An Introduction to the Transverse Structure of Hadrons

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Abstract

I present a brief theoretical introduction to the transverse–spin and transverse–momentum structure of hadrons. This paper is based on the notes for the lectures at the "Niccolò Cabeo" International School of Hadronic Physics (Ferrara, Italy, May 2010).

Contents

1 Historical introduction

Transverse spin and transverse momentum effects are by now recognized as a fundamental sector of high-energy hadronic phenomenology (for reviews, see Refs. [1, 2, 3]).

In the early 90's some authors [4, 5, 6, 7] rediscovered the distribution of transversely polarized quarks in a transversely polarized nucleon first introduced by Ralston and Soper in 1979 [8]. This distribution, called "transversity" and denoted by $h_1(x)$ or by $\Delta_T q$, is a leading-twist quantity that contributes dominantly to the double transverse asymmetry in Drell-Yan (DY) production. Due to its chiral-odd nature, however, h_1 is not measurable in inclusive deep inelastic scattering.

Various proposals were soon put forward to measure the transversity distribution [9, 10, 11, 12, 13, 14, 15]. This work eventually led to to a detailed investigation of transverse momentum phenomena in hadrons. Transverse spin, in fact, couples quite naturally to the intrinsic transverse momentum of quarks. The resulting correlations are described by various transverse-momentum dependent distribution and fragmentation functions, which give rise to a myriad of single-spin and azimuthal asymmetries [16, 17, 11, 18, 19, 20, 21].

It was Sivers who first suggested that single-spin asymmetries could originate, at leading twist, from the intrinsic motion of quarks in the colliding hadrons [16, 17]. His idea was that there exists an azimuthal asymmetry of unpolarized quarks in a transversely polarized hadron (the so-called "Sivers effect"). Although phenomenologically successful [22], this mechanism seemed at first glimpse to violate time-reversal (T) invariance. Collins proposed an alternative mechanism, based on a spin asymmetry in the fragmentation of transversely polarized quarks into an unpolarized hadron (the "Collins effect"), which is not forbidden by T-invariance due to final-state interactions [11]. While the Sivers effect involves a T-odd transverse-momentum dependent distribution function, now commonly called $f_{1T}^{\perp}$, the Collins effect involves a transverse-momentum dependent fragmentation function, $H_1^{\perp}$.

It is known [23] that in inclusive deep inelastic scattering (DIS) transverse single-spin asymmetries (SSA's) are prohibited by time-reversal invariance at lowest order in $\alpha_{\rm em}$. This argument, however, does not hold in semi-inclusive DIS (SIDIS), where there are no first principles forbidding SSA's. These asymmetries are indeed non vanishing, as shown in the last decade by various experiments. In particular, HERMES [24] and COMPASS [25, 26], measuring SIDIS with transversely polarized targets, showed clear evidences of sizable transverse SSA's.

On the theoretical side, Brodsky, Hwang and Schmidt [27, 28] proved by an explicit calculation that final-state interactions in SIDIS, arising from gluon exchange between the struck quark and the nucleon remnant, or initial-state interactions in DY, produce a non-zero Sivers asymmetry. The situation was further clarified by Collins [29] who pointed out that, taking correctly into account the gauge links in the transverse-momentum dependent distributions (TMD's), time-reversal invariance does not imply a vanishing $f_{1T}^{\perp}$, but rather a sign difference between the Sivers distribution measured in SIDIS and the same distribution measured in DY.

Another important asymmetry source is related to the so-called Boer-Mulders function [30], a (naïvely) T-odd TMD that measures the transverse-spin asymmetry of quarks inside an unpolarized hadron. The Boer-Mulders mechanism produces $\cos\phi$ and $\cos 2\phi$ azimuthal modulations in the cross sections of unpolarized SIDIS and DY processes.

In QCD, the TMD description of hard processes is supported by a non-collinear factorization theorem at low P_T (i.e., $P_T \ll Q$), which has been proven for SIDIS and DY [31, 32]. It is also known that twist-3 collinear effects, expressed by quark-gluon correlation functions, can produce single-spin and azimuthal asymmetries at high transverse momenta, $P_T \gg M$ [33, 34, 35]. Thus, there is an overlap region where both the collinear twist-3 factorization and the non-collinear factorization should be valid. The relation between these two pictures, that is, between the T-odd TMD's on one side and the multiparton correlators on the other side, has been clarified in a series of papers [36, 37, 38].

2 Deep inelastic scattering

The reaction that has first contributed to unveiling the structure of hadrons is deep inelastic scattering (DIS). This will be our starting point.

Consider the inclusive lepton–nucleon scattering

$$l(\ell) + N(P) \to l'(\ell') + X(P_X)\,, \tag{1}$$

where X is an undetected hadronic system. In the neutral current case ($l = l' = e, \mu$), if the momentum transfer is not very large, this process is dominated by one-photon exchange (Fig. 1).

The reaction (1) is described by three kinematic variables. One of them (the energy E of the incoming lepton, or equivalently the squared c.m. energy $s = (\ell + P)^2)$) is experimentally fixed. The other two variables can be chosen among the invariants (M is the nucleon mass)

$$q^2 \equiv -Q^2 = (\ell - \ell')^2 \quad \text{(squared momentum transfer)} \tag{2}$$

$$W^2 = (P+q)^2\,, \tag{3}$$

$$\nu = \frac{P \cdot q}{M} = \frac{W^2 + Q^2 - M^2}{2M}\,, \tag{4}$$

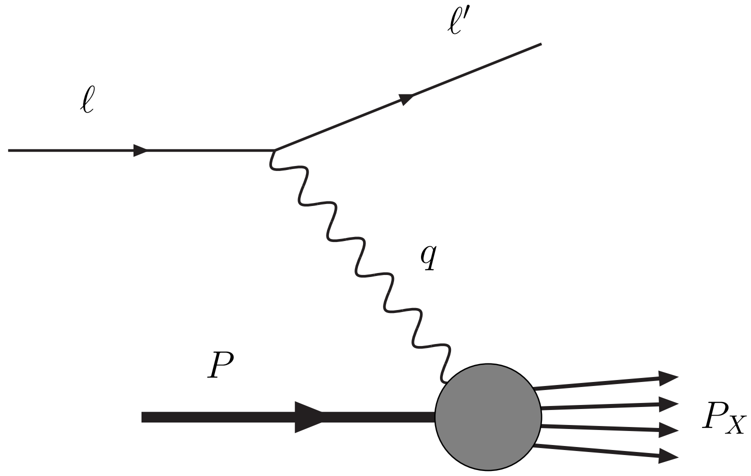

Figure 1: Deep inelastic scattering.

$$x_B = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M\nu} = \frac{Q^2}{Q^2 + W^2 - M^2} \quad \text{(Bjorken's variable)} \,, \tag{5}$$

$$y = \frac{P \cdot q}{P \cdot \ell} = \frac{W^2 + Q^2 - M^2}{s - M^2} \,. \tag{6}$$

In the target rest frame ν is the energy transfer, i.e. $\nu = E - E'$ (E and E' are the incoming and outgoing lepton energies, respectively), and y (sometimes called "inelasticity") is the fraction of the incoming lepton energy carried away by the exchanged photon, $y = \nu/E$. A useful relation is

$$x_B \, y = \frac{Q^2}{s - M^2} \simeq \frac{Q^2}{s} \,. \tag{7}$$

Since $W^2 \geq M^2$ (W is the c.m. energy of the $\gamma^* N$ system, that is, the invariant mass of the hadronic system X), Bjorken's variable x_B takes values between 0 and 1.

The *deep inelastic scattering* regime, or *Bjorken limit*, corresponds to

$$\nu, \, Q^2 \; \to \; \infty, \quad \text{with } x_B = \frac{Q^2}{2M\nu} \; \text{fixed} \,.$$

In this limit all hadronic masses can be neglected.

The DIS cross section reads

$$\begin{aligned} \mathrm{d}^3\sigma \quad &= \quad \frac{1}{4\,(\ell \cdot P)} \sum_{s_{l'}} \sum_{X} \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3 \, 2E_X} \\ &\quad \times (2\pi)^4 \, \delta^4(P + \ell - P_X - \ell') \, |\mathcal{M}|^2 \, \frac{\mathrm{d}^3 \boldsymbol{\ell'}}{(2\pi)^3 \, 2E'} \,. \end{aligned} \tag{8}$$

The squared amplitude in (8) is

$$|\mathcal{M}|^2 = \frac{e^4}{q^4} \sum_{s_{l'}} \left[\overline{u}_{l'}(\ell', s_{l'})\gamma_\mu u_l(\ell, s_l)\right]^* \left[\overline{u}_{l'}(\ell', s_{l'})\gamma_\nu u_l(\ell, s_l)\right] \times \langle X|J^\mu(0)|P,S\rangle^* \langle X|J^\nu(0)|P,S\rangle \, . \tag{9}$$

Note that in eq. (8) and eq. (9) we summed over the final lepton spin ($s_{l'}$) but did not average over the initial lepton spin (s_l), nor sum over the hadron spin (S). Thus we are describing, in general, the scattering of polarized leptons on a polarized target, with no measurement of the outgoing lepton polarization.

We introduce now the hadronic tensor $W^{\mu\nu}$,

$$W^{\mu\nu} = \frac{1}{(2\pi)} \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X} (2\pi)^4 \, \delta^4(P + q - P_X) \times \langle P,S|J^\mu(0)|X\rangle \langle X|J^\nu(0)|P,S\rangle, \tag{10}$$

and the leptonic tensor $L_{\mu\nu}$,

$$\begin{aligned} L_{\mu\nu} &= \sum_{s_{l'}} \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\mu u_l(\ell, s_l)\right]^* \left[\bar{u}_{l'}(\ell', s_{l'})\gamma_\nu u_l(\ell, s_l)\right] \\ &= \mathrm{Tr}\left[(\not\ell + m_l)\,\frac{1}{2}(1+\gamma_5 \not s_l)\,\gamma_\mu\,(\not\ell' + m_l)\,\gamma_\nu\right], \end{aligned} \tag{11}$$

so that the DIS cross section takes the form

$$\mathrm{d}^3\sigma = \frac{1}{4\,\ell{\cdot}P}\,\frac{e^4}{Q^4}\, L_{\mu\nu}\, W^{\mu\nu}\, 2\pi\, \frac{\mathrm{d}^3\boldsymbol{\ell}'}{(2\pi)^3\, 2E'} \, . \tag{12}$$

Using the integral representation of the delta function and translational invariance, the hadronic tensor can be rewritten as

$$W^{\mu\nu} = \frac{1}{2\pi} \int \mathrm{d}^4\xi \; \mathrm{e}^{\mathrm{i}q\cdot\xi} \, \langle P,S|J^\mu(\xi)J^\nu(0)|P,S\rangle \, . \tag{13}$$

It is important to recall that the matrix elements in eq. (13) are connected. Therefore, vacuum transitions of the form $\langle 0|J^\mu(\xi)J^\nu(0)|0\rangle \, \langle P,S|P,S\rangle$ are excluded.

In the target rest frame, where $\ell{\cdot}P = ME$, eq. (12) reads

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{2MQ^4}\,\frac{E'}{E}\, L_{\mu\nu}W^{\mu\nu} \, , \tag{14}$$

where $\mathrm{d}\Omega = \mathrm{d}\cos\vartheta\,\mathrm{d}\varphi$. In terms of the invariants defined above the cross section becomes

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\varphi} = \frac{\alpha_{\mathrm{em}}^2 y}{2Q^4}\, L_{\mu\nu}W^{\mu\nu} \, . \tag{15}$$

The leptonic tensor $L_{\mu\nu}$ can be decomposed into a symmetric and an antisymmetric part under $\mu \leftrightarrow \nu$ interchange

$$L_{\mu\nu} = L^{(\mathrm{S})}_{\mu\nu}(\ell, \ell') + L^{(\mathrm{A})}_{\mu\nu}(\ell, s_l; \ell') \, , \tag{16}$$

and, computing the trace in eq. (11), we obtain (retaining lepton masses)

$$L_{\mu\nu}^{(\mathrm{S})} = 2[\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu - g_{\mu\nu}\,(\ell{\cdot}\ell' - m_l^2)]\,, \tag{17}$$

$$L_{\mu\nu}^{(\mathrm{A})} = 2\mathrm{i}\, m_l\, \epsilon_{\mu\nu\rho\sigma} s_\ell^\rho (\ell - \ell')^\sigma\,. \tag{18}$$

If the incoming lepton is longitudinally polarized, its spin vector is

$$s_l^\mu = \frac{\lambda_l}{m_l}\,\ell^\mu, \quad \lambda_l = \pm 1\,, \tag{19}$$

and eq. (18) becomes

$$L_{\mu\nu}^{(\mathrm{A})} = 2\mathrm{i}\,\lambda_l\,\epsilon_{\mu\nu\rho\sigma}\ell^\rho(\ell - \ell')^\sigma = 2\lambda_l\,\epsilon_{\mu\nu\rho\sigma}\ell^\rho q^\sigma\,. \tag{20}$$

Note that the lepton mass m_l appearing in eq. (18) has been cancelled by the denominator of eq. (19). In contrast, if the lepton is transversely polarized, that is $s_l^\mu = s_{l\perp}^\mu$, no such cancellation occurs and the process is suppressed by a factor m_l/E.

Neglecting, as usual, lepton masses, the leptonic tensor then reads

$$\begin{aligned} L_{\mu\nu} &= L_{\mu\nu}^{(\mathrm{S})} + L_{\mu\nu}^{(\mathrm{A})} \\ &= 2(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu - g_{\mu\nu}\ell\cdot\ell') - 2\mathrm{i}\,\lambda_l\,\varepsilon_{\mu\nu\rho\sigma}\ell^\rho\ell'^\sigma. \end{aligned} \tag{21}$$

The hadronic tensor $W_{\mu\nu}$ admits a similar decomposition

$$W_{\mu\nu} = W_{\mu\nu}^{(\mathrm{S})}(q,P) + W_{\mu\nu}^{(\mathrm{A})}(q;P,S)\,, \tag{22}$$

into a symmetric and an antisymmetric part, which are expressed in terms of two pairs of structure functions, W_1, W_2 and G_1, G_2, as

$$\begin{aligned} \frac{1}{2M}\,W_{\mu\nu}^{(\mathrm{S})} &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1(\nu,q^2) \\ &+ \frac{1}{M^2}\left[\left(P_\mu - \frac{P{\cdot}q}{q^2}\,q_\mu\right)\left(P_\nu - \frac{P{\cdot}q}{q^2}\,q_\nu\right)\right] W_2(\nu,q^2)\,, \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{1}{2M}\,W_{\mu\nu}^{(\mathrm{A})} &= \mathrm{i}\,\epsilon_{\mu\nu\rho\sigma}\,q^\rho \Big\{ MS^\sigma\,G_1(\nu,q^2) \\ &+ \frac{1}{M}\left[P{\cdot}q\,S^\sigma - S{\cdot}q\,P^\sigma\right] G_2(\nu,q^2)\Big\}\,. \end{aligned} \tag{24}$$

Equations (23, 24) are the most general expressions compatible with the requirements of gauge invariance and parity invariance.

Using (16, 22) the cross section (14) becomes

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{2MQ^4}\,\frac{E'}{E}\left[L_{\mu\nu}^{(\mathrm{S})}W^{\mu\nu\,(\mathrm{S})} + L_{\mu\nu}^{(\mathrm{A})}W^{\mu\nu\,(\mathrm{A})}\right]. \tag{25}$$

It is customary to introduce the dimensionless structure functions

$$F_1(x_B, Q^2) \equiv M\, W_1(\nu, Q^2), \quad F_2(x_B, Q^2) \equiv \nu\, W_2(\nu, Q^2)\,. \tag{26}$$
$$g_1(x_B, Q^2) \equiv M^2 \nu\, G_1(\nu, Q^2)\,, \quad g_2(x_B, Q^2) \equiv M \nu^2\, G_2(\nu, Q^2)\,. \tag{27}$$

In the Bjorken limit, the functions F_1, F_2, g_1, g_2 are expected to scale approximately, that is, to depend on x_B only. In terms of F_1 and F_2 the symmetric part of the hadronic tensor reads

$$\begin{aligned} W_{\mu\nu}^{(\mathrm{S})} &= 2\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x_B, Q^2) \\ &+ \frac{2}{P\cdot q}\left[\left(P_\mu - \frac{P\cdot q}{q^2}\, q_\mu\right)\left(P_\nu - \frac{P\cdot q}{q^2}\, q_\nu\right)\right] F_2(x_B, Q^2)\,, \end{aligned} \tag{28}$$

whereas in terms of g_1 and g_2 the antisymmetric part of the hadronic tensor reads

$$\begin{aligned} W_{\mu\nu}^{(\mathrm{A})} &= \frac{2M\, \epsilon_{\mu\nu\rho\sigma}\, q^\rho}{P\cdot q} \Big\{ S^\sigma\, g_1(x_B, Q^2) \\ &\quad + \left[S^\sigma - \frac{S\cdot q}{P\cdot q}\, P^\sigma\right] g_2(x_B, Q^2) \Big\}\,. \end{aligned} \tag{29}$$

Let us focus for definiteness on the unpolarized cross section, which is obtained from eq. (25) by averaging over the spins of the incoming lepton (s_l) and of the nucleon (S), and reads

$$\frac{\mathrm{d}^2\sigma^{\mathrm{unp}}}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{1}{2}\sum_{s_l} \frac{1}{2}\sum_{S} \frac{\mathrm{d}^2\sigma(s_l, S)}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{2MQ^4}\, \frac{E'}{E}\, L_{\mu\nu}^{(\mathrm{S})} W^{\mu\nu\,(\mathrm{S})}\,. \tag{30}$$

Inserting Eqs. (17) and (23) into (30) one obtains the well-known expression

$$\frac{\mathrm{d}^2\sigma^{\mathrm{unp}}}{\mathrm{d}E'\,\mathrm{d}\Omega} = \frac{4\alpha_{\mathrm{em}}^2 E'^2}{Q^4}\left[2W_1\, \sin^2\frac{\vartheta}{2} + W_2\, \cos^2\frac{\vartheta}{2}\right], \tag{31}$$

Expressed in terms of F_1 and F_2, and as a function of x and y, the unpolarized cross section is

$$\frac{\mathrm{d}^2\sigma^{\mathrm{unp}}}{\mathrm{d}x_B\,\mathrm{d}y} = \frac{4\pi\alpha_{\mathrm{em}}^2}{Q^2 x_B y}\left\{x_B y^2\, F_1(x_B, Q^2) + \left(1 - y - \frac{\gamma^2 y^2}{4}\right) F_2(x_B, Q^2)\right\}, \tag{32}$$

where

$$\gamma \equiv \frac{2Mx_B}{Q} \tag{33}$$

is a order $1/Q$ quantity that will often appear in the DIS formulas.

3 The light–cone

The light-cone components of a four-vector a^μ are defined as $a^\pm = (a^0 \pm a^3)/\sqrt{2}$, and grouped in triplets of the form $a^\mu = [a^+, a^-, \boldsymbol{a}_\perp]$, where the transverse bi-vector is $\boldsymbol{a}_\perp = (a^1, a^2)$. The norm of a^μ is given by $a^2 = 2a^+a^- - \boldsymbol{a}_\perp^2$.

