TWO LECTURES ON HEAVY QUARK EFFECTIVE THEORY AND SPECTROSCOPY OF HEAVY-LIGHT MESONS

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I give a basic introduction to the Heavy Quark Effective Theory. I discuss the implications of the heavy quark limit for hadron spectroscopy, devoting particular attention to the classification of heavy-light mesons in doublets. I show how using an approach based on effective lagrangian terms displaying new symmetries that arise in such a limit, it is possible to make predictions for strong decays of heavy-light mesons with the emission of a light meson that are helpful in identifying recently discovered states. Moreover, I consider how weak transitions among members of the various doublets can be expressed in terms of universal form factors. As a last topic, I discuss the case of heavy mesons with hidden flavour, with particular reference to the state $X(3872)$.

Keywords: Heavy Quarks, Hadron Spectroscopy.

1. Introduction

The physics of hadrons containing a single heavy quark can be systematically studied considering the large heavy quark mass limit, which is formalized in the Heavy Quark Effective Theory (HQET), an effective theory developed at the end of the eighties.[1] I would like to start these lectures by giving an intuitive picture of the heavy hadron properties; afterwards I shall recall the systematic formulation of HQET.

As a preliminary, it is necessary to define when a quark can be classified as *heavy*. As is well known, the theory of strong interactions contains a scale parameter, Λ_{QCD}, that can be viewed as the scale separating the strong coupling regime from the regime where there is the onset of asymptotic freedom. This can be understood looking at the one loop expression of the strong coupling constant α_s at a given scale μ:

$$\frac{\alpha_s(\mu)}{4\pi} = \frac{1}{\beta_0 \ln \frac{\mu^2}{\Lambda^2_{QCD}}} \qquad \beta_0 = \frac{11N_c - 2N_f}{3} \, . \tag{1}$$

Eq. (1) stems from solving at leading order the renormalization group equation for the coupling g_s of the gauge group $SU(N_c)$ of strong interactions; N_c is the number of colours, N_f that of *active* flavours, i.e. quarks having mass $m_f \leq \mu$.

Λ_{QCD} is a fundamental parameter of QCD and its value needs to be determined empirically. It can be related to the inverse of the radius of a hadron, where confinement produces the bound state. $R_{had} \simeq 1$ fm corresponds to $\Lambda_{QCD} \simeq 200$ MeV.

One defines a heavy quark Q as that having mass $m_Q \gg \Lambda_{QCD}$. As a result, c, b and t are the heavy quarks, while u, d and s are considered light. Since the top quark decays too fast, and no top bound states are known, HQET actually applies to charm and beauty hadrons.

Here I consider the case of mesons, the treatment of baryons can be done in a similar way. A heavy meson can be described as a hadronic system containing a heavy quark Q and light degrees of freedom consisting of the light antiquark plus the cloud of gluons. A qualitative description of such a system is useful. In analogy with the quantum mechanical spin-orbit interaction, the chromomagnetic interaction between the heavy quark spin s_Q and the light degrees of freedom total angular momentum s_ℓ is proportional to the inverse heavy quark mass. Therefore, when considering the large heavy quark mass limit $m_Q \to \infty$, such an interaction can be neglected and s_Q and s_ℓ decouple: Not only their sum $\vec{J} = \vec{s}_Q + \vec{s}_\ell$ is conserved, but they are also conserved separately. In more formal terms, that I shall introduce formulating the lagrangian of HQET, this is the basis of the *heavy quark spin symmetry* of HQET, which means invariance under rotations of the heavy quark spin.

Furthermore, in the QCD lagrangian it is only the mass term that distinguishes the quark flavor, so that the heavy quark flavor becomes irrelevant for $m_Q \to \infty$. This is analogous to the independence of the chemical properties of a given element on the mass of the nucleus, the various isotopes share the same properties. In the context of HQET, this observation leads to the *heavy quark flavour symmetry.* When the two symmetries are considered together, one concludes that the configuration of the light degrees of freedom in a heavy hadron is independent of the heavy quark flavour as well as of the orientation of the heavy quark spin.

At this point we can already stress one of the limitations of the approach that we are going to describe: HQET is a theory of heavy hadrons that, exploiting symmetries, allows us to derive relations among various quantities involved in heavy hadron decays and to simplify their spectroscopical classification. However, the theory does not say anything about the light degrees of freedom.

An important result, that will be very useful, and that can be intuitively understood, is the *velocity superselection rule*, due to Georgi.[2] Writing the momenta of the heavy hadron H_Q and of the heavy quark Q as $p_H^\mu = m_H\, v^\mu$ and $p_Q^\mu = m_Q\, v^\mu + k^\mu$, respectively, with k a residual momentum of $\mathcal{O}(\Lambda_{QCD})$, and exploiting the relation $p_Q^\mu = m_Q\, v_Q^\mu$, one obtains that, for $m_Q \to \infty$, the equality $v_Q^\mu = v^\mu$ holds: strong interactions do not change the heavy quark four-velocity.

After these preliminary considerations, I now describe how to build an effective theory in large m_Q limit in which the previous intuitive results can be given a systematic and formal basis.

Effective theories are usually built from a more fundamental theory by integrating out those degrees of freedom that are more massive than the energy scale typical of the processes of interest. An example is the Fermi theory, obtained from the Standard Model by integrating out the W boson. This works very well whenever the W appears as intermediate virtual state. However, it is no more possible to use the Fermi theory if one wants to describe, for example, W decays. By the same token, one cannot build HQET just integrating out the heavy quarks, since the aim is to describe the physics of states containing such quarks. For this reason, HQET is built integrating out only the "small" components of the heavy quark spinor. In order to fulfill such a task, one needs to start from the QCD Lagrangian relative to the heavy quark:

$$\mathcal{L}_Q = \bar{Q}(i\not{D} - m_Q)Q \ , \tag{2}$$

where D is the covariant derivative in the fundamental representation, and redefine the heavy quark spinor Q as follows:

$$Q(x) = e^{-im_Q v\cdot x}[h_v(x) + H_v(x)] \ . \tag{3}$$

h_v and H_v are defined applying the velocity projectors,

$$h_v = \frac{1+\not{v}}{2} Q \qquad H_v = \frac{1-\not{v}}{2} Q \ , \tag{4}$$

and consequently they satisfy the relations: $\not{v}\, h_v = h_v$, $\not{v}\, H_v = -H_v$. Substituting Eq. (4) in Eq. (2) the equation of motion for H_v is derived:

$$H_v = \frac{1}{iv\cdot D + 2m_Q} i\not{D}_\perp h_v \ , \tag{5}$$

where $D_\perp^\mu = D^\mu - v\cdot D v^\mu$. Exploiting Eq. (5) one can eliminate H_v from the Lagrangian, obtaining the following one, equivalent to $\mathcal{L}_Q$:

$$\mathcal{L} = \bar{h}_v iv\cdot D h_v + \bar{h}_v i\not{D}_\perp \frac{1}{iv\cdot D + 2m_Q} i\not{D}_\perp h_v \ . \tag{6}$$

