Cosmology: Basics

An introduction to astrophysical cosmology

Alain Blanchard

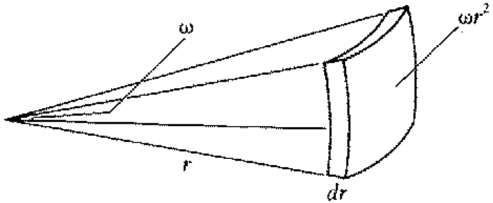
Ferrara, May 22th, 2014

Olbers paradox

Volume element for counts:

Olbers paradox

Volume element for counts:



Olbers paradox

Homogeneous medium of stars:

$$I = \frac{L}{4\pi r^2}$$

Olbers paradox

Homogeneous medium of stars:

$$l = \frac{L}{4\pi r^2}$$

Number of stars between r and $r + dr$: $dN = n_* \times \omega \times r^2 \times dr$

Olbers paradox

Homogeneous medium of stars:

$$l = \frac{L}{4\pi r^2}$$

Number of stars between r and $r + dr$: $dN = n_* \times \omega \times r^2 \times dr$

Number of stars between l and $l + dl$:

$$dN = -\frac{1}{2} n_* \omega \left(\frac{L}{4\pi}\right)^{3/2} \frac{dl}{l^{5/2}}$$

Olbers paradox

Integrated number of stars $> l$:

Olbers paradox

Integrated number of stars $> l$:

$$N(> l) = \frac{1}{3} n_* \omega \left(\frac{L}{4\pi} \right)^{3/2} \frac{1}{l^{3/2}}$$

Olbers paradox

Integrated number of stars $> l$:

$$N(> l) = \frac{1}{3} n_* \omega \left(\frac{L}{4\pi} \right)^{3/2} \frac{1}{l^{3/2}}$$

in term of magnitude ($m = -2.5 \log(l) + \mathrm{cste}$)

$$\log(N(< m)) \propto 0.6 m + \mathrm{cste}$$

Olbers paradox

Integrated luminosity:

$$\phi = \int_0^{+\infty} \frac{dN}{dl} l dl$$

Olbers paradox

Integrated luminosity:

$$\phi = \int_0^{+\infty} \frac{dN}{dl} l dl$$

$l^{-3/2}$ makes the integral diverging!

Olbers paradox

Integrated luminosity:

$$\phi = \int_0^{+\infty} \frac{dN}{dl} l dl$$

$l^{-3/2}$ makes the integral diverging!

Something wrong among:

Olbers paradox

Integrated luminosity:

$$\phi = \int_0^{+\infty} \frac{dN}{dl} l dl$$

$l^{-3/2}$ makes the integral diverging!

Something wrong among:

- The universe is homogeneous

Olbers paradox

Integrated luminosity:

$$\phi = \int_0^{+\infty} \frac{dN}{dl} l dl$$

$l^{-3/2}$ makes the integral diverging!

Something wrong among:

- The universe is homogeneous
- Universe is static and eternal

Olbers paradox

Integrated luminosity:

$$\phi = \int_0^{+\infty} \frac{dN}{dl} l dl$$

$l^{-3/2}$ makes the integral diverging!

Something wrong among:

- The universe is homogeneous
- Universe is static and eternal
- Geometry of space is Euclidian geometry

In retrospect, now that we have reasonably convincing evidence that the universe really is expanding, it is easy to find reasons why a static universe is problematic.

J.Peebles

Olbers paradox

Energetics considerations:

Olbers paradox

Energetics considerations:

- Finite volume =>

Olbers paradox

Energetics considerations:

- Finite volume =>
- Finite number of stars =>

Olbers paradox

Energetics considerations:

- Finite volume =>
- Finite number of stars =>
- Finite amount of energy is available

Olbers paradox

Timescale for energy exhaust:

Olbers paradox

Timescale for energy exhaust:
Order of magnitude for the Sun:

$$L_\odot \sim 4\ 10^{33}\ \ \mathrm{erg/s}$$

$$M_\odot \sim 2\ 10^{33}\ \ \mathrm{g}$$

Olbers paradox

Timescale for energy exhaust:

Order of magnitude for the Sun:

$$L_\odot \sim 4\ 10^{33} \quad \mathrm{erg/s}$$

$$M_\odot \sim 2\ 10^{33} \quad \mathrm{g}$$

Efficiency of nuclear reactions $\epsilon \sim 0.007$

Olbers paradox

Timescale for energy exhaust:
Order of magnitude for the Sun:

$$L_\odot \sim 4\ 10^{33} \quad \mathrm{erg/s}$$

$$M_\odot \sim 2\ 10^{33} \quad \mathrm{g}$$

Efficiency of nuclear reactions $\epsilon \sim 0.007$

$$\tau \sim \frac{\epsilon M_\odot c^2}{L_\odot} = \frac{E_t}{\frac{dE_t}{dt}}$$

$$\sim 3\ 10^{18} s \sim 10^{11} \quad \mathrm{yr}$$

Olbers paradox

Timescale for energy exhaust:
Order of magnitude for the Sun:

$$L_\odot \sim 4\ 10^{33}\quad \mathrm{erg/s}$$

$$M_\odot \sim 2\ 10^{33}\quad \mathrm{g}$$

Efficiency of nuclear reactions $\epsilon \sim 0.007$

$$\tau \sim \frac{\epsilon M_\odot c^2}{L_\odot} = \frac{E_t}{\frac{dE_t}{dt}}$$

$$\sim 3\ 10^{18} s \sim 10^{11}\quad \mathrm{yr}$$

=> the universe cannot remain identical for ever!

Homogeneity

"The universe is homogeneous on large scale"

Einstein cosmological principle

Homogeneity

"The universe is homogeneous on large scale"

Einstein cosmological principle

Can and should be tested from observations.

Homogeneity

"The universe is homogeneous on large scale"

Einstein cosmological principle

Can and should be tested from observations.

$$\lim_{R\to\infty} \overline{\rho}(R) = cste$$

(necessary but not sufficient...)

Homogeneity

"The universe is homogeneous on large scale"

Einstein cosmological principle

Can and should be tested from observations.

$$\lim_{R\to\infty} \overline{\rho}(R) = cste$$

(necessary but not sufficient...)

From galaxies:

$$\log(N(m)) \propto 0.6m\ ?$$

Homogeneity

"The universe is homogeneous on large scale"

Einstein cosmological principle

Can and should be tested from observations.

$$\lim_{R \to \infty} \overline{\rho}(R) = cste$$

(necessary but not sufficient...)

From galaxies:

$$\log(N(m)) \propto 0.6m \text{ ?}$$

Good indication.

Homogeneity

"The universe is homogeneous on large scale"

Einstein cosmological principle

Can and should be tested from observations.

$$\lim_{R \to \infty} \overline{\rho}(R) = cste$$

(necessary but not sufficient...)

From galaxies:

$$\log(N(m)) \propto 0.6m \ ?$$

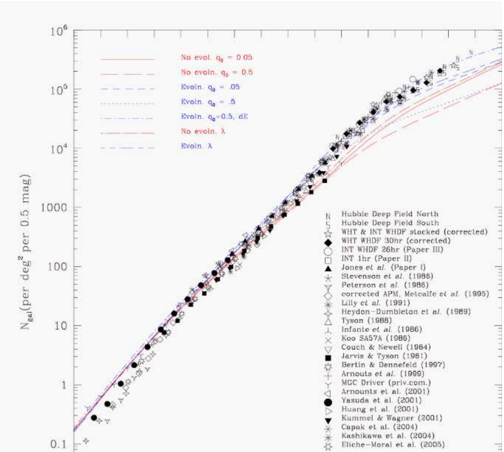
Good indication.

Isotropy

$+$Copernic principle $=>$ homogeneity

Homogeneity

From galaxies number counts:



Geometry

Geometry

- locally we can assign four coordinates to an event (x, y, z, t)

Geometry

- locally we can assign four coordinates to an event (x, y, z, t)
- this does not prejudge of the global *shape* of the universe: Plane ? Sphere ? Torrus ?

Geometry

- locally we can assign four coordinates to an event (x, y, z, t)
- this does not prejudge of the global *shape* of the universe: Plane ? Sphere ? Torrus ?

3D Spherical universe: let's start from 4D (x, y, z, u)

$$x^2 + y^2 + z^2 + u^2 = R^2$$

Geometry

- locally we can assign four coordinates to an event (x, y, z, t)
- this does not prejudge of the global *shape* of the universe: Plane ? Sphere ? Torrus ?

3D Spherical universe: let's start from 4D (x, y, z, u)

$$x^2 + y^2 + z^2 + u^2 = R^2$$

using spherical coordinates $r^2 = x^2 + y^2 + z^2$
from $dl^2 = dx^2 + dy^2 + dz^2 + du^2$ and $u^2 = R^2 - r^2$ one gets:

Geometry

- locally we can assign four coordinates to an event (x, y, z, t)
- this does not prejudge of the global *shape* of the universe: Plane ? Sphere ? Torrus ?

3D Spherical universe: let's start from 4D (x, y, z, u)

$$x^2 + y^2 + z^2 + u^2 = R^2$$

using spherical coordinates $r^2 = x^2 + y^2 + z^2$
from $dl^2 = dx^2 + dy^2 + dz^2 + du^2$ and $u^2 = R^2 - r^2$ one gets:

$$dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad + \quad \frac{dr^2}{1 - \left(\frac{r}{R}\right)^2}$$

Geometry

Geometry

$$dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad + \quad \frac{dr^2}{1 - \left(\frac{r}{R}\right)^2} \text{ (spherical)}$$
$$+ \quad dr^2 \text{ (flat)}$$
$$+ \quad \frac{dr^2}{1 + \left(\frac{r}{R}\right)^2} \text{ (hyperbolic)}$$

Geometry

$$
\begin{aligned}
dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad & + \quad \frac{dr^2}{1-\left(\frac{r}{R}\right)^2} \text{ (spherical)} \\
& + \quad dr^2 \text{ (flat)} \\
& + \quad \frac{dr^2}{1+\left(\frac{r}{R}\right)^2} \text{ (hyperbolic)}
\end{aligned}
$$

The Robertson-Walker line element: $r \rightarrow \frac{r}{R}$

Geometry

$$dl^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad + \quad \frac{dr^2}{1-\left(\frac{r}{R}\right)^2} \text{ (spherical)}$$
$$+ \quad dr^2 \text{ (flat)}$$
$$+ \quad \frac{dr^2}{1+\left(\frac{r}{R}\right)^2} \text{ (hyperbolic)}$$

The Robertson-Walker line element: $r \rightarrow \frac{r}{R}$

$$ds^2 = -c^2dt^2 + R(t)^2[r^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{dr^2}{1-kr^2}]$$

with $k = -1, 0, +1$ accordingly to geometry.

General Geometry

General Geometry

The Robertson-Walker line element: $r = r R_0$

$$ds^2 = -c^2 dt^2 + a(t)^2 [r^2 (d\theta^2 + \sin^2\theta d\phi^2) + \frac{dr^2}{1 - Kr^2}]$$

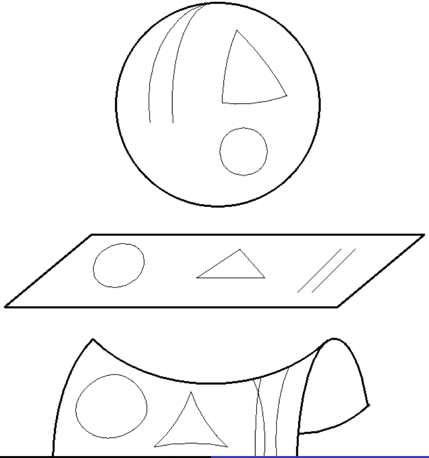
with $K = \frac{k}{R_0^2}$ and $a(t) = \frac{R(t)}{R_0}$.

General Geometry

Three possible geometries:

General Geometry

Three possible geometries:



Topology

The local geometry of space (i.e. the value of k) does not prejudge of the global shape of space i.e. its topology.

Topology

The local geometry of space (i.e. the value of k) does not prejudge of the global shape of space i.e. its topology.

-> The universe is always finite with $k = +1$.

Topology

The local geometry of space (i.e. the value of k) does not prejudge of the global shape of space i.e. its topology.

-> The universe is always finite with $k = +1$.
-> The universe could be finite even with $k = 0, -1$.

Basic Principle

Trajectories of photons = null geodesics

Basic Principle

Trajectories of photons = null geodesics
Observer at $(r = 0, \theta, \phi, t = t_0)$
emitting light source at $(r_{\rm S}, \theta = 0, \phi = 0, t_{\rm S})$

Basic Principle

Trajectories of photons = null geodesics
Observer at $(r = 0, \theta, \phi, t = t_0)$
emitting light source at $(r_{\rm S}, \theta = 0, \phi = 0, t_{\rm S})$
$r(t)$ be the trajectory of the photons emitted. As this trajectory is a null geodesic, we have:

$$c^2 dt^2 - R^2(t)\frac{dr^2}{1 - kr^2} = 0$$

i.e.

Basic Principle

Trajectories of photons = null geodesics
Observer at $(r = 0, \theta, \phi, t = t_0)$
emitting light source at $(r_S, \theta = 0, \phi = 0, t_S)$
$r(t)$ be the trajectory of the photons emitted. As this trajectory is a null geodesic, we have:

$$c^2 dt^2 - R^2(t) \frac{dr^2}{1 - kr^2} = 0$$

i.e.

$$\frac{cdt}{R(t)} = \frac{dr}{\sqrt{1 - kr^2}}$$

General Mattig relation

relation $r_S - t_S$

General Mattig relation

relation $r_S - t_S$

$$\int_{t_S}^{t_0} \frac{cdt}{R(t)} = \int_0^{r_S} \frac{dr}{(1-kr^2)^{1/2}} = S_k^{-1}(r_S)$$

with:

$$S_k(u) = \begin{cases} \sin(u) & \text{if}\, k = +1 \\ u & \text{if}\, k = 0 \\ \sinh(u) & \text{if}\, k = -1 \end{cases}$$

General Mattig relation

relation $r_S - t_S$

$$\int_{t_S}^{t_0} \frac{cdt}{R(t)} = \int_0^{r_S} \frac{dr}{(1-kr^2)^{1/2}} = S_k^{-1}(r_S)$$

with:

$$S_k(u) = \begin{cases} \sin(u) & \mathrm{if}\, k = +1 \\ u & \mathrm{if}\, k = 0 \\ \sinh(u) & \mathrm{if}\, k = -1 \end{cases}$$

When the distance is small in front of R_0 we just have:

$$S_k^{-1}(r) \sim r \text{ and } l.h.s. \sim \frac{c\delta t}{R(t_0)} \equiv \frac{D}{R(t_0)}$$

The Redshift

A source emitting at the frequency ν_S is observed at frequency ν_0

The Redshift

A source emitting at the frequency ν_S is observed at frequency ν_0

We consider the two trajectories of the light ray emitted at the time $t_{\rm S}$ and $t_{\rm S}+\frac{1}{\nu_S}$ arriving at t_0 and $t_0+\frac{1}{\nu_0}$

The Redshift

A source emitting at the frequency ν_S is observed at frequency ν_0

We consider the two trajectories of the light ray emitted at the time t_S and $t_\mathrm{S} + \frac{1}{\nu_S}$ arriving at t_0 and $t_0 + \frac{1}{\nu_0}$

The comoving coordinate r_S of the source remains constant so:

$$S_k^{-1}(r_\mathrm{S}) = \int_{t_\mathrm{S}}^{t_0} \frac{cdt}{R(t)} = \int_{t_\mathrm{S}+1/\nu_S}^{t_0+1/\nu_0} \frac{cdt}{R(t)}$$

The Redshift (2)

so:

$$\frac{c}{R(t_0)}\frac{1}{\nu_0} - \frac{c}{R(t_S)}\frac{1}{\nu_S} = 0$$

leading to the *redshift* z:

$$1 + z = \frac{\nu_s}{\nu_0} = \frac{\lambda_0}{\lambda_E} = \frac{R_0}{R_S}$$

Observed time difference

Let's two events at epoch corresponding to z be separated by Δt_S

Observed time difference

Let's two events at epoch corresponding to z be separated by Δt_S

What will be the observed time difference Δt_0 ?