It is customary to define two light-like vectors $n_+ = [1, 0, \boldsymbol{0}_\perp]$ and $n_- = [0, 1, \boldsymbol{0}_\perp]$, sometimes called "Sudakov vectors", which identify the longitudinal direction and are such that $n_+ \cdot n_- = 1$. Any vector a^μ can be written as

$$a^\mu = a^+ n_+^\mu + a^- n_-^\mu + a_\perp^\mu \,, \tag{34}$$

where $a_\perp^\mu = [0, 0, \boldsymbol{a}_\perp]$. This is the four-dimensional generalization of the familiar decomposition of a three-vector into longitudinal and transverse components with respect to a given direction.

The metric tensor $g_\perp^{\mu\nu}$ which projects onto the plane perpendicular to n_+ and n_- is

$$g_\perp^{\mu\nu} = g^{\mu\nu} - (n_+^\mu n_-^\nu + n_+^\nu n_-^\mu) \,. \tag{35}$$

Another projector onto the transverse plane is

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} n_{+\rho} n_{-\sigma} \,. \tag{36}$$

The reference frame for DIS is chosen so that the nucleon's momentum is purely longitudinal:

$$P^\mu = P^+ n_+^\mu + \frac{M^2}{2P^+} n_-^\mu \simeq P^+ n_+^\mu \,, \tag{37}$$

where the approximate equality means that we are neglecting the nucleon mass (a legitimate approximation in the deep inelastic limit). Note that we are assuming, as usual, that the nucleon moves in the $+z$ direction, at variance with the conventions of Ref. [39], where the nucleon is chosen to move in the $-z$ direction.

The "infinite momentum frame" corresponds to $P^+ \to \infty$. Dominant contributions to DIS are $\mathcal{O}(P^+)$, whereas subleading corrections are suppressed by inverse powers of P^+.

For the analysis of DIS it is convenient to use a class of frames where both the nucleon and the virtual photon have only longitudinal components (the "$\gamma^* N$ collinear frames"). The momentum of the photon is then parametrized as

$$q^\mu \simeq -x_B P^+ n_+^\mu + \frac{Q^2}{2x_B P^+} n_-^\mu. \tag{38}$$

Light-cone variables are useful because DIS probes the parton dynamics on the light-cone (see, e.g., Ref. [40]). To show this let us first rewrite the hadronic tensor as

$$W^{\mu\nu} = \frac{1}{2\pi} \int \mathrm{d}^4\xi \, \mathrm{e}^{\mathrm{i}q\cdot\xi} \, \langle P, S | [J^\mu(\xi), J^\nu(0)] | P, S \rangle, \tag{39}$$

where, with respect to eq. (13), we subtracted a vanishing term.

Causality implies $\xi^2 \geq 0$, i.e. no space-like separations between the two currents in the commutator. In the Bjorken limit one has $q^- = M\nu/P^+ \to \infty$, and large oscillations in the exponential $\mathrm{e}^{\mathrm{i}q\cdot\xi} = \mathrm{e}^{\mathrm{i}(q^+\xi^- + q^-\xi^+)}$ make the integral in eq. (39) vanish unless $\xi^+ \to 0$. Now, $\xi^2 \geq 0$ with $\xi^+ \to 0$ implies $\xi^2 = 0$, that is a light-cone separation.

4 The parton model

Right after the first DIS experiments at SLAC, Feynman proposed the concept of *parton distributions* as the probability densities of finding a parton with a certain momentum fraction inside a nucleon [41]. In its original formulation, Feynman's parton model was based on the observation that the time scale of the interaction between the virtual photon and the partons is $\sim 1/Q$, hence much smaller than the time scale of the binding interactions of partons, which is $\sim 1/M$ in the target rest frame and gets dilated in the infinite-momentum frame. Thus, we can approximately assume that in DIS the lepton interacts elastically with free partons, and define the parton distribution functions as the single-particle longitudinal momentum distributions of the nucleon's constituents.

The parton model can be constructed covariantly in quantum field theory, and we will see that the parton distributions admit a rigorous definition in terms of correlation functions of parton fields taken at two space-time points with a light-like separation [42, 43] (for a modern treatment, see Ref. [44]).

Under the assumption that the virtual photon scatters incoherently off the internal constituents of the nucleon, treated as free particles, the hadronic tensor $W^{\mu\nu}$ is represented by the handbag diagram shown in Fig. 2 and reads (to simplify the discussion, for the moment being we shall consider quarks only)

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{2\pi} \sum_a e_a^2 \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \int \frac{\mathrm{d}^4 \kappa}{(2\pi)^4}\, 2\pi\, \delta(\kappa^2) \\ &\quad \times [\overline{u}(\kappa)\gamma^\mu \phi(k,P,S)]^* \, [\overline{u}(\kappa)\gamma^\nu \phi(k,P,S)] \\ &\quad \times (2\pi)^4\, \delta^4(P-k-P_X)\, (2\pi)^4\, \delta^4(k+q-\kappa)\ , \end{aligned} \tag{40}$$

where $\sum_a$ is a sum over the quark flavors, e_a is the quark charge, and we have introduced the quark-nucleon vertex functions

$$\phi(k,P,S) = \langle X | \psi(0) | P, S \rangle\ . \tag{41}$$

We define the quark-quark correlation matrix $\Phi(k,P,S)$ as

$$\Phi(k,P,S) = \frac{1}{(2\pi)^4} \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2P_X^0}\, (2\pi)^4\, \delta^4(P-k-P_X)\, \phi(k,P,S)\, \overline{\phi}(k,P,S)\ , \tag{42}$$

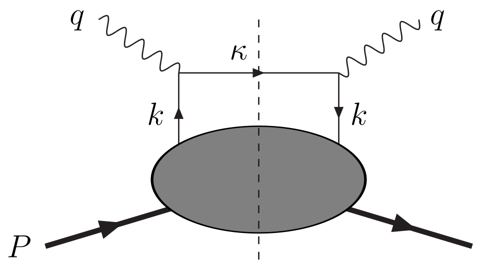

Figure 2: The handbag diagram.

so that the hadronic tensor can be rewritten in the form (we neglect quark masses)

$$
\begin{aligned}
W^{\mu\nu} &= \sum_a e_a^2 \int \mathrm{d}^4 k \int \mathrm{d}^4 \kappa \, \delta(\kappa^2) \, \delta^4(k+q-\kappa) \, \mathrm{Tr}\,[\Phi \, \gamma^\mu \, \not{\kappa} \, \gamma^\nu] \\
&= \sum_a e_a^2 \int \mathrm{d}^4 k \, \delta((k+q)^2) \, \mathrm{Tr}\,[\Phi \, \gamma^\mu \, (\not{k} + \not{q}) \, \gamma^\nu] \, . \qquad (43)
\end{aligned}
$$

Using the definition (41) and the completeness of the $|X\rangle$ states allows us to reexpress the correlation matrix Φ in the more transparent form

$$
\Phi_{ij}(k, P, S) = \frac{1}{(2\pi)^4} \int \mathrm{d}^4 \xi \; \mathrm{e}^{ik\cdot\xi} \, \langle P, S | \overline{\psi}_j(0) \psi_i(\xi) | P, S \rangle \, . \qquad (44)
$$

Once more we remind that the matrix elements in eq. (44) are connected.

The Sudakov decomposition of the quark momentum is[1]

$$
k^\mu = x P^+ n_+^\mu + \frac{k^2 + k_\perp^2}{2 x P^+} \, n_-^\mu + k_\perp^\mu \, . \qquad (45)
$$

In the parton model one assumes that the handbag diagram contribution to the hadronic tensor is dominated by small values of k^2 and $k_\perp^2$. Thus we can approximately write k^μ as

$$
k^\mu \simeq x P^+ n_+^\mu \simeq x P^\mu \, . \qquad (46)
$$

Notice that in the infinite momentum frame ($P^+ \to \infty$), k^μ reduces automatically to (46). That is why the infinite momentum frame is a convenient – but not mandatory – frame for the parton model.

The on-shellness of the outgoing quark, in the light of eq. (46), implies

$$
\delta((k+q)^2) \simeq \delta(-Q^2 + 2 \, x \, P \cdot q) = \frac{1}{2 \, P \cdot q} \, \delta(x - x_B) \, , \qquad (47)
$$

that is

$$
k^\mu \simeq x_B \, P^\mu \, . \qquad (48)
$$

[1]From now on the modulus of a transverse two-vector $\boldsymbol{a}_\perp$ will be denoted as $a_\perp \equiv |\boldsymbol{a}_\perp|$.

Thus the Bjorken variable x_B emerges as the fraction of the longitudinal proton momentum carried by the struck quark:

$$x_B = x = \frac{k^+}{P^+} \; . \tag{49}$$

Coming back to the hadronic tensor (43), the identity

$$\gamma^\mu \gamma^\rho \gamma^\nu = g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho} - g^{\mu\nu} g^{\rho\sigma} - i\varepsilon^{\mu\rho\nu\sigma}\, \gamma_\sigma \gamma^5 \; , \tag{50}$$

allows splitting $W^{\mu\nu}$ into a symmetric (S) and an antisymmetric (A) part under $\mu \leftrightarrow \nu$ interchange. $W^{(\mathrm{S})}_{\mu\nu}$, which contributes to unpolarized DIS, is given by

$$\begin{aligned} W^{(\mathrm{S})}_{\mu\nu} \;&=\; \frac{1}{2(P\cdot q)} \sum_a e_a^2 \int \mathrm{d}^4 k \, \delta(x_B - k^+/P^+) \\ &\quad \times \left[(k_\mu + q_\mu) \,\mathrm{Tr}\,(\Phi\, \gamma_\nu) + (k_\nu + q_\nu)\,\mathrm{Tr}\,(\Phi\,\gamma_\mu) \right. \\ &\quad \left. - \, g_{\mu\nu} (k^\rho + q^\rho)\,\mathrm{Tr}\,(\Phi\,\gamma_\rho) \right] \; . \end{aligned} \tag{51}$$

From eq. (38) and eq. (48) we get $k^\mu + q^\mu \simeq (P\cdot q)\, n_-^\mu / P^+$, and eq. (51) becomes

$$\begin{aligned} W^{(\mathrm{S})}_{\mu\nu} \;&=\; \frac{1}{2P^+} \sum_a e_a^2 \int \mathrm{d}^4 k \, \delta(x - k^+/P^+) \\ &\quad \times \left[n_{-\mu} \,\mathrm{Tr}\,(\Phi\gamma_\nu) + n_{-\nu}\,\mathrm{Tr}\,(\Phi\gamma_\mu) - g_{\mu\nu} n_-^\rho \,\mathrm{Tr}\,(\Phi\gamma_\rho) \right] \; . \end{aligned} \tag{52}$$

For later convenience we introduce the following notation. We call $\Phi^{[\Gamma]}$, where Γ is a Dirac matrix, the quantity

$$\begin{aligned} \Phi^{[\Gamma]}(x_B) \;&\equiv\; \frac{1}{2P^+} \int \mathrm{d}^4 k \, \delta(x_B - k^+/P^+)\,\mathrm{Tr}\,(\Gamma\,\Phi) \\ &=\; \frac{P^+}{2} \int \frac{\mathrm{d}\xi^-}{2\pi}\, \mathrm{e}^{\mathrm{i} x_B P^+ \xi^-} \, \langle P,S | \overline{\psi}(0)\, \Gamma\, \psi(0,\xi^-,\mathbf{0}_\perp)|P,S\rangle . \end{aligned} \tag{53}$$

Hence $W^{(\mathrm{S})}_{\mu\nu}$ reads

$$W^{(\mathrm{S})}_{\mu\nu} = \sum_a e_a^2 \, \left[n_{-\mu}\, \Phi^{[\gamma_\nu]} + n_{-\nu}\, \Phi^{[\gamma_\mu]} - g_{\mu\nu}\, n_-^\rho \, \Phi^{[\gamma_\rho]} \right] \; . \tag{54}$$

We now have to parametrize $\Phi^{[\gamma^\mu]}$, which is a vector quantity. At leading twist, that is considering contributions $\mathcal{O}(P^+)$ in the infinite momentum frame, the only vector at our disposal is $P^\mu \simeq x_B P^+ n_+^\mu$ (recall that $k^\mu \simeq x_B P^\mu$). Thus we can write

$$\Phi^{[\gamma^\mu]}(x_B) = \frac{1}{2P^+} \int \mathrm{d}^4 k \, \delta(x_B - k^+/P^+)\,\mathrm{Tr}\,(\gamma^\mu\,\Phi) = f_1(x_B)\, n_+^\mu \tag{55}$$

where the coefficient of n_+^μ, that we called $f_1(x_B)$, is the number density of quarks, as it will become clear later on. Multiplication of (55) by $n_{-\mu}$ gives $f_1(x_B)$ in the explicit form

$$f_1(x_B) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}x_B P^+\xi^-} \, \langle P,S|\overline{\psi}(0)\gamma^+\psi(0,\xi^-,\mathbf{0}_\perp)|P,S\rangle \; , \tag{56}$$

where $\gamma^+ = (\gamma^0 + \gamma^3)/\sqrt{2}$.

Inserting (55) in (54) yields

$$W_{\mu\nu}^{(\mathrm{S})} = \sum_a e_a^2 \left(n_{-\mu} n_{+\nu} + n_{-\nu} n_{+\mu} - g_{\mu\nu}\right) f_{1a}(x)\,. \tag{57}$$

The structure functions F_1 and F_2 can be extracted from $W^{\mu\nu}$ by means of two projectors (terms of order $1/Q^2$ are neglected)

$$F_1 \;=\; \mathcal{P}_1^{\mu\nu} W_{\mu\nu} = \frac{1}{4}\left(\frac{4x^2}{Q^2}\, P^\mu P^\nu - g^{\mu\nu}\right) W_{\mu\nu}\,, \tag{58}$$

$$F_2 \;=\; \mathcal{P}_2^{\mu\nu} W_{\mu\nu} = \frac{x}{2}\left(\frac{12x^2}{Q^2}\, P^\mu P^\nu - g^{\mu\nu}\right) W_{\mu\nu}\,. \tag{59}$$

Since $(P^\mu P^\nu/Q^2)\, W_{\mu\nu} = \mathcal{O}(M^2/Q^2)$ we find that F_1 and F_2 are proportional to each other (the so-called Callan-Gross relation) and are given by

$$F_2(x_B) = 2x_B F_1(x_B) = -\frac{x_B}{2}\, g^{\mu\nu} W_{\mu\nu} = \sum_a e_a^2\, x_B\, f_{1a}(x_B)\,, \tag{60}$$

which is the well known parton model expression for the unpolarized structure functions. This justifies the identification of (56) with the unpolarized quark distribution function (i.e., the number density of quarks). To get the full expression of F_1 and F_2, one should simply add to (59) the antiquark distributions $\bar{f}_1(x_B)$, which were left aside in the above discussion. They read (the role of ψ and $\overline{\psi}$ is interchanged with respect to the quark distributions)

$$\bar{f}_1(x_B) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}x_B P^+ \xi^-}\, \langle P, S|\mathrm{Tr}\,[\gamma^+ \psi(0)\overline{\psi}(0,\xi^-,\mathbf{0}_\perp)]|P, S\rangle\,, \tag{61}$$

and the structure functions F_1, F_2 are

$$F_2(x_B) = 2x_B F_1(x_B) = \sum_a e_a^2\, x_B\, \left[f_{1a}(x_B) + \bar{f}_{1a}(x_B)\right]\,. \tag{62}$$

5 The quark correlation matrix

Quark distribution functions are contained in the correlation matrix Φ (Fig. 3), defined as

$$\Phi_{ij}(k, P, S) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\, \mathrm{e}^{\mathrm{i}k\cdot\xi}\, \langle P, S|\bar{\psi}_j(0)\, \mathcal{W}[0,\xi]\, \psi_i(\xi)|P, S\rangle\,. \tag{63}$$

where $|P, S\rangle$ is the nucleon state of momentum P^μ and polarization vector S^μ, i and j are Dirac indices and a summation over colour is implicit. In QCD one has to insert between the quark fields, taken at different spacetime points, a gauge link, or Wilson line, $\mathcal{W}$, which guarantees the gauge invariance of the correlator. This quantity

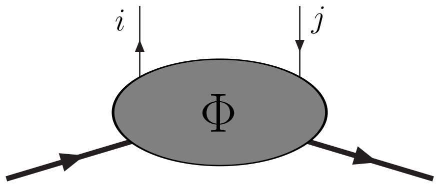

Figure 3: The quark correlation matrix Φ.

arises from multigluon final–state interactions between the struck quark and the target spectators, which factorize to form a path-ordered exponential of the gluon field

$$\mathcal{W}[0,\xi] = \mathcal{P}\,\exp\left(-\mathrm{i}g\int_0^{\xi} \mathrm{d}z_\mu\, A^\mu(z)\right), \tag{64}$$

where $A^\mu \equiv A_a^\mu t_a$ (t_a is the generator of colour SU(3) in the fundamental representation). The presence of this gauge link introduces in principle a path-dependence in Φ, which in some cases turns out to be highly non trivial (Sec. 8).

Integrating $\Phi(k,P,S)$ over the quark momentum, with the condition $x = k^+/P^+$ that defines x as the fraction of the longitudinal momentum of the nucleon carried by the quark, yields

$$\begin{aligned}\Phi(x) &= \int \mathrm{d}^4k\; \Phi(k,P,S)\,\delta(k^+ - xP^+)\\ &= \int \frac{\mathrm{d}\xi^-}{2\pi}\,\mathrm{e}^{\mathrm{i}xP^+\xi^-}\,\langle P,S|\bar\psi(0)\,\mathcal{W}^-[0,\xi]\,\psi(\xi)|P,S\rangle|_{\xi^+=0,\boldsymbol{\xi}_\perp=\mathbf{0}_\perp}\,,\end{aligned} \tag{65}$$

where the Wilson line $\mathcal{W}^-[0,\xi]$ connects $(0,0,\mathbf{0}_\perp)$ to $(0,\xi^-,\mathbf{0}_\perp)$ along the n_- direction and reads

$$\mathcal{W}^-[0,\xi] = \mathcal{P}\,\exp\left(-\mathrm{i}g\int_0^{\xi^-}\mathrm{d}z^-\, A^+(0,z^-,\mathbf{0}_\perp)\right). \tag{66}$$

In the light-cone gauge, $A^+ = 0$, the Wilson link reduces to unity and can be omitted. The situation is more complicated in the case of transverse-momentum distributions, which are defined in terms of field separations of the type $(0,\xi^-,\boldsymbol{\xi}_\perp)$: we shall return to this issue in Sec. 8.

$\Phi(x)$ contains the collinear (i.e., $k_\perp$-integrated) quark distribution functions. Notice that these distributions depend in general on x. It is only in the parton model, neglecting order $1/Q^2$ corrections, that x coincides with the variable x_B.