Expanding the second term in powers of $1/m_Q$ yields

$$\mathcal{L} = \bar{h}_v iv\cdot D h_v + \frac{1}{2m_Q}\bar{h}_v (i\not{D}_\perp)^2 h_v + \frac{1}{2m_Q}\bar{h}_v \frac{g_s \sigma_{\alpha\beta} G^{\alpha\beta}}{2} h_v + \mathcal{O}\left(\frac{1}{m_Q}\right)^2 \ . \tag{7}$$

The HQET Lagrangian is the first term of the previous equation, obtained at leading order in the heavy quark expansion:

$$\mathcal{L}_{HQET} = \bar{h}_v iv\cdot D h_v \ . \tag{8}$$

The other two terms appearing at order $1/m_Q$ represent the kinetic energy of the heavy quark due to its residual momentum k, and the chromomagnetic coupling of the heavy quark spin to the gluon field, respectively. The $1/m_Q$ suppression of this term is the origin of the heavy quark spin symmetry.

From Eq. (8) the Feynman rule for the heavy quark propagator can be derived:

$$\frac{i}{v\cdot k}\frac{1+\not{v}}{2} \ , \tag{9}$$

as well as that for the quark-gluon vertex,

$$i\ g_s T^a v_\mu \ . \tag{10}$$

T^a are the $SU(3)$ generators, $T^a = \lambda^a/2$, and λ^a the Gell-Mann matrices. The absence of gamma matrices in Eq. (10) guarantees invariance under rotations of the heavy quark spin and leads to the heavy quark spin symmetry; moreover, the independence of $\mathcal{L}_{HQET}$ of m_Q shows that flavor symmetry holds. As we shall see in the following, exploiting such symmetries provides us with a number of important results. The predictive power of this approach can be further improved by assessing the role of symmetry-breaking terms which requires the inclusion of subleading terms in the heavy quark expansion. In order to do so, we consider Eq. (5) and derive the expression for the heavy quark field Q in QCD at the order $1/m_Q$:

$$Q(x) = e^{-im_Q v\cdot x}\left(1 + \frac{i\not{D}_\perp}{2m_Q}\right) h_v \ . \tag{11}$$

Using this result, we are now able to write each QCD operator containing the field Q as an expansion in powers of $1/m_Q$, too. As a consequence, also the matrix elements of currents can be expressed as an expansion in $1/m_Q$.

However, one should consider that also the states on which the operators act depend on m_Q, as a consequence of the fact that, at the next to leading order in the heavy quark expansion, h_v does not satisfy the equation of motion stemming from Eq. (8). In order to avoid this problem, one assumes that h_v is the heavy quark field in HQET, i.e. it satisfies exactly the equation of motion deriving from the HQET Eq. (8): $\not{v}\, h_v = h_v$. The terms appearing in Eq. (7) at $\mathcal{O}(1/m_Q)$ are then treated as perturbations: the results obtained when including the perturbation are expressed in terms of the unperturbed quantities.

As an example of how an asymptotic relation could be modified by $1/m_Q$ corrections, I consider the heavy hadron masses. Spin symmetry allows to predict that hadrons differing only in the orientation of the heavy quark spin should be degenerate in mass. In the infinite heavy quark limit such a mass should coincide with that of the heavy quark. Consider indeed the pseudoscalar and vector heavy mesons. The difference between the measured masses of such mesons is, in the case of charm: $m_{D^*} - m_D \simeq 140$ MeV, while for beauty: $m_{B^*} - m_B \simeq 47$ MeV. This confirms that heavy quark symmetry works better in the b case.

We can go further, including $\mathcal{O}\left(1/m_Q\right)$ corrections in terms of the matrix elements of the subleading terms in Eq. (7). Their expectation values on a hadron H_Q is defined as follows:

$$\mu_\pi^2 = \frac{1}{2m_Q}\ \langle H_Q|\bar{h}_v(i\ \not{D}_\perp)^2 h_v|H_Q\rangle \ , \tag{12}$$

$$\mu_G^2 = \frac{1}{2m_Q}\ \langle H_Q|\bar{h}_v \frac{g_s\sigma_{\alpha\beta}G^{\alpha\beta}}{2} h_v|H_Q\rangle \ . \tag{13}$$

Since the states are normalized to $2m_Q$, both μ_π^2 and μ_G^2 are independent of m_Q. In terms of these quantities, the mass of H_Q reads:

$$M_{H_Q} = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_Q} + \mathcal{O}\left(\frac{1}{m_Q^2}\right) \ . \tag{14}$$

Let me discuss in turn the parameters entering the previous equation. $\bar{\Lambda}$ is $\mathcal{O}(\Lambda_{QCD})$, and represents the mass difference between the heavy hadron and the heavy quark in the $m_Q \to \infty$ limit. μ_G^2 is the matrix element of the chromomagnetic operator, responsible for the spin symmetry breaking, so that it can be related to the mass difference of the hadrons which differ only in the orientation of the heavy quark spin. In particular, one can write $\mu_G^2 = -2\left[J(J+1) - \frac{3}{2}\right]\lambda_2$, where λ_2, as well as $\bar{\Lambda}$ and μ_π^2, does not depend on m_Q. In the case of the lowest lying beauty mesons its value is

$$\lambda_2 = \frac{1}{4}(M_{B^*}^2 - M_B^2) \simeq 0.12\ \text{GeV}^2 \, .$$

As for μ_π^2, no consensus has been reached about its value, mainly due to the difficulty of extracting this parameter from experimental data.

In the following section, I discuss the implications of heavy quark spin-flavour symmetries for the spectroscopic classification of heavy-light mesons.

2. Heavy mesons: Classification of the states and strong decays

I consider heavy-light $Q\bar{q}$ mesons, where $\bar{q}$ is a light antiquark: $q = u, d, s$. In the previous section I have pointed out that in the heavy quark (HQ) limit $m_Q \to \infty$ the heavy quark decouples from the light degrees of freedom, so that the spin s_Q of the heavy quark and the total angular momentum of the light degrees of freedom s_ℓ are separately conserved in strong interactions. As a consequence, heavy hadrons can be classified according to the value of s_ℓ and collected in doublets formed by two states with total spin $J = s_\ell \pm \frac{1}{2}$ and parity $P = (-1)^{\ell+1}$. ℓ is the orbital angular momentum of the light degrees of freedom; denoting by s_q the light antiquark spin, one has that $\vec{s}_\ell = \vec{\ell} + \vec{s}_q$. Spin symmetry predicts that the two states in a doublet are degenerate; on the other hand, flavour symmetry allows to relate the properties of the states with the same quantum numbers which differ for the flavour of the heavy quark. In the following I focus on the doublets built when $\ell = 0, 1, 2$ (in the quark model language they are referred to as $s-$, $p-$ and $d-$wave states).