Observed time difference

Let's two events at epoch corresponding to z be separated by Δt_S

What will be the observed time difference Δt_0 ?

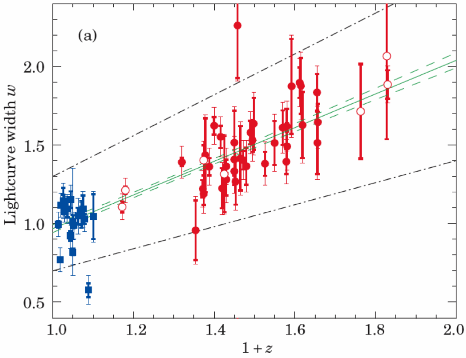
$$\frac{\Delta t_0}{\Delta t_S} = 1 + z$$

Observed time difference

Using distant supernovae

Observed time difference

Using distant supernovae



Nature of cosmological redshift

Interpretation?

Nature of cosmological redshift

Interpretation?

Doppler?

Nature of cosmological redshift

Interpretation?

Doppler?
Gravitational?

Nature of cosmological redshift

Interpretation?

Doppler?
Gravitational?
(It is not the same!)

The Redshift (3)

If the "distance" changes with time:

$$v = \frac{\Delta l}{\Delta t}$$

and if :

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \text{ (first order)}$$

this could be qualified as a purely Doppler shift.

The proper distance

Distance obtained as a sum of rulers:

$$dl^2 = ds^2 = R(t)^2 \frac{dr^2}{1 - kr^2}$$

so that the proper distance is :

$$D = \int_0^S dl = R(t) S_k^{-1}(r_{\rm S})$$

The proper distance

Distance obtained as a sum of rulers:

$$dl^2 = ds^2 = R(t)^2 \frac{dr^2}{1 - kr^2}$$

so that the proper distance is :

$$D = \int_0^S dl = R(t) S_k^{-1}(r_{\mathrm{S}})$$

This distance varries with time:

$$\dot{D} = \dot{R} S_k^{-1}(r_S)$$

Hubble law

So that the source is recessing at speed:

$$v = \frac{\dot{R}}{R} D = HD$$

Hubble law

So that the source is recessing at speed:

$$v = \frac{\dot{R}}{R} D = HD$$

This is the **Hubble law**.

Nature of Redshift

The redshift from expansion:

$$\frac{\nu_0}{\nu_s} = \frac{R(t_S)}{R(t_0)} \sim \frac{R(t_0) + \dot{R}(t_S - t_0)}{R(t_0)}$$

Nature of Redshift

The redshift from expansion:

$$\frac{\nu_0}{\nu_s} = \frac{R(t_S)}{R(t_0)} \sim \frac{R(t_0) + \dot{R}(t_S - t_0)}{R(t_0)}$$

So that:

$$\frac{\nu_S - \nu_0}{\nu_s} = \frac{\delta\nu}{\nu} = \frac{\dot{R}}{R}\delta t = H\frac{D}{c} = \frac{v}{c}$$

Nature of Redshift

The redshift from expansion:

$$\frac{\nu_0}{\nu_s} = \frac{R(t_S)}{R(t_0)} \sim \frac{R(t_0) + \dot{R}(t_S - t_0)}{R(t_0)}$$

So that:

$$\frac{\nu_S - \nu_0}{\nu_s} = \frac{\delta\nu}{\nu} = \frac{\dot{R}}{R}\delta t = H\frac{D}{c} = \frac{v}{c}$$

so it is a **Doppler shift.**

Distances...

when $r << 1$ space can be regarded as being flat.
i.e. $R(t_s) \sim R(t_0)$ or

$$z \ll 1$$

Distances...

when $r << 1$ space can be regarded as being flat.
i.e. $R(t_s) \sim R(t_0)$ or

$$z \ll 1$$

when $z \geq 1$ this is not true anymore
A "distance measurement" needs a precise experimental procedure.

Distances...

when $r << 1$ space can be regarded as being flat.
i.e. $R(t_s) \sim R(t_0)$ or

$$z \ll 1$$

when $z \geq 1$ this is not true anymore
A "distance measurement" needs a precise experimental procedure.
Different procedures lead to different answers.

Distances...

Distances...

- Angular distance : $\theta = \frac{d}{D}$

Distances...

- Angular distance : $\theta = \frac{d}{D}$
- Luminosity distance : $l = \frac{L}{4\pi D^2}$

Distances...

- Angular distance : $\theta = \frac{d}{D}$
- Luminosity distance : $l = \frac{L}{4\pi D^2}$
- Paralax distance : $\pi = \frac{R_T}{D}$

Distances...

- Angular distance : $\theta = \frac{d}{D}$
- Luminosity distance : $l = \frac{L}{4\pi D^2}$
- Paralax distance : $\pi = \frac{R_T}{D}$
- ...

Angular Distance

Take a ruler : size d seen from epoch t_{S}

Angular Distance

Take a ruler : size d seen from epoch t_{S}
Observer: $(r = 0, 0, 0, t = t_0)$

Angular Distance

Take a ruler : size d seen from epoch t_{S}
Observer: $(r = 0, 0, 0, t = t_0)$
ruler: $(r_{\mathrm{S}}, 0, 0, t_{\mathrm{S}})$ and $(r_{\mathrm{S}}, \theta, 0, t_{\mathrm{S}})$

Angular Distance

Take a ruler : size d seen from epoch t_{S}
Observer: $(r = 0, 0, 0, t = t_0)$
ruler: $(r_{\mathrm{S}}, 0, 0, t_{\mathrm{S}})$ and $(r_{\mathrm{S}}, \theta, 0, t_{\mathrm{S}})$
Proper length:

$$d^2 = ds^2 = R^2(t_S)r^2\theta^2$$

by definition: $\theta = \frac{d}{D_{\mathrm{ang}}} = \frac{d}{R(t_S)r}$
so:

Angular Distance

Take a ruler : size d seen from epoch t_{S}
Observer: $(r = 0, 0, 0, t = t_0)$
ruler: $(r_{\mathrm{S}}, 0, 0, t_{\mathrm{S}})$ and $(r_{\mathrm{S}}, \theta, 0, t_{\mathrm{S}})$
Proper length:

$$d^2 = ds^2 = R^2(t_S)r^2\theta^2$$

by definition: $\theta = \frac{d}{D_{\mathrm{ang}}} = \frac{d}{R(t_S)r}$
so:

$$D_{\mathrm{ang}} = R(t_S)r$$

Surface Brightness

Energy going through a surface dA, during dt, in the frequency range $\nu, \nu + d\nu$:

Surface Brightness

Energy going through a surface dA, during dt, in the frequency range $\nu, \nu + d\nu$:

$$du = i(\nu)\ d\nu\ dA\ dt\ d\Omega$$

i : specific intensity.

Surface Brightness

Energy going through a surface dA, during dt, in the frequency range $\nu, \nu + d\nu$:

$$du = i(\nu)\ d\nu\ dA\ dt\ d\Omega$$

i : specific intensity.
In terms of the distribution function of photons:

$$du = f(p)\ p^2\ dp\ d\Omega\ dA\ cdt\ pc$$

$(p = h\nu/c)$

Surface Brightness

Energy going through a surface dA, during dt, in the frequency range $\nu, \nu + d\nu$:

$$du = i(\nu)\ d\nu\ dA\ dt\ d\Omega$$

i : specific intensity.
In terms of the distribution function of photons:

$$du = f(p)\ p^2\ dp\ d\Omega\ dA\ cdt\ pc$$

$(p = h\nu/c)$
so:

$$i(\nu) \propto f(p)\ p^3$$

Surface Brightness

Liouville's theorem: $f(p)$ is conserved during propagation, so:

$$\frac{i(\nu)}{p^3} \propto \frac{i(\nu)}{\nu^3} = \mathrm{cste\ i.e.\ i}(\nu_0) = \frac{\mathrm{i}(\nu_\mathrm{S})}{(1+\mathrm{z})^3}$$

Surface Brightness

Liouville's theorem: $f(p)$ is conserved during propagation, so:

$$\frac{i(\nu)}{p^3} \propto \frac{i(\nu)}{\nu^3} = \mathrm{cste\ i.e.\ i}(\nu_0) = \frac{\mathrm{i}(\nu_\mathrm{S})}{(1+\mathrm{z})^3}$$

$$\frac{\nu_S}{\nu_0} = 1 + z = \frac{d\nu_S}{d\nu_0}$$

Surface Brightness

Liouville's theorem: $f(p)$ is conserved during propagation, so:

$$\frac{i(\nu)}{p^3} \propto \frac{i(\nu)}{\nu^3} = \mathrm{cste\ i.e.\ i}(\nu_0) = \frac{\mathrm{i}(\nu_\mathrm{S})}{(1+\mathrm{z})^3}$$

$$\frac{\nu_S}{\nu_0} = 1 + z = \frac{d\nu_S}{d\nu_0}$$

Integrated surface brightness:

$$\int_0^{+\infty} i(\nu_0)\ d\nu_0 = \frac{1}{(1+z)^4} \int_0^{+\infty} i(\nu_S)\ d\nu_S$$

Surface Brightness

Liouville's theorem: $f(p)$ is conserved during propagation, so:

$$\frac{i(\nu)}{p^3} \propto \frac{i(\nu)}{\nu^3} = \mathrm{cste\ i.e.\ i}(\nu_0) = \frac{\mathrm{i}(\nu_\mathrm{S})}{(1+\mathrm{z})^3}$$

$$\frac{\nu_S}{\nu_0} = 1 + z = \frac{d\nu_S}{d\nu_0}$$

Integrated surface brightness:

$$\int_0^{+\infty} i(\nu_0)\ d\nu_0 = \frac{1}{(1+z)^4} \int_0^{+\infty} i(\nu_S)\ d\nu_S$$

test of expansion (Tolman, 1931; Sandage and Perulmuter, 1991)

Luminosity distance

Telescope with diameter $2d$ observes a point source of luminosity L
2θ is the angle of the telescope *seen from the source*

$$d = R(t_0)\ r\ \theta$$

l: the apparent luminosity of the source

Luminosity distance

Telescope with diameter $2d$ observes a point source of luminosity L
2θ is the angle of the telescope *seen from the source*

$$d = R(t_0)\ r\ \theta$$

l: the apparent luminosity of the source

$$l = L\ \frac{\pi\theta^2}{4\pi}\ \frac{1}{1+z}\ \frac{1}{1+z}\ \frac{1}{\pi\ d^2}$$

Luminosity distance

Telescope with diameter $2d$ observes a point source of luminosity L
2θ is the angle of the telescope *seen from the source*

$$d = R(t_0)\ r\ \theta$$

l: the apparent luminosity of the source

$$l = L\ \frac{\pi\theta^2}{4\pi}\ \frac{1}{1+z}\ \frac{1}{1+z}\ \frac{1}{\pi\ d^2}$$

$$l = \frac{L}{4\pi(R(t_0)\ r)^2}\ \frac{1}{(1+z)^2} = \frac{L}{4\pi\ D_{\rm lum}^2}$$

Luminosity distance

We get the luminosity distance:

$$\begin{aligned} D_{\mathrm{lum}} &= R(t_0)\ r\ (1+z) \\ &= R(t_S)\ r\ (1+z)^2 \\ &= D_{\mathrm{ang}}\ (1+z)^2 \end{aligned}$$

Luminosity distance

We get the luminosity distance:

$$\begin{aligned} D_{\mathrm{lum}} &= R(t_0)\ r\ (1+z) \\ &= R(t_S)\ r\ (1+z)^2 \\ &= D_{\mathrm{ang}}\ (1+z)^2 \end{aligned}$$

This last relation is always valid.

Distance along the line of sight

$$dl = cdt$$

Distance along the line of sight

$$dl = cdt$$

$1 + z = R_0/R(t)$ one gets $dz = -H(z)(1+z)dt$ so:

$$dl = -\frac{c}{H(z)}\frac{dz}{1+z}$$

Distance along the line of sight

$$dl = cdt$$

$1 + z = R_0/R(t)$ one gets $dz = -H(z)(1+z)dt$ so:

$$dl = -\frac{c}{H(z)}\frac{dz}{1+z}$$

From RW, it is also:

$$dl = R(t)\frac{dr}{\sqrt{1-kr^2}}$$

Volume element

$$
\begin{aligned}
dV &= d\Omega\ D_{ang}^2\ dl \\
&= -d\Omega (R(t_S)\ r)^2 \frac{c}{H(z)}\ \frac{dz}{1+z} \\
&= d\Omega\ (R(t_S)\ r)^2 R(t_S) \frac{dr}{\sqrt{1-kr^2}}
\end{aligned}
$$

Volume element

$$
\begin{aligned}
dV &= d\Omega\ D_{ang}^2\ dl \\
&= -d\Omega (R(t_S)\ r)^2 \frac{c}{H(z)}\ \frac{dz}{1+z} \\
&= d\Omega\ (R(t_S)\ r)^2 R(t_S) \frac{dr}{\sqrt{1-kr^2}}
\end{aligned}
$$

$\rightarrow$ useful for number counts.

Describing gravity

Describing gravity

Based on Einstein's G.R.

Describing gravity

Based on Einstein's G.R.

- $R_{ij} - 1/2 g_{ij} R = 8\pi G T_{ij}$

Describing gravity

Based on Einstein's G.R.

- $R_{ij} - 1/2 g_{ij} R = 8\pi G T_{ij}$
- rest frame :

$$T_{ij} = \begin{bmatrix} \rho & & & \\ & P & & \\ & & P & \\ & & & P \end{bmatrix}$$

Describing gravity

Based on Einstein's G.R.