Introducing the longitudinal and transverse components of the the polarization vector of the nucleon,

$$S^\mu = \frac{S_\parallel}{M}\left(P^+ n_+^\mu - \frac{M^2}{2P^+}\,n_-^\mu\right) + S_\perp^\mu \simeq \frac{S_\parallel}{M}\,P^+ n_+^\mu + S_\perp^\mu\,, \tag{67}$$

where $S_\parallel^2 + S_\perp^2 \leq 1$ (the equality sign applies to pure states), the expression of $\Phi(x)$ at leading twist, that is at leading order in P^+, is

$$\Phi(x) = \frac{1}{2}\left\{ f_1(x)\, \not{n}_+ + S_\parallel\, g_1(x)\, \gamma_5 \not{n}_+ + h_1(x)\, \gamma_5\, \frac{[\not{S}_\perp, \not{n}_+]}{2} \right\} . \tag{68}$$

Here one sees the three leading-twist distribution functions: the number density $f_1(x)$, already introduced, the helicity distribution $g_1(x)$ and the transversity distribution $h_1(x)$, first identified by Ralston and Soper [8]. The quark distributions can be extracted from (68) by tracing Φ with some Dirac matrix Γ. We will use the notation $\Phi^{[\Gamma]}(x) \equiv \frac{1}{2}\,\mathrm{Tr}\,[\Phi(x)\,\Gamma]$. The explicit expressions of the leading-twist distributions are (the transverse Dirac matrix $\gamma_\perp$ is either γ^1 or γ^2):

$$f_1(x) = \Phi^{[\gamma^+]}(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle P, S|\bar{\psi}(0)\mathcal{W}^-\gamma^+\psi(\xi)|P, S\rangle|_{\xi^+=0,\, \boldsymbol{\xi}_\perp=\mathbf{0}_\perp}, \tag{69}$$

$$g_1(x) = \Phi^{[\gamma^+\gamma_5]}(x) = \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle P, S|\bar{\psi}(0)\mathcal{W}^-\gamma^+\gamma_5\psi(\xi)|P, S\rangle|_{\xi^+=0,\, \boldsymbol{\xi}_\perp=\mathbf{0}_\perp}, \tag{70}$$

$$\begin{aligned} h_1(x) &= \Phi^{[\gamma^+\gamma_\perp\gamma_5]}(x) \\ &= \int \frac{\mathrm{d}\xi^-}{4\pi}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-} \langle P, S|\bar{\psi}(0)\mathcal{W}^-\gamma^+\gamma_\perp\gamma_5\psi(\xi)|P, S\rangle|_{\xi^+=0,\, \boldsymbol{\xi}_\perp=\mathbf{0}_\perp}. \end{aligned} \tag{71}$$

In QCD the operators appearing in (69-71) are ultraviolet divergent, so they have to be renormalised. This introduces a scale dependence into the distribution functions, $f_1(x) \to f_1(x, \mu)$, etc., which is governed by the renormalisation group equations, the well known DGLAP equations [45, 46, 47].

If the quarks are perfectly collinear with the parent hadron, the three distribution functions we have mentioned so far, $f_1(x)$, $g_1(x)$, $h_1(x)$, exhaust the information on the internal dynamics of hadrons at leading twist, i.e., at zeroth order in $1/Q$ (for an operational definition of twist, see Ref. [40]). If instead we admit a non negligible quark transverse momentum, the number of distribution functions considerably increases. At leading twist, there are eight of them. In order to understand their origin and meaning, it is necessary to adopt a more systematic approach.

Notation In the Jaffe-Ji nomenclature of distribution functions [6], extended to transverse momentum dependent distributions by Mulders and collaborators [20, 30], $f_1(x)$, $g_1(x)$, $h_1(x)$ are the unpolarized ("number"), longitudinally polarized ("helicity") and transversely polarized ("transversity") distribution functions, respectively, with the subscript 1 denoting leading-twist quantities. The main disadvantage of this nomenclature is the use of g_1 to denote a *distribution function* whereas the same notation is adopted for one of the two *structure functions* of polarized deep inelastic scattering. We label the contribution of a specific flavor by a subscript or superscript q, or a. Bars indicate antiquark distribution (and fragmentation) functions.

Other common names in the literature are $q(x)$, or $f_q(x)$, for the unpolarized distribution, $\Delta q(x)$, or $\Delta f_q(x)$, for the helicity distribution, $\Delta_T q(x)$, or $\Delta_T f_q(x)$, for the transversity distribution (which is also called sometimes δq: I reserve this name to the tensor charge, i.e., the first moment of $h_1^q - \bar{h}_1^q$). The Mulders et al. classification scheme for the $k_\perp$-dependent distributions [20, 30] is illustrated in detail in Sec. 8.

6 Probabilistic interpretation of distribution functions

Distribution functions are essentially the probability densities for finding partons with a given momentum fraction and a given polarization inside a hadron.

Focusing on quarks, $f_1(x)$ is the number density of quarks carrying a fraction x of the longitudinal momentum of the nucleon, that is the probability of finding a quark with a longitudinal momentum fraction x, irrespective of its polarization.

The helicity distribution function $g_1(x)$ is the helicity asymmetry of quarks in a longitudinally polarized nucleon, that is, the number density $q_+(x)$ of quarks with momentum fraction x and polarization parallel to that of the nucleon minus the number density $q_-(x)$ of quarks with the same momentum fraction but antiparallel polarization: $g_1(x) = q_+(x) - q_-(x)$. In terms of $q_\pm$ the unpolarized distribution $f_1(x)$ is simply the sum of the two probability densities: $f_1(x) = q_+(x) + q_-(x)$.

The case of transverse polarization can be treated in a similar way: for a transversely polarized nucleon the transversity distribution $h_1(x)$ is defined as the number density of quarks with momentum fraction x and polarization parallel to that of the hadron, minus the number density of quarks with the same momentum fraction and antiparallel polarization, that is, denoting transverse polarizations by arrows, $h_1(x) = q_\uparrow(x) - q_\downarrow(x)$. In a basis of transverse polarization states, $h_1(x)$ too has a probabilistic interpretation. In the helicity basis, in contrast, it has no simple meaning, being related to an off-diagonal quark-hadron amplitude.

We shall now see how the probabilistic interpretation comes about from the field-theoretical definitions of quark (and antiquark) distribution functions presented above.

Let us first of all decompose the quark fields into "good" and "bad" components:

$$\psi = \psi_{(+)} + \psi_{(-)} \,, \tag{72}$$

where

$$\psi_{(\pm)} = \frac{1}{2} \gamma^{\mp} \gamma^{\pm} \psi \,. \tag{73}$$

The usefulness of this procedure lies in the fact that "bad" components are not dynamically independent: using the equations of motion, they can be eliminated in favour of "good" components and terms containing quark masses and gluon fields. Since in the $P^+ \to \infty$ limit $\psi_{(+)}$ dominates over $\psi_{(-)}$, the presence of "bad" components in a parton distribution function signals higher twists. Using the relations

$$\begin{aligned}
\bar{\psi}\,\gamma^+\,\psi &= \sqrt{2}\,\psi^\dagger_{(+)}\,\psi_{(+)}\,, && (74)\\
\bar{\psi}\,\gamma^+\gamma_5\,\psi &= \sqrt{2}\,\psi^\dagger_{(+)}\,\gamma_5\,\psi_{(+)}\,, && (75)\\
\bar{\psi}\,\mathrm{i}\sigma^{i+}\gamma_5\,\psi &= \sqrt{2}\,\psi^\dagger_{(+)}\,\gamma^i\gamma_5\,\psi_{(+)}\,. && (76)
\end{aligned}$$

the leading-twist distributions can be re-expressed as

$$\begin{aligned}
f_1(x) &= \int \frac{\mathrm{d}\xi^-}{2\sqrt{2}\,\pi}\mathrm{e}^{\mathrm{i}xP^+\xi^-}\,\langle P,S|\psi^\dagger_{(+)}(0)\psi_{(+)}(0,\xi^-,\mathbf{0}_\perp)|P,S\rangle\,, && (77)\\
g_1(x) &= \int \frac{\mathrm{d}\xi^-}{2\sqrt{2}\,\pi}\mathrm{e}^{\mathrm{i}xP^+\xi^-}\,\langle P,S|\psi^\dagger_{(+)}(0)\gamma_5\psi_{(+)}(0,\xi^-,\mathbf{0}_\perp)|P,S\rangle\,, && (78)\\
h_1(x) &= \int \frac{\mathrm{d}\xi^-}{2\sqrt{2}\,\pi}\mathrm{e}^{\mathrm{i}xP^+\xi^-}\,\langle P,S|\psi^\dagger_{(+)}(0)\gamma_\perp\gamma_5\psi_{(+)}(0,\xi^-,\mathbf{0}_\perp)|P,S\rangle. && (79)
\end{aligned}$$

Note that, as anticipated, only "good" components appear. It is the peculiar structure of the quark-field bilinears in eqs. (77–79) that allows us to put the distributions in a form that renders their probabilistic nature transparent.

A remark on the support of the distribution functions is now in order. In principle, nothing in the definitions of the distribution functions constrains the ratio $x \equiv k^+/P^+$ to take on values between 0 and 1. The correct support of the distributions emerges, along with their probabilistic content, if one inserts into (77–79) a complete set of intermediate states $\{|n\rangle\}$ [48] (see Fig. 4). Considering, for instance, the unpolarized distribution we obtain from eq. (77)

$$f_1(x) = \frac{1}{\sqrt{2}}\sum_n \delta\big((1-x)P^+ - P_n^+\big)\,|\langle P,S|\psi_{(+)}(0)|n\rangle|^2\,, \qquad (80)$$

where $\sum_n$ incorporates the integration over the phase space of the intermediate states. Equation (80) clearly gives the probability of finding inside the nucleon a quark with longitudinal momentum k^+/P^+, irrespective of its polarization. Since the states $|n\rangle$ are physical we must have $P_n^+ \geq 0$, that is $E_n \geq |\boldsymbol{P}_n|$, and therefore $x \leq 1$. Moreover, if we exclude semi-connected diagrams like that in Fig. 4b, which correspond to $x < 0$, we are left with the connected diagram of Fig. 4a and with the correct support $0 \leq x \leq 1$. A similar reasoning applies to antiquarks.

Let us turn now to the polarized distributions. Using the Pauli–Lubanski projectors $\mathcal{P}_\pm = \frac{1}{2}\,(1 \pm \gamma_5)$ (for helicity) and $\mathcal{P}_{\uparrow\downarrow} = \frac{1}{2}(1 \pm \gamma^1\gamma_5)$ (for transverse polarization), we obtain

$$\begin{aligned}
g_1(x) &= \frac{1}{\sqrt{2}}\sum_n \delta\big((1-x)P^+ - P_n^+\big)\\
&\quad \times \Big\{\big|\langle P,S|\mathcal{P}_+\psi_{(+)}(0)|n\rangle\big|^2 - \big|\langle P,S|\mathcal{P}_-\psi_{(+)}(0)|n\rangle\big|^2\Big\}\,, && (81)
\end{aligned}$$

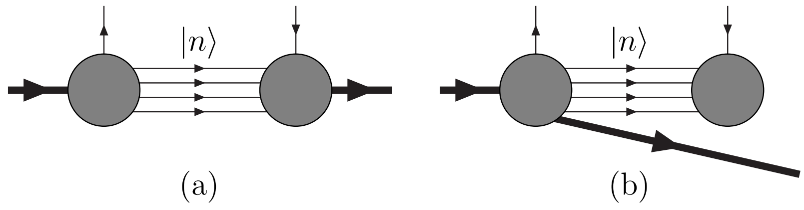

Figure 4: (a) A connected matrix element with the insertion of a complete set of intermediate states and (b) a semi-connected matrix element.

$$
\begin{aligned}
h_1(x) &= \frac{1}{\sqrt{2}} \sum_n \delta\left((1-x)P^+ - P_n^+\right) \\
&\quad \times \left\{ \left|\langle P, S|\mathcal{P}_\uparrow \psi_{(+)}(0)|n\rangle\right|^2 - \left|\langle P, S|\mathcal{P}_\downarrow \psi_{(+)}(0)|n\rangle\right|^2 \right\} . \qquad (82)
\end{aligned}
$$

These expressions exhibit the probabilistic content of the leading-twist polarizzed distributions $g_1(x)$ and $h_1(x)$: $g_1(x)$ is the number density of quarks with helicity $+$ minus the number density of quarks with helicity $-$ (assuming the parent nucleon to have helicity $+$); $h_1(x)$ is the number density of quarks with transverse polarization $\uparrow$ minus the number density of quarks with transverse polarization $\downarrow$ (assuming the parent nucleon to have transverse polarization $\uparrow$). It is important to notice that h_1 admits an interpretation in terms of probability densities only in the transverse polarization basis.

7 The transversity distribution

Let us focus on the "third" parton density, the transversity distribution h_1, eq. (71). Its main properties are: *i)* it is *chirally-odd* and therefore does not appear in the handbag diagram of inclusive DIS, which cannot flip the chirality; in order to measure h_1, the chirality must be flipped twice, so one always needs two hadrons, both in the initial state, or one in the initial state and one in the final state, and at least one of them must be transversely polarized ; *ii)* there is no gluon transversity distribution: this would imply a helicity-flip gluon-nucleon amplitude, which does not exist since gluons have helicity ± 1 and the nucleon cannot undergo an helicity change of two units.

The DGLAP equations for h_1 have been worked out at leading order by Artru and Mekhfi [4], and years later at next-to-leading order by various authors [49, 50, 51]. There are two noteworthy features of the evolution of h_1: first of all, since there is no gluon transversity distribution, h_1 does not mix with gluons and evolves as a non-singlet density [4]; second, at low x, h_1 is suppressed by the evolution with respect to g_1 [52]. This has important consequences for those observables that involve h_1 at low x and large Q^2, such as the Drell-Yan double transverse asymmetry at collider energies

[53].

The transversity distribution satisfies a bound discovered by Soffer [54]:

$$|h_1(x)| \leq \frac{1}{2}\left[f_1(x) + g_1(x)\right]. \tag{83}$$

This inequality, which is derived in the context of the parton model from the expressions of the distribution functions in terms of quark-nucleon forward amplitudes, is strictly preserved in leading-order QCD [52, 55]. At next-to-leading order, parton densities are not univoquely defined, but a regularization scheme can be chosen such that the Soffer inequality is still valid [51].

The integral of $\Phi(x)$ over x gives the local matrix element $\langle P, S|\bar{\psi}(0)\psi(0)|P, S\rangle$, which can be parametrised in terms of the vector, axial and tensor charge of the nucleon. In particular, the tensor charge (that we call δq, for the flavour q) is given by the matrix element of the operator $\bar{\psi}\mathrm{i}\sigma^{\mu\nu}\gamma_5\psi$,

$$\langle P, S|\bar{\psi}_q(0)\mathrm{i}\sigma^{\mu\nu}\gamma_5\psi_q(0)|P, S\rangle = 2\,\delta q\,(S^\mu P^\nu - S^\nu P^\mu)\,, \tag{84}$$

and is related to the transversity distributions as follows

$$\int_0^1 \mathrm{d}x\,[h_1^q(x) - \bar{h}_1^q(x)] = \delta q\,. \tag{85}$$

Note that, due to the charge-conjugation properties of $\bar{\psi}\mathrm{i}\sigma^{\mu\nu}\gamma_5\psi$, which is a C-odd operator, the tensor charge is the first moment of a flavour non-singlet combination (quarks minus antiquarks).

It is now time to discuss the crucial distinction between *transverse spin* and *transverse polarization* [5]. The transverse spin operator (i.e., the generator of rotations) for a quark is $\Sigma_\perp = \gamma_5\gamma_0\gamma_\perp$, and does not commute with the free quark Hamiltonian $H_0 = \alpha_z p_z$. Thus, there are no common eigenstates of $\Sigma_\perp$ and H_0: said otherwise, in a transversely polarized nucleon quarks cannot be in a definite *transverse spin* state (the distribution related to $\Sigma_\perp$, called $g_T(x)$, is a twist-three quantity that reflects a complicated quark-gluon dynamics with no partonic interpretation). On the other hand, the transversity distribution h_1 carries information about the *transverse polarization* of quarks inside a transversely polarized nucleon. The transverse polarization operator is $\Pi_\perp = \frac{1}{2}\gamma_0\Sigma_\perp$, and commutes with H_0, owing to the presence of an extra γ_0. Therefore, in a transversely polarized nucleon, quarks may exist in a definite transverse polarization state, and a simple partonic picture applies to h_1.

The argument above shows that the integral $\int \mathrm{d}x\,(h_1^q + \bar{h}_1^q)$ does not represent the quark + antiquark contribution to the transverse spin of the nucleon. A transverse spin sum rule containing the first moment of $h_1 + \bar{h}_1$ has been derived in Ref. [56] within the parton model, but, in the light of what we have just said and of other general considerations, is quite controversial (see the discussion in Ref. [57]). A sum rule for the total angular momentum of transversely polarized quarks in an unpolarized hadron [58, 59], involving the generalized parton distributions, will be introduced in Sec. 14.

8 Transverse-momentum dependent distribution functions (TMD's)

In the quark momentum

$$k^\mu = xP^+ n_+^\mu + \frac{k^2 + \boldsymbol{k}_\perp^2}{2xP^+}\, n_-^\mu + k_\perp^\mu, \tag{86}$$

$k^+ = xP^+$ is the dominant component, whereas $\boldsymbol{k}_\perp$ and k^- are suppressed by one and two powers of $1/P^+$, respectively. Retaining the subleading transverse part, the quark momentum can be written as

$$k^\mu \simeq xP^\mu + k_\perp^\mu\,. \tag{87}$$

We will see that at leading twist there are eight transverse-momentum distributions (TMD's): three of them, once integrated over $\boldsymbol{k}_\perp$, yield f_1, g_1, h_1; the remaining five are new and vanish upon $\boldsymbol{k}_\perp$ integration.

Integrating $\Phi(k, P, S)$ over k^+ and k^- only, one obtains the $\boldsymbol{k}_\perp$-dependent correlation matrix

$$\Phi(x, \boldsymbol{k}_\perp) = \int \mathrm{d}k^+ \int \mathrm{d}k^-\, \Phi(k, P, S)\, \delta(k^+ - xP^+)\,, \tag{88}$$

which contains the TMD's. The field-theoretical expression of $\Phi(x, \boldsymbol{k}_\perp)$ [60] turns out to be quite complicated due to the structure of the gauge link, which now connects two space-time points with a transverse separation. One has [61, 62]

$$\begin{aligned}\Phi(x, \boldsymbol{k}_\perp) \;=\; & \int \frac{\mathrm{d}\xi^-}{2\pi} \int \frac{\mathrm{d}^2\boldsymbol{\xi}_\perp}{(2\pi)^2}\, \mathrm{e}^{\mathrm{i}xP^+\xi^-}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp} \\ & \times \langle P, S|\bar{\psi}(0)\, \mathcal{W}^-[0,\infty]\, \mathcal{W}^\perp[0_\perp, \infty_\perp]\, \mathcal{W}^\perp[\infty_\perp, \xi_\perp]\, \mathcal{W}^-[\infty, \xi]\psi(\xi)|P, S\rangle|_{\xi^+=0}, \end{aligned} \tag{89}$$

with two longitudinal Wilson lines directed along n^-,

$$\mathcal{W}^-[0,\infty] = \mathcal{P}\,\exp\left(-\mathrm{i}g \int_0^{+\infty} \mathrm{d}z^- A^+(0, z^-, \mathbf{0}_\perp)\right)\,, \tag{90}$$

$$\mathcal{W}^-[\infty,\xi] = \mathcal{P}\,\exp\left(-\mathrm{i}g \int_{+\infty}^{\xi^-} \mathrm{d}z^- A^+(0, z^-, \boldsymbol{\xi}_\perp)\right)\,, \tag{91}$$

from $(0, 0, \mathbf{0}_\perp)$ to $(0, \infty, \mathbf{0}_\perp)$ and from $(0, \infty, \boldsymbol{\xi}_\perp)$ to $(0, \xi^-, \boldsymbol{\xi}_\perp)$, and two Wilson lines $\mathcal{W}^\perp$ at $\xi^- = \infty$ containing the transverse gluon field $A_\perp^\mu$ (Fig. 5).

$$\mathcal{W}^T[0_\perp, \infty_\perp] = \mathcal{P}\,\exp\left(-\mathrm{i}g \int_{0_\perp}^{\infty_\perp} \mathrm{d}z_\perp \cdot A_\perp(0, \infty, \boldsymbol{z}_\perp)\right)\,, \tag{92}$$

$$\mathcal{W}^T[\infty_\perp, \xi_\perp] = \mathcal{P}\,\exp\left(-\mathrm{i}g \int_{\infty_\perp}^{\xi_\perp} \mathrm{d}z_\perp \cdot A_\perp(0, \infty, \boldsymbol{z}_\perp)\right)\,. \tag{93}$$

This link structure, with the longitudinal Wilson lines $\mathcal{W}^-$ running to $\xi^- = +\infty$, applies to semi-inclusive deep inelastic scattering. In Drell-Yan processes, the Wilson

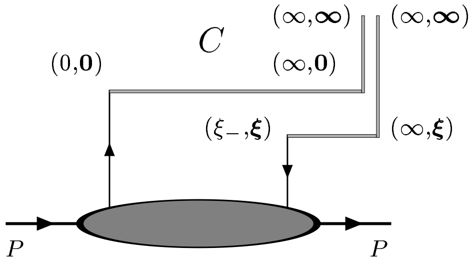

Figure 5: The gauge-link structure of TMD distributions in SIDIS.

line runs to $-\infty$ and this may change the sign of the distributions, as we will discuss later (Sec. 9).