When $\ell = 0$ one has $s_\ell^P = \frac{1}{2}^-$; the corresponding doublet, consisting of the lowest lying $Q\bar{q}$ mesons, comprises two states with spin-parity $J^P = (0^-, 1^-)$ that I denote as (P, P^*). For $\ell = 1$ s_ℓ can take two values: either $s_\ell^P = \frac{1}{2}^+$ or $s_\ell^P = \frac{3}{2}^+$, corresponding to the two doublets. I denote the members of the $J^P_{s_\ell} = (0^+, 1^+)_{1/2}$ doublet as (P_0^*, P_1'), and those of the $J^P_{s_\ell} = (1^+, 2^+)_{3/2}$ doublet as (P_1, P_2^*). $\ell = 2$ gives either $s_\ell^P = \frac{3}{2}^-$ or $s_\ell^P = \frac{5}{2}^-$; the members of such doublets are denoted as (P_1^*, P_2) and $(P_2'^*, P_3)$, respectively. An analogous notation can be used to denote the radial excitations of these states, having radial quantum number $n = 2$. I distinguish their fields by a tilde ($\tilde{P}$, $\tilde{P}^*$, ...).

In order to study the phenomenology of the states belonging to these doublets it is very convenient to introduce the effective fields that describe them in the HQ limit. I collect below the expressions of such fields: H_a ($a = u, d, s$ a light flavour index) corresponds to the doublet with $s_\ell^P = \frac{1}{2}^-$; S_a and T_a to $s_\ell^P = \frac{1}{2}^+$ and $s_\ell^P = \frac{3}{2}^+$, respectively; X_a is the effective field corresponding to the doublet with $s_\ell^P = \frac{3}{2}^-$, and X'_a to the $s_\ell^P = \frac{5}{2}^-$ doublet:

$$H_a = \frac{1+\not{v}}{2}[P^*_{a\mu}\gamma^\mu - P_a\gamma_5] \tag{15}$$

$$S_a = \frac{1+\not{v}}{2}\left[P'^{\mu}_{1a}\gamma_\mu\gamma_5 - P^*_{0a}\right] \tag{16}$$

$$T^\mu_a = \frac{1+\not{v}}{2}\left\{P^{\mu\nu}_{2a}\gamma_\nu - P_{1a\nu}\sqrt{\frac{3}{2}}\gamma_5\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\right]\right\} \tag{17}$$

$$X^\mu_a = \frac{1+\not{v}}{2}\left\{P^{*\mu\nu}_{2a}\gamma_5\gamma_\nu - P'^*_{1a\nu}\sqrt{\frac{3}{2}}\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu + v^\mu)\right]\right\} \tag{18}$$

$$X'^{\mu\nu}_a = \frac{1+\not{v}}{2}\left\{P^{\mu\nu\sigma}_{3a}\gamma_\sigma - P^{*'\alpha\beta}_{2a}\sqrt{\frac{5}{3}}\gamma_5\left[g^\mu_\alpha g^\nu_\beta - \frac{1}{5}\gamma_\alpha g^\nu_\beta(\gamma^\mu - v^\mu) - \frac{1}{5}\gamma_\beta g^\mu_\alpha(\gamma^\nu - v^\nu)\right]\right\}. \tag{19}$$

The various operators in Eqs.(15-19) include the factor $\sqrt{m_Q}$ and have dimension 3/2. They annihilate mesons of four velocity v, conserved in strong interaction processes.

Table 1: Observed open charm and open beauty mesons, classified in HQ doublets. States with uncertain assignment are indicated with $\star$ and classified according to the scheme proposed in this study.

doublet	s_ℓ^P	J^P	$c\bar{q}$ (n=1)	$c\bar{q}$ (n=2)	$c\bar{s}$ (n=1)	$c\bar{s}$ (n=2)	$b\bar{q}$ (n=1)	$b\bar{s}$ (n=1)
H	$\frac{1}{2}^-$	0^-	$D(1869)$	$D(2550)$ $\star$	$D_s(1968)$		$B(5279)$	$B_s(5366)$
		1^-	$D^*(2010)$	$D^*(2600)$ $\star$	$D_s^*(2112)$	$D^*_{s1}(2700)$	$B^*(5325)$	$B_s^*(5415)$
S	$\frac{1}{2}^+$	0^+	$D_0^*(2400)$		$D^*_{s0}(2317)$			
		1^+	$D_1'(2430)$		$D'_{s1}(2460)$	$D_{sJ}(3040)$ $\star$		
T	$\frac{3}{2}^+$	1^+	$D_1(2420)$		$D_{s1}(2536)$	$D_{sJ}(3040)$ $\star$	$B_1(5721)$	$B_{s1}(5830)$
		2^+	$D_2^*(2460)$		$D^*_{s2}(2573)$		$B_2^*(5747)$	$B^*_{s2}(5840)$
X	$\frac{3}{2}^-$	1^-						
		2^-						
X'	$\frac{5}{2}^-$	2^-	$D(2750)$ $\star$					
		3^-	$D(2760)$ $\star$		$D_{sJ}(2860)$ $\star$			

In Table 1 I collect the observed charmed $c\bar{q}$ and $c\bar{s}$, and beauty $b\bar{q}$ and $b\bar{s}$ (with $q = u, d$) mesons, adopting the classification previously introduced within the heavy quark doublet scheme. In that Table there are states not yet classified: I have placed them according to the assignment of their quantum numbers[3] that I review below. A $\star$ mark has been put in correspondence of such mesons. In the case of beauty there are no observations of candidates of states with $n = 2$.

The fact that there exist a number of newly observed heavy mesons that need to be properly identified calls for a theoretical investigation about how to distinguish among the various assignments. A possibility is to look at the strong decays of heavy mesons with the emission of a light meson. The most important among these processes are those with a light pseudoscalar meson in the final state.