- $R_{ij} - 1/2 g_{ij} R = 8\pi G T_{ij}$
- rest frame :
$$T_{ij} = \begin{bmatrix} \rho & & & \\ & P & & \\ & & P & \\ & & & P \end{bmatrix}$$
- Source of gravity : $\rho + 3P/c^2$

Describing gravity

Based on Einstein's G.R.

- $R_{ij} - 1/2 g_{ij} R = 8\pi G T_{ij}$
- rest frame : $$T_{ij} = \begin{bmatrix} \rho & & & \\ & P & & \\ & & P & \\ & & & P \end{bmatrix}$$
- Source of gravity : $\rho + 3P/c^2$
- Birkoff's theorem : analog of Gauss theorem

Describing gravity

Based on Einstein's G.R.

- $R_{ij} - 1/2 g_{ij} R = 8\pi G T_{ij}$
- rest frame : $$T_{ij} = \begin{bmatrix} \rho & & & \\ & P & & \\ & & P & \\ & & & P \end{bmatrix}$$
- Source of gravity : $\rho + 3P/c^2$
- Birkoff's theorem : analog of Gauss theorem

For spherical distribution only $\rho(r < R)$ matters for the solution within $r < R$.

Dynamics from Newtonian argument

Inside a sphere of Radius a

$$\ddot{a} = g$$

Dynamics from Newtonian argument

Inside a sphere of Radius a

$$\ddot{a} = g$$

Source is $\rho + 3P/c^2$:

$$\ddot{a} = -\frac{GM}{a^2} = -\frac{4\pi G}{3}(\rho + 3P/c^2)a \qquad (1)$$

Dynamics from Newtonian argument

Inside a sphere of Radius a

$$\ddot{a} = g$$

Source is $\rho + 3P/c^2$:

$$\ddot{a} = -\frac{GM}{a^2} = -\frac{4\pi G}{3}(\rho + 3P/c^2)a \tag{1}$$

Energy conservation

E_t total energy of the sphere :

$$\begin{aligned} d(E_t) &= d(\rho V c^2) = -PdV \\ &= c^2(Vd\rho + \rho dV) = -PdV \end{aligned}$$

Dynamics from Newtonian argument

leading to :

$$\dot{\rho} = -(\rho + P/c^2)\frac{\dot{V}}{V} = -3(\rho + P/c^2)\frac{\dot{a}}{a} \quad (2)$$

(1) and (2) allow to eliminate P:

Dynamics from Newtonian argument

leading to :

$$\dot{\rho} = -(\rho + P/c^2)\frac{\dot{V}}{V} = -3(\rho + P/c^2)\frac{\dot{a}}{a} \qquad (2)$$

(1) and (2) allow to eliminate P:

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3P/c^2)a$$

Dynamics from Newtonian argument

leading to :

$$\dot{\rho} = -(\rho + P/c^2)\frac{\dot{V}}{V} = -3(\rho + P/c^2)\frac{\dot{a}}{a} \quad (2)$$

(1) and (2) allow to eliminate P:

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3P/c^2)a$$

$$\ddot{a} = -\frac{4\pi G}{3}(3\rho + 3P/c^2)a + 2\frac{4\pi G}{3}\rho a$$

Dynamics from Newtonian argument

leading to :

$$\dot{\rho} = -(\rho + P/c^2)\frac{\dot{V}}{V} = -3(\rho + P/c^2)\frac{\dot{a}}{a} \tag{2}$$

(1) and (2) allow to eliminate P:

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3P/c^2)a$$

$$\ddot{a} = -\frac{4\pi G}{3}(3\rho + 3P/c^2)a + 2\frac{4\pi G}{3}\rho a$$

$$\ddot{a} = +\frac{4\pi G}{3}\frac{a\dot{\rho}}{\dot{a}}a + 2\frac{4\pi G}{3}\rho a$$

Dynamics from Newtonian argument

multiplying by $\dot{a}$:

$$\dot{a}\ddot{a} = +\frac{4\pi G}{3}a^2\dot{\rho} + \frac{8\pi G}{3}\rho a\dot{a}$$

Dynamics from Newtonian argument

multiplying by $\dot{a}$:

$$\dot{a}\ddot{a} = +\frac{4\pi G}{3}a^2\dot{\rho} + \frac{8\pi G}{3}\rho a\dot{a}$$

$$(\dot{a}^2)' = \left(\frac{8\pi G a^2\rho}{3}\right)'$$

Dynamics from Newtonian argument

multiplying by $\dot{a}$:

$$\dot{a}\ddot{a} = +\frac{4\pi G}{3}a^2\dot{\rho} + \frac{8\pi G}{3}\rho a\dot{a}$$

$$(\dot{a}^2)' = \left(\frac{8\pi G a^2 \rho}{3}\right)'$$

that is :

$$\dot{a}^2 = \frac{8\pi G a^2 \rho}{3} + cste$$

Dynamics from Newtonian argument

multiplying by $\dot{a}$:

$$\dot{a}\ddot{a} = +\frac{4\pi G}{3}a^2\dot{\rho} + \frac{8\pi G}{3}\rho a\dot{a}$$

$$(\dot{a}^2)' = \left(\frac{8\pi G a^2 \rho}{3}\right)'$$

that is :

$$\dot{a}^2 = \frac{8\pi G a^2 \rho}{3} + cste$$

For $R(t)$:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2}$$

Equation of state

Solution -> needs an equation of state $F(\rho, P) = 0$

Equation of state

Solution -> needs an equation of state $F(\rho, P) = 0$

Notation : $P = w\rho$

Equation of state

Solution -> needs an equation of state $F(\rho, P) = 0$

Notation : $P = w\rho$

The density ρ reads:

$$\rho = \sum_i \int \frac{E_i}{c^2} f(p_i) dp_i$$

Equation of state

Solution -> needs an equation of state $F(\rho, P) = 0$

Notation : $P = w\rho$

The density ρ reads:

$$\rho = \sum_i \int \frac{E_i}{c^2} f(p_i) dp_i$$

the pressure P:

$$P = \sum_i \int \frac{1}{3} \frac{p_i^2}{E_i} f(p_i) dp_i$$

Equation of state

Two important regimes:

Equation of state

Two important regimes:

$\rightarrow$ matter dominated: $p << mc$ i.e. $P = 0$

$\rho = \int m$ and $g \propto \rho$

$\dot{\rho} = -3\rho\dot{a}/a$ $(a \propto R)$ so :

$$\rho a^3 = \mathrm{cste}$$

Equation of state

Two important regimes:

→ matter dominated: $p << mc$ i.e. $P = 0$

$\rho = \int m$ and $g \propto \rho$

$\dot{\rho} = -3\rho\dot{a}/a$ ($a \propto R$) so :

$$\rho a^3 = \mathrm{cste}$$

→ pressure (radiation) dominated:

$p >> mc$ so $\rho = \int p/c...$ and $P = \int 1/3\ p\ c...$

$$P = \frac{1}{3}\rho c^2$$

$\dot{\rho} = -4\rho\dot{a}/a$ so :

$$\rho a^4 = \mathrm{cste}$$

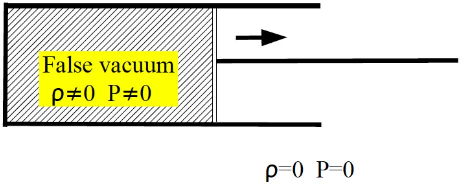
Vacuum

Vacuum

Naively : $\rho_v = 0$ and $P_v = 0$

Vacuum

Naively : $\rho_v = 0$ and $P_v = 0$



But take a box with vacuum in it:

$$d(E_t) = d(\rho_v V c^2) = \rho_v c^2 dV = -P_v dV$$

Vacuum

Vacuum

so we get the equation of state of vacuum:

$$P_v = -\rho_v c^2$$

Vacuum

so we get the equation of state of vacuum:

$$P_v = -\rho_v c^2$$

Or look for a fluid Lorentz invariant.

Vacuum

so we get the equation of state of vacuum:

$$P_v = -\rho_v c^2$$

Or look for a fluid Lorentz invariant.
Introducing the cosmological constant:

$$\Lambda = 8\pi G \rho_v$$

Summary

Space is described by RW metric.

Summary

Space is described by RW metric.
Einstein-Friedmann-Lemaître (EFL) equations:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} + \frac{\Lambda}{3}$$

Summary

Space is described by RW metric.
Einstein-Friedmann-Lemaître (EFL) equations:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} + \frac{\Lambda}{3}$$

and

$$\dot{\rho} = -3\left(\frac{P}{c^2} + \rho\right)\frac{\dot{R}}{R}$$

Summary

Space is described by RW metric.
Einstein-Friedmann-Lemaître (EFL) equations:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} + \frac{\Lambda}{3}$$

and

$$\dot{\rho} = -3\left(\frac{P}{c^2} + \rho\right)\frac{\dot{R}}{R}$$

$$2\frac{\ddot{R}}{R} = -\frac{8\pi G}{3}(\rho + 3P/c^2) + \frac{2\Lambda}{3}$$

Notations

Notations

$H = \frac{\dot{R}}{R}$, the Hubble parameter,

Notations

$H = \frac{\dot{R}}{R}$, the Hubble parameter,
$\Omega_M = \Omega = \frac{8\pi G\rho}{3H^2}$ the density parameter,

Notations

$H = \frac{\dot{R}}{R}$, the Hubble parameter,
$\Omega_M = \Omega = \frac{8\pi G\rho}{3H^2}$ the density parameter,
$q = -\frac{\ddot{R}R}{\dot{R}^2}$, the deceleration parameter,

Notations

$H = \frac{\dot{R}}{R}$, the Hubble parameter,
$\Omega_M = \Omega = \frac{8\pi G\rho}{3H^2}$ the density parameter,
$q = -\frac{\ddot{R}R}{\dot{R}^2}$, the deceleration parameter,
$\Omega_{\rm vac} = \Omega_\lambda = \lambda = \frac{\Lambda}{3H^2}$, the (reduced) cosmological constant,

Notations

$H = \frac{\dot{R}}{R}$, the Hubble parameter,
$\Omega_M = \Omega = \frac{8\pi G\rho}{3H^2}$ the density parameter,
$q = -\frac{\ddot{R}R}{\dot{R}^2}$, the deceleration parameter,
$\Omega_{\rm vac} = \Omega_\lambda = \lambda = \frac{\Lambda}{3H^2}$, the (reduced) cosmological constant,
$\Omega_k = -\alpha = -\frac{kc^2}{H^2R^2}$, the curvature parameter.

Notations

$H = \frac{\dot{R}}{R}$, the Hubble parameter,
$\Omega_M = \Omega = \frac{8\pi G\rho}{3H^2}$ the density parameter,
$q = -\frac{\ddot{R}R}{\dot{R}^2}$, the deceleration parameter,
$\Omega_{\rm vac} = \Omega_\lambda = \lambda = \frac{\Lambda}{3H^2}$, the (reduced) cosmological constant,
$\Omega_k = -\alpha = -\frac{kc^2}{H^2R^2}$, the curvature parameter.
Quantities are labeled by 0 when they are referred to their present value: Ω_0, q_0, ...

Notations

$H = \frac{\dot{R}}{R}$, the Hubble parameter,
$\Omega_M = \Omega = \frac{8\pi G\rho}{3H^2}$ the density parameter,
$q = -\frac{\ddot{R}R}{\dot{R}^2}$, the deceleration parameter,
$\Omega_{\rm vac} = \Omega_\lambda = \lambda = \frac{\Lambda}{3H^2}$, the (reduced) cosmological constant,
$\Omega_k = -\alpha = -\frac{kc^2}{H^2R^2}$, the curvature parameter.
Quantities are labeled by 0 when they are referred to their present value: Ω_0, q_0, ...
E.F.L. :

$$\Omega_k + \Omega_M + \Omega_\lambda = 1$$

Matter domination area

$$\ddot{a} = g = -\frac{GM}{a^2} \text{ and } + \ \rho \ a^3 = \text{cste}$$

Matter domination area

$$\ddot{a} = g = -\frac{GM}{a^2} \text{ and} + \ \rho \ a^3 = \text{cste}$$

from this we have derived:

$$\dot{a}^2 - \frac{8\pi \ G \ \rho \ a^2}{3} = \dot{a}^2 - \frac{2GM}{a} = -k \ c^2$$

Matter domination area

$$\ddot{a} = g = -\frac{GM}{a^2} \text{ and} + \ \rho \ a^3 = \text{cste}$$

from this we have derived:

$$\dot{a}^2 - \frac{8\pi \ G \ \rho \ a^2}{3} = \dot{a}^2 - \frac{2GM}{a} = -k \ c^2$$

This is exactly the equation of a test particle in the field of one mass in Newtonian theory!

$$E_c + E_p = cste$$

Matter domination area

$$\ddot{a} = g = -\frac{GM}{a^2} \text{ and} + \ \rho \ a^3 = \text{cste}$$

from this we have derived:

$$\dot{a}^2 - \frac{8\pi \ G \ \rho \ a^2}{3} = \dot{a}^2 - \frac{2GM}{a} = -k \ c^2$$

This is exactly the equation of a test particle in the field of one mass in Newtonian theory!

$$E_c + E_p = cste$$

Solutions:

Matter domination area

$$\ddot{a} = g = -\frac{GM}{a^2} \text{ and} + \ \rho \ a^3 = \text{cste}$$

from this we have derived:

$$\dot{a}^2 - \frac{8\pi \ G \ \rho \ a^2}{3} = \dot{a}^2 - \frac{2GM}{a} = -k \ c^2$$

This is exactly the equation of a test particle in the field of one mass in Newtonian theory!

$$E_c + E_p = cste$$

Solutions:

- $k = -1$ unbound hyperbolic solution

Matter domination area

$$\ddot{a} = g = -\frac{GM}{a^2} \text{ and } + \ \rho \ a^3 = \text{cste}$$

from this we have derived:

$$\dot{a}^2 - \frac{8\pi \ G \ \rho \ a^2}{3} = \dot{a}^2 - \frac{2GM}{a} = -k \ c^2$$

This is exactly the equation of a test particle in the field of one mass in Newtonian theory!

$$E_c + E_p = cste$$

Solutions:

- $k = -1$ unbound hyperbolic solution
- $k = 0$ parabolic solution

Matter domination area

$$\ddot{a} = g = -\frac{GM}{a^2} \text{ and } + \ \rho \ a^3 = \text{cste}$$

from this we have derived:

$$\dot{a}^2 - \frac{8\pi \ G \ \rho \ a^2}{3} = \dot{a}^2 - \frac{2GM}{a} = -k \ c^2$$

