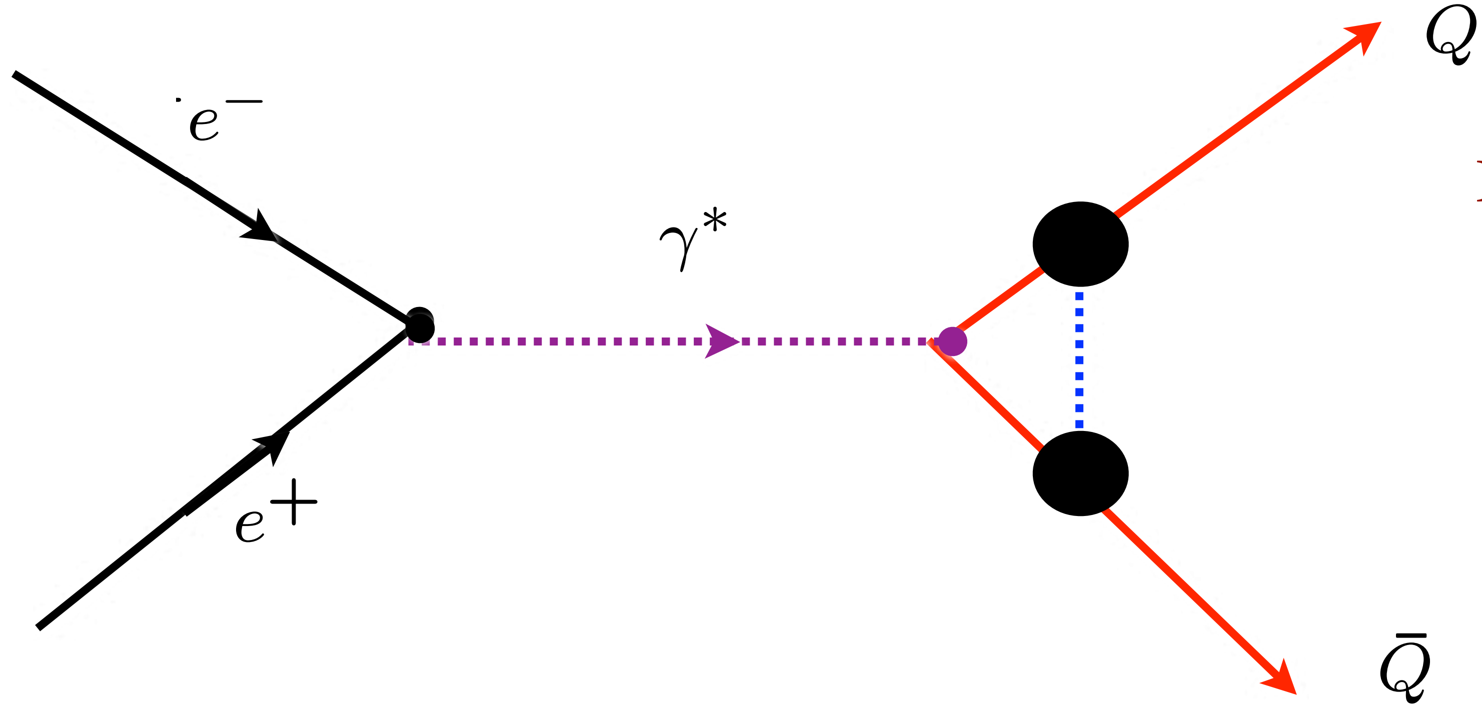
Di Giustino, Wu, sjb

- **All terms associated with nonzero beta function summed into running coupling**
- **Standard procedure in QED**
- **Resulting series identical to conformal series**
- **Renormalon n! growth of PQCD coefficients from beta function eliminated!**
- *Scheme Independent* **!!!**
- **In general, BLM/PMC scales depend on all invariants**
- **Single Effective PMC scale at NLO**

Principle of Maximum Conformality (PMC)

- Sets pQCD renormalization scale correctly at every finite order
- Predictions are scheme-independent
- Satisfies all principles of the renormalization group
- Agrees with Gell Mann-Low procedure for pQED in Abelian limit
- Shifts all β terms into α_s, leaving conformal series
- Automatic procedure: R_δ scheme
- Number of flavors n_f set
- Eliminates n! renormalon growth
- Choice of initial scale irrelevant
- Eliminates unnecessary systematic error -- conventional guess is scheme-dependent, disagrees with QED
- Reduces disagreement with pQCD for top/anti-top asymmetry at Tevatron from 3σ to 1σ

Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB



Hoang, Kuhn, Teubner, sjb

$$F_1 + F_2 = \left[1 - 2\frac{\alpha_s(se^{3/4}/4)}{\pi}\right] \times \left[1 + \frac{\pi\alpha_s(sv^2)}{4v}\right]$$

Angular distributions of massive quarks close to threshold.