In order to describe the octet of light pseudoscalar mesons it is convenient to introduce the definition $\xi = e^{\frac{i\mathcal{M}}{f_\pi}}$, $\Sigma = \xi^2$, with the matrix $\mathcal{M}$ containing π, K and η fields ($f_\pi = 132$ MeV):

$$\mathcal{M} = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} .$$

The decays $F \to HM$ ($F = H, S, T, X, X'$, M denoting generically a light pseudoscalar meson) can be described by effective Lagrangians written in terms of the fields introduced in (15-19) that hold at leading order in the heavy quark mass and light meson momentum expansion:[4]

$$\begin{aligned}
\mathcal{L}_H &= g\,Tr\Big[\bar{H}_a H_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba}\Big] \\
\mathcal{L}_S &= h\,Tr\Big[\bar{H}_a S_b \gamma_\mu \gamma_5 \mathcal{A}^\mu_{ba}\Big] + h.c. \\
\mathcal{L}_T &= \frac{h'}{\Lambda_\chi} Tr\Big[\bar{H}_a T^\mu_b (iD_\mu \not{\mathcal{A}} + i\not{D}\mathcal{A}_\mu)_{ba}\gamma_5\Big] + h.c. \\
\mathcal{L}_X &= \frac{k'}{\Lambda_\chi} Tr\Big[\bar{H}_a X^\mu_b (iD_\mu \not{\mathcal{A}} + i\not{D}\mathcal{A}_\mu)_{ba}\gamma_5\Big] + h.c. \\
\mathcal{L}_{X'} &= \frac{1}{{\Lambda_\chi}^2} Tr\Big[\bar{H}_a X'^{\mu\nu}_b \left[k_1\{D_\mu, D_\nu\}\mathcal{A}_\lambda + k_2(D_\mu D_\lambda \mathcal{A}_\nu + D_\nu D_\lambda \mathcal{A}_\mu)\right]_{ba}\gamma^\lambda\gamma_5\Big] + h.c.
\end{aligned} \tag{20}$$

$\Lambda_\chi \simeq 1$ GeV represents the chiral symmetry-breaking scale. g, h, h' k', k_1, k_2 (I put $k = k_1 + k_2$) are effective coupling constants. The decays described by $\mathcal{L}_S$ and $\mathcal{L}_T$ occur in $s-$ and $d-$ wave, respectively; those described by $\mathcal{L}_X$ and $\mathcal{L}_{X'}$ occur in $p-$ and $f-$ wave. The structure of the analogous Lagrangian terms for radial excitations of the various doublets is the same, except that one has to replace the various coupling constants by new ones, denoted by $\tilde{g}$, $\tilde{h}$, ...

It is possible to determine these constants using theoretical approaches or to fix them from experiment. g could be determined from the transitions $D^*_{(s)} \to D\pi(K)$. Using experimental data[5] for the branching fractions and the total width of $D^{*\pm}$ one obtains the value $g = 0.64 \pm 0.075$, a value larger than theoretical predictions derived in the HQ limit.[6–8] QCD sum rules in the HQ limit[7] as well as lattice QCD[8] provide determinations of h in fair agreement with each other. Predictions for the other couplings can be found in a recent work.[3]

I can now discuss how the calculation of strong decay rates in the HQ limit, performed using the effective lagrangians just introduced, can help identifying the newly discovered $c\bar{s}$ and $c\bar{q}$ mesons.

2.1. *Exploiting the heavy quark limit to classify* $D_{sJ}(2860)$, $D_{sJ}(2700)$ *and* $D_{sJ}(3040)$

I start the discussion of the most recently discovered $c\bar{s}$ mesons, considering the two states $D_{sJ}(2860)$ and $D_{sJ}(2700)$. $D_{sJ}(2860)$ was observed in 2006 by BaBar Collaboration[9] decaying to D^0K^+ and D^+K_S. Its mass and width were experimentally fixed to: $M = 2856.6 \pm 1.5 \pm 5.0$ MeV and $\Gamma = 47 \pm 7 \pm 10$. Shortly after, $D_{sJ}(2700)$ was discovered by Belle Collaboration[10] as a peak in the D^0K^+ invariant mass distribution in $B^+ \to \bar{D}^0D^0K^+$. Mass and width measurements were also provided: $M = 2708 \pm 9^{+11}_{-10}$ MeV, $\Gamma = 108 \pm 23^{+36}_{-31}$ MeV, while spin-parity was fixed to $J^P = 1^-$.

The strong decay widths of $D_{sJ}(2860)$ and $D_{sJ}(2710)$ have been calculated in the HQ limit in the effective lagrangian framework described above.[11,12] This calculation was performed in correspondence to different assignments for the quantum numbers of such mesons. Comparison of the predictions in the various cases against experimental outcome was suggested as an effective way to identify these states. I present here a summary of this strategy.

Table 2: Predicted ratios R_1 and R_2 (see text for definitions) for the various classifications of $D_{sJ}(2860)$ and $D_{sJ}(2710)$.

$D_{sJ}(2860)$	R_1	R_2
$s_\ell^p = \frac{1}{2}^-$, $J^P = 1^-$, $n = 2$	1.23	0.27
$s_\ell^p = \frac{1}{2}^+$, $J^P = 0^+$, $n = 2$	0	0.34
$s_\ell^p = \frac{3}{2}^+$, $J^P = 2^+$, $n = 2$	0.63	0.19
$s_\ell^p = \frac{3}{2}^-$, $J^P = 1^-$, $n = 1$	0.06	0.23
$s_\ell^p = \frac{5}{2}^-$, $J^P = 3^-$, $n = 1$	0.39	0.13
$D_{sJ}(2710)$	R_1	R_2
$s_\ell^p = \frac{1}{2}^-$, $J^P = 1^-$, $n = 2$	0.91	0.20
$s_\ell^p = \frac{3}{2}^-$, $J^P = 1^-$, $n = 1$	0.043	0.163

In order to identify $D_{sJ}(2860)$ it should be considered that it decays to DK. Among the states with radial quantum number $n = 1$ this is possible only for the $J^P = 1^-$ state of the $s_\ell^P = \frac{3}{2}^-$ doublet, or for the $J^P = 3^-$ state of the $s_\ell^P = \frac{5}{2}^-$ one. As for radial excitations with $n = 2$, those for which this decay is allowed are the first radial excitation of D_s^* ($J^P = 1^-$ $s_\ell^P = \frac{1}{2}^-$) or of $D_{sJ}(2317)$ ($J^P = 0^+$ $s_\ell^P = \frac{1}{2}^+$) or of $D_{s2}^*(2573)$ ($J^P = 2^+$ $s_\ell^P = \frac{3}{2}^+$).

In the case of $D_{sJ}(2710)$ it is possible to exploit the information that it has $J^P = 1^-$, so that it can be identified only with the first radial excitation in the $s_\ell^P = \frac{1}{2}^-$ doublet ($D_s^{*\prime}$) or with the low lying state with $s_\ell^P = \frac{3}{2}^-$ (D_{s1}^*).

