Poissonian, binomial and asymmetric errors

Asymmetric errors

Confidence limits and errors to be assigned to small numbers obtained from counting experiments. The limits given are upper and lower confidence limits for confidence coefficient 0.8413. This results in a confidence interval of probability $1-2(1-0.8413) = 0.6826$. The error is the difference between the confidence limit and the number of counts. In the following table we report the asymmetric errors for a number of counts in the interval:

$$0 \leqslant N_{count} \leqslant 25$$

Number of counts	Errors	Confidence limits	Percent errors	Number of counts	Errors	Confidence limits	Percent errors
0	+1.841 −−−	1.841 −−−	−−− −−−	13	+4.70 −3.56	17.70 9.44	+36.2 −27.4
1	+2.299 −0.827	3.299 0.1727	+229.9 −82.7	14	+4.83 −3.70	18.83 10.30	+34.5 −26.4
2	+2.637 −1.292	4.637 0.7083	+131.8 −64.6	15	+4.96 −3.83	19.96 11.17	+33.1 −−25.5
3	+2.918 −1.632	5.918 1.368	+97.3 −54.4	16	+5.06 −3.96	21.06 12.04	+31.6 −24.8
4	+3.162 −1.914	7.162 2.086	+79.0 −47.8	17	+5.20 −4.09	22.20 12.91	+30.6 −24.1
5	+3.382 −2.158	8.382 2.842	+67.6 −43.2	18	+5.32 −4.20	23.32 13.80	+29.6 −23.3
6	+3.583 −2.377	9.583 3.623	+59.7 −39.6	19	+5.43 −4.32	24.43 14.68	+28.6 −22.7
7	+3.770 −2.579	10.770 4.421	+53.9 −36.8	20	+5.54 −4.43	25.54 15.57	+27.7 −22.2
8	+3.944 −2.766	11.944 5.234	+49.3 −34.6	21	+5.65 −4.54	26.65 16.46	+26.9 −21.6
9	+4.110 −2.942	13.110 6.058	+45.7 −32.7	22	+5.76 −4.65	27.76 17.35	+26.2 −21.1
10	+4.27 −3.10	14.27 6.90	+42.7 −31.0	23	+5.86 −4.76	28.86 18.24	+25.5 −20.7
11	+4.41 −3.27	15.41 7.73	+40.1 −29.7	24	+5.97 −4.86	29.97 19.14	+24.9 −20.2
12	+4.56 −3.41	16.56 8.59	+38.0 −28.4	25	+6.07 −4.97	31.07 20.03	+24.3 −19.9

For high numbers, grater than 25, it is possible to calculate the asymmetric errors with good approximation using the following expression:

$$\text{For } N > 25 \implies N \, {}^{+\sqrt{N}+1}_{-\sqrt{N}}$$

Binomial and Poissonian errors

The errors of the number of events n selected from a sample of N events is:

$$\textbf{Binomial} \implies \delta n = \sqrt{n\left(1 - \frac{n}{N}\right)}$$

For $N \to \infty$, or simply for $\frac{n}{N} \to 0$ the error of n becomes Poissonian:

$$\textbf{Poissonian} \implies \delta n = \sqrt{n}$$