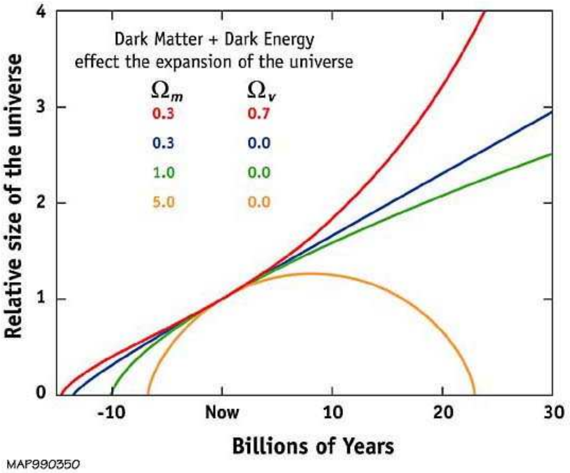
This is exactly the equation of a test particle in the field of one mass in Newtonian theory!

$$E_c + E_p = cste$$

Solutions:

- $k = -1$ unbound hyperbolic solution
- $k = 0$ parabolic solution
- $k = +1$ bound elliptic solution

General solutions



Matter domination area: case $k = 0$ $\Lambda = 0$

$$\dot{R}^2 = \frac{8\pi \; G \; \rho \; R^2}{3} \text{ and } \rho R^3 = \rho_0 R_0^3$$

Matter domination area: case $k = 0$ $\Lambda = 0$

$$\dot{R}^2 = \frac{8\pi\ G\ \rho\ R^2}{3} \text{ and } \rho R^3 = \rho_0 R_0^3$$

First Eq. implies:

$$\Omega = \frac{8\pi\ G\ \rho}{3\ H^2} = 1 = \Omega_0$$

(present-day) critical density :

$$\rho_c = \frac{3\ H_0^2}{8\pi\ G}$$

Matter domination area: case $k = 0$ $\Lambda = 0$

$$\dot{R}^2 = \frac{8\pi\ G\ \rho\ R^2}{3} \text{ and } \rho R^3 = \rho_0 R_0^3$$

First Eq. implies:

$$\Omega = \frac{8\pi\ G\ \rho}{3\ H^2} = 1 = \Omega_0$$

(present-day) critical density :

$$\rho_c = \frac{3\ H_0^2}{8\pi\ G}$$

Second Eq. implies:

$$\dot{R}^2 = \frac{8\pi\ G\ \rho_0\ R_0^3}{3\ R} = H_0^2 \frac{R_0^3}{R}$$

Matter domination area: case $k = 0$ $\Lambda = 0$

$\Omega_m = 1$: Einstein-de Sitter solution.

Matter domination area: case $k = 0$ $\Lambda = 0$

$\Omega_m = 1$: Einstein-de Sitter solution. Solution:

$$R(t) = R_0 \left(\frac{3}{2} H_0 \ t\right)^{2/3} = R_0 \left(t/t_0\right)^{2/3}$$

with :

$$t_0 = \frac{2}{3} \ H_0^{-1} = \frac{1}{\sqrt{6\pi \ G \ \rho_c}}$$

Matter domination area: case $k = 0$ $\Lambda = 0$

$\Omega_m = 1$: Einstein-de Sitter solution. Solution:

$$R(t) = R_0 \left(\frac{3}{2} H_0 \ t\right)^{2/3} = R_0 \left(t/t_0\right)^{2/3}$$

with :

$$t_0 = \frac{2}{3} \ H_0^{-1} = \frac{1}{\sqrt{6\pi \ G \ \rho_c}}$$

This solution goes through 0 in the past...

Matter domination area: case $k = 0$ $\Lambda = 0$

$\Omega_m = 1$: Einstein-de Sitter solution. Solution:

$$R(t) = R_0 \left(\frac{3}{2} H_0 \ t\right)^{2/3} = R_0 \left(t/t_0\right)^{2/3}$$

with :

$$t_0 = \frac{2}{3} \ H_0^{-1} = \frac{1}{\sqrt{6\pi \ G \ \rho_c}}$$

This solution goes through 0 in the past...
The solution has an “Initial” singularity.

Initial singularity

$$2\frac{\ddot{R}}{R} = -\frac{8\ \pi\ G}{3}(\rho + 3P/c^2)$$

and :

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\ \pi\ G\ \rho}{3} - \frac{k\ c^2}{R^2}$$

Initial singularity

$$2\frac{\ddot{R}}{R} = -\frac{8\ \pi\ G}{3}(\rho + 3P/c^2)$$

and :

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\ \pi\ G\ \rho}{3} - \frac{k\ c^2}{R^2}$$

so if: $(\rho + 3P/c^2) > 0$ R will go through 0 (in the past) in a finite time t_0.

Initial singularity

$$2\frac{\ddot{R}}{R} = -\frac{8\ \pi\ G}{3}(\rho + 3P/c^2)$$

and :

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\ \pi\ G\ \rho}{3} - \frac{k\ c^2}{R^2}$$

so if: $(\rho + 3P/c^2) > 0$ R will go through 0 (in the past) in a finite time t_0.
There is a theorem more general than this.

Initial singularity

$$2\frac{\ddot{R}}{R} = -\frac{8\ \pi\ G}{3}(\rho + 3P/c^2)$$

and :

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\ \pi\ G\ \rho}{3} - \frac{k\ c^2}{R^2}$$

so if: $(\rho + 3P/c^2) > 0$ R will go through 0 (in the past) in a finite time t_0.

There is a theorem more general than this.

When $R \to 0$ than $\left(\frac{\dot{R}}{R}\right)^2 \sim \frac{8\ \pi\ G\ \rho}{3}$ i.e. $\Omega \sim 1$

Behavior of Ω (matter and $\Lambda = 0$):

previous second Eq. implies $-\Omega_k = \Omega_0 - 1$ so :

$$H^2 = H_0^2[\Omega_0(1+z)^3 + (1-\Omega_0)(1+z)^2]$$

Behavior of Ω (matter and $\Lambda = 0$):

previous second Eq. implies $-\Omega_k = \Omega_0 - 1$ so :

$$H^2 = H_0^2[\Omega_0(1+z)^3 + (1-\Omega_0)(1+z)^2]$$

so:

$$H^2 = H_0^2(1+z)^2(1+\Omega_0 z)$$

Behavior of Ω (matter and $\Lambda = 0$):

previous second Eq. implies $-\Omega_k = \Omega_0 - 1$ so :

$$H^2 = H_0^2[\Omega_0(1+z)^3 + (1-\Omega_0)(1+z)^2]$$

so:

$$H^2 = H_0^2(1+z)^2(1+\Omega_0 z)$$

and:

$$\Omega(z) = \frac{8\pi\ G\ \rho}{3H^2} = \frac{8\pi\ G\ \rho_0}{3H_0^2}\frac{(1+z)^3}{(1+z)^2(1+\Omega_0 z)}$$

Behavior of Ω (matter and $\Lambda = 0$):

previous second Eq. implies $-\Omega_k = \Omega_0 - 1$ so :

$$H^2 = H_0^2[\Omega_0(1+z)^3 + (1-\Omega_0)(1+z)^2]$$

so:

$$H^2 = H_0^2(1+z)^2(1+\Omega_0 z)$$

and:

$$\Omega(z) = \frac{8\pi \ G \ \rho}{3H^2} = \frac{8\pi \ G \ \rho_0}{3H_0^2} \frac{(1+z)^3}{(1+z)^2(1+\Omega_0 z)}$$

so:

$$\Omega(z) = \Omega_0 \frac{(1+z)}{(1+\Omega_0 z)}$$

Mattig relation $\Lambda = 0$

Along a light ray:

$$\frac{dr^2}{1-kr^2} = \frac{c^2 dt^2}{R^2(t)} = \frac{c^2 dR^2}{R^2(t)\dot{R}^2(t)}$$

Mattig relation $\Lambda = 0$

Along a light ray:

$$\frac{dr^2}{1-kr^2} = \frac{c^2dt^2}{R^2(t)} = \frac{c^2dR^2}{R^2(t)\dot{R}^2(t)}$$

From this, setting $v = \frac{\alpha_0}{\Omega_0 R_0}$ R in the right hand side, one can derive (...):

$$R_0\ r = \frac{c}{H_0}\ \frac{2}{\Omega_0^2}\ \frac{\Omega_0(1+z)+2-2\Omega_0-(2-\Omega_0)\sqrt{1+\Omega_0\ z}}{1+z}$$

Mattig relation $\Lambda = 0$

Along a light ray:

$$\frac{dr^2}{1-kr^2} = \frac{c^2 dt^2}{R^2(t)} = \frac{c^2 dR^2}{R^2(t)\dot{R}^2(t)}$$

From this, setting $v = \frac{\alpha_0}{\Omega_0 R_0}$ R in the right hand side, one can derive (...):

$$R_0\ r = \frac{c}{H_0}\ \frac{2}{\Omega_0^2}\ \frac{\Omega_0(1+z) + 2 - 2\Omega_0 - (2-\Omega_0)\sqrt{1+\Omega_0\ z}}{1+z}$$

when $z << 1$ $R_0\ r \sim \frac{c}{H_0} z$
when $z >> 1$ $R_0\ r \sim \frac{c}{H_0} \frac{2}{\Omega_0}$

Matter domination area: case $k = -1$ $\Lambda = 0$

$$\begin{aligned}\dot{R}^2 &= \frac{8\pi\ G\ \rho R^2}{3} - kc^2 \\ &= H_0^2\Omega_0\ R_0^2\ (1+z) + (1-\Omega_0)\ H_0^2\ R_0^2\end{aligned}$$

Matter domination area: case $k = -1$ $\Lambda = 0$

$$\begin{aligned}\dot{R}^2 &= \frac{8\pi\ G\ \rho R^2}{3} - kc^2 \\ &= H_0^2\Omega_0\ R_0^2\ (1+z) + (1-\Omega_0)\ H_0^2\ R_0^2\end{aligned}$$

so when $1+z >> \frac{1-\Omega_0}{\Omega_0}$ one has : $R \propto t^{2/3}$

while when $1+z << \frac{1-\Omega_0}{\Omega_0}$ $\dot{R} \sim cste$ one has $R \propto t$

Matter domination area: case $k = -1$ $\Lambda = 0$

$$\dot{R}^2 = \frac{8\pi\ G\ \rho R^2}{3} - kc^2$$
$$= H_0^2\Omega_0\ R_0^2\ (1+z) + (1-\Omega_0)\ H_0^2\ R_0^2$$

so when $1 + z >> \frac{1-\Omega_0}{\Omega_0}$ one has : $R \propto t^{2/3}$
while when $1 + z << \frac{1-\Omega_0}{\Omega_0}$ $\dot{R} \sim cste$ one has $R \propto t$ $R(t)$ can be developped:

$$H_0\ t = \frac{\Omega_0}{2(1-\Omega_0)^{3/2}}(\sinh(\psi) - \psi)$$
$$\frac{1}{1+z} = \frac{R(t)}{R_0} = \frac{\Omega_0}{2(1-\Omega_0)}(\cosh(\psi) - 1)$$

Allows analytical expression of $H_0\ t(z)$

Matter domination area: case $k = +1$ $\Lambda = 0$

The expression:

$$\dot{R}^2 = H_0^2 \Omega_0 \ R_0^2 \ (1+z) + (1-\Omega_0) \ H_0^2 \ R_0^2$$

allows to find R_m so that $\dot{R} = 0$

$$R_m = R_0 \ \frac{\Omega_0}{\Omega_0 - 1}$$

Matter domination area: case $k = +1$ $\Lambda = 0$

The expression:

$$\dot{R}^2 = H_0^2 \Omega_0 \; R_0^2 \; (1+z) + (1-\Omega_0) \; H_0^2 \; R_0^2$$

allows to find R_m so that $\dot{R} = 0$

$$R_m = R_0 \; \frac{\Omega_0}{\Omega_0 - 1}$$

$R(t)$ can be developped as well:

$$\begin{aligned} H_0 \; t &= \frac{\Omega_0}{2(\Omega_0 - 1)^{3/2}} (\phi - \sin(\phi)) \\ \frac{1}{1+z} &= \frac{R(t)}{R_0} = \frac{\Omega_0}{2(\Omega_0 - 1)} (1 - \cos(\phi)) \end{aligned}$$

Matter domination area: case $k = +1$ $\Lambda = 0$

At the maximum:

$$R_m = c\frac{2\ t_m}{\pi}$$

$$\rho_m = \frac{3\pi}{32\ G\ t_m^2}$$

$$t_m = \frac{1}{H_0}\ \frac{\Omega_0}{(\Omega_0 - 1)^{3/2}}\ \pi$$

(useful for structure formation)

Matter domination area: cases $\Lambda \neq 0$

$$2\ddot{R} = -\frac{8\,\pi G}{3}(\rho + \frac{3P}{c^2})R + \frac{2\Lambda}{3}R$$

If $\Lambda < 0$ it is an attractive force
If $\Lambda > 0$ it is a repulsive force, in which case $R(t)$ might not go through $R = 0$.

Matter domination area: cases $\Lambda \neq 0$

$$2\ddot{R} = -\frac{8\ \pi G}{3}(\rho + \frac{3P}{c^2})R + \frac{2\Lambda}{3}R$$

If $\Lambda < 0$ it is an attractive force

If $\Lambda > 0$ it is a repulsive force, in which case $R(t)$ might not go through $R = 0$.

Case $P = 0$

$$2\ddot{R} = H_0^2 R_0[\frac{2\ \lambda_0}{(1+z)} - \Omega_0\ (1+z)^2]$$

$$\dot{R}^2 = H_0^2 R_0^2[\frac{\lambda_0}{(1+z)^2} + (1 - \Omega_0 - \lambda_0) + \Omega_0(1+z)]$$

Matter domination area: cases $\Lambda \neq 0$

$$2\ddot{R} = -\frac{8\,\pi G}{3}(\rho + \frac{3P}{c^2})R + \frac{2\Lambda}{3}R$$

If $\Lambda < 0$ it is an attractive force
If $\Lambda > 0$ it is a repulsive force, in which case $R(t)$ might not go through $R = 0$.
Case $P = 0$

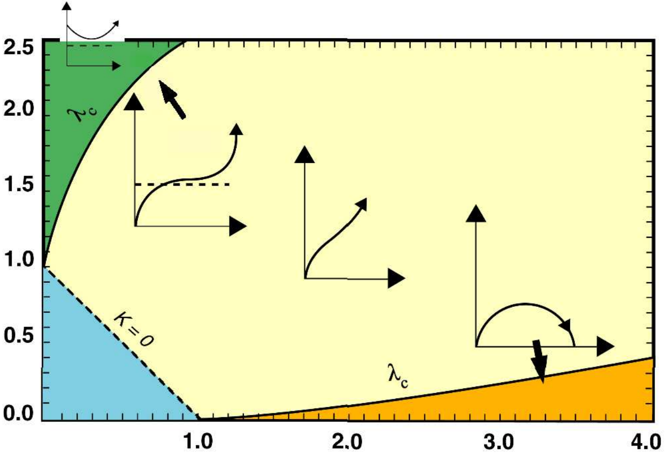
$$2\ddot{R} = H_0^2 R_0[\frac{2\;\lambda_0}{(1+z)} - \Omega_0\;(1+z)^2]$$

$$\dot{R}^2 = H_0^2 R_0^2[\frac{\lambda_0}{(1+z)^2} + (1-\Omega_0-\lambda_0) + \Omega_0(1+z)]$$

setting $u = 1 + z$ one gets:

$$\dot{R}^2 \propto \frac{\lambda_0}{u^2} + (1-\Omega_0-\lambda_0) + \Omega_0\;u = f(u)$$

Matter domination area: cases $\Lambda \neq 0$



Matter domination area: cases $\Lambda \neq 0$

The "useful" relations $R_0 r$, $t(z)$, ... are not analytical.