Example of Multiple BLM Scales

Need QCD coupling at small scales at low relative velocity v

Principle of Maximum Conformality (PMC)

QCD Observables

$$\mathcal{O} = C(\alpha_s(\mu_0^2)) + B(\beta \log \frac{Q^2}{\mu_0^2}) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Scale-Free Conformal Series

Running Coupling Effects

Higher Twist from Hadron Dynamics

Intrinsic Heavy Quarks

Light by Light Loops

BLM/PMC: Absorb β-terms into running coupling

$$\mathcal{O} = C(\alpha_s(Q^{*2})) + D(\frac{m_q^2}{Q^2}) + E(\frac{\Lambda_{QCD}^2}{Q^2}) + F(\frac{\Lambda_{QCD}^2}{m_Q^2}) + G(\frac{m_q^2}{m_Q^2})$$

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

$Choose\ renormalization\ scheme;\ e.g.\ \alpha_s^R(\mu_R^{\rm init})$

↓

$Choose\ \mu_R^{init};\ arbitrary\ initial\ renormalization\ scale$

↓

$Identify\ \{\beta_i^R\} - terms\ using\ n_f - terms$

$through\ the\ PMC - BLM\ correspondence\ principle$

↓

$Shift\ scale\ of\ \alpha_s\ to\ \mu_R^{\rm PMC}\ to\ eliminate\ \{\beta_i^R\} - terms$

