Radiative corrections to lepton-hadron interactions

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PART I
What are Radiative Corrections (RC)?

Types of RC and the present status of their calculation

Application of RC to specific processes

RC in Monte Carlo codes

One-loop RC: the scheme of calculation
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- RC is the subject of a tight collaboration between theoreticians and experimentalists.
Physics is a *natural science*:

Experiment $\oplus$ Theory
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- Theoretical accuracy should be adequate, so we need more and more RC
Why do we need RC? (II)

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- LHC scenarios . . .
- RC for discrimination of various new physics models
Radiative Corrections

Perturbative, Non-perturbative, Re-summed perturbative

$O(\alpha) \equiv \text{one-loop, two-loop, etc.}$

According to interaction types:

QED Electroweak (EW) QCD non-SM

In practice we always have a mixture of ALL types of RC

One of our tasks is to disentangle the mixture

- All relevant perturbative RC are already computed

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- All relevant perturbative RC are already computed
- it’s a MYTH! (even for QED RC)

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Fit of the Higgs boson mass from LEP data [arXiv:0712.0929(hep-ex)]

$$m_{\text{H}} \text{ [GeV]}$$

$${Dc}^2 = D_c(5)$$

$$0.02758 \pm 0.00035$$

$$0.02749 \pm 0.00012$$

incl. low $Q^2$ data

Theory uncertainty

$$m_{\text{Limit}} = 144 \text{ GeV}$$

ZFITTER, TOPAZO, ... Remind LEP's fit $m_t \approx 172 \text{ GeV}$
As a small parameter we can take the fine structure constant

\[ \alpha \approx \frac{1}{137} \]

J. Schwinger: the proper parameter is

\[ \frac{\alpha}{2\pi} \approx 0.12\% \text{, } \left( \frac{\alpha}{2\pi} \right)^2 \approx 1.3 \cdot 10^{-4} \% \]

So perturbative QED RC as series in \( \alpha \) should (???) converge rapidly

In practice (and theory) it is not so simple. There can be other small parameters (e.g. \( m^2/Q^2 \ll 1 \)) and large enhancement factors due to kinematics or large logarithms.
Logarithmic approximations

For large energies we have Large Logs:

$$L = \ln\left(\frac{Q^2}{\mu_0^2}\right) \quad \mu = m_e, \Lambda_{\text{QCD}}, \ldots$$

e.g., \(\ln\left(\frac{M_Z^2}{m_e^2}\right) \approx 24, \quad \ln\left(\frac{m_{\mu}^2}{m_e^2}\right) \approx 11\)

Double log approximation (Sudakov logs):
\(O\left(\alpha^n L^{2n}\right), \quad n = 0, 1, 2, \ldots\)

Leading log approximation (“LLA” in QED and “LO” in QCD):
\(O\left(\alpha^n L^n\right), \quad n = 0, 1, 2, \ldots\)

Next-to-Leading log approximation (“NLO”):
\(O\left(\alpha^n L^{n-1}\right), \quad n = 1, 2, \ldots\)
Small angle Bhabha scattering (SABS) was used at LEP to measure the luminosity.

So the theoretical uncertainty in SABS description contributed to the errors in all LEP results.

\[
\sigma^{\text{Corrected}}(x_c) = \sigma^{\text{Born}}(x_c) \left[ 1 + \sum \delta_i / 100\% \right]
\]

In spite of our efforts, the theoretical precision was worse than the experimental one!

It should be improved before ILC
(There are already some updates: new 2-loop results)
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Still application of (even) well known results to a concrete case is rather non-trivial:
- old analytic calculations can have obsolete approximations
- different effects should be combined properly
- experimental conditions should be taken into account
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Dedicated MC (for a specific process) vs General purpose MC (PYTHIA, HERWIG, PHOTOS)
One-loop RC (I)

$\mathcal{O}(\alpha)$ correction is a very standard thing: "MUST HAVE"

Typically we decompose it into 3 parts:

- Virtual (loop) corrections: involved calculations
- Soft photon radiation: easy to compute
- Hard photon radiation: keep completely differential for MC

$$\sigma^{1\text{-loop corr.}} = \sigma^{\text{Born}} + \sigma^{\text{Virt}}(\lambda) + \sigma^{\text{Soft}}(\lambda, \bar{\omega}) + \sigma^{\text{Hard}}(\bar{\omega})$$

$\lambda$ is an infrared regulator, e.g. fictitious photon mass ($\lambda \ll m, E$)

$\bar{\omega}$ is a soft-hard separator (phase space splitting) the maximal energy of a soft photon $\equiv$ minimal energy of a hard one in a certain reference frame ($\lambda \ll \bar{\omega} \ll E$)
Results for QED RC in $\mathcal{O}(\alpha)$ are known practically for all Standard Model processes (with up-to 6 legs).