Matter domination area: cases $\Lambda \neq 0$

The "useful" relations $R_0 r$, $t(z)$, ... are not analytical.

$$\begin{aligned}\dot{R}^2 &= \frac{8\,\pi G\;\rho\;R^2}{3} - kc^2 + \frac{\Lambda R^2}{3} \\ &= H_0^2 R_0^2 [\frac{\Omega_\Lambda}{(1+z)^2} - \Omega_k + \Omega_0(1+z)]\end{aligned}$$

Matter domination area: cases $\Lambda \neq 0$

The "useful" relations $R_0 r$, $t(z)$, ... are not analytical.

$$\dot{R}^2 = \frac{8\,\pi G\;\rho\;R^2}{3} - kc^2 + \frac{\Lambda R^2}{3}$$
$$= H_0^2 R_0^2 [\frac{\Omega_\Lambda}{(1+z)^2} - \Omega_k + \Omega_0(1+z)]$$

Mattig relation

$$S_k^{-1}(r) = \int_{t(z)}^{t_0} \frac{c\;dt}{R(t)} = |\Omega_k|^{1/2} \int_1^{1+z} \frac{d\;\;u}{(\Omega_0 u^3 - \Omega_k u^2 + \Omega_\Lambda)^{1/2}}$$

Matter domination area: cases $\Lambda \neq 0$

The "useful" relations $R_0 r$, $t(z)$, ... are not analytical.

$$\begin{aligned}\dot{R}^2 &= \frac{8\,\pi G\ \rho\ R^2}{3} - kc^2 + \frac{\Lambda R^2}{3} \\ &= H_0^2 R_0^2 [\frac{\Omega_\Lambda}{(1+z)^2} - \Omega_k + \Omega_0(1+z)]\end{aligned}$$

Mattig relation

$$S_k^{-1}(r) = \int_{t(z)}^{t_0} \frac{c\ dt}{R(t)} = |\Omega_k|^{1/2} \int_1^{1+z} \frac{d\ u}{(\Omega_0 u^3 - \Omega_k u^2 + \Omega_\Lambda)^{1/2}}$$

Age:

$$t_0 - t(z) = \int_{t(z)}^{t_0} dt = \int_1^{1+z} \frac{1}{H_0} \frac{d\ u}{u(\Omega_0 u^3 - \Omega_k u^2 + \Omega_\Lambda)^{1/2}}$$

Matter dominated cases $\Lambda \neq 0$: Applications

Matter dominated cases $\Lambda \neq 0$: Applications

- Mattig relation : $R_0\ r(z)$

Matter dominated cases $\Lambda \neq 0$: Applications

- Mattig relation : $R_0\ r(z)$
- Angular distance : $\theta = \frac{d}{D_{ang}(z)}$
  $\rightarrow$ minimum at some z then increases!

Matter dominated cases $\Lambda \neq 0$: Applications

- Mattig relation : $R_0 \; r(z)$
- Angular distance : $\theta = \frac{d}{D_{ang}(z)}$
  $\rightarrow$ minimum at some z then increases!
- Look back time: $H_0(t_0 - t(z))$

$\rightarrow$ at $z \sim 1$ the universe is significantly younger:

$$\Omega \sim 0. \quad \Omega_\Lambda = 0. \qquad z = 1 \leftrightarrow t_1 \sim 0.5 \quad t_0$$
$$\Omega = 1. \quad \Omega_\Lambda = 0. \qquad z = 1 \leftrightarrow t_1 \sim 0.35 \; t_0$$
$$\Omega = 0.3 \; \Omega_\Lambda = 0.7 \qquad z = 1 \leftrightarrow t_1 \sim 0.35 \; t_0$$

Matter dominated cases $\Lambda \neq 0$: Applications

- Mattig relation : $R_0 \; r(z)$
- Angular distance : $\theta = \frac{d}{D_{ang}(z)}$
  $\rightarrow$ minimum at some z then increases!
- Look back time: $H_0(t_0 - t(z))$

$\rightarrow$ at $z \sim 1$ the universe is significantly younger:

$$\Omega \sim 0. \quad \Omega_\Lambda = 0. \qquad z = 1 \leftrightarrow t_1 \sim 0.5 \quad t_0$$
$$\Omega = 1. \quad \Omega_\Lambda = 0. \qquad z = 1 \leftrightarrow t_1 \sim 0.35 \; t_0$$
$$\Omega = 0.3 \; \Omega_\Lambda = 0.7 \qquad z = 1 \leftrightarrow t_1 \sim 0.35 \; t_0$$

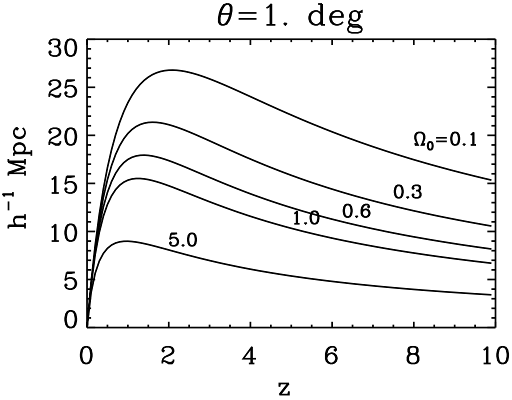
Models with $(\Omega, \Omega_\Lambda > 0)$ are older than with $(\Omega, \Omega_\Lambda = 0)$, the difference being important only when $\Omega_\Lambda \sim \lambda_c$.

Angular distance

$D_{ang} = R(t)r$ pour 1°

Angular distance

$D_{ang} = R(t)r$ pour 1°

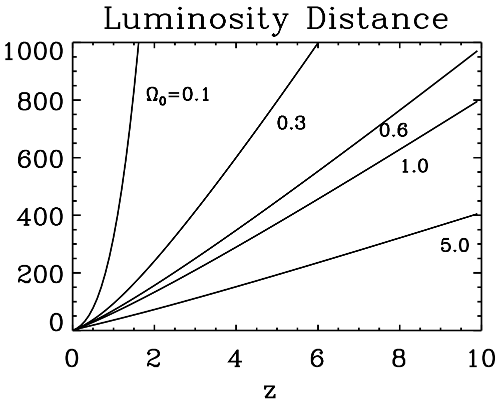


Luminosity distance

Arbitrary units

Luminosity distance

Arbitrary units



Radiation dominated case

$$P = \frac{1}{3}\rho_\gamma\ c^2 \quad \text{and} \quad \rho_\gamma\ R^4 = \text{cste}$$

Radiation dominated case

$$P = \frac{1}{3}\rho_\gamma \; c^2 \quad \text{and} \quad \rho_\gamma \; R^4 = \text{cste}$$

E.F.L. Equations:

$$\begin{array}{ccccccc} \left(\frac{\dot{R}}{R}\right)^2 & = & \frac{8\,\pi G}{3}(\rho_\gamma & + \rho_m) & - \frac{kc^2}{R^2} & + & \frac{\Lambda}{3} \\ & \propto & \frac{1}{R^4} & \frac{1}{R^3} & \frac{1}{R^2} & & \text{cste} \end{array}$$

Radiation dominated case

$$P = \frac{1}{3}\rho_\gamma \ c^2 \quad \text{and} \quad \rho_\gamma \ R^4 = \text{cste}$$

E.F.L. Equations:

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\ \pi G}{3}(\rho_\gamma + \rho_m) - \frac{kc^2}{R^2} + \frac{\Lambda}{3}$$

$$\propto \quad \frac{1}{R^4} \quad \frac{1}{R^3} \quad \frac{1}{R^2} \quad \text{cste}$$

$\rightarrow$ The radiation term is dominant at high redshift: $\dot{R} = \frac{\text{cste}}{R}$

Radiation dominated case

$$P = \frac{1}{3}\rho_\gamma \; c^2 \quad \text{and} \quad \rho_\gamma \; R^4 = \text{cste}$$

E.F.L. Equations:

$$\begin{array}{ccccccc} \left(\frac{\dot{R}}{R}\right)^2 & = & \frac{8\;\pi G}{3}(\rho_\gamma & + \rho_m) & - \frac{kc^2}{R^2} & + & \frac{\Lambda}{3} \\ & \propto & \frac{1}{R^4} & \frac{1}{R^3} & \frac{1}{R^2} & & \text{cste} \end{array}$$

$\rightarrow$ The radiation term is dominant at high redshift: $\dot{R} = \frac{\text{cste}}{R}$

Solution:

$$R = R_1 \left(\frac{t}{\tau}\right)^{1/2} \quad \text{with} \quad \tau^2 = \frac{3}{32\;\pi\;G\;\rho_1}$$

The development of RG cosmological models

The development of RG cosmological models

A. Einstein: 1916: GR + first consistent cosmological model. Einstein cosmological principle: The universe is homogeneous on large scale.

The development of RG cosmological models

A. Einstein: 1916: GR + first consistent cosmological model.
Einstein cosmological principle: The universe is homogeneous on large scale.

W. De Sitter: 1919 GR $+\Lambda$ with $\rho = 0$.
Static but particles move. Redshift $\propto D$.

The development of RG cosmological models

A. Einstein: 1916: GR + first consistent cosmological model. Einstein cosmological principle: The universe is homogeneous on large scale.

W. De Sitter: 1919 GR $+\Lambda$ with $\rho = 0$. Static but particles move. Redshift $\propto D$.

A. Friedmann: 1922-1924: G.R. general solutions with positive and negative curvature. Polemic with Einstein.

The Discovery of expansion

The Discovery of expansion

Lemaître 1925: De Sitter world = expanding world.

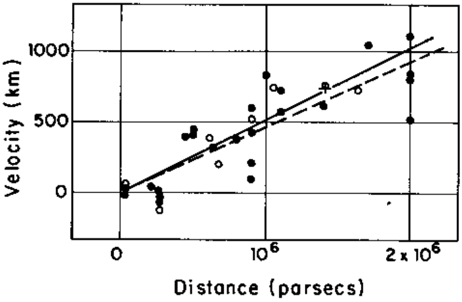
The Discovery of expansion

Lemaître 1925: De Sitter world = expanding world.
1927: expanding solution with $\rho \neq 0$.

The Discovery of expansion

Lemaître 1925: De Sitter world = expanding world.
1927: expanding solution with $\rho \neq 0$.

Hubble 1929: The linear relation between D and v



The golden age: 1933-1964

The golden age: 1933-1964

Zwicky Missing mass in Coma.

The golden age: 1933-1964

Zwicky Missing mass in Coma.

Lemaître Beginning ? Singularity ? How did structures originate ?

The golden age: 1933-1964

Zwicky Missing mass in Coma.

Lemaître Beginning ? Singularity ? How did structures originate ?

Gamov 1942-1948: Origin of elements $\rightarrow T$

The golden age: 1933-1964

Zwicky Missing mass in Coma.

Lemaître Beginning ? Singularity ? How did structures originate ?

Gamov 1942-1948: Origin of elements $\rightarrow T$

Penzias, Wilson, Dicke's group 1964: Discovery and interpretation of the CMB.

Classical Cosmology

Classical established physics

Classical Cosmology

Classical established physics

- Expansion

Classical Cosmology

Classical established physics

- Expansion
- Abundance of light elemnts

Classical Cosmology

Classical established physics

- Expansion
- Abundance of light elemnts
- Existence and properties of the CMB radiation

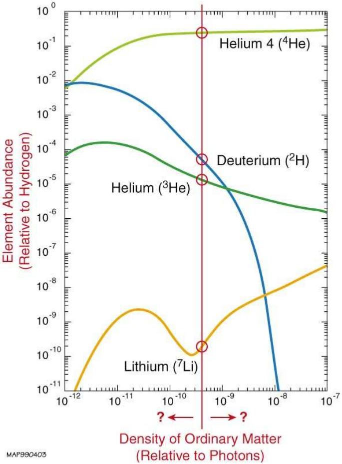
Classical Cosmology

Classical established physics

- Expansion
- Abundance of light elemnts
- Existence and properties of the CMB radiation

Physics is known up to $E \sim 10$ TeV, i.e. $t \sim 10^{-14}$ s.

Primordial Nucleosynthesis



Primordial Nucleosynthesis

After few minutes, BBN is set up.

Primordial Nucleosynthesis

After few minutes, BBN is set up. Only one parameter:

Primordial Nucleosynthesis

After few minutes, BBN is set up. Only one parameter:

$$\eta_{10} = 10^{10}\eta = 10^{10}\frac{n_{p+n}}{n_\gamma}$$

For a "standard" model ($N_\nu = 3$, no exotic physics, ...), very predictive.

Primordial Nucleosynthesis

After few minutes, BBN is set up. Only one parameter:

$$\eta_{10} = 10^{10}\eta = 10^{10}\frac{n_{p+n}}{n_\gamma}$$

For a "standard" model ($N_\nu = 3$, no exotic physics, ...), very predictive.
Calculation, reaction network are simple.
Code publicly available:
www-thphys.physics.ox.ac.uk/users/SubirSarkar/bbn.html

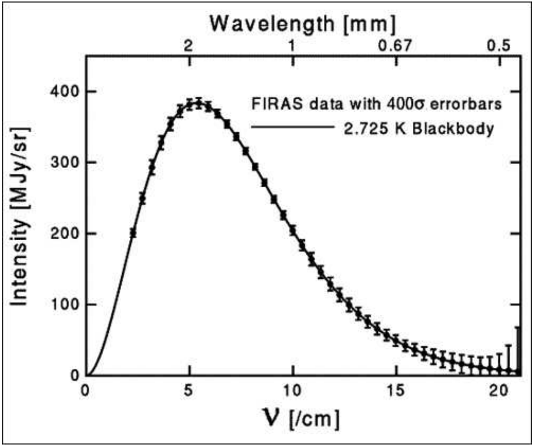
Primordial Nucleosynthesis

After few minutes, BBN is set up. Only one parameter:

$$\eta_{10} = 10^{10}\eta = 10^{10}\frac{n_{p+n}}{n_\gamma}$$

For a "standard" model ($N_\nu = 3$, no exotic physics, ...), very predictive.
Calculation, reaction network are simple.
Code publicly available:
www-thphys.physics.ox.ac.uk/users/SubirSarkar/bbn.html
Limitations:
- neutron lifetime
- nuclear reaction rate
- primordial abundances estimations

CMB spectrum



Thermal equilibrium $t \sim 10$ days

Determine observed values for the model parameters $(H_0, \Omega_0, \lambda_0, q_0, \alpha_0, t_0, \Omega_b,$ topology, ...)