Using the effective lagrangian (21), it is convenient to compute the ratios $R_1 = \frac{\Gamma(D_{sJ}\to D^*K)}{\Gamma(D_{sJ}\to DK)}$ and $R_2 = \frac{\Gamma(D_{sJ}\to D_s\eta)}{\Gamma(D_{sJ}\to DK)}$ ($D^{(*)}K = D^{(*)+}K_S + D^{(*)0}K^+$), when the decaying meson D_{sJ} is either $D_{sJ}(2860)$ or $D_{sJ}(2710)$, since these quantities depend on the quantum numbers of the decaying particle, but are independent on the coupling constants in the effective lagrangians, which reduces the model dependence of the result.

The results[11,12] are reported in Table 2. The identification with the state with $J^P = 0^+$ has been excluded when the process $D_{sJ}(2860) \to D^*K$ was observed,[13] since the decay to D^*K is not possible for such a state[a]. Looking at the other possibilities in Table 2, the identification that seems the most likely one for $D_{sJ}(2860)$ is that with the state D_{s3}, having $s_\ell^p = \frac{5}{2}^-$, $J^P = 3^-$, $n = 1$. This would attribute the small DK width to the kaon momentum suppression factor: $\Gamma(D_{sJ} \to DK) \propto q_K^7$, which reflects the fact that the transition occurs in f-wave. This argument is supported by the following observation. If $D_{sJ}(2860)$ has $J^P = 3^-$, it is not expected to be produced in $B \to DD_{sJ}(2860)$ decays and indeed no signal of $D_{sJ}(2860)$ was found in the analysis of the $B^+ \to \bar{D}^0 D^0 K^+$ Dalitz plot.[10]

However, the measurement[13] of the ratio R_1

$$\frac{BR(D_{sJ}(2860) \to D^*K)}{BR(D_{sJ}(2860) \to DK)} = 1.10 \pm 0.15_{stat} \pm 0.19_{syst} \tag{21}$$

does not agree with the prediction reported in Table 2 for this state. An interesting consideration has been put forward[3] by considering the spin partner of $D_{sJ}(2860)$. Identifying the latter with the $J^P = 3^-$ state, such a partner has $J^P = 2^-$ and is denoted by $D_{s2}^{*\prime}$. Using the HQ limit, one can predict its mass: $M(D_{s2}^{*\prime}) = 2851 \pm 7$ MeV, and it also decays to D^*K. Being the two states very close to each other, the experimental separation in the common D^*K decay channel could be difficult. Therefore, the measurement in Eq.(21), attributed only to $D_{sJ}(2860)$, might suffer of the contamination of the decay $D_{s2}^{*\prime} \to D^*K$, so that it possible that what are actually measured are the $D^{(*)}K$ pairs produced from both the states:

$$\bar{R}(2860) = \frac{\Gamma(D_{sJ}(2860) \to D^*K) + \Gamma(D_{s2}^{*\prime}(2851) \to D^*K)}{\Gamma(D_{sJ}(2860) \to DK)} \quad . \tag{22}$$

The HQ prediction for this ratio, considering the two contributions, is[3]

$$\bar{R}(2860) = 0.99 \pm 0.05 \tag{23}$$

in agreement with the datum Eq.(21).

[a]The possibility that two overlapping structures with $J^P = 0^+$ and $J^P = 2^+$ (both with $n = 2$) exist in the mass range about 2860 MeV has also been proposed.[14]

Table 3: Features of the decay modes of $D_{sJ}(3040)$ in correspondence to the four proposed assignments.

decay modes	$\tilde{D}'_{s1}$ (n=2) ($J^P_{s_\ell} = 1^+_{1/2}$)	$\tilde{D}_{s1}$ (n=2) ($J^P_{s_\ell} = 1^+_{3/2}$)	D_{s2} (n=1) ($J^P_{s_\ell} = 2^-_{3/2}$)	$D^{*\prime}_{s2}$ (n=1) ($J^P_{s_\ell} = 2^-_{5/2}$)
D^*K, $D^*_s\eta$	$s-$ wave	$d-$ wave	$p-$ wave	$f-$ wave
R_1	0.34	0.20	0.245	0.143
D^*_0K, $D^*_{s0}\eta$, D'_1K	$p-$ wave	$p-$ wave	$d-$ wave	$d-$ wave
D_1K	$p-$ wave	$p-$ wave	-	$d-$ wave
D^*_2K	$p-$ wave	$p-$ wave	$s-$ wave	$d-$ wave
DK^*, $D_s\phi$	$s-$ wave	$s-$ wave	$p-$ wave	$p-$ wave
	$\Gamma \simeq 140$ MeV	$\Gamma \simeq 20$ MeV	negligible	negligible

Further checks of the proposed identification for $D_{sJ}(2860)$ could be the observation of its non-strange partner, D_3, having also $J^P = 3^-$, as well as the corresponding beauty mesons $B_{(s)3}$ with or without strangeness, with analogous features. In particular, all such states should be narrow, too.[3,15]

I now consider the case of $D_{sJ}(2710)$. From the results in Table 2 it can be argued that an effective way to distinguish between the two possible identifications for this meson is to measure the ratio R_1. Comparison of the prediction in Table 2 with the result provided by BaBar Collaboration[13]

$$\frac{BR(D_{sJ}(2710) \to D^*K)}{BR(D_{sJ}(2710) \to DK)} = 0.91 \pm 0.13_{stat} \pm 0.12_{syst}$$

leads to conclude that $D_{sJ}(2710)$ is most likely $D^{*\prime}_s$.

The most recently observed[13] $c\bar{s}$ meson is $D_{sJ}(3040)$, a broad state decaying to D^*K and not to DK with mass $M = 3044 \pm 8_{stat}(^{+30}_{-5})_{syst}$ MeV and width $\Gamma = 239 \pm 35_{stat}(^{+46}_{-42})_{syst}$ MeV. On the basis of the observed decay mode one can deduce that it has unnatural parity: $J^P = 1^+, 2^-, 3^+, \cdots$, so that among the states with $n = 1$ it might be identified only with one of the two states D_{s2} and D'^*_{s2} having both $J^P = 2^-$ that belong to the doublets with $s_\ell = 3/2$ and $s_\ell = 5/2$, respectively. Among radial excitations ($n = 2$) the two $J^P = 1^+$ mesons $\tilde{D}'_{s1}$, belonging to the doublet with $s_\ell = 1/2$ and $\tilde{D}_{s1}$, belonging to the doublet with $s_\ell = 3/2$ are suitable candidates. It should be noticed that if $D_{sJ}(2860)$ were experimentally confirmed as the $J^P_{s_\ell} = 3^-_{5/2}$ meson, $D_{sJ}(3040)$ could hardly be its spin partner $D^{*\prime}_{s2}$ with $J^P_{s_\ell} = 2^-_{5/2}$, since this would require an unlikely mass inversion in a spin doublet. The same argument would disfavor also the identification with D_{s2}, even though in that case the two mesons would belong to different doublets.