Note the presence in (89) of the transverse links, which survive in the light-cone gauge $A^+ = 0$, enforcing gauge invariance under residual gauge transformations. These transverse lines are responsible for the final-state (or initial-state) interactions that generate some TMD distributions otherwise forbidden by time-reversal invariance (the so-called T-odd distributions, see below). In non singular gauges, on the contrary, the gauge potential vanishes at infinity and one is left with the longitudinal links. It is known that in this case there are light-cone logarithmic divergences arising in the limit $z^+ \to 0$ [60] due to contributions of virtual gluons with zero plus momentum, i.e., with infinitely negative rapidity. One way to avoid these singularities is to use Wilson lines slightly displaced from the light-like direction. This introduces a dependence of the TMD distributions on a new scalar quantity, $\zeta^2 = (2P \cdot v)^2/v^2$ (v is a vector slightly off the light-cone), acting as a rapidity cutoff. The light-cone divergences now appear as large logarithms of ζ, which are resummed by the so-called Collins-Soper equation [60, 63]. A lucid presentation of this subject is contained in Ref. [44]

Let us come back to the quark correlator. At leading twist, $\Phi(x, \boldsymbol{k}_\perp)$ has the following structure [20, 30]

$$\begin{aligned}\Phi(x, \boldsymbol{k}_\perp) \;&=\; \frac{1}{2}\Bigg\{ f_1 \, \not{n}_+ - f_{1T}^\perp \, \frac{\epsilon_\perp^{ij} k_{\perp i} S_{\perp j}}{M} \, \not{n}_+ + \left(S_\parallel g_{1L} + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M} \, g_{1T} \right) \gamma_5 \not{n}_+ \\ &+ h_{1T} \frac{[\not{S}_\perp, \not{n}_+]\gamma_5}{2} + \left(S_\parallel h_{1L}^\perp + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M} \, h_{1T}^\perp \right) \frac{[\not{k}_\perp, \not{n}_+]\gamma_5}{2M} + \mathrm{i} h_1^\perp \frac{[\not{k}_\perp, \not{n}_+]}{2M} \Bigg\}, \quad (94)\end{aligned}$$

where $\epsilon_\perp^{ij}$ is the two-dimensional antisymmetric Levi-Civita tensor, with $\epsilon_\perp^{12} = 1$. By tracing $\Phi(x, \boldsymbol{k}_\perp)$ with Dirac matrices, $\Phi^{[\Gamma]} \equiv \frac{1}{2}\mathrm{Tr}\,(\Gamma\Phi)$, one gets

$$
\begin{aligned}
\Phi^{[\gamma^+]} &= f_1(x, k_\perp^2) - \frac{\epsilon_\perp^{ij} k_{\perp i} S_{\perp j}}{M} f_{1T}^\perp(x, k_\perp^2)\,, &(95)\\
\Phi^{[\gamma^+\gamma_5]} &= S_\parallel g_{1L}(x, k_\perp^2) + \frac{\boldsymbol{k}_\perp \cdot \boldsymbol{S}_\perp}{M} g_{1T}(x, k_\perp^2)\,, &(96)\\
\Phi^{[\mathrm{i}\sigma^{i+}\gamma_5]} &= S_\perp^i h_1(x, k_\perp^2) + S_\parallel \frac{k_\perp^i}{M} h_{1L}^\perp \\
&\quad - \frac{k_\perp^i k_\perp^j + \frac{1}{2} k_\perp^2 g_\perp^{ij}}{M^2} S_{Tj}\, h_{1T}^\perp(x, k_\perp^2) - \frac{\epsilon_\perp^{ij} k_{\perp j}}{M} h_1^\perp(x, k_\perp^2)\,, \quad i = 1, 2 &(97)
\end{aligned}
$$

where $h_1 \equiv h_{1T} + (k_\perp^2/2M) h_{1T}^\perp$. The three quantities $\Phi^{[\gamma^+]}$, $\Phi^{[\gamma^+\gamma_5]}$ and $\Phi^{[\mathrm{i}\sigma^{i+}\gamma_5]}$ represent the probabilities of finding an unpolarized, a longitudinally polarized and a transversely polarized quark, respectively, with momentum fraction x and transverse momentum $\boldsymbol{k}_\perp$. In eqs. (95-97) we count eight independent TMD's: $f_1, f_{1T}^\perp, g_1, g_{1T}, h_1, h_{1L}^\perp, h_{1T}^\perp, h_1^\perp$. Upon integration over $\boldsymbol{k}_\perp$, only three of these, $f_1(x, k_\perp^2), g_1(x, k_\perp^2), h_1(x, k_\perp^2)$, survive, yielding the x-dependent leading-twist distributions $f_1(x)$, $g_1(x)$, $h_1(x)$.

A remark about the notation is in order. In the "Amsterdam classification" of TMD's [20], the letters f, g, h refer to unpolarized, longitudinally polarized, and transversely polarized distributions, respectively (as first proposed by Jaffe and Ji [5, 6]). The subscript 1 labels the leading twist. Subscripts L and T indicate that the parent hadron is longitudinally or transversely polarized. A superscript $\perp$ signals the presence of $k_\perp^i$ factors in the quark correlation function.

From eq. (97) we see that the spin asymmetry of transversely polarized quarks inside a transversely polarized nucleon is given not only by the unintegrated transversity $h_1(x, k_\perp^2)$, but also by the TMD $h_{1T}^\perp(x, k_\perp^2)$, which has been given the name of "pretzelosity", as it is somehow related to the non-sphericity of the nucleon shape [64] (for a review of the properties of $h_{1T}^\perp$, see Ref. [65]). Both h_1 and $h_{1T}^\perp$ contribute to single-spin asymmetries in SIDIS with a transversely polarized target, but with different angular distributions, $\sin(\phi_h + \phi_S)$ the former, $\sin(3\,\phi_h - \phi_S)$ the latter, where ϕ_h and ϕ_S are the azimuthal orientations of the final hadron momentum and of the nucleon spin vector, respectively. Note that, due to the intrinsic transverse motion, quarks can also be transversely polarized in a longitudinally polarized nucleon ($h_{1L}^\perp$), and longitudinally polarized in a transversely polarized nucleon (g_{1T}).

9 The T-odd couple: Sivers and Boer-Mulders distributions

Equation (95) shows that the probability of finding an unpolarized quark with longitudinal momentum fraction x and transverse momentum $\boldsymbol{k}_\perp$ inside a transversely polarized nucleon ($\hat{\boldsymbol{P}} \equiv \boldsymbol{P}/|\boldsymbol{P}|$) is

$$f_{q/N^\uparrow}(x, \boldsymbol{k}_\perp) = f_1(x, k_\perp^2) - \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}_\perp}{M} f_{1T}^\perp(x, k_\perp^2)\,, \tag{98}$$

where $\hat{\boldsymbol{P}} \equiv \boldsymbol{P}/|\boldsymbol{P}|$. From eq. (98) we can construct the azimuthal asymmetry

$$f_{q/N^\uparrow}(x, \boldsymbol{k}_\perp) - f_{q/N^\uparrow}(x, -\boldsymbol{k}_\perp) = -2\,\frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}_\perp}{M} f_{1T}^\perp(x, k_\perp^2)\,, \tag{99}$$

which is proportional to $f_{1T}^\perp$, called the *Sivers function* [16, 17]. A non vanishing $f_{1T}^\perp$ signals that unpolarized quarks in a transversely polarized nucleon have a preferential motion direction: in particular, $f_{1T}^\perp > 0$ means that in a nucleon moving along $+\hat{z}$ with transverse polarization in the $+\hat{y}$ direction, unpolarized quarks tend to move to the right, i.e. towards $-\hat{x}$.

Specularly, the distribution of transversely polarized quarks inside an unpolarized nucleon is [66]

$$f_{q^\uparrow/N}(x, \boldsymbol{k}_\perp) = \frac{1}{2}\left[f_1(x, k_\perp^2) - \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}_{q\perp}}{M} h_1^\perp(x, k_\perp^2)\right]\,, \tag{100}$$

and from this we get a spin asymmetry of the form

$$f_{q^\uparrow/N}(x, \boldsymbol{k}_\perp) - f_{q^\downarrow/N}(x, \boldsymbol{k}_\perp) = -\frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}_{q\perp}}{M} h_1^\perp(x, k_\perp^2)\,, \tag{101}$$

which is proportional to $h_1^\perp$, the *Boer–Mulders distribution* [30]. Positivity bounds for $f_{1T}^\perp$ and $h_1^\perp$ were derived in Ref. [67]. Note that in the literature (see Ref. [2] and bibliography therein) one also encounters the notation

$$\Delta^N f_{q/p^\uparrow} \equiv -\frac{2|\boldsymbol{k}_\perp|}{M} f_{1T}^{\perp q}\,, \quad \Delta^N f_{q^\uparrow/p} \equiv -\frac{|\boldsymbol{k}_\perp|}{M} h_1^{\perp q}\,. \tag{102}$$

The Sivers and Boer-Mulders functions are associated with the time-reversal (T) odd correlations $(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}_\perp$ and $(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}_{q\perp}$, hence the name of "$T$-odd distributions". To see the implications of time-reversal invariance, let us recall the operator definition of these distributions, e.g., the Sivers function:

$$\begin{aligned} f_{1T}^\perp(x, k_\perp^2) \quad \sim \quad & \int \mathrm{d}\xi^- \int \mathrm{d}^2\boldsymbol{\xi}_\perp\, \mathrm{e}^{\mathrm{i}xP^+\xi^- - \mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp} \\ & \times \langle P, S_\perp | \overline{\psi}(0)\gamma^+ \mathcal{W}[0,\xi]\psi(\xi)|P, S_\perp\rangle|_{\xi^+=0}. \end{aligned} \tag{103}$$

If the overall Wilson link $\mathcal{W}$ is naïvely set to unity, the matrix element in (103) changes sign under time reversal, and the Sivers function must therefore be zero [11]. On the other hand, a direct calculation [27] in a spectator model shows that $f_{1T}^\perp$ is non vanishing: gluon exchange between the struck quark and the target remnant generates a non-zero Sivers asymmetry (the presence of a quark transverse momentum smaller than Q ensures that this asymmetry is proportional to $M/k_\perp$, rather than to M/Q, and therefore is a leading-twist observable). The puzzle is solved by carefully considering the Wilson line in eq. (103) [29]. We have seen in fact that $\mathcal{W}$ includes transverse links at infinity that do not reduce to unity in the light-cone gauge [62]. Since time reversal changes a future-pointing Wilson line into a past-pointing Wilson line, T-invariance, rather than constraining $f_{1T}^{\perp}$ to zero, gives a relation between processes that probe Wilson lines pointing in opposite time directions. In particular, since in SIDIS the Sivers asymmetry arises from the interaction between the spectator and the outgoing quark, whereas in Drell-Yan production it arises from the interaction between the spectator and an incoming quark, one gets

$$f_{1T}^{\perp}(x, k_\perp^2)_{\mathrm{SIDIS}} = -f_{1T}^{\perp}(x, k_\perp^2)_{\mathrm{DY}} \,. \tag{104}$$

A similar relation holds for the Boer-Mulders function $h_1^{\perp}$. Eq, (104) is an example of the “time-reversal modified universality” of distribution functions in SIDIS, Drell-Yan production and e^+e^- annihilation studied in Ref. [68] The relation (104) is a direct consequence of the gauge structure of parton distribution functions, and its experimental check would be extremely important.

Gauge link patterns of hadroproduction processes are more complicated and do not result in a simple sign flip of TMD's [69, 70, 71, 72]. An assumption often made is that the transverse-momentum factorization holds with TMD's containing process-dependent Wilson lines (the so-called “generalized transverse-momentum factorization, to be compared with standard transverse-momentum factorization, where TMD's are strictly universal quantities). Recent studies show that not only the standard factorization, but also the generalized factorization, does not hold in hadroproduction of back-to-back jets or hadrons [73].

10 Gluonic TMD's

So far we have only discussed quark (and antiquark) distributions. Let us now consider gluons. If the transverse momenta are integrated over, there are gluonic f_1 and g_1 distributions[2], but not a gluonic h_1, for the reason explained in Sec. 5 (at twist three, on the other hand, there is a polarized gluon distribution in transversely polarized hadrons, analogous to $g_T(x)$ [74]).

The panorama of transverse-momentum dependent gluon distributions is of course much richer. It was explored by Mulders and Rodrigues [75] and can be summarised as follows. There are eight gluon TMD's: four of them, labelled by the letter G, are diagonal in the gluon helicities and represent unpolarized or circularly polarized gluons;

[2]In the DIS context, they are usually called $g(x)$, or $G(x)$, and $\Delta g(x)$, or $\Delta G(x)$, respectively.

the remaining four, labelled by the letter H, flip the gluon helicity, and describe linearly polarized gluons in unpolarized or polarized hadrons.

One of these TMD's, $G_T(x, k_\perp^2)$, corresponds exactly to the Sivers function, that is, represents the distribution of unpolarized gluons inside a transversely polarized hadron. Thus, instead of $G_T(x, k_\perp^2)$, it is more often denoted by $f_{1T}^{\perp g}(x, k_\perp^2)$.

Two other noteworthy TMD gluon distributions are: $H^\perp(x, k_\perp^2)$, the distribution of linearly polarized gluons in an unpolarized hadron, somehow similar (but not exactly equivalent) to the Boer-Mulders function; $\Delta H_T'(x, k_\perp^2)$, the distribution of linearly polarized gluons in a transversely polarized hadron, which plays a rôle similar to transversity.

Polarized TMD gluon distributions are probed at leading order in hadroproduction processes.

11 Constraints on the TMD's

Non trivial positivity bounds for the TMD's were derived in Ref. [67]. They read

$$\frac{k_\perp^2}{M^2}\left[(f_{1T}^\perp)^2 + (g_{1T})^2\right] \le (f_1)^2 - (g_{1L})^2 \,, \tag{105}$$

$$\frac{k_\perp^2}{M^2}\left[(h_1^\perp)^2 + (h_{1L}^\perp)^2\right] \le (f_1)^2 - (g_{1L})^2 \,, \tag{106}$$

$$|h_{1T}^\perp| \le \frac{1}{2}\,(f_1 - g_{1L})\,. \tag{107}$$

Eliminating some TMD's and relaxing the bounds, one gets the intuitive constraints (remember that $f_{1T}^\perp$, $\boldsymbol{h}_1^\perp$ and $h_{1T}^\perp$ are asymmetries, that is differences of parton densities)

$$\frac{k_\perp}{M}\,|f_{1T}^\perp| \le f_1 \,, \quad \frac{k_\perp}{M}\,|h_1^\perp| \le f_1 \,, \quad |h_{1T}^\perp| \le f_1 \,. \tag{108}$$

A sum rule for the Sivers function was derived in QCD by Burkardt [76, 77], who showed that the sum of all contributions to the average transverse momentum of unpolarized partons in a transversely polarized target (that is, the average transverse momentum induced by the Sivers effect), must vanish:

$$\sum_{a=q,\bar{q},g} \langle \boldsymbol{k}_\perp^a \rangle|_{\rm Sivers} = 0\,. \tag{109}$$

In terms of the Sivers function, the condition (109) becomes [78]

$$\sum_{a=q,\bar{q},g} \int_0^1 \mathrm{d}x\, f_{1T}^{\perp(1)\,a}(x) = 0\,, \tag{110}$$

where we have introduced the first $k_\perp^2$-moment of $f_{1T}^\perp$,

$$f_{1T}^{\perp(1)}(x) \equiv \int \mathrm{d}^2\boldsymbol{k}_\perp\, \frac{k_\perp^2}{2M^2}\, f_{1T}^\perp(x, k_\perp^2)\,. \tag{111}$$

Although some QCD aspects, such as ultraviolet divergences and light-cone singularities, were not considered in the original derivation, the result (111) is likely to be valid in general. In Ref. [79] it has been shown that the Burkardt sum rule is fulfilled for a quark target in perturbative QCD at one-loop order.

From a phenomenological viewpoint, the importance of eq. (110) is that one can infer the size of the gluon Sivers function from fits to SIDIS observables involving the quark and antiquark Sivers functions [80, 81].

12 Higher-twist distributions

At twist three (suppressed by $1/Q$, i.e., by $1/P^+$ in the infinite momentum frame, with respect to leading twist), the quark correlator $\Phi(x)$ admits the general decomposition [5, 6]

$$\Phi(x)|_{\text{twist}\,3} = \frac{M}{2P^+}\left\{ e(x) + g_T(x)\gamma_5 \not{S}_\perp + S_\parallel\, h_L(x)\, \frac{[\not{n}_+, \not{n}_-]\gamma_5}{2} \right\}, \tag{112}$$

displaying the three distribution functions $e(x), g_T(x), h_L(x)$. In particular, $g_T(x)$ contributes to the polarized DIS structure function $g_2(x, Q^2)$ (see, e.g., Ref. [82]). Higher-twist distributions do not have a probabilistic interpretation. They involve in fact both good and bad components of the quark fields, so the procedure leading to expressions such as eqs. (80-82) cannot be applied. We will see in Sec. 13 that higher-twist effects reflect quark-gluon correlations.