↓

$Conformal\ Series$

$Result\ is\ independent\ of\ \mu_R^{\rm init}\ and\ scheme\ at\ fixed\ order$

Principle of Maximum Conformality

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

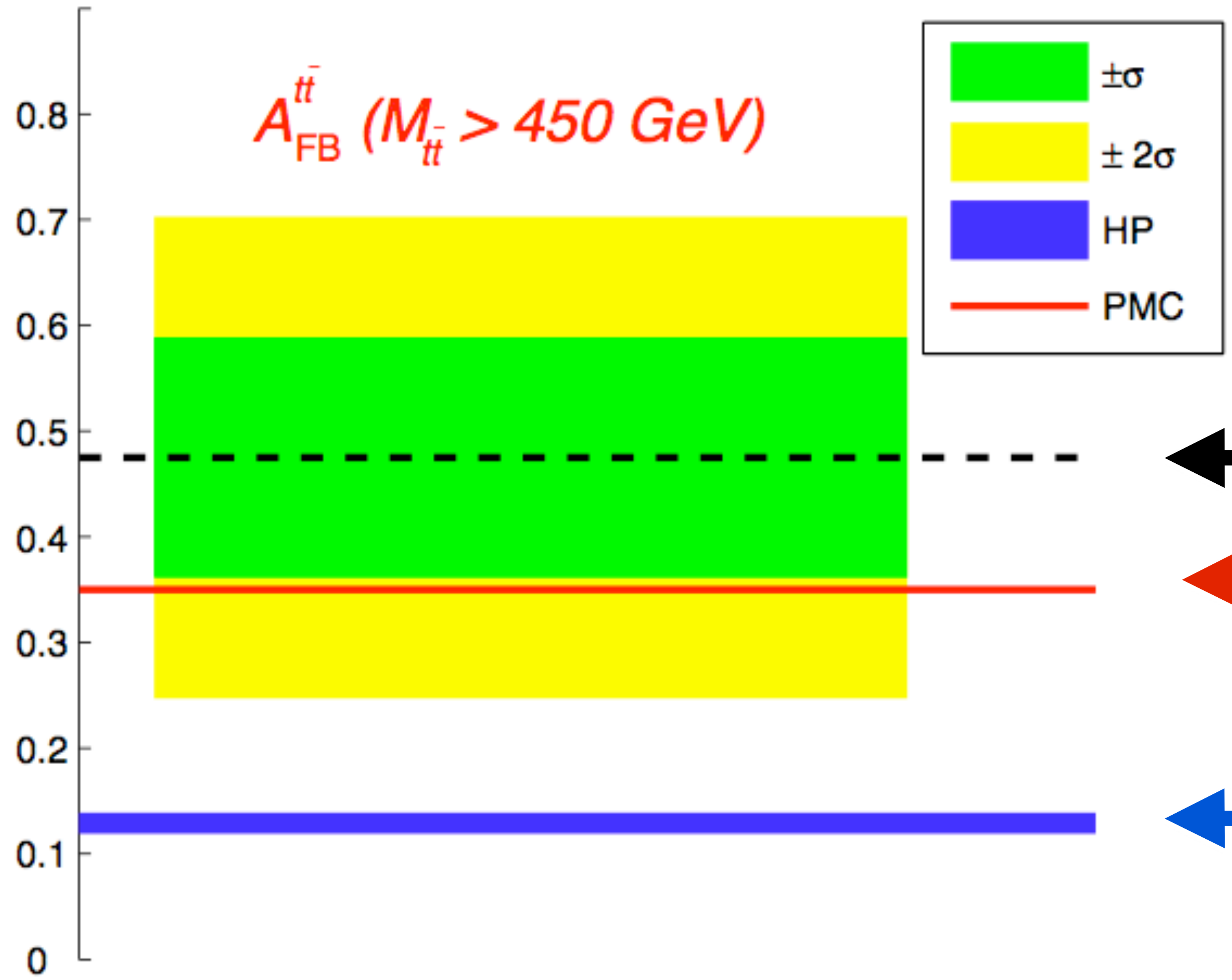
QED Scale Setting at N_C=0

Eliminates unnecessary systematic uncertainty

δ -Scheme automatically identifies β -terms!

Xing-Gang Wu, Matin Mojaza Leonardo di Giustino, SJB

The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)