Due to its large mass, several decay modes are allowed for $D_{sJ}(3040)$. The effective Lagrangians in Eq.(21) can be used to compute the decay widths to a charmed meson and a light pseudoscalar one and, consequently, the ratio $R_1 = \frac{\Gamma(D_{sJ}(3040) \to D^*_s\eta)}{\Gamma(D_{sJ}(3040) \to D^*K)}$ ($D^*K = D^{*0}K^+ + D^{*^+}K^0_S$).

Other possible decay modes are to $(D^*_0, D'_1)K$, $(D_1, D^*_2)K$ and $D^*_{s0}\eta$, as well as to DK^* or $D_s\phi$ for which one can also use effective lagrangian approaches.[16] Predictions[17] for the various transitions obtained in the four possible identifications for $D_{sJ}(3040)$ are summarized in Table 3. Paying attention to the wave in which the decays proceed, one can realize that the two $J^P = 1^+$ should be broader than the two $J^P = 2^+$ states, so that identification of $D_{sJ}(3040)$ with one of the two axial-vector mesons seems more likely. Furthermore, it is possible to distinguish between the two by considering the widths to the DK^* and $D_s\phi$ final states, that are larger for $\tilde{D}'_{s1}$ than for $\tilde{D}_{s1}$.

2.2. *Newly discovered $c\bar{q}$ mesons*

There are recent observations of charmed mesons without strangeness. Indeed, looking at the process $e^+e^- \to c\bar{c} \to D^{(*)}\pi X$, BaBar Collaboration[18] observed four new structures: $D^0(2550)$ decaying to $D^{*+}\pi^-$; $D^{*0}(2600)$ decaying to $D^+\pi^-$ and $D^{*+}\pi^-$ and $D^{*+}(2600)$ decaying to $D^0\pi^+$; $D^{*0}(2760)$ decaying to $D^+\pi^-$ and $D^{*+}(2760)$ decaying to $D^0\pi^+$; $D^{*0}(2750)$ decaying to $D^{*+}\pi^-$. Furthermore, the following ratio was measured:

$$\frac{\mathcal{B}(D^{*0}(2600) \to D^+\pi^-)}{\mathcal{B}(D^{*0}(2600) \to D^{*+}\pi^-)} = 0.32 \pm 0.02 \pm 0.09 \ . \tag{24}$$

In the case of the final state $D^{*+}\pi^-$, studying the distribution in $\cos\theta_H$, where θ_H is the helicity angle between the primary pion π^- and the slow pion π^+ produced in the D^{*+} decay, BaBar finds that in the case of $D^*(2600)$ such a distribution favours the assignment of natural parity to this state, also consistent with the observation of both $D\pi$

and $D^*\pi$ final states; in the case of $D^0(2550)$ the helicity angle distribution behaves like $\sim\cos^2\theta_H$, suggesting that it is a $J^P=0^+$ state.

As a consequence, BaBar Collaboration concludes[18] that ($D(2550)$, $D^*(2600)$) are most likely the members of the $J^P=(0^-,1^-)$ doublet of $n=2$ radial excitations of (D, D^*) mesons, while ($D(2750)$, $D^*(2760)$) might be identified with $\ell=2$, $n=1$ states. I consider in more details the case of $D^*(2600)$. Since it decays both to $D\pi$ and $D^*\pi$, it could be identified with one of the four states for which both processes are allowed. As I have stressed above, BaBar Collaboration has suggested the identification of this meson with $\tilde{D}^*$, i.e. the non strange partner of $D_{sJ}(2700)$, identified with $\tilde{D}^*_s$. Computing the ratio in (24) for $\tilde{D}^*$, once the mass of 2600 MeV has been assigned to it and exploiting the effective lagrangian approach previously reviewed, one obtains:[3]

$$\frac{\mathcal{B}(\tilde{D}^{*0}(2600)\to D^+\pi^-)}{\mathcal{B}(\tilde{D}^{*0}(2600)\to D^{*+}\pi^-)}=0.822\pm 0.003\ , \tag{25}$$

which does not agree with the datum (24). The other states that can decay to both $D\pi$ and $D^*\pi$ with which it is possible to identify $D^*(2600)$ are: D_1^* in X, D_3 in X' and $\tilde{D}_2^*$ in $\tilde{T}$. However, also in these cases the calculation of the ratio in (24) in the HQ limit does not reproduce the experimental datum. It can be argued that either corrections to the HQ limit should be included in this case or that the experimental analysis needs to be revised.

3. Weak decays of heavy mesons and universal form factors

The best known application of the HQ symmetries concerns weak decays of heavy mesons. Consider, for example, the elastic scattering of a pseudoscalar heavy meson $P(v)$ induced by an external vector current coupled to the heavy quark. Due to the decoupling of the heavy quark, this process can be described as if the current were acting only on Q:

$$\langle P(v')|\bar{h}_{v'}\gamma^\mu h_v|P(v)\rangle=\langle Q,v',s'_Q|\bar{h}_v\gamma^\mu h_v|Q,v,s_Q\rangle\ \langle L,s'_\ell|L,s_\ell\rangle\ . \tag{26}$$

The last term in Eq. (26) represents the overlap of the light degrees of freedom, denoted by L. If $v=v'$, due to spin symmetry, the light degrees of freedom remain unchanged and the overlap is just δ_{s_ℓ,s'_ℓ}. This is true even if the heavy quark in the final state is a different one, due to the flavor symmetry.

If $v\neq v'$, the decoupling of the light degrees of freedom still allows us to factorize the previous matrix element. However, the configuration of the light degrees of freedom in the final state will be different with respect to the initial state so that the overlap is a function $\xi(v\cdot v')$ that does depend on the spin and the flavor of the heavy quark, and is normalized to 1 for $v\cdot v'=1$. This function is the celebrated Isgur-Wise universal form factor.[19] It is defined as *universal* since it is possible to describe the semileptonic decays $B\to D$ or $B\to D^*$ in terms of this single function, hence greatly simplifying the theoretical analysis of these processes that usually require 2 and 4 form factors, respectively.

I have introduced the Isgur-Wise function in an intuitive way. However, it is possible to systematically define a number of analogous universal functions considering again the transitions among members of the various heavy meson doublets. Heavy quark symmetries allow to introduce one universal function for each transition of whatever a member of one doublet to another member of another doublet. The Isgur-Wise function is the form factor describing the decay of an element of lowest lying doublet to another element of the same doublet. In the case of the states in one of the two doublets with $s_\ell^P=1/2^+$ and $s_\ell^P=3/2^+$ the transitions towards the lowest lying doublet are described by introducing two universal functions that are referred to as $\tau_{1/2}(v\cdot v')$ and $\tau_{3/2}(v\cdot v')$, respectively. It should be again stressed what a great simplification this result represents, since these transitions are usually described in terms of 14 form factors. In order to derive the structure of a given matrix element in terms of the universal functions, it is convenient to use the trace formalism, in which the heavy doublets are represented again by the effected fields already introduced.