The structure of the $\boldsymbol{k}_\perp$-dependent quark correlator $\Phi(x, \boldsymbol{k}_\perp)$ at twist three has been studied by various authors [20, 30, 83, 84]. It is now known that there are 16 twist-three TMD's (we use the nomenclature of [85]): e, $e_T^\perp$, e_L, e_T, f_T, $f_L^\perp$, $f_T^\perp$, $f^\perp$, g_T, $g_L^\perp$, $g_T^\perp$, g^T, h_L, h_T, h, $h_T^\perp$. Among these, the T-odd functions are: $e_T^\perp$, e_L, e_T, f_T, $f_L^\perp$, $f_T^\perp$, $g^\perp$, h.

Four distributions, namely $g^\perp, e_T^\perp, f_T, f_T^\perp$, not identified in earlier studies, exist because the Wilson line in the quark correlator provides an extra independent vector (n_-) for the Lorentz decomposition of $\Phi(x, \boldsymbol{k}_\perp)$.

If we integrate $\Phi(x, \boldsymbol{k}_\perp)$ over $\boldsymbol{k}_\perp$, the only non vanishing distributions are the three T-even functions in eq. (112). In fact, the TMD's with a $\perp$ superscript give zero due to the presence of $k_\perp^i$ factors, while the T-odd functions f_T, e_L, h vanish due to the time-reversal invariance of QCD [11, 84].

Concerning twist four, the integrated parton distributions were first identified in Refs. [5, 6, 86]. More recently, the complete description of the $\boldsymbol{k}_\perp$-dependent correlator $\Phi(x, \boldsymbol{k}_\perp)$ has been presented by Goeke, Metz and Schlegel [84], who have shown that up to twist four there are in total 32 TMD's. The unintegrated correlation matrix Φ is also composed of 32 Lorentz-scalar structures: 12 amplitudes associated to the four-vectors k, P, S and 20 amplitudes associated to n_-. The number of distribution functions being equal to the number of amplitudes of Φ, all the TMD's are independent and there are no general relations among them. In earlier studies [87, 20], some "Lorentz-invariance relations" (LIR's) were derived from an expansion of Φ that did not take into account the amplitudes associated to the gauge link vector n_-. Two of these relations are

$$g_T(x) = g_1(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{\perp(1)}(x)\,, \qquad h_L(x) = h_1(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)}(x)\,, \tag{113}$$

where the transverse moments of g_{1T} and $h_{1L}^{\perp}$ are defined as in eq. (111). The presence of the n_--dependent amplitudes invalidate the LIR's, which are not valid in QCD [88, 89]. However, they approximately hold as as far as quark-gluon interactions (and quark mass terms) are neglected [90]. This is known as the "generalized Wandzura-Wilczek (WW) approximation" [87, 20, 91, 90] and represents an extension of the original WW approximation [92], which relates the polarized DIS structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$ [93, 82, 94].

A general remark about higher-twist distributions is in order. While the distributions $e(x)$, $g_T(x)$, $h_L(x)$ – or, to be precise, the corresponding quark-gluon correlators – enter into the collinear twist-three factorization theorem of QCD [33, 34], the $k_\perp$-dependent higher-twist distributions are employed in factorization formulas that lack a solid QCD foundation. Thus, they should rather be interpreted as a way to model subleading effects.

13 Quark-gluon correlations

Higher-twist effects in hard processes are determined by quark-gluon correlations inside the hadrons [95, 96]. Referring to Fig. 6, we denote by k and k' the momenta of the outgoing and of the incoming quark, respectively, and by $x = k^+/P^+$ and $x' = k'^+/P^+$ their longitudinal momentum ratios. The momentum of the gluon is $k_g = k' - k$, with $x_g = x' - x = k_g^+/P^+$. Let us introduce the quark-gluon correlation matrix $\Phi_D^\mu(x, x')$ [6] containing the transverse covariant derivative $D_\perp^\mu = \partial_\perp^\mu - \mathrm{i}gA_\perp^\mu$,

$$\begin{aligned}\Phi_D^\mu(x, x') \;&=\; \int \frac{\mathrm{d}\xi^-}{2\pi} \int \frac{\mathrm{d}\eta^-}{2\pi} \mathrm{e}^{\mathrm{i}xP^+\xi^-} \mathrm{e}^{\mathrm{i}(x'-x)P^+\eta^-} \\ &\quad \times \langle P, S| \bar{\psi}(0)\, \mathcal{W}^-[0,\eta]\, \mathrm{i}D_\perp^\mu(\eta) \mathcal{W}^-[\eta,\xi]\, \psi(\xi) |P, S\rangle\,, \end{aligned} \tag{114}$$

with $\xi^+ = \eta^+ = 0$ and $\boldsymbol{\xi}_\perp = \boldsymbol{\eta}_\perp = \mathbf{0}$. The general decomposition of Φ_D^μ is ($\epsilon_\perp^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma} n_{+\rho} n_{-\sigma}$)

$$\begin{aligned}\Phi_D^\mu(x, x') \;&=\; \frac{M}{2P^+} \Big\{ G_D(x, x') \mathrm{i}\epsilon_\perp^{\mu\nu} S_{T\nu} \not{n}_+ + \widetilde{G}_D(x, x') S_\perp^\mu \gamma_5 \not{n}_+ \\ &\quad + H_D(x, x') S_\parallel \gamma_5 \gamma_\perp^\mu \not{n}_+ + E_D(x, x') \gamma_\perp^\mu \not{n}_+ \Big\}\,, \end{aligned} \tag{115}$$

and displays the four twist-three quark-gluon correlation functions G_D, $\widetilde{G}_D$, H_D, E_D. Time-reversal invariance implies that they are real. By hermiticity, $\widetilde{G}_D$ and H_D are

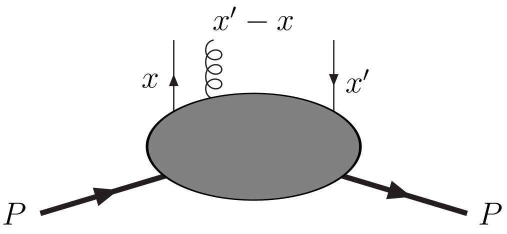

Figure 6: The quark–gluon correlation matrix.

symmetric, whereas G_D and E_D are antisymmetric under interchange of x and x'. Integrating $\Phi_D^\mu(x,x')$ over x', one gets a quark-gluon correlation matrix $\Phi_D^\mu(x)$, where one of the quark fields and the covariant derivative are evaluated at the same space-time point. By means of the QCD equations of motion, this matrix can be related to the quark correlation matrix $\Phi(x)$ at twist three, eq. (112), with the following identifications

$$\begin{aligned} g_T(x) &= \frac{1}{x}\int \mathrm{d}x' \, [G_D(x,x') + \widetilde{G}_D(x,x')] \,, && (116)\\ h_L(x) &= \frac{2}{x}\int \mathrm{d}x' \, H_D(x,x') \,, && (117)\\ e(x) &= \frac{2}{x}\int \mathrm{d}x' \, E_D(x,x') \,. && (118) \end{aligned}$$

Here we have neglected quark mass (m) corrections, which would give additional contributions of the type $(m/M)h_1$, $(m/M)g_1$, $(m/M)f_1$, to g_T, h_L, e, respectively. From eqs. (116-118) we see that the two-point functions G_D, $\widetilde{G}_D$, H_D, E_D are more fundamental than the one-point functions g_T, h_L, e, introduced in Sec. 12.

One can introduce another set of quark-gluon correlation functions, related to the D-type functions of eq. (115). They are contained in the correlation matrix $\Phi_F^\mu(x,x')$, defined as Φ_D^μ with the covariant derivative $\mathrm{i}D_\perp^\mu$ replaced by the gluon field strength $gF^{+\mu}$. The decomposition of Φ_F^μ is

$$\begin{aligned} \Phi_F^\mu(x,x') &= \frac{M}{2}\Big\{ G_F(x,x')\epsilon_\perp^{\nu\mu} S_{\perp\nu} \not{n}_+ + \tilde{G}_F(x,x')\mathrm{i}S_\perp^\mu \gamma_5 \not{n}_+ \\ &\quad + H_F(x,x')\mathrm{i}S_\parallel \gamma_5 \gamma_\perp^\mu \not{n}_+ + E_F(x,x')\mathrm{i}\gamma_\perp^\mu \not{n}_+ \Big\} \,, \qquad (119) \end{aligned}$$

The F-type correlators[3] G_F, $\widetilde{G}_F$, H_F, E_F are real and symmetric (G_F and E_F) or antisymmetric ($\widetilde{G}_F$ and H_F) functions of their arguments.

It is this set of correlators that appear in the twist-three factorization formulas. The F-type functions are not independent: it is possible to show in fact [99] that they

[3]In the literature [33, 35, 36, 97, 98], G_F, $\widetilde{G}_F$, H_F, E_F are also called T_F, $\widetilde{T}_F$, $\widetilde{T}_F^{(\sigma)}$, $T_F^{(\sigma)}$, respectively, but their normalisation varies from paper to paper.

are related to the D-type functions as follows

$$\begin{aligned} G_D(x,x') &= P\,\frac{1}{x-x'}\,G_F(x,x')\,, && (120)\\ \tilde{G}_D(x,x') &= P\,\frac{1}{x-x'}\,\tilde{G}_F(x,x') + \delta(x-x')\,g_{1T}^{(1)}(x)\,, && (121)\\ E_D(x,x') &= P\,\frac{1}{x-x'}\,E_F(x,x')\,, && (122)\\ \tilde{H}_D(x,x') &= P\,\frac{1}{x-x'}\,\tilde{H}_F(x,x') + \delta(x-x')\,h_{1L}^{\perp(1)}(x)\,, && (123) \end{aligned}$$

where P stands for the principal value and $g_{1T}^{(1)}, h_{1L}^{\perp(1)}$ are the first transverse moments of g_{1T} and $h_{1L}^{\perp}$, defined as in eq. (111). If we integrate eqs. (120-123) over x', we get [20]

$$g_T(x) = \frac{g_{1T}^{(1)}(x)}{x} + \tilde{g}_T(x)\,, \quad h_L(x) = -2\,\frac{h_{1L}^{\perp(1)}(x)}{x} + \tilde{h}_L(x)\,, \quad e(x) = \tilde{e}(x)\,. \qquad (124)$$

having denoted by a tilde some genuinely twist-three distributions related to the F-type correlation functions.

Ignoring the contributions of tilde functions (and of quark mass terms) is what we have called the generalized Wandzura-Wilczek (WW) approximation, which relates twist-three distributions to twist-two distributions. This approximation has been worked out by various authors [87, 20, 91, 90] and also applied in phenomenological analyses [100].

In QCD the quark-gluon correlation functions acquire a dependence on a scale μ. The equations governing the evolution in μ have been recently written down and solved [101, 97].

14 Generalized parton distributions

The generalized parton distributions (GPD's), which are related to non-forward quark-quark (or gluon-gluon) correlators, emerge in the description of hard exclusive processes, such as deeply-virtual Compton scattering and exclusive meson production, characterized by a non-zero momentum transfer to the target nucleon [102, 103, 104, 105, 106, 107]. Here we will be mostly concerned with the relations existing between the GPD's and the transverse spin distributions (for more details, see Ref. [108]).

The kinematics of GPD's is represented in fig. 7 (we follow the conventions of [105]). The momenta of the incoming and the outgoing nucleon are $p = P - \frac{1}{2}\Delta$ and $p' = P + \frac{1}{2}\Delta$, respectively. The momentum transfer squared is $t = \Delta^2$. The GPD's depend on t and on two light-cone momentum ratios: $x = k^+/P^+$ and $\xi = -\Delta^+/2P^+$. The variable ξ is sometimes called "skewness", and the GPD's are also known as "skewed parton distributions".

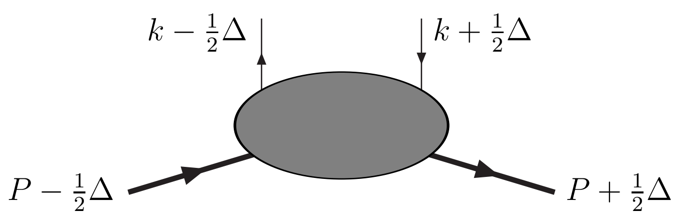

Figure 7: Kinematics of GPD's.

The quark GPD's are defined through the correlator in the helicity basis

$$F^{[\Gamma]}(x, P, \Delta; \lambda, \lambda') = \int \frac{dz^-}{4\pi}\, e^{ixP^+z^-}\, \langle p', \lambda' | \bar{\psi}(z_1) \Gamma\, \mathcal{W}^-[z_1, z_2] \psi(z_2) | p, \lambda \rangle\,, \tag{125}$$

where Γ is a Dirac matrix, $\mathcal{W}^-$ a longitudinal Wilson line connecting $z_1 \equiv (0^+, -\frac{1}{2}z^-, \mathbf{0}_\perp)$ to $z_2 \equiv (0^+, \frac{1}{2}z^-, \mathbf{0}_\perp)$, and λ (λ') is the helicity of the incoming (outgoing) nucleon. At leading twist, there are 8 GPD's [109]:

$$F^{[\gamma^+]} = \frac{1}{2P^+}\, \bar{u}(p', \lambda') \left(\gamma^+ H(x, \xi, t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}\, E(x, \xi, t) \right) u(p, \lambda)\,, \tag{126}$$

$$F^{[\gamma^+\gamma_5]} = \frac{1}{2P^+}\, \bar{u}(p', \lambda') \left(\gamma^+\gamma_5\, \widetilde{H}(x, \xi, t) + \frac{\Delta^+\gamma_5}{2M}\, \widetilde{E}(x, \xi, t) \right) u(p, \lambda)\,, \tag{127}$$

$$\begin{aligned} F^{[i\sigma^{j+}\gamma_5]} = & -\frac{i\epsilon_\perp^{ij}}{2P^+}\, \bar{u}(p', \lambda') \Bigg(i\sigma^{+i} H_T(x, \xi, t) + \frac{\gamma^+\Delta_\perp^i - \Delta^+\gamma_\perp^i}{2M}\, E_T(x, \xi, t) \\ & + \frac{P^+\Delta_\perp^i - \Delta^+P_\perp^i}{M^2}\, \widetilde{H}_T(x, \xi, t) + \frac{\gamma^+P_\perp^i - P^+\gamma_\perp^i}{M}\, \widetilde{E}_T(x, \xi, t) \Bigg) u(p, \lambda)\,. \end{aligned} \tag{128}$$

The first four, $H(x, \xi, t), E(x, \xi, t), \widetilde{H}(x, \xi, t), \widetilde{E}(x, \xi, t)$, are chirally even and are related to the familiar form factors. Integrating H, E, $\widetilde{H}$, $\widetilde{E}$ over x, in fact, one gets the Dirac, Pauli, axial and pseudoscalar form factors, respectively. The quantity

$$\int dx\, E^q(x, 0, 0) = \kappa^q\,, \tag{129}$$

is the contribution of the flavour q to the anomalous magnetic moment of the nucleon, that is, to the Pauli form factor F_2 at $t = 0$. The GPD's $H, \widetilde{H}, H_T$, taken at $\xi = t = 0$, coincide with the integrated quark distributions f_1, g_1, h_1:

$$H(x, 0, 0) = f_1(x)\,,\ \widetilde{H}(x, 0, 0) = g_1(x)\,,\ H_T(x, 0, 0) = h_1(x)\,. \tag{130}$$

The original interest in GPD's was prompted by Ji's sum rule relating the total angular momentum of quarks (in a nucleon with polarization vector $\boldsymbol{S}$) to the second moment of H and E [103]:

$$\langle J_q^i \rangle = S^i \int dx\, x\, [H(x, 0, 0) + E(x, 0, 0)]\,. \tag{131}$$

Burkardt has derived a similar decomposition for the angular momentum of quarks with transverse polarization vector $\boldsymbol{S}_q$ in an unpolarized nucleon [58, 59]:

$$\langle J_q^i(\boldsymbol{S}_q)\rangle = \frac{S_q^i}{4}\int \mathrm{d}x\, x\,[H_T(x,0,0) + 2\widetilde{H}_T(x,0,0) + E_T(x,0,0)]\,. \tag{132}$$

Here $H_T(x,0,0)$ coincides, as we have seen, with transversity, whereas the combination $2\widetilde{H}_T + E_T$ appears in the impact-parameter description of the Boer-Mulders effect (see Sec. 15).

Note in conclusion that there are no direct and model-independent connections between the GPD's and the TMD distributions, as stressed in Refs. [108, 110]. GPD's are instead directly related to the distribution functions in the impact-parameter space.

15 Distribution functions in the impact-parameter space

In the impact-parameter space one can get a more intuitive picture of some transverse spin and transverse momentum effects. To define the impact-parameter dependent distributions (IPD's), we first introduce nucleon states localized at a transverse position $\boldsymbol{R}_\perp$, by means of an inverse Peierls-Yoccoz projection:

$$|P^+, \boldsymbol{R}_\perp; S\rangle = \mathcal{N}\int \frac{\mathrm{d}^2\boldsymbol{P}_\perp}{(2\pi)^2}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{P}_\perp\cdot\boldsymbol{R}_\perp}\,|P, S\rangle\,, \tag{133}$$

where $\mathcal{N}$ is a normalization factor. The IPD's are light-cone correlations in these transverse-position nucleon eigenstates. For instance, the unpolarized IPD is given by

$$q(x, b_\perp^2) = \int \frac{\mathrm{d}z^-}{4\pi}\,\mathrm{e}^{\mathrm{i}xP^+z^-}\,\langle P^+, \boldsymbol{0}_\perp; S|\bar{\psi}(z_1)\,\mathcal{W}^-[z_1, z_2]\gamma^+\psi(z_2)|P^+, \boldsymbol{0}_\perp; S\rangle\,, \tag{134}$$

with $z_{1,2} = (0^+, \mp\frac{1}{2}z^-, \boldsymbol{b}_\perp)$. This is the number density of quarks with momentum fraction x and transverse position $\boldsymbol{b}_\perp$ inside an unpolarized hadron. The polarized IPD's are obtained by inserting in the matrix element of eq. (134), instead of γ^+, the matrices $\gamma^+\gamma_5$ and $\mathrm{i}\sigma^{i+}\gamma_5$.