But for certain cases we have to revise results, even for “classic processes”, e.g.

$$\mu + A \rightarrow \mu + \gamma + A$$

for COMPASS experiment

$$\mu \rightarrow e + \nu_\mu + \bar{\nu}_e$$

for TWIST experiment
RC to High Energy Lepton Bremsstrahlung on Heavy Nuclei

Several relevant effects beyond the Born approximation:

- $\mathcal{O}(\alpha)$ corrections to the lepton tensor
- vacuum polarization
- multiple photon exchange with the nucleus
- electromagnetic nuclear elastic and inelastic form factors
- screening of the nucleus by the electrons surrounding it
- inelastic interactions with the atomic electrons
Results for COMPASS conditions ($E_\mu = 190\text{GeV}, \quad Z = 82, \ldots$)

<table>
<thead>
<tr>
<th>$\omega/E_1$</th>
<th>Born</th>
<th>Virtual</th>
<th>Soft$_1$</th>
<th>Hard$_1$</th>
<th>$\delta_1$, %</th>
<th>Soft$_2$</th>
<th>Hard$_2$</th>
<th>$\delta_2$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>15677(1)</td>
<td>76.8(4)</td>
<td>-260.1(1)</td>
<td>226.9(3)</td>
<td>+0.28</td>
<td>-307.0(1)</td>
<td>273.7(3)</td>
<td>+0.28</td>
</tr>
<tr>
<td>0.5</td>
<td>10836(1)</td>
<td>77.9(2)</td>
<td>-319.0(1)</td>
<td>280.0(3)</td>
<td>+0.36</td>
<td>-377.4(1)</td>
<td>338.1(3)</td>
<td>+0.36</td>
</tr>
<tr>
<td>0.7</td>
<td>7337.7(1)</td>
<td>76.9(2)</td>
<td>-363.3(1)</td>
<td>297.1(2)</td>
<td>+0.15</td>
<td>-430.9(1)</td>
<td>364.8(2)</td>
<td>+0.15</td>
</tr>
<tr>
<td>0.9</td>
<td>1267.4(1)</td>
<td>20.5(1)</td>
<td>-111.1(2)</td>
<td>65.9(1)</td>
<td>−1.95</td>
<td>-132.4(2)</td>
<td>87.2(1)</td>
<td>−1.95</td>
</tr>
</tbody>
</table>

Rule: **the more you cut the more you get**
\[ \frac{d^2 \Gamma^{(1)}}{dx dc} = \Gamma_0 x^2 \beta \frac{\alpha}{2\pi} \left( f_1(x) + c \xi g_1(x) \right), \quad L = \ln \frac{m_{\mu}^2}{m_e^2}, \quad \beta = \sqrt{1 - \frac{m_e^2}{E_e^2}} \]

\[ f_1(x) = f^{\text{Born}}(x) \left( \frac{2}{\beta} A + \frac{x^2 (1 - \beta^2) - 4 (1 + x \beta)}{2 x \beta} \ln \frac{q^2}{m_{\mu}^2} + \frac{4 - x^2 (1 - \beta^2)}{x \beta} \ln \frac{2 - x (1 - \beta)}{2} \right) \]

\[ + \frac{1}{\beta} \left( L + 2 \ln x + 2 \ln \frac{1 + \beta}{2} \right) \left\{ \frac{5 x^4}{384} (1 - \beta^2)^3 - \frac{x^3}{4} (1 - \beta^2)^2 + \frac{3 x^2}{32} (3 - 12 \beta + \beta^2) (1 - \beta^2) \right\} \]

\[ + x \left[ \frac{2}{3} + 2 \beta + (1 - \beta^2) \left( \frac{3}{2} + \beta \right) \right] + \frac{1}{8} [-20 - 12 \beta - 19 (1 - \beta^2)] + \frac{2}{x} + \frac{5}{6 x^2} \]

\[ + \left( \ln x + \ln \frac{1 + \beta}{2} \right) \left[ \frac{9}{4} x^2 (1 - \beta^2) + 2 x (\beta^2 - 3) + 3 \right] + f^{\text{Born}}(x) \left[ -\frac{11}{18} x (1 - \beta^2) + \frac{22}{27} \beta^2 - \frac{2}{9} \right] \]

\[ + x \left( -\frac{22}{27} \beta^4 + \frac{\beta^2}{2} - \frac{11}{6} \right) + \frac{22}{9} (3 - \beta^2) - \frac{22}{3 x}, \]

\[ A = L \left( \ln \frac{q^2}{m_{\mu}^2} - \ln x + \ln \frac{1 + \beta}{2 \beta} + \ln \frac{2 - x (1 - \beta)}{2 \beta} \right) + \left[ \ln \frac{q^2}{m_{\mu}^2} - 2 \ln x + 2 \ln \frac{1 + \beta}{2} \right. \]

\[ + 4 \ln \frac{2 - x (1 - \beta)}{2 \beta} \left( \ln x + \ln \frac{1 + \beta}{2} \right) + 2 \text{Li}_2 \left( \frac{(1 - \beta)(2 - x (1 + \beta))}{(1 + \beta)(2 - x (1 - \beta))} \right) - 2 \text{Li}_2 \left( \frac{2 - x (1 + \beta)}{2 - x (1 - \beta)} \right), \]

\[ g_1(x) = \ldots \]
Modern approach to $\mathcal{O}(\alpha)$ RC

Semi-automatic analytic $\oplus$ numeric calculations
some representatives:

**FeynArts** — a Mathematica package for generation and visualization of Feynman diagrams

**FeynCalc** — a Mathematica package for algebraic calculations in elementary particle physics

**LoopTools** — a package for evaluation of scalar and tensor one-loop integrals

**GRACE-loop** — a generic automated package for the calculation of Feynman diagrams at one-loop

**SANC** — ...