For definiteness, I consider the matrix element of an external current coupled to the heavy quarks, $J=h_{v'}^{Q'}\Gamma h_v^Q$ (Γ being a product of Dirac matrices) between two heavy mesons $P(v)$ and $P'(v')$. The most general decomposition for the transition matrix element, satisfying the symmetry requirements of Lorentz covariance and invariance under heavy quark symmetry and parity transformations is

$$\langle P'(v')|J|P(v)\rangle=Tr[\Xi(v,v')\bar{P}'(v')\Gamma P(v)]\ \ . \tag{27}$$

$P(v)$ and $P'(v')$ are effective fields the expression of which depends on the doublet to which the states belong. $\Xi(v,v')$ is a matrix containing the overlap of the light degrees of freedom. Its tensorial structure is such to contract free indices in the expressions of $P(v)$ and $P'(v')$ in order to reproduce the correct Lorentz structure of the matrix element.

Coming back to $B \to D^{(*)}$ semileptonic transitions, one has $\Xi(v, v') = -\xi(v \cdot v')$, and the following results hold:

$$\langle D(v')|\bar{h}^c_{v'}\gamma^\mu h^b_v|\bar{B}(v)\rangle = \xi(v \cdot v')\ \sqrt{m_D m_B}\ (v^\mu + v'^\mu)\ , \tag{28}$$

$$\langle D^*(\epsilon', v')|\bar{h}^c_{v'}\gamma^\mu(1-\gamma_5)h^b_v|\bar{B}(v)\rangle = i\xi(v \cdot v')\ \sqrt{m_D m_B}\,[\epsilon^{\nu\alpha\mu\beta}\epsilon'^*_\nu v'_\alpha v_\beta + (1 + v \cdot v')\ \epsilon'^{*\mu} - \epsilon'^{*\nu} v_\nu v'^\mu]. \tag{29}$$

Using the previous parametrization in the theoretical description of the $B \to D^*\ell\nu$ semileptonic decay, it turns out that the differential decay rate for this process can be written as follows:

$$\frac{d\Gamma}{dy}(B \to D^*\ell\nu) = \frac{G_F^2}{48\pi^3}(m_B - m_{D^*})^2 m^3_{D^*}\sqrt{y^2-1}(y+1)^2\left[1 + \frac{4y}{y+1}\frac{m_B^2 - 2y m_B m_{D^*} + m^2_{D^*}}{(m_B - m_{D^*})^2}\right]|V_{cb}|^2\mathcal{F}^2(y)\ . \tag{30}$$

V_{cb} is the element of the Cabibbo-Kobayashi-Maskawa mixing matrix which weights the $b \to c$ transition. The function $\mathcal{F}^2(y)$ coincides with the Isgur-Wise function in the HQ limit. Short distance and $\mathcal{O}(m_Q^{-1})$ corrections are known[20–22] and can be included. Therefore, fitting the experimental data for the differential distribution in (30) provides the value of the product $|V_{cb}|^2\mathcal{F}(1)$, after a suitable extrapolation to the zero recoil point due to the vanishing of the spectrum. Most recent experimental analyses aimed at determining V_{cb} are based on this method.[23]

Let me conclude this section stressing again what I have already underlined in the Introduction. On the one hand HQET provides us with relations among different quantities that enter in the description of heavy hadron processes, on the other it does not allow to compute such quantities within the same framework. In the case of form factors, for example, one essentially deals with non perturbative quantities that need to be computed through suitable approaches. I consider just an example, the method of QCD sum rules.[24] Within this approach the functions $\xi(v \cdot v')$ and $\tau_{1/2}(v \cdot v')$ have been computed at $\mathcal{O}(\alpha_s)$.[25,26] As for $\tau_{3/2}(v \cdot v')$, the calculation has been performed at leading order.[27] Finally, the analogous universal functions describing semileptonic decays of the states in the $s_\ell^P = \frac{3}{2}^-$ and $s_\ell^P = \frac{5}{2}^-$ doublets to $D^{(*)}\ell\nu_\ell$ have been computed, at the leading order.[15]

4. An excursion to hidden charm mesons: The case of X(3872)

As I have reviewed in the previous sections, there are a number of newly observed open charm mesons and the application of the heavy quark effective theory helps classifying such states. One may wonder whether the same framework can be applied also to the case of mesons with hidden flavour, such as charmonium or bottomonium. Indeed this would be useful, since the spectroscopy of charmonium-like states presents striking novelties, due to discovery of states that are still to be identified.[28] There are also recent observations of bottomonium-like states, but they are less copious. Among the many new charmonium-like states, I discuss the case of $X(3872)$, showing that it is again possible to exploit the heavy quark symmetries (in a modified version, as I shall show below) to try to assign the right quantum numbers to this state.

$X(3872)$ was discovered in 2003 by Belle Collaboration in $B^\pm \to K^\pm X \to K^\pm J/\psi\pi^+\pi^-$ decays.[29] Afterwards, other experiments confirmed this observation.[30] Its average mass and width are[5] $M(X) = 3871.57 \pm 0.25$ MeV and $\Gamma(X) < 2.3$ MeV (90/% C.L.). No charged partners in the $J/\psi\pi^\pm\pi^0$ channel were found,[31] while the observation of the decay $X \to J/\psi\gamma$ has allowed to fix $C = +1$. X mainly decays into final states with open charm mesons as the measure[32] $\frac{B(X \to D^0\bar{D}^0\pi^0)}{B(X \to J/\psi\pi^+\pi^-)} = 9 \pm 4$ shows.

The natural identification of X with an ordinary charmonium state is debated mainly due to the measure[33] of the branching fraction ratio $\frac{B(X \to J/\psi\pi^+\pi^-\pi^0)}{B(X \to J/\psi\pi^+\pi^-)} = 1.0 \pm 0.4 \pm 0.3$ which implies, if the two modes proceed through ρ^0 and ω intermediate states, isospin violation. However, as Suzuki has pointed out,[34] the large ratio $\frac{B(X \to J/\psi\pi^+\pi^-\pi^0)}{B(X \to J/\psi\pi^+\pi^-)}$ is also the result of the very different phase space effects in two and three pion modes. Indeed, the isospin violating amplitude is about 20% of the isospin conserving one.

Another open question is the spin-parity to X, since the assignment $J^P = 1^+$ is favoured by the angular analysis in $X \to J/\psi\pi^+\pi^-$, while the study[35] of the three pion distribution in $X \to J/\psi\omega \to J/\psi\pi\pi\pi$ seems to indicate $J^P = 2^-$. As a consequence, if X is an ordinary charmonium state, it could be identified either with the state $\chi_{c1}(2P)$, the first radial excitation of χ_{c1} having $J^{PC} = 1^{++}$, or with the state η_{c2} having $J^{PC} = 2^{-+}$.