IPD's are Fourier transforms not of the TMD's, but of the GPD's. We define the impact-parameter transform of a generic GPD X for $\xi = 0$ (which implies $\Delta^2 = -\Delta_\perp^2$) as

$$\mathcal{X}(x, b_\perp^2) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\,X(x, 0, -\Delta_\perp^2)\,. \tag{135}$$

It is straightforward to show [111] that the unpolarized IPD $q(x, b_\perp^2)$ coincides with the Fourier transform of $H(x, 0, -\boldsymbol{\Delta}_\perp)$, that is

$$q(x, b_\perp^2) = \mathcal{H}_q(x, b_\perp^2)\,. \tag{136}$$

The impact-parameter density of unpolarized quarks in a transversely polarized nucleon ($N^\uparrow$) is [111, 112, 108]

$$q_{N^\uparrow}(x, b_\perp) = \mathcal{H}_q(x, b_\perp^2) + \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{b}_\perp) \cdot \boldsymbol{S}_\perp}{M} \mathcal{E}_q'(x, b_\perp^2) \,, \tag{137}$$

with

$$\mathcal{E}_q'(x, b_\perp^2) \equiv \frac{\partial}{\partial b_\perp^2} \mathcal{E}_q(x, b_\perp^2) \,, \tag{138}$$

where $\mathcal{E}(x, b_\perp^2)$ is the Fourier transform of $E(x, 0, -\Delta_\perp^2)$. Notice the formal similarity with eq. (98) and the correspondence $f_{1T}^\perp \leftrightarrow -\mathcal{E}'$. . Due to the $\mathcal{E}_q'$ term, which can be regarded as the $\boldsymbol{b}_\perp$-space analogue of the Sivers distribution, $q_{N^\uparrow}(x, \boldsymbol{b}_\perp)$ is not axially symmetric and describes a spatial distortion of the quark distribution in the transverse plane. Final-state interactions can translate this position-space asymmetry into a momentum-space asymmetry. For instance, if the nucleon moves (as usual) in the $+\hat{z}$ directions and is polarized in the $+\hat{x}$ direction, a positive $\mathcal{E}_q'$ implies that quarks tend to be displaced in the $-\hat{y}$ direction, and final-state interactions, which is expected to be attractive on average, convert this transverse distortion into a momentum asymmetry in the $+\hat{y}$ direction. This is the intuitive explanation of the Sivers effect in the impact-parameter picture [76, 113]. A measure of the space distortion is given by the flavour dipole moment

$$d_q^i = \int \mathrm{d}x \int \mathrm{d}^2\boldsymbol{b}_\perp \, b_\perp^i \, q_{N^\uparrow}(x, \boldsymbol{b}_\perp) = -\frac{\epsilon_\perp^{ij} S_\perp^j}{2M} \int \mathrm{d}x \, E_q(x, 0, 0) = -\frac{\epsilon_\perp^{ij} S_\perp^j}{2M} \, \kappa^q \,, \tag{139}$$

where κ^q is the contribution of the quark flavour q to the anomalous magnetic moment of the nucleon, see eq. (129. The argument developed so far is summarised by the following qualitative relation between the Sivers function $f_{1T}^\perp$ and κ^q [114, 115, 113] (any quantitative relation between these two quantities is necessarily model-dependent [108, 110])

$$f_{1T}^{\perp q} \sim -\kappa^q \,, \tag{140}$$

where the minus sign is a consequence of attractive final-state interactions that transform a preferential direction in the $\boldsymbol{b}_\perp$-space into the opposite direction in $\boldsymbol{k}_\perp$. Eq. (140) leads to an immediate prediction: since the quark contributions to the anomalous magnetic moment of the proton κ^p (extracted from the experimental value of κ^p using SU(2) flavour simmetry) are $\kappa^u \simeq 1.7, \kappa^d = -2.0$, one expects $f_{1T}^{\perp u} < 0$ and $f_{1T}^{\perp d} > 0$. This prediction has been corroborated by the SIDIS experiments.

Consider now the case of transversely polarized quarks inside an unpolarized nucleon. Their impact-parameter distribution is [112]

$$q^\uparrow(x, \boldsymbol{b}_\perp) = \frac{1}{2} \left\{ \mathcal{H}_q(x, b_\perp^2) + \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{b}_\perp) \cdot \boldsymbol{S}_{q\perp}}{M} [\mathcal{E}_{Tq}'(x, b_\perp^2) + 2\widetilde{\mathcal{H}}_{Tq}'(x, b_\perp^2)] \right\} \,, \tag{141}$$

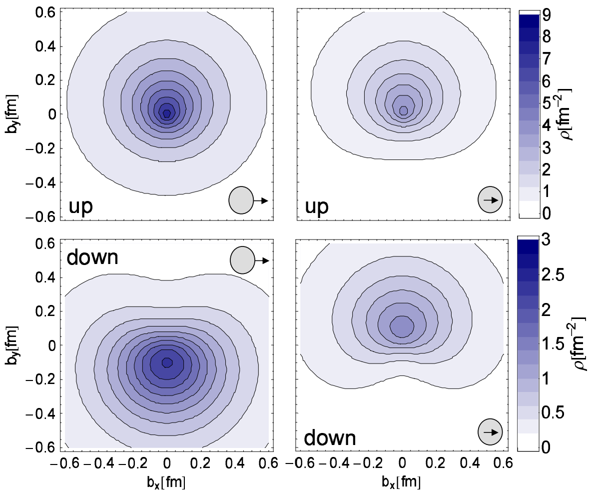

Figure 8: First x-moments of the densities of unpolarized quarks in a transversely polarized nucleon (left) and transversely polarized quarks in an unpolarized nucleon (right) for u (upper plots) and d (lower plots) quarks. Quark spins (inner arrows) and nucleon spins (outer arrows) are oriented in the transverse plane as indicated. From Ref. [116].

The term $\mathcal{E}'_T + 2\widetilde{\mathcal{H}}'_T$ is the analogue of the Boer-Mulders function in the $\boldsymbol{b}_\perp$-space – see eq. (100). Again, we see that transverse spin (of quarks, in this case) causes a spatial distortion of the distribution, which is at the origin of the Boer-Mulders effect. One can repeat the same reasoning developed for the Sivers effect and introduce a transverse anomalous moment κ_T^q, defined by

$$\kappa_T^q \equiv \int \mathrm{d}x \, [E_T^q(x,0,0) + 2\widetilde{H}_T^q(x,0,0)] \, . \tag{142}$$

The Boer-Mulders function is expected to scale with this quantity,

$$h_1^{\perp q} \sim -\kappa_T^q \, . \tag{143}$$

where the minus sign has the same meaning as before. Unfortunately, no data exist for κ_T^q. This quantity, however, and the impact-parameter distributions have been calculated in lattice QCD [116, 117] (Fig. 8). The result for κ_T is: $\kappa_T^u = 3.0$, $\kappa_T^d = 1.9$. Thus, at variance with $f_{1T}^\perp$, we expect the u and d components of $h_1^\perp$ to have the same sign, and in particular to be both negative. Moreover, assuming simple proportionality between the ratio $h_1^\perp/f_{1T}^\perp$ and κ/κ_T, the u component of $h_1^\perp$ should be approximately twice as large as the corresponding component of $f_{1T}^\perp$, while $h_1^{\perp d}$ and $f_{1T}^{\perp d}$ should have a comparable magnitude and opposite sign. These predictions are well supported by a phenomenological analysis of SIDIS data [118].

16 Model calculations of TMD's

Models and other non-perturbative approaches (e.g., lattice calculations) play a very important rôle when the experimental information about distribution functions is scarce or lacking at all. This is the case of the TMD's, which are still essentially unknown. As we said, even the existence of some of them (the T-odd distributions) was for longtime quite uncertain. So, it is not surprising that a considerable effort has been made to compute the TMD's in various models of the nucleon and by lattice QCD. Here we will be not be able to give an exhaustive account of all this work (still largely in progress), and we will limit ourselves to a general discussion (for a recent review of model results see Ref. [119]).

The first calculation of TMD's was performed in a quark-diquark spectator model [120]. This class of models, with various quark-diquark vertex functions, has been subsequently used by many authors. In particular, Brodsky, Hwang and Schmidt [27] used a simple scalar spectator model with gluon exchange to show explicitly that the Sivers function is non vanishing. Since Wilson links, representing gluon insertions, are crucial in order to guarantee the existence of the T-odd distribution functions, these can only be computed in models containing gluonic degrees of freedom. Following Ref. [27], more refined calculations of the Sivers and Boer-Mulders functions were performed in spectator models with both scalar and axial-vector diquarks and various quark-diquark vertices [121, 122, 123, 79, 124, 125, 126]. Other models used to evaluate the T-odd functions include the MIT bag model [127, 128, 129, 130], the constituent quark model [131, 130] and a light-cone model [132]. In Ref. [128] final state interactions were assumed to be induced by instanton effects.

What emerges from models is that the Sivers function, although quite variable in magnitude, is negative for u quarks and positive for d quarks (a different sign of $f_{1T}^{\perp d}$ is however found in the model of Ref. [128]). As for the Boer-Mulders function, the general prediction (with the exception of Ref. [123]) is that both the u and the d distributions are negative. These signs for $f_{1T}^{\perp}$ and $h_1^{\perp}$ are also expected in the impact-parameter picture [111, 114, 113, 115, 58], in the large-N_c approach (which predicts the isoscalar component of $f_{1T}^{\perp}$ and the isovector component of $h_1^{\perp}$ to be suppressed) [133] and in chiral models [134].

Spectator models have been also used [135, 136] to calculate T-odd twist-3 distributions, in particular $g^{\perp}$, which contributes to the longitudinal beam spin asymmetry in SIDIS.

Models without gluonic degrees of freedom can be used to compute T-even TMD's only. These distributions have been calculated in a spectator model [125], in light-cone quark models [137, 138, 139], in a covariant parton model with orbital motion [140] and in the bag model [65, 141]. In particular, Ref. [141] presents a systematic study of leading and subleading twist TMD's and of the relations among them.

In any quark model without gluons, the Lorentz–invariance relations, obtained by neglecting the amplitudes of the quark-quark correlator related to the gauge link (Sec. 12), must obviously be valid. There are also a number of other relations that hold in some specific models. We just mention [65]

$$g_1(x, k_\perp^2) - h_1(x, k_\perp^2) = \frac{k_\perp^2}{2M^2}\, h_{1T}^\perp(x, k_\perp^2)\,. \tag{144}$$

According to this relation, $h_{1T}^\perp$ can be interpreted as a measure of the relativistic effects in the nucleon, which are known to be responsible for the difference between the helicity and the transversity distributions [142]. Other model-dependent relations involving the TMD's are listed in Refs. [119, 141].

Finally, one should keep in mind that models provide a dynamical picture of the nucleon at some fixed, very low, scale $\mu^2 < 1$ GeV^2 [143, 144, 145, 146]. The quark distributions that one gets are therefore valid at this unrealistic scale and must be evolved to the experimental scales. The evolution of the TMD's has been unknown until very recently and is therefore neglected, or approximated, in current phenomenological analyses.

17 Semi-inclusive deep inelastic scattering

Semi-inclusive deep inelastic scattering (SIDIS) is the process, $\ell(l) + N(P) \to \ell'(l') + h(P_h) + X(P_X)$, where ℓ (ℓ') is the incoming (outgoing) lepton, N the nucleon target, h the detected hadron, and the correponding momenta are given in parentheses. In the following we will denote by S^μ the spin four-vector of the target and by λ_ℓ the longitudinal polarization of the incident lepton.

SIDIS is usually described in terms of the invariant variables

$$x_B = \frac{Q^2}{2\,P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z_h = \frac{P \cdot P_h}{P \cdot q}, \tag{145}$$

with $q = l - l'$ and $Q^2 \equiv -q^2$. In the deep inelastic limit, Q^2 is much larger than the hadronic masses (the mass M of the nucleon and the mass M_h of the final hadron). Hereafter mass corrections will be neglected unless otherwise stated.

To parametrize the SIDIS cross section in terms of structure functions we adopt a $\gamma^* N$ collinear frame. In this class of frames the final hadron has a transverse momentum $\boldsymbol{P}_{h\perp}$. The decomposition of the target spin vector is $S^\mu = S_\parallel^\mu + S_\perp^\mu$. All azimuthal angles are referred to the lepton scattering plane: ϕ_h is the azimuthal angle of the hadron h, ϕ_S is the azimuthal angle of the nucleon spin $S_\perp^\mu$. The phase space of the process contains another angle, ψ, which is the azimuthal angle of the outgoing lepton around the beam axis with respect to an arbitrary fixed direction, which is chosen to be given by the target spin. Up to corrections of order M^2/Q^2 one has $\mathrm{d}\psi \simeq \mathrm{d}\phi_S$ [147].

We consider the case of a spinless or unpolarized detected hadron. The SIDIS differential cross section in the six variables $x_B, y, z_h, \phi_S, |\boldsymbol{P}_{h\perp}|, \phi_h$ is given by

$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}x_B \mathrm{d}y \mathrm{d}z_h \mathrm{d}\phi_S \mathrm{d}\phi_h \mathrm{d}\boldsymbol{P}_{h\perp}^2} = \frac{\alpha_{\mathrm{em}}^2}{8Q^4} \frac{y}{z_h} L_{\mu\nu} W^{\mu\nu} , \tag{146}$$

where $L_{\mu\nu}$ is the usual DIS leptonic tensor and $W^{\mu\nu}$ is the hadronic tensor

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{(2\pi)^4} \sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3 2E_X} (2\pi)^4 \delta^4(P + q - P_X - P_h) \\ &\quad \times \langle P, S | J^\mu(0) | X; P_h, S_h \rangle \langle X; P_h, S_h | J^\nu(0) | P, S \rangle . \end{aligned} \tag{147}$$

The complete SIDIS cross section can be parametrized in terms of 18 structure functions, in the following way (we neglect M^2/Q^2 corrections) [147, 85]

$$\begin{aligned} &\frac{\mathrm{d}^6\sigma}{\mathrm{d}x_B \mathrm{d}y \mathrm{d}z_h \mathrm{d}\phi_h \mathrm{d}\boldsymbol{P}_{h\perp}^2 \, \mathrm{d}\phi_S} = \frac{\alpha_{\mathrm{em}}^2}{x_B y Q^2} \Big\{ (1 - y + \frac{1}{2} y^2) \, F_{UU,T} + (1-y) \, F_{UU,L} \\ &+(2-y)\sqrt{1-y} \, \cos\phi_h F_{UU}^{\cos\phi_h} + (1-y) \, \cos 2\phi_h \, F_{UU}^{\cos 2\phi_h} + \lambda_\ell \, y \sqrt{1-y} \, \sin\phi_h \, F_{LU}^{\sin\phi_h} \\ &+S_\parallel \left[(2-y)\sqrt{1-y} \, \sin\phi_h \, F_{UL}^{\sin\phi_h} + (1-y) \, \sin 2\phi_h \, F_{UL}^{\sin 2\phi_h} \right] \\ &+S_\parallel \lambda_\ell \left[y(1 - \frac{1}{2} y) \, F_{LL} + y\sqrt{1-y} \cos\phi_h \, F_{LL}^{\cos\phi_h} \right] \\ &+|\boldsymbol{S}_\perp| \left[\sin(\phi_h - \phi_S) \, \left((1 - y + \frac{1}{2} y^2) \, F_{UT,T}^{\sin(\phi_h - \phi_S)} + (1-y) \, F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\ &+(1-y) \, \sin(\phi_h + \phi_S) \, F_{UT}^{\sin(\phi_h + \phi_S)} + (1-y) \, \sin(3\phi_h - \phi_s) \, F_{UT}^{\sin(3\phi_h - \phi_S)} \\ &\left. + \, (2-y)\sqrt{1-y} \, \sin\phi_S \, F_{UT}^{\sin\phi_S} + (2-y)\sqrt{1-y} \, \sin(2\phi_h - \phi_S) \, F_{UT}^{\sin(2\phi_h - \phi_S)} \right] \\ &+|\boldsymbol{S}_\perp| \lambda_\ell \left[y(1 - \frac{1}{2} y) \, \cos(\phi_h - \phi_S) \, F_{LT}^{\cos(\phi_h - \phi_S)} + y\sqrt{1-y} \, \cos\phi_S \, F_{LT}^{\cos\phi_S} \right. \\ &\left. + \, y\sqrt{1-y} \, \cos(2\phi_h - \phi_S) \, F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \Big\} \, . \end{aligned} \tag{148}$$

The structure functions depend on $x_B, y, z_h, \boldsymbol{P}_{h\perp}^2$. Their first and second subscript denote the polarization of the beam and of the target, respectively (U = unpolarized, L = longitudinally polarized, T = transversely polarized), whereas the third subscript refer to the polarization of the virtual photon.

If we integrate (148) over $\boldsymbol{P}_{h\perp}$, only five structure functions survive: $F_{UU,T}$, $F_{UU,L}$, F_{LL}, $F_{LT}^{\cos\phi_S}$, $F_{UT}^{\sin\phi_S}$. The first two, upon a further integration in z and a sum over all outgoing hadrons, yield the unpolarized DIS structure functions $F_T(x_B, Q^2) = 2xF_1(x_B, Q^2)$ and $F_L(x_B, Q^2) = F_2(x_B, Q^2) - 2x_B F_1(x_B, Q^2)$, the second two lead to combinations of the structure functions $g_1(x_B, Q^2)$ and $g_2(x_B, Q^2)$ of longitudinally polarized DIS, the fifth one vanishes. The fact that

$$\sum_h \int \mathrm{d}z_h \, z_h \int \mathrm{d}^2\boldsymbol{P}_{h\perp} \, F_{UT}^{\sin\phi_S} = 0 \tag{149}$$

is a consequence of time-reversal invariance [147] and is another way to express the Christ-Lee theorem [23], according to which there cannot be transverse spin asymmetries in inclusive DIS. The proof of this theorem is instructive and goes as follows [35]. The DIS cross section has the form

$$\mathrm{d}\sigma \sim L^{\mu\nu} W_{\mu\nu}(\boldsymbol{S}_\perp)\,, \tag{150}$$

where the leptonic tensor $L^{\mu\nu}$ is symmetric (the lepton is assumed to be unpolarized) and the hadronic tensor is related to matrix elements of the electromagnetic current,

$$W_{\mu\nu}(\boldsymbol{S}_\perp) \sim \langle P, \boldsymbol{S}_\perp | J_\mu(\xi)\, J_\nu(0) | P, \boldsymbol{S}_\perp \rangle\,. \tag{151}$$

Applying parity, time-reversal and translational invariance, one gets

$$\langle P, \boldsymbol{S}_\perp | J_\mu(\xi)\, J_\nu(0) | P, \boldsymbol{S}_\perp \rangle = \langle P, -\boldsymbol{S}_\perp | J_\nu(\xi)\, J_\mu(0) | P, -\boldsymbol{S}_\perp \rangle\,, \tag{152}$$

and therefore

$$W_{\mu\nu}(\boldsymbol{S}_\perp) = W_{\nu\mu}(-\boldsymbol{S}_\perp)\,. \tag{153}$$

The spin asymmetry is

$$\Delta\sigma(\boldsymbol{S}_\perp) \sim L^{\mu\nu}[W_{\mu\nu}(\boldsymbol{S}_\perp) - W_{\mu\nu}(-\boldsymbol{S}_\perp)]\,, \tag{154}$$

that is, using (153)

$$\Delta\sigma(\boldsymbol{S}_\perp) \sim L^{\mu\nu}[W_{\mu\nu}(\boldsymbol{S}_\perp) - W_{\nu\mu}(\boldsymbol{S}_\perp)] = 0 \tag{155}$$

due to the symmetry of $L^{\mu\nu}$.

This proof does *not* work in SIDIS. The reason is simply that the SIDIS hadronic tensor (147) does not reduce to the simple form (151), due to the presence of the detected particle, besides the unresolved X system, in the final state. Therefore, in SIDIS no first principle forbids the existence of transverse spin asymmetries.

18 SIDIS in the extended parton model

In the current fragmentation region, which we are going to focus on, the virtual photon strikes a quark (or an antiquark) that successively fragments into a hadron h. The process is represented by the diagram in Fig. 9. We will take transverse momenta of quarks into account and refer to this description as the "extended parton model".