Determine observed values for the model parameters $(H_0, \Omega_0, \lambda_0, q_0, \alpha_0, t_0, \Omega_b$, topology, ...)

Tests of the model:

Determine observed values for the model parameters
$(H_0, \Omega_0, \lambda_0, q_0, \alpha_0, t_0, \Omega_b,$ topology, ...)

Tests of the model:

- Hubble diagram $\rightarrow H_0$

Determine observed values for the model parameters
$(H_0, \Omega_0, \lambda_0, q_0, \alpha_0, t_0, \Omega_b,$ topology, ...)

Tests of the model:

- Hubble diagram $\rightarrow H_0$
- Age : $H_0 t_0 = F(\Omega_0, \lambda_0)$

Determine observed values for the model parameters $(H_0, \Omega_0, \lambda_0, q_0, \alpha_0, t_0, \Omega_b,$ topology, ...)

Tests of the model:

- Hubble diagram $\rightarrow H_0$
- Age : $H_0 t_0 = F(\Omega_0, \lambda_0)$
- Equation EFL: $\alpha_0 = \Omega_0 + \lambda_0 - 1$

Determine observed values for the model parameters $(H_0, \Omega_0, \lambda_0, q_0, \alpha_0, t_0, \Omega_b,$ topology, ...)

Tests of the model:

- Hubble diagram $\rightarrow H_0$
- Age : $H_0 t_0 = F(\Omega_0, \lambda_0)$
- Equation EFL: $\alpha_0 = \Omega_0 + \lambda_0 - 1$
- ...

Hubble constant : H_0

Hubble constant : H_0

- $V = H_0 D$

Hubble constant : H_0

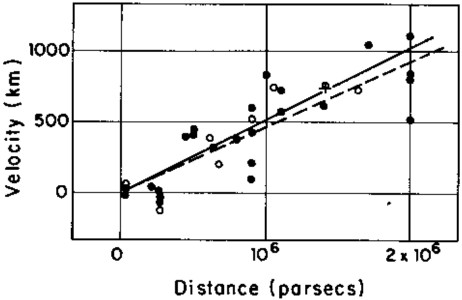
- $V = H_0 D$
- $H_0 = 100h\mathrm{km/s/Mpc}$

Hubble constant : H_0

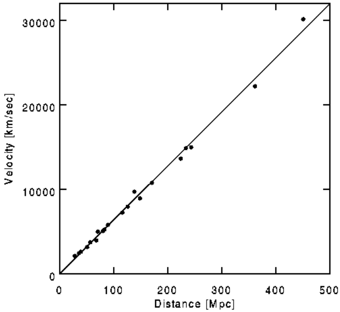
- $V = H_0 D$
- $H_0 = 100h\mathrm{km/s/Mpc}$
- Need unbiased distances measurements.

Hubble constant : H_0

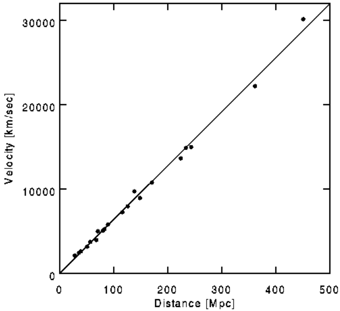
- $V = H_0 D$
- $H_0 = 100h\mathrm{km/s/Mpc}$
- Need unbiased distances measurements.



Hubble constant : H_0

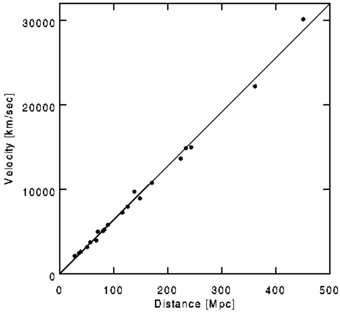


Hubble constant : H_0



- "Best Value" : HST $72 \pm 4 \pm 8$ km/s/Mpc (Freedman et al., 1998)

Hubble constant : H_0



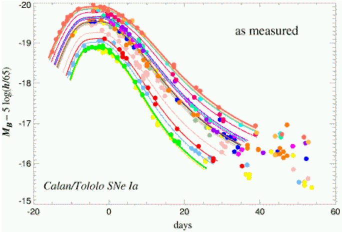
- "Best Value" : HST $72 \pm 4 \pm 8$ km/s/Mpc (Freedman et al., 1998)
- Different techniques $\rightarrow$ different answers (SZ, gravitational time delay...)...

SNIa Hubble diagram

The stretch miracle...

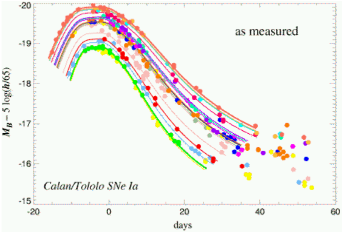
SNIa Hubble diagram

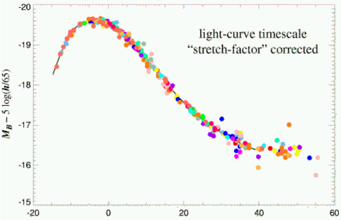
The stretch miracle...



SNIa Hubble diagram

The stretch miracle...



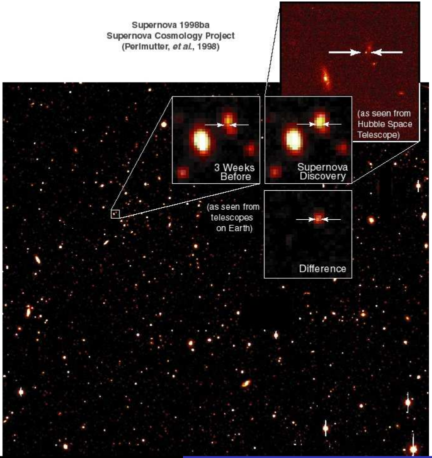


SNIa Hubble diagram

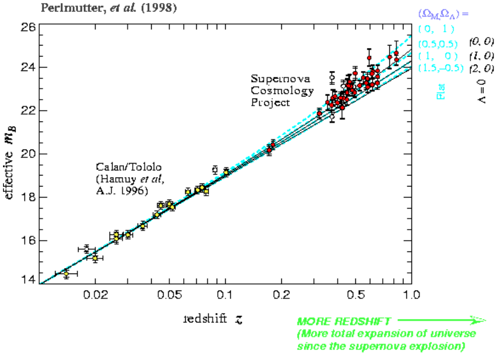
Looking for distant supernovae...

SNIa Hubble diagram

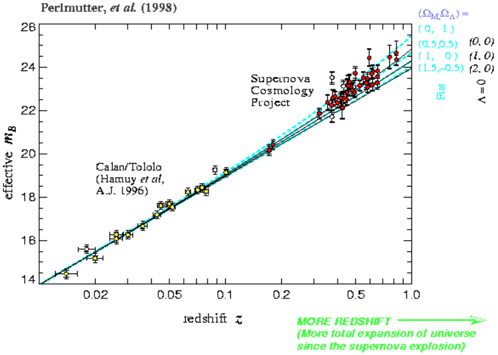
Looking for distant supernovae...



SNIa Hubble diagram

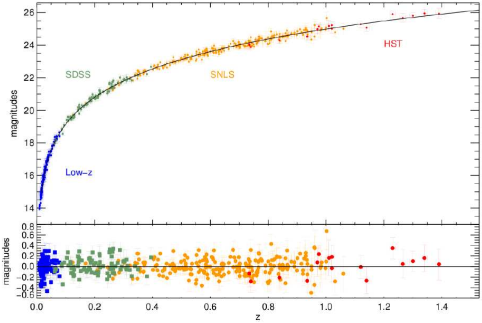


SNIa Hubble diagram



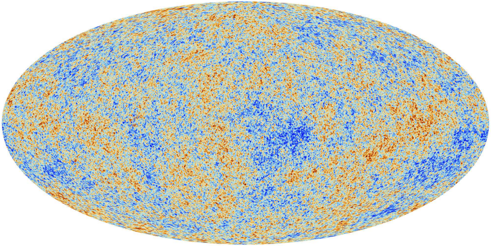
→ Acceleration!

SNIa Hubble diagram



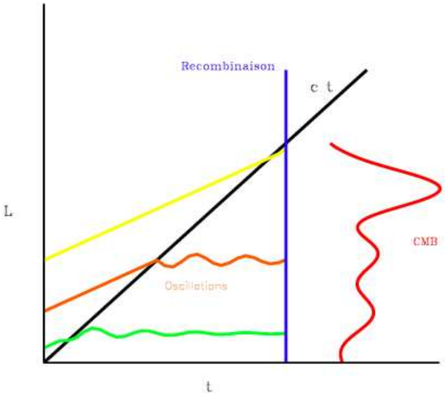
Lattest

Cosmological parameters from CMB



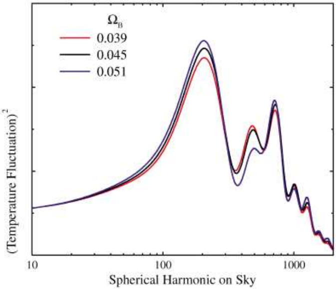
PLANCK

Cosmological parameters from CMB



Dynamics of fluctuations

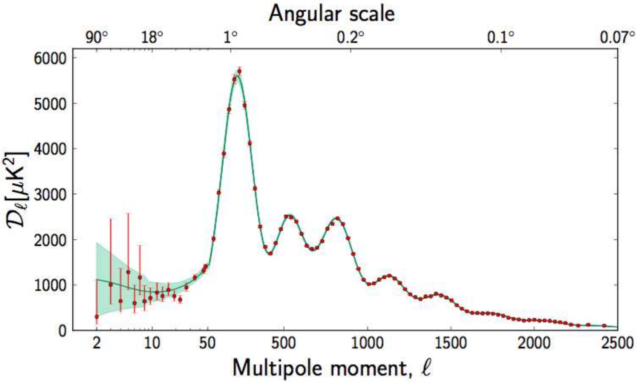
Cosmological parameters from CMB



Dynamics of fluctuations

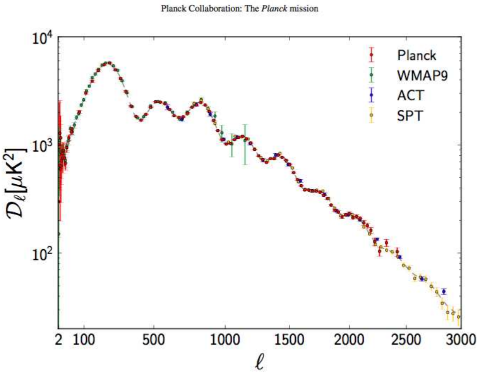
Cosmological parameters from CMB

Planck Collaboration: The *Planck* mission



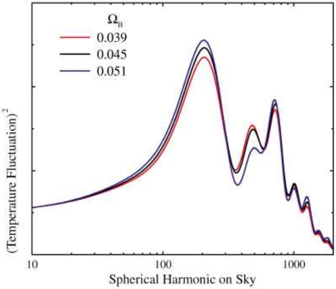
PLANCK

Cosmological parameters from CMB

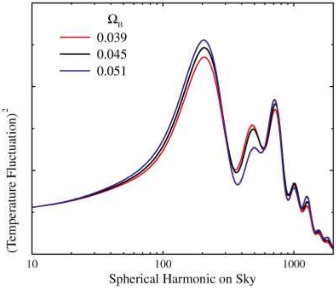


PLANCK

Cosmological parameters from CMB



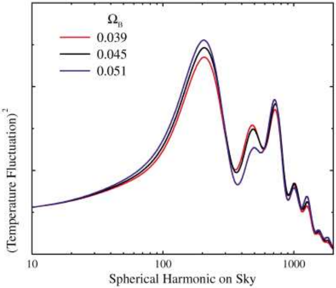
Cosmological parameters from CMB



Best Values (PLANCK):

$$\eta_{10} = 6.315 \pm 0.085 (1.3\%)$$

Cosmological parameters from CMB



Best Values (PLANCK):

$$\eta_{10} = 6.315 \pm 0.085 (1.3\%)$$

i.e.

$$\Omega_b = 0.049 \pm 0.00065$$

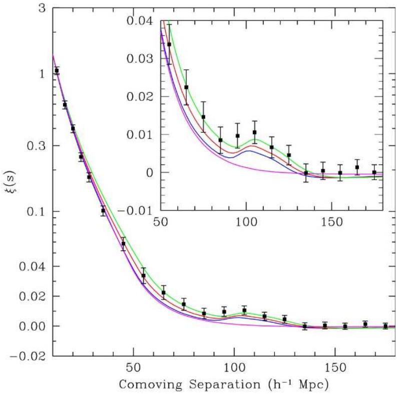
(depends on some asumptions)