Several exotic interpretations have also been proposed, which suggest that the structure of X differs from the simple $c\bar{c}$ quark content of a charmonium meson. Among such suggestions, considering the coincidence between the mass $M(D^{*0}\overline{D}^0) = 3871.2 \pm 1.0$ MeV and the mass of X, the possibility that $X(3872)$ could be a $D^{*0}\overline{D}^0$ molecule[36,37] has been put forward. Since in this case the wave function of $X(3872)$ would consist of several hadronic components[38] X would have no definite isospin. However, the molecular binding mechanism still needs to be clearly understood.

It is interesting to consider what the HQ limit predicts for an ordinary charmonium state for a quantity that has been experimentally measured and is sensitive to the structure of the decaying meson, i.e. the ratio of the radiative decay rates of $X(3872)$ to $J/\psi\gamma$ and $\psi(2S)\gamma$. In order to do so, I use an effective lagrangian approach[39] in which only spin symmetry for heavy $Q\bar{Q}$ states can be exploited since in heavy quarkonia there is no heavy flavour symmetry.[40] A heavy $Q\bar{Q}$ state ($Q = c, b$) with parity $P = (-1)^{L+1}$ and charge-conjugation $C = (-1)^{L+s}$ can be classified using the notation $n^{2s+1}L_J$, where n is the radial quantum number, L the orbital angular momentum, s the spin and J the total angular momentum.

I assume that $X(3872)$ is the state $\chi_{c1}(2P)$, so that it belongs to the $L = 1$ multiplet. As for heavy-light mesons, I can describe such a multiplet by an effective field:

$$P^{(Q\bar{Q})\mu} = \left(\frac{1+\not{v}}{2}\right)\left(\chi_2^{\mu\alpha}\gamma_\alpha + \frac{1}{\sqrt{2}}\epsilon^{\mu\alpha\beta\gamma}v_\alpha\gamma_\beta\chi_{1\gamma} + \frac{1}{\sqrt{3}}(\gamma^\mu - v^\mu)\chi_0 + h_1^\mu\gamma_5\right)\left(\frac{1-\not{v}}{2}\right). \tag{31}$$

The spin triplet with $J^{PC} = 2^{++}, 1^{++}, 0^{++}$ is described by the fields χ_2, χ_1, χ_0, while the spin singlet with $J^{PC} = 1^{+-}$ by the field h_1.

J/ψ and $\psi(2S)$ are the states with $J^P = 1^-$ and radial quantum number $n = 1$ and $n = 2$, respectively. therefore, they are both described by the H_1 component of the doublet:

$$J = \frac{1+\not{v}}{2}\left[H_1^\mu\gamma_\mu - H_0\gamma_5\right]\frac{1-\not{v}}{2}\,. \tag{32}$$

It is possible to write an effective Lagrangian describing radiative transitions among members of the P wave and of the S wave multiplets just introduced:[39]

$$\mathcal{L}_{nP\leftrightarrow mS} = \delta_Q^{nPmS}Tr\left[\bar{J}(mS)J_\mu(nP)\right]v_\nu F^{\mu\nu} + \text{h.c.}\,. \tag{33}$$

$F^{\mu\nu}$ the electromagnetic field strength tensor. As in all the lagrangian terms already seen, there is a single constant δ_Q^{nPmS} which describes all the transitions among the elements of the nP multiplet and those of the mS one.

Using the effective lagrangian (33) it is possible to compute the ratios $R_J^{(b)} = \frac{\Gamma(\chi_{bJ}(2P)\to\Upsilon(2S)\,\gamma)}{\Gamma(\chi_{bJ}(2P)\to\Upsilon(1S)\,\gamma)}$, proportional to $R_J^{(b)} = \frac{\delta_b^{2P1S}}{\delta_b^{2P2S}}$ ($J = 0, 1, 2$). Since the decay rates entering in such ratios have been measured,[5] the ratio of the coupling constants can be predicted, with the average result[41] $R^{(b)} = 8.8 \pm 0.7$. The fact that flavour symmetry does not hold any more implies that the couplings might be different for beauty mesons and for charm ones. On the other hand, it is reasonable to assume that their ratios stay stable so that one can use the value of $R^{(b)}$ in the case of $\chi_{c1}(2P)$ with the result:[41]

$$R_1^{(c)} = \frac{\Gamma(\chi_{c1}(2P)\to\psi(2S)\,\gamma)}{\Gamma(\chi_{c1}(2P)\to\psi(1S)\,\gamma)} = 1.64 \pm 0.25\,. \tag{34}$$

This should be compared to the experimental result obtained by BaBar Collaboration:[42]

$$R_X = \frac{\Gamma(X(3872)\to\psi(2S)\,\gamma)}{\Gamma(X(3872)\to\psi(1S)\,\gamma)} = 3.5 \pm 1.4\,, \tag{35}$$

while Belle Collaboration only provides an upper limit:[43] $R_X < 2.1$ (at 90% C.L.). Considering the underlying approximation, the outcome in (34) seems consistent with experiment, leading to conclude that the identification $X(3872) = \chi_{c1}(2P)$ cannot be ruled out, in contrast with scenarios that suggest a composite structure for X which predict a suppression of the rate of $X(3872) \to \psi(2S)\,\gamma$ with respect to $X(3872) \to \psi(1S)\,\gamma$.[44,45]

5. Conclusions

I have given an overview of the Heavy Quark Effective Theory and of some of its applications. In keeping with the subject of the 2012 *Niccoló Cabeo School*, I paid particular attention to spectroscopic implications of the heavy quark limit. I have shown how heavy-light mesons can be classified in doublets, and described how the observed states fit in this classification. For those states that have not been identified yet, I have shown how predictions for strong decay widths obtained using an effective lagrangian approach in the HQ limit can help assigning the proper quantum numbers. I have also reviewed the implications of the HQ limit in semileptonic decays of heavy-light mesons, of outmost importance for the determination of the elements of the quark mixing matrix. As a final topic, I have briefly considered the case of hidden charm mesons in the HQ limit and, in particular, of the still puzzling state $X(3872)$. Even though I have mainly considered the case of charmed mesons, predictions for beauty mesons can be analogously developed.

I warmly thank the organizers of the 2012 *Niccoló Cabeo School* for inviting me to give these lectures. A particular acknowledgment is due to Paola Ferretti Dalpiatz, since without her efforts the school would not have taken place after the earthquake in Ferrara. I also thank P. Colangelo, R. Ferrandes, F. Giannuzzi, S. Nicotri, M. Rizzi for collaboration on some of the topics discussed in this paper.

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