For the partonic description of SIDIS we work in a reference frame where the momenta of the target nucleon and of the outgoing hadron are collinear and define the longitudinal direction. In this class of "hN collinear frames", one has $P^\mu = P^+ n_+^\mu$ and $P_h^\mu = P^- n_-^\mu$, whereas the virtual photon momentum acquires a transverse component $\boldsymbol{q}_T$. The incoming quark momentum is $k^\mu = xP^\mu + k_T^\mu$, with $x = k^+/P^+$; the fragmenting quark momentum is $\kappa^\mu = P_h^\mu/z + \kappa_T^\mu$, with $z = P_h^-/\kappa^-$. Notice that the transverse

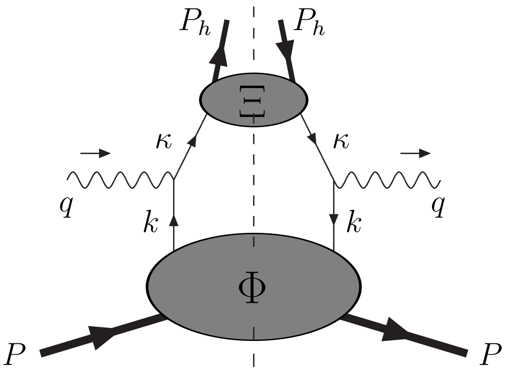

Figure 9: Diagram contributing to SIDIS in the parton model.

quantities in the hN collinear frame (labelled by the subscript T) differ from the transverse quantities in the $\gamma^* N$ collinear frame (labelled by the subscript $\perp$) by terms suppressed at least as $1/Q$. In particular, $\boldsymbol{q}_T$ is related to $\boldsymbol{P}_{h\perp}$ by $\boldsymbol{q}_T = -\boldsymbol{P}_{h\perp}/z_h$, up to $1/Q^2$ corrections.

Referring to Fig. 9 for the notation, the hadronic tensor is given by (for simplicity we consider only the quark contribution)

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{(2\pi)^4} \sum_a e_a^2 \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X} \sum_{X'} \int \frac{\mathrm{d}^3 \boldsymbol{P}'_X}{(2\pi)^3\, 2E'_X} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \int \frac{\mathrm{d}^4 \kappa}{(2\pi)^4} \\ &\quad \times (2\pi)^4\, \delta^4(P - k - P_X)\, (2\pi)^4\, \delta^4(k + q - \kappa)\, (2\pi)^4 \delta^4(\kappa - P_h - P_{X'}) \\ &\quad \times \left[\bar{\chi}(\kappa; P_h, S_h)\gamma^\mu \phi(k; P, S)\right]^* \left[\bar{\chi}(\kappa; P_h, S_h)\gamma^\nu \phi(k; P, S)\right], \end{aligned} \tag{156}$$

where $\phi(k; P, S)$ and $\chi(\kappa; P_h, S_h)$ are matrix elements of the quark field ψ, defined as

$$\phi(k; P, S) = \langle X | \psi(0) | PS \rangle\,, \tag{157}$$

$$\chi(\kappa; P_h, S_h) = \langle 0 | \psi(0) | P_h S_h, X \rangle\,. \tag{158}$$

We now introduce the quark–quark correlation matrices

$$\begin{aligned} \Phi_{ij}(k; P, S) &= \frac{1}{(2\pi)^4} \sum_{X'} \int \frac{\mathrm{d}^3 \boldsymbol{P}'_X}{(2\pi)^3\, 2E'_X}\, (2\pi)^4\, \delta^4(P'_X + k - P)\, \phi_i(k; P, S)\, \bar{\phi}_j(k; P, S) \\ &= \int \mathrm{d}^4\xi\, \mathrm{e}^{\mathrm{i}k\cdot\xi}\, \langle P, S | \bar{\psi}_j(0) \psi_i(\xi) | P, S \rangle, \end{aligned} \tag{159}$$

and

$$\begin{aligned} \Xi_{ij}(\kappa; P_h, S_h) &= \frac{1}{(2\pi)^4} \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X}\, (2\pi)^4\, \delta^4(P_h + P_X - \kappa) \\ &\qquad \times \chi_i(\kappa; P_h, S_h)\, \bar{\chi}_j(\kappa; P_h, S_h) \\ &= \frac{1}{(2\pi)^4} \sum_X \int \frac{\mathrm{d}^3 \boldsymbol{P}_X}{(2\pi)^3\, 2E_X} \int \mathrm{d}^4\xi\, \mathrm{e}^{\mathrm{i}\kappa\cdot\xi} \\ &\qquad \times \langle 0 | \psi_i(\xi) | P_h, S_h; X \rangle \langle P_h, S_h; X | \bar{\psi}_j(0) | 0 \rangle\,, \end{aligned} \tag{160}$$

Here Φ is the matrix already encountered in inclusive DIS, which incorporates the quark distribution functions. Ξ is a new quark–quark correlation matrix (called fragmentation, or decay, matrix), which contains the fragmentation functions of quarks into a hadron h. An average over colors is included in Ξ. Inserting eqs. (159, 160) into eq. (156) yields

$$W^{\mu\nu} = \sum_a e_a^2 \int \mathrm{d}^4 k \int \mathrm{d}^4 \kappa \, \delta^4(k + q - \kappa) \, \mathrm{Tr} \left[\Phi \, \gamma^\mu \, \Xi \gamma^\nu \right] . \tag{161}$$

It is an assumption of the parton model that k^2, $k{\cdot}P$, κ^2 and $\kappa{\cdot}P_h$ are much smaller than Q^2. Stated differently, when these quantities become large, Φ and Ξ are strongly suppressed. In a hN collinear frame, the photon momentum is

$$q^\mu \simeq -x_B P^\mu + \frac{1}{z_h} P_h^\mu + q_T^\mu . \tag{162}$$

The quark momenta are

$$\begin{aligned} k^\mu &\simeq x \, P^\mu + k_T^\mu , &(163)\\ \kappa^\mu &\simeq \frac{1}{z} P_h^\mu + \kappa_T^\mu . &(164) \end{aligned}$$

If the fragmenting quark has a transverse momentum $\boldsymbol{\kappa}_T$ with respect to the final hadron, the hadron has a transverse momentun $\boldsymbol{p}_T = -z\boldsymbol{\kappa}_T$ with respect to the quark.

The delta function in eq. (161) can be decomposed as

$$\begin{aligned} \delta^4(k + q - \kappa) &= \delta(k^+ + q^+ - \kappa^+) \, \delta(k^- + q^- - \kappa^-) \, \delta^2(\boldsymbol{k}_T + \boldsymbol{q}_T - \boldsymbol{\kappa}_T) \\ &\simeq \delta(k^+ - x_B P^+) \, \delta(\kappa^- - P_h^-/z_h) \, \delta^2(\boldsymbol{k}_T + \boldsymbol{q}_T - \boldsymbol{\kappa}_T) . \end{aligned} \tag{165}$$

which implies $x_B = x = k^+/P^+$ and $z_h = z = P_h^-/\kappa^-$.

Exploiting the delta functions in the longitudinal momenta, the hadronic tensor (161) then becomes

$$\begin{aligned} W^{\mu\nu} &= 2 z_h \sum_a e_a^2 \int \mathrm{d}^2 \boldsymbol{k}_T \int \mathrm{d}^2 \boldsymbol{\kappa}_T \; \delta^2(\boldsymbol{k}_T + \boldsymbol{q}_T - \boldsymbol{\kappa}_T) \\ &\quad \times \mathrm{Tr} \left[\Phi(x_B, \boldsymbol{k}_T) \, \gamma^\mu \, \Xi(z_h, \boldsymbol{\kappa}_T) \gamma^\nu \right] , \end{aligned} \tag{166}$$

where $\Xi_{ij}(z, \boldsymbol{\kappa}_T)$ is the fragmentation analogue of $\Phi_{ij}(x, \boldsymbol{k}_T)$:

$$\begin{aligned} \Xi_{ij}(z, \boldsymbol{\kappa}_T) &= \frac{1}{2z} \int \mathrm{d}\kappa^+ \, \mathrm{d}\kappa^- \, \Xi(\kappa) \, \delta(\kappa^- - P_h^-/z) \\ &= \frac{1}{2z} \sum_X \int \frac{\mathrm{d}\xi^+}{2\pi} \int \frac{\mathrm{d}^2 \boldsymbol{\xi}_T}{(2\pi)^3} \, \mathrm{e}^{\mathrm{i} P_h^- \xi^+ / z} \, \mathrm{e}^{-\mathrm{i} \boldsymbol{\kappa}_T \cdot \boldsymbol{\xi}_T} \\ &\quad \times \langle 0 | \mathcal{W}[+\infty, \xi] \, \psi_i(\xi) | P_h, S_h; X \rangle \langle P_h, S_h; X | \bar{\psi}_j(0) \, \mathcal{W}[0, +\infty] | 0 \rangle |_{\xi^- = 0} . \end{aligned} \tag{167}$$

Each Wilson line (that we do not specify in detail: see, e.g., Ref. [85]) includes a longitudinal link along n_+ and a transverse link at infinity. In the case of fragmentation one has the same gauge structure in SIDIS and in e^+e^- annihilation, which means that there is no difference between the fragmentation functions of these processes (they are universal quantities in a full sense) [68].

19 Fragmentation functions

The integrated fragmentation correlator is given at leading twist by

$$\begin{aligned}\Xi(z) &= z^2 \int \mathrm{d}^2\boldsymbol{\kappa}_T\, \Xi(z, \boldsymbol{\kappa}_T) \\ &= \frac{1}{2}\left\{ D_1(z) \not{n}_- + S_L\, G_1(z)\gamma_5 \not{n}_- + H_1(z)\, \frac{[\not{S}_T, \not{n}_-]\gamma_5}{2} \right\}. \end{aligned} \tag{168}$$

D_1, G_1 and H_1 are the integrated leading-twist fragmentation functions (FF's): D_1 is the ordinary unpolarized fragmentation function, whereas H_1 is the analogue of the transversity distribution and describes the fragmentation of a transversely polarized quark into a transversely polarized hadron.

To compute the azimuthal asymmetries we need the transverse-momentum dependent FF's. For simplicity, we limit ourselves to listing the FF's of main phenomenological interest. The traces of the fragmentation matrix corresponding to unpolarized and transversely polarized quarks are [20]

$$\Xi^{[\gamma^-]}(z, \boldsymbol{\kappa}_T) = D_1(z, \kappa_T^2) + \frac{\epsilon_{Tij} \kappa_T^i S_{hT}^j}{M_h}\, D_{1T}^{\perp}(z, \kappa_T^2)\,, \tag{169}$$

$$\Xi^{[\mathrm{i}\sigma^{i-}\gamma_5]}(z, \boldsymbol{\kappa}_T) = S_{hT}^i H_1(z, \kappa_T^2) + \frac{\epsilon_T^{ij} p_{Tj}}{M_h}\, H_1^{\perp}(z, \kappa_T^2) + \ldots \tag{170}$$

$D_1(z, \kappa_T^2)$ and $H_1(z, \kappa_T^2)$ are the κ_T-dependent unpolarized and transversely polarized fragmentation functions, respectively, yielding $D_1(z)$ and $H_1(z)$ once integrated over the transverse momentum according to eq. (168).

$D_{1T}^{\perp}$ is analogous to the Sivers distribution function and describes the production of transversely polarized hadrons from unpolarized quarks (for this reason it is called "polarizing fragmentation function" [148]).

The most noteworthy FF appearing in (170) is $H_1^{\perp}(z, \kappa_T^2)$, the so-called *Collins function*, describing the fragmentation of a transversely polarized quark into an unpolarized hadron [11]. The resulting transverse-momentum asymmetry of hadrons, expressed in terms of the hadron transverse momentum $\boldsymbol{p}_T$ with respect to the fragmenting quark, is

$$D_{h/q^{\uparrow}}(z, \boldsymbol{p}_T) - D_{h/q^{\uparrow}}(z, -\boldsymbol{p}_T) = 2\, \frac{(\hat{\boldsymbol{\kappa}} \times \boldsymbol{p}_T) \cdot \boldsymbol{S}'_{qT}}{z M_h} H_1^{\perp}(z, p_T^2)\,, \tag{171}$$

where $\boldsymbol{S}'_q$ is the spin vector of the fragmenting quark. From the structure of the correlation $(\hat{\boldsymbol{\kappa}} \times \boldsymbol{p}_T) \cdot \boldsymbol{S}'_{qT}$ one sees that a positive $H_1^{\perp}$ corresponds to a preference of the hadron to be emitted on the left side of the jet if the quark spin points upwards. Through this mechanism the transverse momentum of the produced hadron with respect to the jet direction acts as a quark polarimeter.

The Collins function satisfies a sum rule arising from the conservation of the intrinsic transverse momentum during quark fragmentation. This sum rule, discovered by

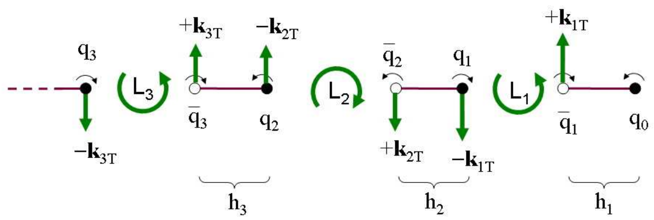

Figure 10: The fragmentation process in the string model.

Schäfer and Teryaev [149], reads

$$\sum_h \int \mathrm{d}z \, z \, H_1^{\perp(1)q}(z) = 0 \,, \tag{172}$$

with

$$H_1^{\perp(1)}(z) \equiv z^2 \int \mathrm{d}^2\boldsymbol{\kappa}_T \, \frac{\kappa_T^2}{2M_h^2} \, H_1^{\perp}(z, \kappa_T^2) \,. \tag{173}$$

A simple qualitative explanation of the Collins effect is provided by Artru's string model [10, 150]. Suppose that a quark q_0, polarized in the $+\hat{y}$ direction (i.e., out of the page in Fig. 10), fragments into a pion with an antiquark $\bar{q}_1$ created by string breaking. If we assume that the $q_1\bar{q}_1$ pair is in a $^3P_0$ state, the orbital angular momentum of the pair is $L = 1$, and the pion, inheriting the transverse momentum of $\bar{q}_1$, moves in the $+\hat{x}$ direction. The quark q_1, with the subleading pion that contains it, moves in the opposite direction. This model predicts opposite Collins asymmetries for π^+ and π^-, and a positive (negative) sign for the favored (unfavored) Collins function (where "favored" refers to the fragmentation of a quark or an antiquark belonging to the valence component of the final hadron, e.g. $u \to \pi^+$, $d \to \pi^-$, $\bar{d} \to \pi^+$, etc.).

The Collins function for pions has been computed in various fragmentation models [151, 152, 153, 154, 155]. What is common to these approaches is that $H_1^{\perp}$ arises from the interference between a tree level amplitude and loop corrections that provide the necessary imaginary parts. The differences reside in the pion-quark couplings and in the nature of the virtual particles in the loops (pions or gluons). An assessment of model calculations of the Collins function is contained in Ref. [156].

20 SIDIS structure functions

Inserting the expressions of Φ and Ξ into eq. (166) and contracting $W^{\mu\nu}$ with $L_{\mu\nu}$ leads to the SIDIS structure functions. With the following notation for the transverse momenta convolutions

$$
\begin{aligned}
\mathcal{C}\,[wfD] \;&=\; \sum_a e_a^2\, x_B \int \mathrm{d}^2\boldsymbol{k}_T \int \mathrm{d}^2\boldsymbol{\kappa}_T\, \delta^2(\boldsymbol{k}_T - \boldsymbol{\kappa}_T - \boldsymbol{P}_{h\perp}/z_h) \\
&\quad \times w(\boldsymbol{k}_T, \boldsymbol{\kappa}_T)\, f^a(x_B, k_T^2) D^a(z_h, \kappa_T^2)\,, \qquad (174)
\end{aligned}
$$

the non vanishing structure functions at leading twist are ($\hat{\boldsymbol{h}} \equiv \boldsymbol{P}_{h\perp}/|\boldsymbol{P}_{h\perp}|$) [85]:
<u>Unpolarized target</u>

$$
F_{UU,T} = \mathcal{C}\,\left[f_1 D_1\right], \qquad (175)
$$

$$
F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_T) - \boldsymbol{k}_T\cdot\boldsymbol{\kappa}_T}{M M_h}\, h_1^\perp\, H_1^\perp\right]. \qquad (176)
$$

<u>Longitudinally polarized target</u>

$$
F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_T) - \boldsymbol{k}_T\cdot\boldsymbol{\kappa}_T}{M M_h}\, h_{1L}^\perp\, H_1^\perp\right], \qquad (177)
$$

$$
F_{LL} = \mathcal{C}\,\left[g_{1L} D_1\right]. \qquad (178)
$$

<u>Transversely polarized target</u>

$$
F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M}\, f_{1T}^\perp D_1\right], \qquad (179)
$$

$$
F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_T}{M_h}\, h_1 H_1^\perp\right], \qquad (180)
$$

$$
\begin{aligned}
&F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad = \mathcal{C}\left[\frac{2(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_T)(\boldsymbol{k}_T\cdot\boldsymbol{\kappa}_T) + \boldsymbol{k}_T^2(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_T) - 4(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)^2(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_T)}{2M^2 M_h}\, h_{1T}^\perp H_1^\perp\right], \quad (181)
\end{aligned}
$$

$$
F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M}\, g_{1T} D_1\right]. \qquad (182)
$$

The structure function $F_{UU,T}$ gives the dominant contribution to the unpolarized cross section integrated over ϕ_h, which reads

$$
\begin{aligned}
\frac{\mathrm{d}^4\sigma}{\mathrm{d}x_B \mathrm{d}y \mathrm{d}z_h \mathrm{d}\boldsymbol{P}_{h\perp}^2} \;&=\; \frac{4\pi\alpha_{\rm em}^2}{x_B y Q^2} \sum_a e_a^2 x_B \left(1 - y + \frac{1}{2}y^2\right) \\
&\quad \times \int \mathrm{d}^2\boldsymbol{k}_T \int \mathrm{d}^2\boldsymbol{\kappa}_T\; \delta^2(\boldsymbol{k}_T - \boldsymbol{\kappa}_T - \boldsymbol{P}_{h\perp}/z)\, f_1^a(x_B, k_T^2) D_1^a(z_h, \kappa_T^2). \quad (183)
\end{aligned}
$$

Concerning the angular distributions, the $\sin(\phi_h + \phi_S)$ and $\sin(\phi_h - \phi_S)$ terms are particularly important: the first is related to the Collins effect, the second to the Sivers effect (both leading-twist mechanisms requiring a transversely polarized target). Notice that also in the unpolarized case there can be an azimuthal modulation, of the type $\cos 2\phi_h$, due to the Boer-Mulders distribution $h_1^\perp$.