Cosmological parameters from CMB

Planck Collaboration: Cosmological parameters

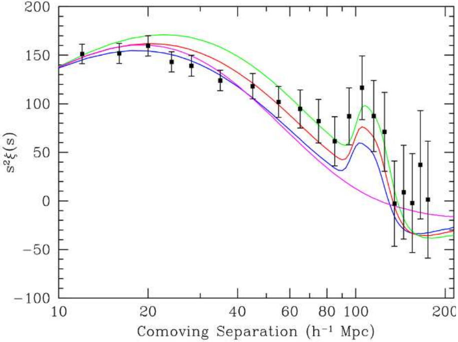
Parameter	*Planck* Best fit	*Planck* 68% limits	*Planck*+lensing Best fit	*Planck*+lensing 68% limits	*Planck*+WP Best fit	*Planck*+WP 68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033	0.022242	0.02217 ± 0.00033	0.022032	0.02205 ± 0.00028
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031	0.11805	0.1186 ± 0.0031	0.12038	0.1199 ± 0.0027
$100\theta_{MC}$	1.04122	1.04132 ± 0.00068	1.04150	1.04141 ± 0.00067	1.04119	1.04131 ± 0.00063
τ	0.0925	0.097 ± 0.038	0.0949	0.089 ± 0.032	0.0925	$0.089^{+0.012}_{-0.014}$
n_s	0.9624	0.9616 ± 0.0094	0.9675	0.9635 ± 0.0094	0.9619	0.9603 ± 0.0073
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072	3.098	3.085 ± 0.057	3.0980	$3.089^{+0.024}_{-0.027}$
Ω_Λ	0.6825	0.686 ± 0.020	0.6964	0.693 ± 0.019	0.6817	$0.685^{+0.018}_{-0.016}$
Ω_m	0.3175	0.314 ± 0.020	0.3036	0.307 ± 0.019	0.3183	$0.315^{+0.016}_{-0.018}$
σ_8	0.8344	0.834 ± 0.027	0.8285	0.823 ± 0.018	0.8347	0.829 ± 0.012
z_{re}	11.35	$11.4^{+4.0}_{-2.8}$	11.45	$10.8^{+3.1}_{-2.5}$	11.37	11.1 ± 1.1
H_0	67.11	67.4 ± 1.4	68.14	67.9 ± 1.5	67.04	67.3 ± 1.2
$10^9 A_s$	2.215	2.23 ± 0.16	2.215	$2.19^{+0.12}_{-0.14}$	2.215	$2.196^{+0.051}_{-0.060}$
$\Omega_m h^2$	0.14300	0.1423 ± 0.0029	0.14094	0.1414 ± 0.0029	0.14305	0.1426 ± 0.0025
$\Omega_m h^3$	0.09597	0.09590 ± 0.00059	0.09603	0.09593 ± 0.00058	0.09591	0.09589 ± 0.00057
Y_P	0.247710	0.24771 ± 0.00014	0.247785	0.24775 ± 0.00014	0.247695	0.24770 ± 0.00012
Age/Gyr	13.819	13.813 ± 0.058	13.784	13.796 ± 0.058	13.8242	13.817 ± 0.048
z_*	1090.43	1090.37 ± 0.65	1090.01	1090.16 ± 0.65	1090.48	1090.43 ± 0.54
r_*	144.58	144.75 ± 0.66	145.02	144.96 ± 0.66	144.58	144.71 ± 0.60
$100\theta_*$	1.04139	1.04148 ± 0.00066	1.04164	1.04156 ± 0.00066	1.04136	1.04147 ± 0.00062
z_{drag}	1059.32	1059.29 ± 0.65	1059.59	1059.43 ± 0.64	1059.25	1059.25 ± 0.58
r_{drag}	147.34	147.53 ± 0.64	147.74	147.70 ± 0.63	147.36	147.49 ± 0.59
k_D	0.14026	0.14007 ± 0.00064	0.13998	0.13996 ± 0.00062	0.14022	0.14009 ± 0.00063
$100\theta_D$	0.161332	0.16137 ± 0.00037	0.161196	0.16129 ± 0.00036	0.161375	0.16140 ± 0.00034
z_{eq}	3402	3386 ± 69	3352	3362 ± 69	3403	3391 ± 60
$100\theta_{eq}$	0.8128	0.816 ± 0.013	0.8224	0.821 ± 0.013	0.8125	0.815 ± 0.011
$r_{drag}/D_V(0.57)$	0.07130	0.0716 ± 0.0011	0.07207	0.0719 ± 0.0011	0.07126	0.07147 ± 0.00091

Cosmological parameters from LSS



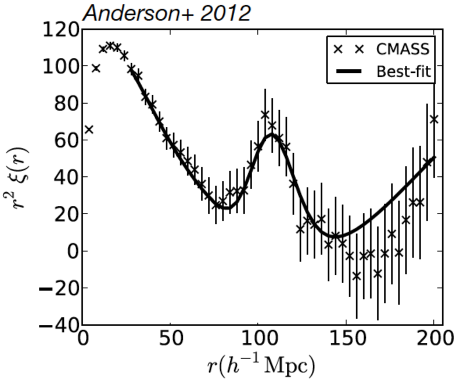
SDSS

Cosmological parameters from LSS



SDSS

Cosmological parameters from LSS



BOSS2

Summary at this point

The homogenous Big Bang is **extremely successful!**

Summary at this point

The homogenous Big Bang is **extremely successful!**

The structure formation within Big Bang picture is **extremely successful!**

Summary at this point

Troubles/Questions

Summary at this point

Troubles/Questions

- Asymmetry matter-anti-matter

Summary at this point

Troubles/Questions

- Asymmetry matter-anti-matter
- The model relies on the existence of non-baryonic matter

Summary at this point

Troubles/Questions

- Asymmetry matter-anti-matter
- The model relies on the existence of non-baryonic matter
- The model suffers from an "initial condition" problem.

Summary at this point

Troubles/Questions

- Asymmetry matter-anti-matter
- The model relies on the existence of non-baryonic matter
- The model suffers from an “initial condition” problem.
- The expansion is accelerating!

Summary at this point

Troubles/Questions

- Asymmetry matter-anti-matter
- The model relies on the existence of non-baryonic matter
- The model suffers from an "initial condition" problem.
- The expansion is accelerating!

This is calling for Physics beyond "knwon Physics"

The non-baryonic dark matter issue

The non-baryonic dark matter issue

- Baryons are not sufficient: $\Omega_B \sim 0.05$ while $\Omega_m \sim 0.3$

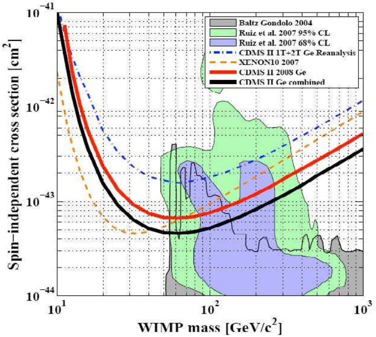
The non-baryonic dark matter issue

- Baryons are not sufficient: $\Omega_B \sim 0.05$ while $\Omega_m \sim 0.3$
- The CDM model is doing very well with structures formation.

The non-baryonic dark matter issue

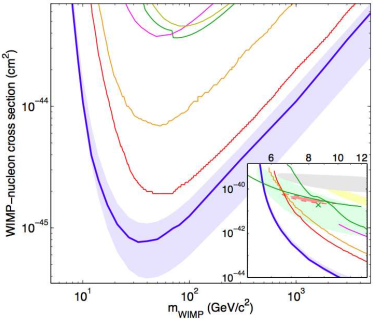
- Baryons are not sufficient: $\Omega_B \sim 0.05$ while $\Omega_m \sim 0.3$
- The CDM model is doing very well with structures formation.
- Alternative are said to exist (WDM, MOND...)

The non-baryonic dark matter issue



CDMS2011

The non-baryonic dark matter issue



LUX2014

The initial condition issue

What inflation solves (A.Guth)

The initial condition issue

What inflation solves (A.Guth)

- No magnetic monopole...

The initial condition issue

What inflation solves (A.Guth)

- No magnetic monopole...
- Curvature is surprisingly close to zero

The initial condition issue

What inflation solves (A.Guth)

- No magnetic monopole...
- Curvature is surprisingly close to zero
- The observable universe contains many regions which were not causaly connected (and still are synchronized!)

The initial condition issue

What inflation solves (A.Guth)

- No magnetic monopole...
- Curvature is surprisingly close to zero
- The observable universe contains many regions which were not causaly connected (and still are synchronized!)
- The mechanism provides an origin for the initial fluctuations

Horizon and Inflation

Horizon and Inflation

horizon in standard FL dynamics:

$$R(t) = R_0(t/t_0)^{1/2}$$

(neglecting matter dominated phase...)

Horizon and Inflation

horizon in standard FL dynamics:

$$R(t) = R_0(t/t_0)^{1/2}$$

(neglecting matter dominated phase...) thus :

$$R_0 r_H(t) = R_0 \int_0^t \frac{cdt}{R(t)} = 2Ct1/2t_0^{1/2}$$

Horizon and Inflation

horizon in standard FL dynamics:

$$R(t) = R_0(t/t_0)^{1/2}$$

(neglecting matter dominated phase...) thus :

$$R_0 r_H(t) = R_0 \int_0^t \frac{cdt}{R(t)} = 2Ct1/2t_0^{1/2}$$

i.e. at the Planck time (t_P):

$$(R_0 r_H(t_0))^3 \approx 10^{90}(R_0 r_H(t_P))^3$$

Horizon and Inflation

horizon in standard FL dynamics:

$$R(t) = R_0(t/t_0)^{1/2}$$

(neglecting matter dominated phase...) thus :

$$R_0 r_H(t) = R_0 \int_0^t \frac{cdt}{R(t)} = 2Ct1/2t_0^{1/2}$$

i.e. at the Planck time (t_P):

$$(R_0 r_H(t_0))^3 \approx 10^{90}(R_0 r_H(t_P))^3$$

This is just crazy...

The acceleration from a scalar field

The acceleration from a scalar field

In GR $\ddot{R} = -\frac{4\pi G}{3}(\rho + 3P/c^2)R$

$$P = w\rho$$

The acceleration from a scalar field

In GR $\ddot{R} = -\frac{4\pi G}{3}(\rho + 3P/c^2)R$

$$P = w\rho$$

For a scalar field, Φ, the density is:

$$\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi)$$

The acceleration from a scalar field

In GR $\ddot{R} = -\frac{4\pi G}{3}(\rho + 3P/c^2)R$

$$P = w\rho$$

For a scalar field, Φ, the density is:

$$\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi)$$

and the pressure P:

$$P_\Phi = \frac{1}{2}\dot{\Phi}^2 - V(\Phi)$$

The acceleration from a scalar field

In GR $\ddot{R} = -\frac{4\pi G}{3}(\rho + 3P/c^2)R$

$$P = w\rho$$

For a scalar field, Φ, the density is:

$$\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi)$$

and the pressure P:

$$P_\Phi = \frac{1}{2}\dot{\Phi}^2 - V(\Phi)$$

The condition $P < -1/3\rho$ reads $\dot{\Phi}^2 < V(\Phi)$

Evolution

Go back to energy conservation :

Evolution

Go back to energy conservation :

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$

Evolution

Go back to energy conservation :

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$

Slow roll condition : $\dot{\Phi}^2 \ll V(\Phi)$ i.e. $H^2 = 1/3V$ $(8\pi G = 1)$

Evolution

Go back to energy conservation :

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$

Slow roll condition : $\dot{\Phi}^2 \ll V(\Phi)$ i.e. $H^2 = 1/3V$ $(8\pi G = 1)$
so $\ddot{\Phi} \ll V'(\Phi)$ $(' = d/d\phi)$.

Evolution

Go back to energy conservation :

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$

Slow roll condition : $\dot{\Phi}^2 \ll V(\Phi)$ i.e. $H^2 = 1/3V$ $(8\pi G = 1)$
so $\ddot{\Phi} \ll V'(\Phi)$ $(' = d/d\phi)$.

$$3H\dot{\phi} = -V'(\phi)$$

Evolution

Go back to energy conservation :

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$

Slow roll condition : $\dot{\Phi}^2 \ll V(\Phi)$ i.e. $H^2 = 1/3V$ $(8\pi G = 1)$
so $\ddot{\Phi} \ll V'(\Phi)$ $(' = d/d\phi)$.

$$3H\dot{\phi} = -V'(\phi)$$

so slow roll condition becomes:

$$\frac{V'^2}{H^2} = \frac{V'^2}{V} \ll V \text{ i.e. } \left(\frac{V'}{V}\right)^2 \ll 1$$

Dark energy: Quintessence

Dark energy: Quintessence

Today acceleration $\rightarrow$ scalar field!

Dark energy: Quintessence

Today acceleration $\rightarrow$ scalar field!
Peebles-Ratra:

$$V(\Phi) \propto \Phi^{-n}$$

other potentials come from other theories (SUGRA, ...)

Dark energy: Quintessence

Today acceleration → scalar field!
Peebles-Ratra:

$$V(\Phi) \propto \Phi^{-n}$$

other potentials come from other theories (SUGRA, ...)

$$\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi) \text{ and } P/c^2 = \frac{1}{2}\dot{\Phi}^2 - V(\Phi)$$

Dark energy: Quintessence

Today acceleration $\rightarrow$ scalar field!
Peebles-Ratra:

$$V(\Phi) \propto \Phi^{-n}$$

other potentials come from other theories (SUGRA, ...)

$$\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi) \text{ and } P/c^2 = \frac{1}{2}\dot{\Phi}^2 - V(\Phi)$$

Allowing $-1 \leq w \leq 0$.

Dark energy: Quintessence

Today acceleration → scalar field!
Peebles-Ratra:

$$V(\Phi) \propto \Phi^{-n}$$

other potentials come from other theories (SUGRA, ...)

$$\rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi) \text{ and } P/c^2 = \frac{1}{2}\dot{\Phi}^2 - V(\Phi)$$

Allowing $-1 \leq w \leq 0$. Even $w \leq -1$ is possible...

Modified gravity

Modified gravity

Take a Lagrangian extending Einstein-Hilbert's one.

Modified gravity

Take a Lagrangian extending Einstein-Hilbert's one.

Have look at Amendola et al. 2013, arXiv1206.1225A

Modified gravity

Take a Lagrangian extending Einstein-Hilbert's one.

Have look at Amendola et al. 2013, arXiv1206.1225A

Build EUCLID...

Vacuum

Vacuum

From quantum field point of view:
in vacuum a non zero electric field can exist for some duration Δt provide it does not violate Heisenberg.

Vacuum

From quantum field point of view:
in vacuum a non zero electric field can exist for some duration Δt
provide it does not violate Heisenberg.
In GR energy gravitates.

Vacuum

From quantum field point of view:
in vacuum a non zero electric field can exist for some duration Δt provide it does not violate Heisenberg.
In GR energy gravitates.
when $\Delta t \to 0$ $\Delta E \to +\infty$...

Vacuum

From quantum field point of view:
in vacuum a non zero electric field can exist for some duration Δt provide it does not violate Heisenberg.
In GR energy gravitates.
when $\Delta t \rightarrow 0$ $\Delta E \rightarrow +\infty$...
Harmonic oscillator:

$$E_n = (n + \frac{1}{2})h\nu$$

zero point energy: $\frac{1}{2}h\nu$ contributes to ρ_V.

Historical aspects

Λ was introduced by Einstein

Historical aspects

Λ was introduced by Einstein

Lemaître (1934) made the comment that Λ is equivalent to a Lorentz invariant non-zero vacuum, i.e.

$$P = -\rho \tag{1}$$

Historical aspects

Λ was introduced by Einstein

Lemaître (1934) made the comment that Λ is equivalent to a Lorentz invariant non-zero vacuum, i.e.

$$P = -\rho \tag{1}$$

Is there an experimental difference between Λ and L.I.V.?

Historical aspects

Λ was introduced by Einstein

Lemaître (1934) made the comment that Λ is equivalent to a Lorentz invariant non-zero vacuum, i.e.

$$P = -\rho \tag{1}$$

Is there an experimental difference between Λ and L.I.V.?

Nerst (1916) and Pauli discussed the possible contribution of zero-point energy to the density of the Universe ($\rightarrow$ Kragh `arXiv:1111.4623`)

Historical aspects

Λ was introduced by Einstein

Lemaître (1934) made the comment that Λ is equivalent to a Lorentz invariant non-zero vacuum, i.e.

$$P = -\rho \quad (1)$$

Is there an experimental difference between Λ and L.I.V.?

Nerst (1916) and Pauli discussed the possible contribution of zero-point energy to the density of the Universe ($\rightarrow$ Kragh `arXiv:1111.4623`)

So is this the origin of the acceleration ?

Historical aspects

No!