Xing-Gang Wu
SJB

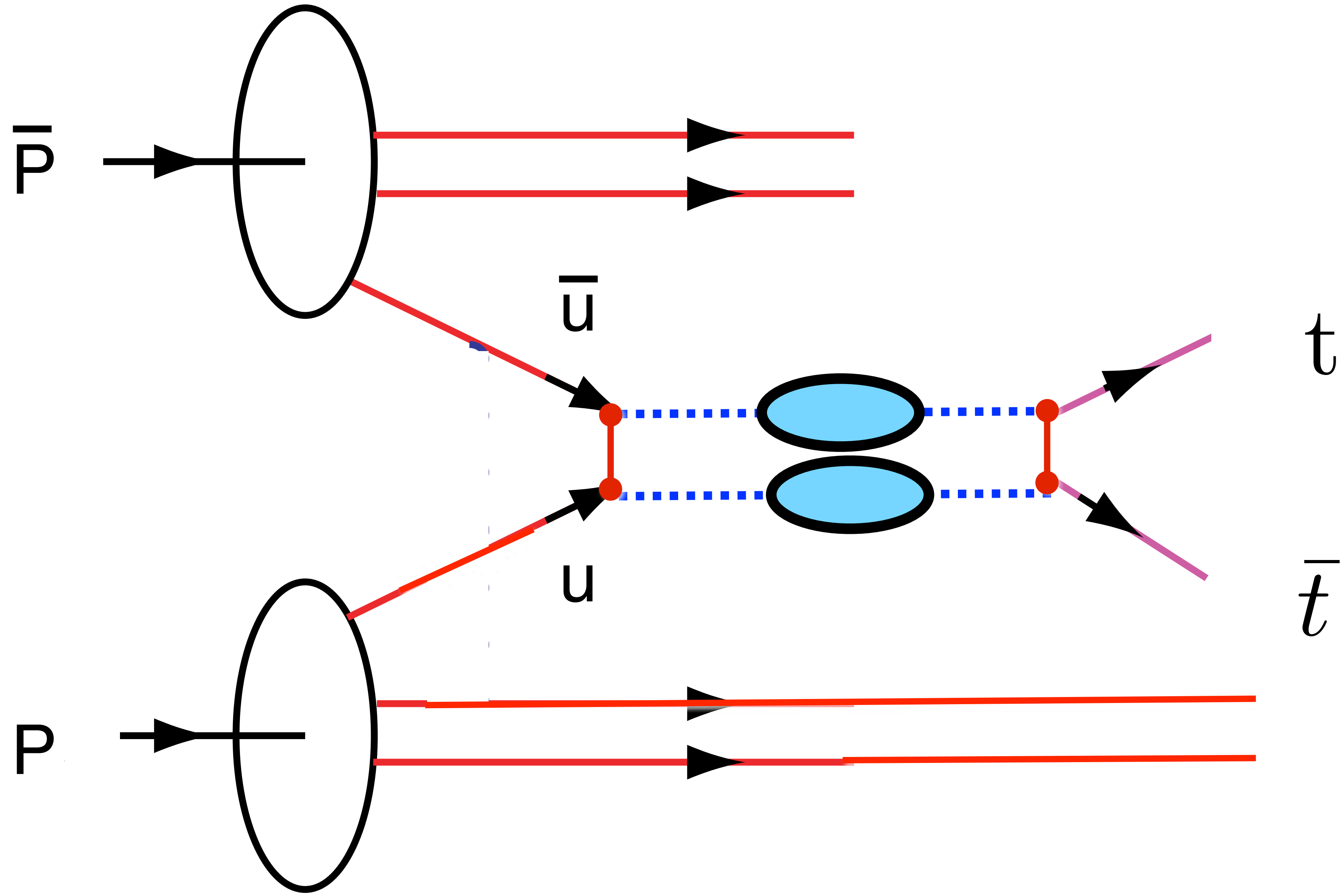
Experimental asymmetry

PMC Prediction

Conventional guess for renormalization scale and range

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

Contributes to the $\bar{p}p \to \bar{t}tX$ asymmetry at the Tevatron



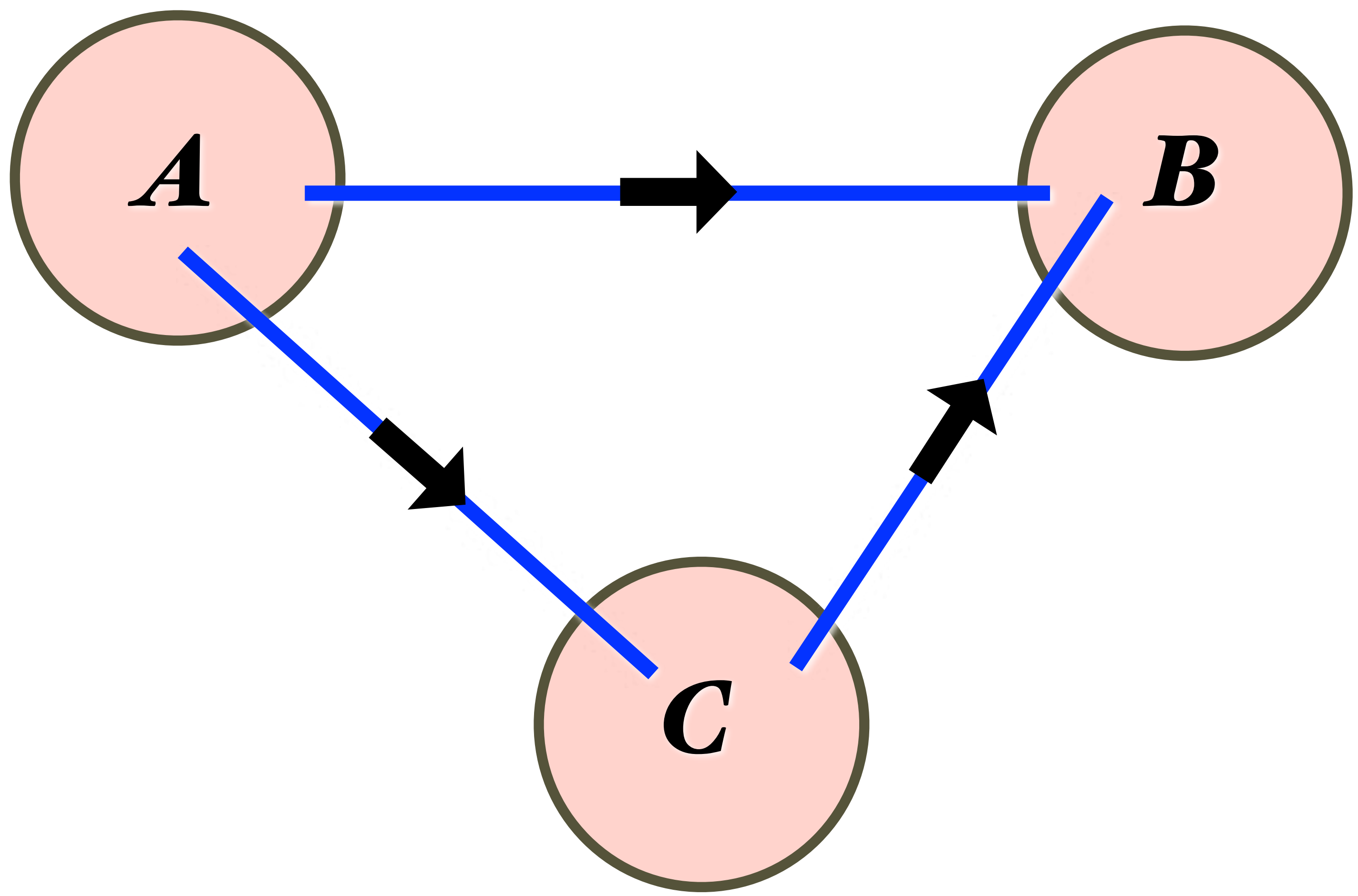
Interferes with Born term.

Small value of renormalization scale increases asymmetry

Xing-Gang Wu, sjb

Transitivity Property of Renormalization Group

Relation of observables must be independent of intermediate scheme



A → C   C → B   identical to   A → B

Violated by PMS!