Going to twist three, i.e. to order $1/Q$, it turns out that the leading-twist structure functions (175-181) do not acquire any extra contribution, but there appear other non vanishing structure functions. Among them, of particular phenomenological importance are those related to the $\cos\phi_h$ and $\sin\phi_h$ modulations (the complete list is in Ref. [85]). Ignoring, in the spirit of the parton model, interaction-dependent terms, that is quark-gluon correlations, and quark mass contributions (the generalized Wandzura–Wilczek approximation) one finds [85]

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_T)\,k_T^2}{M_h M^2}\,h_1^\perp\,H_1^\perp - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M}\,f_1 D_1\right], \tag{184}$$

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{(\hat{\boldsymbol{h}}\cdot\boldsymbol{\kappa}_T)\,k_T^2}{M_h M^2}\,h_{1L}^\perp\,H_1^\perp\right]. \tag{185}$$

In the same approximation we get $F_{LU}^{\sin\phi_h} = 0$. Thus a deviation of the beam-spin $\sin\phi_h$ asymmetry from zero might signal the relevance of interaction effects in the nucleon (one should recall however that at high transverse momenta $F_{LU}^{\sin\phi_h}$ is non zero in next-to-leading order QCD). The term in $F_{UU}^{\cos\phi_h}$ containing the product of the unpolarized functions $f_1 D_1$ is a purely kinematical contribution arising from the intrinsic transverse motion of quarks, with no relation to spin. This contribution was discovered longtime ago by Cahn [157, 158], and the existence of the corresponding azimuthal asymmetry is referred to as the ($\cos\phi_h$) "Cahn effect". A similar contribution emerges at twist four, that is at order $1/Q^2$, in the $\cos 2\phi_h$ term:

$$F_{UU,\mathrm{Cahn}}^{\cos 2\phi_h} = \frac{M^2}{Q^2}\,\mathcal{C}\left[\frac{(2(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)^2 - k_T^2)}{M^2}\,f_1\,D_1\right]. \tag{186}$$

21 The helicity approach

The parton-model results of the previous Sections have been obtained using the most general decompositions of the correlation matrices Φ and Ξ, and inserting them into the SIDIS hadronic tensor $W^{\mu\nu}$. There is an alternative approach (the so-called "generalized parton model" approach), which relies on the helicity formalism and expresses the cross section as a convolution of helicity amplitudes of elementary subprocesses with partonic distribution and fragmentation functions, taking fully into account non collinear kinematics [159]. In this picture, the basic factorization formula for the SIDIS cross section is (we consider for simplicity an unpolarized lepton beam and a spinless or unpolarized final hadron)

$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}x_B\mathrm{d}y\mathrm{d}z_h\mathrm{d}^2\boldsymbol{P}_{h\perp}\mathrm{d}\phi_S} = \sum_{q_i}\sum_{\lambda_{q_i}\lambda'_{q_i}}\int\mathrm{d}^2\boldsymbol{k}_\perp\int\mathrm{d}^2\boldsymbol{p}_\perp\,\rho^{q_i}_{\lambda_{q_i}\lambda'_{q_i}}\,f_{q_i}(x,\boldsymbol{k}_\perp) \times\left(\frac{\mathrm{d}\hat{\sigma}_{\lambda_{q_i}\lambda'_{q_i}}}{\mathrm{d}y}\right)D^{h/q_f}(z,\boldsymbol{p}_T)\,\delta^2(z\boldsymbol{k}_\perp-\boldsymbol{p}_\perp-\boldsymbol{P}_{h\perp}) \quad (187)$$

where the λ's are helicity indices, $f_{q_i}(x,\boldsymbol{k}_\perp)$ is the probability of finding a quark q_i with momentum fraction x and transverse momentum $\boldsymbol{k}_\perp$ inside the target nucleon, $\rho^q_{\lambda_{q_i}\lambda'_{q_i}}$ is the helicity density matrix of the quark q_i, $D^{h/q_f}(z,\boldsymbol{p}_\perp)$ is the fragmentation function of the struck quark q_f into the hadron h (having transverse momentum $\boldsymbol{p}_\perp$ with respect to the fragmenting quark) and $\mathrm{d}\hat{\sigma}_{\lambda_{q_i}\lambda'_{q_i}}\sim\hat{M}_{\lambda_{\ell'}\lambda_{q_f};\lambda_\ell\lambda_{q_i}}\hat{M}^*_{\lambda_{\ell'}\lambda_{q_f};\lambda_\ell\lambda'_{q_i}}$ is the cross section of lepton-quark scattering $\ell q_i\to\ell' q_f$ at tree level. Note that, whereas in the collinear case the produced hadron is constrained to have $\boldsymbol{P}_{h\perp}=0$ and the entire process takes place in the scattering plane, the intrinsic transverse momentum of quarks introduces a non planar geometry. The elementary scattering amplitudes $\hat{M}$'s take into account this non collinear and out-of-plane kinematics.

The quantity $\rho^{q_i}_{\lambda_{q_i}\lambda'_{q_i}}f_{q_i}(x,\boldsymbol{k}_\perp)$ contains all the leading-twist TMD's. Writing explicitly the helicity density matrix ρ^{q_i} in terms of the components of the quark polarization vector one can easily identify the eight TMD's. Another method is to introduce the helicity amplitudes $\hat{\mathcal{F}}$ for the process $N\to q_i+X$ [160] and relate the helicity density matrix of the quark to that of the parent nucleon as follows

$$\begin{aligned}\rho^{q_i}_{\lambda_{q_i}\lambda'_{q_i}}f_{q_i}(x,\boldsymbol{k}_\perp) &= \sum_{\lambda_N\lambda'_N}\rho^N_{\lambda_N\lambda'_N}\sum\!\!\!\!\!\!\!\int_{X,\lambda_X}\hat{\mathcal{F}}_{\lambda_{q_i},\lambda_X;\lambda_N}(x,\boldsymbol{k}_\perp)\hat{\mathcal{F}}^*_{\lambda'_{q_i},\lambda_X;\lambda'_N}(x,\boldsymbol{k}_\perp)\\ &\equiv \sum_{\lambda_N\lambda'_N}\rho^N_{\lambda_N\lambda'_N}\,\hat{F}^{\lambda_{q_i}\lambda'_{q_i}}_{\lambda_N\lambda'_N}(x,\boldsymbol{k}_\perp).\end{aligned} \quad (188)$$

The dependence of the amplitudes $\hat{\mathcal{F}}$ on $\boldsymbol{k}_\perp=|\boldsymbol{k}_\perp|\mathrm{e}^{\mathrm{i}\phi}$ (ϕ is the azimuthal angle in a plane orthogonal to the quark momentum) is

$$\hat{\mathcal{F}}_{\lambda_{q_i},\lambda_X;\lambda_N}(x,\boldsymbol{k}_\perp)=\mathcal{F}_{\lambda_{q_i},\lambda_X;\lambda_N}(x,k_\perp^2)\,\exp(\mathrm{i}\lambda_N\phi)\,, \quad (189)$$

so that

$$\hat{F}^{\lambda_{q_i}\lambda'_{q_i}}_{\lambda_N\lambda'_N}(x,\boldsymbol{k}_\perp)=F^{\lambda_{q_i}\lambda'_{q_i}}_{\lambda_N\lambda'_N}(x,k_\perp^2)\,\exp[\mathrm{i}(\lambda_N-\lambda'_N)\phi]\,. \quad (190)$$

Due to rotational and parity invariance, only eight real and/or imaginary parts of the functions $F^{\lambda_{q_i}\lambda'_{q_i}}_{\lambda_N\lambda'_N}$ are independent and are related to the leading-twist TMD's. The explicit correspondence between the components of $\rho^{q_i}_{\lambda_{q_i}\lambda'_{q_i}}f_{q_i}(x,\boldsymbol{k}_\perp)$, or the functions $F^{\lambda_{q_i}\lambda'_{q_i}}_{\lambda_N\lambda'_N}(x,k_\perp^2)$, and the TMD's defined in Sec. 8 can be found in Ref. [159].

In principle, generalized parton model formulas incorporate transverse motion effects at all orders in k_T/Q. Neglecting $\mathcal{O}(k_T^2/Q^2)$ contributions, the description becomes simpler, since in this case the longitudinal momentum fractions coincide with the kinematic invariants: $x=x_B, z=z_h$.

Despite their apparent dissimilarity, the two parton model approaches described so far (the approach based on quark correlation matrices and eq. (166) and the generalized parton model approach based on the helicity formalism) are perfectly equivalent, as far as parton interactions are ignored. In other terms, all the leading-twist asymmetries listed in eqs. (175-182) can be exactly reobtained from eq. (187) [161], whereas at twist three the results of the two approaches are identical if one neglects the "tilde" distribution and fragmentation functions arising from quark-gluon correlations.

22 QCD factorization schemes

So far, we have been working in the framework of the parton model. One may wonder whether the results we have presented have any solid QCD foundation. The answer to this question is positive, at least in a particular kinematic regime. Semi-inclusive processes are characterized by two scales, besides the confinement scale Λ_{QCD}: the momentum transfer Q and the transverse momentum of the final hadron $P_{h\perp}$ (or, equivalently, the transverse momentum of the virtual photon in the hN collinear frame, $Q_T \equiv |\boldsymbol{q}_T|$).

Extending the pioneering work of Collins and Soper on back-to-back jet production [60], Ji, Ma and Yuan [32, 31] (see also Ref. [162]) proved a TMD factorization theorem for SIDIS and DY, valid in the low transverse-momentum region, $P_{h\perp}(Q_T) \ll Q$. In this framework the unpolarized SIDIS structure function is written as

$$\begin{aligned} F_{UU,T}(x_B, z_h, Q^2, Q_T^2) &= \sum_a e_a^2\, x \int \mathrm{d}^2\boldsymbol{k}_T \int \mathrm{d}^2\boldsymbol{\kappa}_T \int \mathrm{d}^2\boldsymbol{l}_T\, \delta^2(\boldsymbol{k}_T - \boldsymbol{\kappa}_T + \boldsymbol{l}_T + \boldsymbol{q}_T) \\ &\quad \times H(Q^2)\, f_1^a(x, k_T^2)\, D_1^a(z, \kappa_T^2)\, U(l_T^2)\,. \end{aligned} \tag{191}$$

For simplicity we have omitted the dependence of the distribution functions on $\zeta^2 = (2v \cdot P)^2/v^2$ and of the fragmentation function on $\zeta_h^2 = (2\tilde{v} \cdot P)^2/\tilde{v}^2$, where v and $\tilde{v}$ are vectors off the light-cone. The variables ζ and ζ_h serve to regulate the light-cone singularities, as explained in Sec. 8. H is a perturbative hard factor written as a series in powers of α_s. The soft factor U arises from the radiation of soft gluons (of transverse momentum $\boldsymbol{l}_T$) and is a matrix element of Wilson lines in the QCD vacuum. Also not displayed in eq. (191) is the dependence of all quantities on the renormalization scale μ and on the soft-gluon rapidity cut-off $\rho = \sqrt{(2v \cdot \tilde{v})^2/v^2\tilde{v}^2}$. Of course, the physical observable F does not depend on any of these regulators.

The generalization of eq. (191) to the polarized structure functions, in particular to those generating transverse SSA's, has been proposed in Refs. [163, 164]. The parton model expressions of Sec. 18 are recovered at tree level, i.e. $\mathcal{O}(\alpha_s^0)$, since $H^{(0)} = 1$ and $U^{(0)}(\boldsymbol{l}_T) = \delta^2(\boldsymbol{l}_T)$.

At high transverse momenta, $Q_T \gg \Lambda_{QCD}$, SIDIS structure functions can be described in collinear QCD. The azimuthal angular dependence of hadrons in leptoproduction was proposed longtime ago by Georgi and Politzer as a test of perturbative

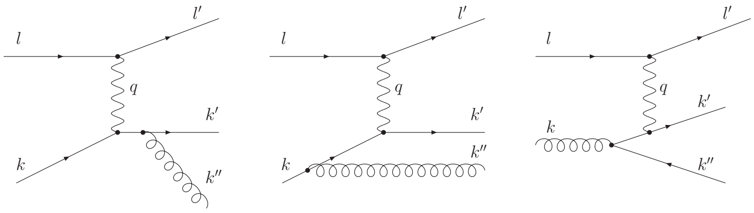

Figure 11: Feynman diagrams of the elementary processes contributing to SIDIS at first order in α_s.

QCD [165]. In collinear factorization, transverse momenta are generated by gluon radiation. At first order in α_s the hard elementary processes shown in Fig. 11 contribute to the four unpolarized SIDIS structure functions F_{UU} and to the two double-longitudinal structure functions F_{LL}. Introducing the partonic variables $\hat{x}$ and $\hat{z}$, defined as $\hat{x} = Q^2/2k \cdot q = x_B/x$, $\hat{z} = k \cdot k'/k \cdot q = z_h/z$, where k and k' are the four-momenta of the incident and fragmenting partons, respectively, and x and z are the usual light-cone momentum fractions, i.e. $k = xP$ and $k' = P_h/z$, one has for the F_{UU}'s at leading order in α_s and leading twist [166, 167, 164]

$$\begin{aligned} F_{UU}(x, Q^2) \;=\; & \frac{\alpha_s}{4\pi^2\, z^2 q^2} \sum_a e_a^2 x_B \int_{x_B}^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \int_{z_h}^1 \frac{\mathrm{d}\hat{z}}{\hat{z}}\, \delta\left(\frac{Q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \\ & \times \left[f_1^a\left(\frac{x_B}{\hat{x}}\right) D_1^a\left(\frac{z_h}{\hat{z}}\right) C_{UU}^{\gamma^* q \to qg} + f_1^a\left(\frac{x_B}{\hat{x}}\right) D_1^g\left(\frac{z_h}{\hat{z}}\right) C_{UU}^{\gamma^* q \to gq} \right. \\ & \left. + f_1^g\left(\frac{x_B}{\hat{x}}\right) D_1^a\left(\frac{z_h}{\hat{z}}\right) C_{UU}^{\gamma^* g \to q\bar{q}} \right] , \end{aligned} \tag{192}$$

and analogous formulas for the F_{LL}'s. The Wilson coefficients C represent elementary cross sections and are listed in Ref. [164].

The structure function $F_{LU}^{\sin\phi_h}$ encountered in Sec. 18, which produces a beam-spin asymmetry and vanishes in the parton model, gets a non zero perturbative QCD contribution at leading twist and order α_s^2 [168, 169].

On the contrary, the transversely polarized structure functions F_{UT}, which vanish at leading twist in collinear factorization, since there is no chirally-odd fragmentation function, emerge at twist three, as the result of quark-gluon correlations. A twist-three collinear factorization theorem valid at large transverse momenta was proven by Qiu and Sterman [33, 34, 35], following early work by Efremov and Teryaev [170, 171, 172]. In this approach the cross section for SIDIS with a transversely polarized target has the general form [163, 173, 174]

$$\mathrm{d}\sigma \sim G_F(x, x') \otimes \mathrm{d}\hat{\sigma} \otimes D_1(z) + h_1(x) \otimes \mathrm{d}\hat{\sigma}' \otimes \hat{E}_F(z, z') , \tag{193}$$

where the first term contains a quark-gluon correlation function for the transversely polarized nucleon and the ordinary unpolarized fragmentation function for the final hadron, whereas the second term combines the transversity distribution with a twist-three fragmentation function. Let us focus on the first contribution (twist-three effects in the initial state). The hadronic tensor can then be written as

$$W_{\mu\nu}(P,q,P_h) = \sum_a \int \frac{\mathrm{d}z}{z}\, w^a_{\mu\nu}(P,q,P_h/z)\, z D^a_1(z)\,, \tag{194}$$

where the partonic tensor $w_{\mu\nu}$ contributing to the transversely polarized structure functions is (see Fig. 12)

$$w_{\mu\nu}(P,q,P_h) = \int \mathrm{d}^4k \int \mathrm{d}^4k' \;\mathrm{Tr}\left[\Phi_A(k,k')\, H_{\mu\nu}(k,k',q,P_h)\right]\,. \tag{195}$$

In this expression Φ_A is the quark-gluon correlator

$$\begin{aligned}\Phi_A(k,k') \;\; &= \;\; \int \frac{\mathrm{d}^4\xi}{(2\pi)^4} \int \frac{\mathrm{d}^4\eta}{(2\pi)^4}\, \mathrm{e}^{\mathrm{i}k\cdot\xi}\, \mathrm{e}^{\mathrm{i}(k'-k)\cdot\eta} \\ &\quad \times \langle P,S|\bar{\psi}(0)\, \mathcal{W}^-[0,\eta]\, gA^+(\eta)\, \mathcal{W}^-[\eta,\xi]\, \psi(\xi)|P,S\rangle\,,\end{aligned} \tag{196}$$

and $H_{\mu\nu}$ represents the perturbatively calculable partonic hard scattering. By means of the collinear expansion [34] (we display the dependence on the quark momenta only)

$$H(k,k') = H(x,x') + \left.\frac{\partial H}{\partial k_\alpha}\right|_{x,x'} (k_\alpha - xP_\alpha) + \left.\frac{\partial H}{\partial k'_\alpha}\right|_{x,x'} (k'_\alpha - x'P_\alpha)\,, \tag{197}$$

one finally ends up with

$$w_{\mu\nu} = \mathrm{i} \int \mathrm{d}x \int \mathrm{d}x' \;\mathrm{Tr}\left[\Phi^\alpha_F(x,x')\, \frac{\partial H(x,x')}{\partial k'^\alpha}\right]\,, \tag{198}$$

where one recognizes the quark-gluon correlator Φ^α_F introduced in eq. (119). It is easy to verify that, due to the structure of Φ^α_F, the hadronic tensor receives contributions only from the imaginary part of the hard blob, arising from internal propagator poles.

Considering for definiteness the Sivers contribution to the cross section, its explicit expression is [163, 173]

$$\begin{aligned}\mathrm{d}\sigma|_{\mathrm{Siv}} \;\; \sim \;\; & \int \frac{\mathrm{d}x}{x} \int \frac{\mathrm{d}z}{z}\, \delta\left(\frac{Q_T^2}{Q^2} - \left(1 - \frac{x}{x_B}\right)\left(1 - \frac{z}{z_h}\right)\right) \\ & \times \sum_a e_a^2 \left[x\, \frac{\mathrm{d}G^a_F(x,x)}{\mathrm{d}x}\, \hat{\sigma}_D + G^a_F(x,x)\, \hat{\sigma}_G + G^a_F(x,0)\, \hat{\sigma}_F \right. \\ & \left. + \, G^a_F(x,x_B)\, \hat{\sigma}_H \right] D^a_1(z) + \ldots \end{aligned} \tag{199}$$

The first two terms represent the so-called "soft-gluon pole" contribution ($x_g = x' - x = 0$), the third term is the "soft-fermion pole" contribution ($x' = 0$), the fourth term is

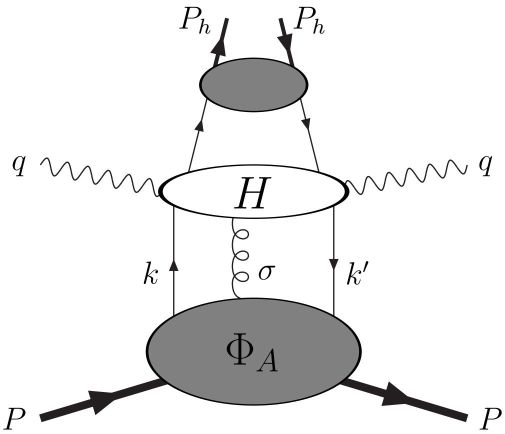

Figure 12: General diagram contributing to SIDIS SSA's in the twist-three factorization.

the "hard pole" contribution ($x' = x_B$). The dots represent the contributions of $\tilde{G}_F$ and of the gluonic correlation functions.

In the intermediate transverse-momentum region, i.e. $\Lambda_{QCD} \ll Q_T^2(P_{h\perp}^2) \ll Q^2$, one expects that both the TMD and the twist-three pictures should hold. This has been explicitly verified in Refs. [36, 38]. The output of these important works is a set of relations that connect the T-odd TMD's (Sivers and Boer-Mulders functions) on one side, with the quark-gluon correlations on the other side.

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