Historical aspects

No!

The Vacuum catastroph (Weinberg, 1989):

$$\rho_v = \langle 0 | T^{00} | 0 \rangle = \frac{1}{(2\pi)^3} \int_0^{+\infty} \frac{1}{2} \hbar\omega \, \mathrm{d}^3\mathbf{k}$$

with $\omega^2 = k^2 + m^2$

Historical aspects

No!

The Vacuum catastroph (Weinberg, 1989):

$$\rho_v = \langle 0 | T^{00} | 0 \rangle = \frac{1}{(2\pi)^3} \int_0^{k_c} \frac{1}{2} \hbar\omega \, \mathrm{d}^3\mathbf{k}$$

with $\omega^2 = k^2 + m^2$ highly divergent:

$$\rho_v(k_c) \propto \frac{k_c^4}{16\pi^2}$$

(for $k_c \gg m$).

Equation of state

The pressure (massless field):

$$P_v = (\mathbf{1}/\mathbf{3}) \sum_i \langle 0 | T^{ii} | 0 \rangle = \frac{1}{3} \frac{1}{2(2\pi)^3} \int_0^{+\infty} k \, \mathrm{d}^3\mathbf{k}$$

Equation of state

The pressure (massless field):

$$P_v = (\mathbf{1/3}) \sum_i \langle 0 | T^{ii} | 0 \rangle = \frac{1}{3} \frac{1}{2(2\pi)^3} \int_0^{+\infty} k \, \mathrm{d}^3\mathbf{k}$$

So that any regularization that is applied to both quantities leads to the e.o.s.:

Equation of state

The pressure (massless field):

$$P_v = (\mathbf{1}/\mathbf{3}) \sum_i \langle 0| T^{ii} |0\rangle = \frac{1}{3} \frac{1}{2(2\pi)^3} \int_0^{+\infty} k \, \mathrm{d}^3\mathbf{k}$$

So that any regularization that is applied to both quantities leads to the e.o.s.:

$$P = \frac{1}{3}\rho \qquad (2)$$

Equation of state

The pressure (massless field):

$$P_v = (\mathbf{1}/\mathbf{3}) \sum_i \langle 0 | T^{ii} | 0 \rangle = \frac{1}{3} \frac{1}{2(2\pi)^3} \int_0^{+\infty} k \, \mathrm{d}^3\mathbf{k}$$

So that any regularization that is applied to both quantities leads to the e.o.s.:

$$P = \frac{1}{3}\rho \tag{2}$$

i.e. eq. (1) + eq. (2) leads to :

$$P_v = \rho_v = 0$$

Equation of state

The pressure (massless field):

$$P_v = (\mathbf{1/3}) \sum_i \langle 0 | T^{ii} | 0 \rangle = \frac{1}{3} \frac{1}{2(2\pi)^3} \int_0^{+\infty} k \, \mathrm{d}^3\mathbf{k}$$

So that any regularization that is applied to both quantities leads to the e.o.s.:

$$P = \frac{1}{3}\rho \qquad (2)$$

i.e. eq. (1) + eq. (2) leads to :

$$P_v = \rho_v = 0$$

$\rightarrow$ usual conclusion on zero-point energy contribution (for instance by dimensional regularization).

Equation of state

Does not hold for a massive field (Zeldovich 1968, ...):

$$P_v = -\rho_v$$

Equation of state

Does not hold for a massive field (Zeldovich 1968, ...):

$$P_v = -\rho_v$$

But

$$\rho_v = m^4(...)$$

Equation of state

Does not hold for a massive field (Zeldovich 1968, ...):

$$P_v = -\rho_v$$

But

$$\rho_v = m^4(...)$$

cf Review by J.Martin 2012 (astro-ph/1205.3365).

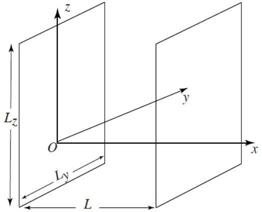
Everything You Always Wanted To Know About
The Cosmological Constant Problem (But Were Afraid To Ask)

Casimir effect

Where is there vacuum contribution in laboratory physics?

Casimir effect

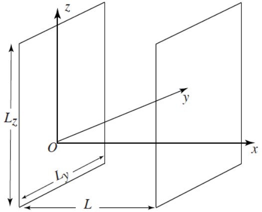
Where is there vacuum contribution in laboratory physics?



Casimir effect

Casimir effect

Where is there vacuum contribution in laboratory physics?



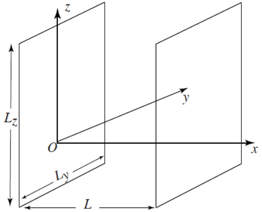
Casimir effect

with:

$$P_x = 3\rho$$

Casimir effect

Where is there vacuum contribution in laboratory physics?



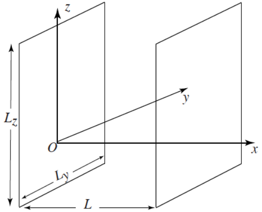
Casimir effect

with:

$$P_x = 3\rho < 0$$

Casimir effect

Where is there vacuum contribution in laboratory physics?



Casimir effect

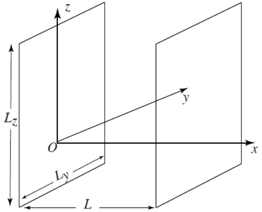
with:

$$P_x = 3\rho < 0$$

and ...

Casimir effect

Where is there vacuum contribution in laboratory physics?



Casimir effect

with:

$$P_x = 3\rho < 0$$

and ...

$$P_{//} = -\rho$$

Brown & Maclay (1968)

Casimir effect from higher dimension

Assume there is an additional compact dimension.

Casimir effect from higher dimension

Assume there is an additional compact dimension.

Standard physics in 3+1 D (brane), gravity in 3+1+1D (Bulk).

Casimir effect from higher dimension

Assume there is an additional compact dimension.

Standard physics in 3+1 D (brane), gravity in 3+1+1D (Bulk).

The quantification of gravitational field modes in the bulk leads to a Casimir energy (Appelquist & Chodos, 1983).

Casimir effect from higher dimension

Assume there is an additional compact dimension.

Standard physics in 3+1 D (brane), gravity in 3+1+1D (Bulk).

The quantification of gravitational field modes in the bulk leads to a Casimir energy (Appelquist & Chodos, 1983).

This result can be established by evaluating zero mode contributions (Rohrlich 1984).
Dispersion relation:

$$\omega^2 = k^2 + \frac{n^2}{R^2}$$

Casimir effect from higher dimension

Assume there is an additional compact dimension.

Standard physics in 3+1 D (brane), gravity in 3+1+1D (Bulk).

The quantification of gravitational field modes in the bulk leads to a Casimir energy (Appelquist & Chodos, 1983).

This result can be established by evaluating zero mode contributions (Rohrlich 1984).
Dispersion relation:

$$\omega^2 = k^2 + \frac{n^2}{R^2}$$

This (permanent) contribution can be evaluated by mean of dimensional regularization.

Casimir effect: the Hubble radius

Casimir effect: the Hubble radius

Assumption 1: At high energy, only modes with λ smaller than ct have to be taken into account i.e.:

$$\rho_v = \frac{5\hbar c}{8\pi^3 R} \int_{\omega > \omega_H}^{\infty} k^2 \mathrm{d}k \left[\sum_{n=-\infty}^{\infty} \left(k^2 + \frac{n^2}{R^2} \right)^{1/2} \right]$$

Casimir effect: the Hubble radius

Assumption 1: At high energy, only modes with λ smaller than ct have to be taken into account i.e.:

$$\rho_v = \frac{5\hbar c}{8\pi^3 R} \int_{\omega > \omega_H}^{\infty} k^2 \mathrm{d}k \left[\sum_{n=-\infty}^{\infty} \left(k^2 + \frac{n^2}{R^2} \right)^{1/2} \right]$$

Assumption 2: as long as $ct \ll \pi R$ gravitational vacuum should be that of a massless field in a 4+1D space time i.e.:

$$\rho_v = 0$$

Space Isotropy ends...

Space Isotropy ends...

when $ct \sim \pi R \; \omega_H \sim \frac{1}{R}$, this is the last time at which symetries ensure $\rho_v = 0$. Then

$$\rho_v = \frac{5\hbar c}{8\pi^3 R} \int_{1/R}^{\infty} k^2 \mathrm{d}k \, [...] = 0$$

Space Isotropy ends...

when $ct \sim \pi R \; \omega_H \sim \frac{1}{R}$, this is the last time at which symetries ensure $\rho_v = 0$. Then

$$\rho_v = \frac{5\hbar c}{8\pi^3 R} \int_{1/R}^{\infty} k^2 \mathrm{d}k \, [...] = 0$$

Later, when $ct \gg \pi R$ i.e. $\omega_H \sim 0$

$$\rho_v = \frac{5\hbar c}{8\pi^3 R} \int_{0}^{\infty} k^2 \mathrm{d}k \, [...] = \frac{5\hbar c}{8\pi^3 R} \int_{0}^{1/R} k^2 \mathrm{d}k \, [...]$$

with :

$$[...] = \left[\sum_{n=-\infty}^{\infty} \left(k^2 + \frac{n^2}{R^2} \right)^{1/2} \right]$$

Isotropy ends...

The condition :

$$\omega = \sqrt{k^2 + \frac{n^2}{R^2}} < \frac{1}{R}$$

ensured only if $n = 0$, so:

$$\rho_v = \frac{5\hbar c}{8\pi^3 R} \int_0^{1/R} k^3 \mathrm{d}k = \frac{5\hbar c}{32\pi^3 R^5}$$

Isotropy ends...

The condition :

$$\omega = \sqrt{k^2 + \frac{n^2}{R^2}} < \frac{1}{R}$$

ensured only if $n = 0$, so:

$$\rho_v = \frac{5\hbar c}{8\pi^3 R} \int_0^{1/R} k^3 \mathrm{d}k = \frac{5\hbar c}{32\pi^3 R^5}$$

In the brane:

$$\rho_v = \frac{5\hbar c}{16\pi^2 R^4}$$

Dark energy emerges...

Pressure:

$$P_v^{\perp} = 4\rho_0 = \frac{20\hbar c}{32\pi^3 R^5}$$

Along the brane, using the fact that the $T^{\mu\nu}$ is traceless and integrating along the 4th spatial dimension:

$$P_v^{\parallel} = -\frac{5\hbar c}{16\pi^2 R^4} = -\rho_v$$

Dark energy emerges...

Pressure:

$$P_v^{\perp} = 4\rho_0 = \frac{20\hbar c}{32\pi^3 R^5}$$

Along the brane, using the fact that the $T^{\mu\nu}$ is traceless and integrating along the 4th spatial dimension:

$$P_v^{\parallel} = -\frac{5\hbar c}{16\pi^2 R^4} = -\rho_v$$

so:

$$R = \left(\frac{5\hbar G}{2\pi c \Lambda}\right)^{\frac{1}{4}}$$

Dark energy emerges...

Pressure:

$$P_v^{\perp} = 4\rho_0 = \frac{20\hbar c}{32\pi^3 R^5}$$

Along the brane, using the fact that the $T^{\mu\nu}$ is traceless and integrating along the 4th spatial dimension:

$$P_v^{||} = -\frac{5\hbar c}{16\pi^2 R^4} = -\rho_v$$

so:

$$R = \left(\frac{5\hbar G}{2\pi c\Lambda}\right)^{\frac{1}{4}}$$

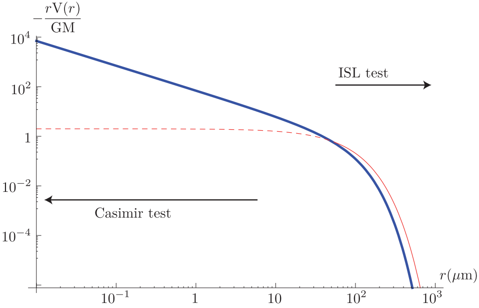
$\Omega_v \sim 0.7 \Rightarrow R \sim 35\mu\mathrm{m}$ fits data. Corresponding to $E \sim 1\,TeV$

Consequences

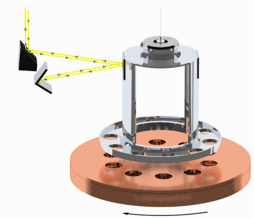
Acceleration is due to vacuum: GR $+ \; w = -1$

Consequences

The presence of additional compact "large" dimension ($\sim 35\mu\mathrm{m}$) can be tested by experiment on gravitational inverse square law on short scale. Additional term:



Consequences



Present day limit (Adelberger et al. 2009) :

$$R < 46\mu\mathrm{m}$$

Conclusion

Conclusion

- Casimir effect from quantized scalar field in additional compact dimension can produce a non-zero vacuum contribution to the density of the universe with the correct equation of state for a cosmological constant. i.e. "usual" physics for DE.

Conclusion

- Casimir effect from quantized scalar field in additional compact dimension can produce a non-zero vacuum contribution to the density of the universe with the correct equation of state for a cosmological constant. i.e. "usual" physics for DE.

- Acceleration could be the direct manifestation of the quantum gravitational vacuum.

Conclusion

- Casimir effect from quantized scalar field in additional compact dimension can produce a non-zero vacuum contribution to the density of the universe with the correct equation of state for a cosmological constant. i.e. "usual" physics for DE.
- Acceleration could be the direct manifestation of the quantum gravitational vacuum.
- With $R \sim 35 \mu\mathrm{m}$ it produces a cosmological constant as observed.

Conclusion

- Casimir effect from quantized scalar field in additional compact dimension can produce a non-zero vacuum contribution to the density of the universe with the correct equation of state for a cosmological constant. i.e. "usual" physics for DE.

- Acceleration could be the direct manifestation of the quantum gravitational vacuum.

- With $R \sim 35\mu\mathrm{m}$ it produces a cosmological constant as observed. $\rightarrow$ gravitation is modified on scales $\leq 45\mu\mathrm{m}$

General Conclusion on Cosmology

General Conclusion on Cosmology

- A simple model for the universe based on known physics was build and was successful

General Conclusion on Cosmology

- A simple model for the universe based on known physics was build and was successful
- A model for structure formation the universe based on somewhat unknown physics was build (ΛCDM) and was successful

General Conclusion on Cosmology

- A simple model for the universe based on known physics was build and was successful
- A model for structure formation the universe based on somewhat unknown physics was build (ΛCDM) and was successful
- It is likely that more and more astrophysical data are needed and in some case could be the only way out for progresses

Few references

- Gravitation and Cosmology, S. Weinberg
- Principles of Physical Cosmology, J.Peebles, Princeton university press.
- Primordial Cosmology, P.Peter J.P. Uzan
- Physical Foundations of Cosmology, V.Mukhanov's
- General Relativity: An Introduction for Physicists, M. P. Hobson, G. P. Efstathiou, A. N. Lasenby
- Neutrino cosmology J. Lesgourgues, G.Mangero