Electromagnetic Interactions

Introduction to Elementary Particle Physics

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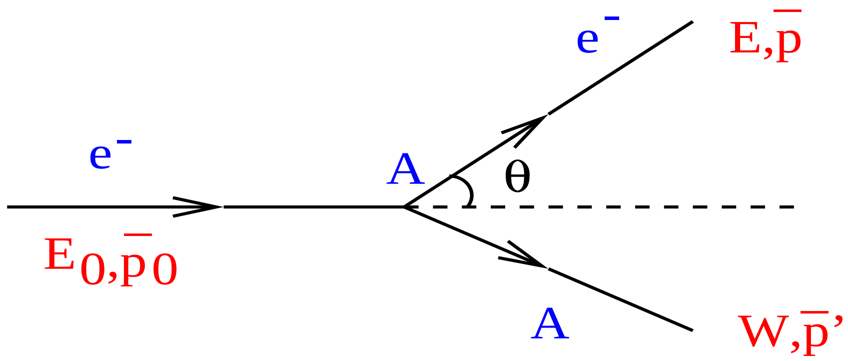
Outline

- Electron-nucleus scattering
- Rutherford formula
  - Mott cross-section
- Electron-nucleon scattering
- Rosenbluth formula
  - Hadron form factors
- The process $e^+e^- \rightarrow \mu^+\mu^-$
- Bhabha scattering $e^+e^- \rightarrow e^+e^-$
- Magnetic moments of leptons
  - Measurement of the muon g-2.

e-nucleus Scattering (neglecting spin)



The transition probability (per unit time) is given by **the Fermi golden rule:**

$$W = \frac{2\pi}{\hbar}\left|M_{if}\right|^2 \rho_f$$

For a potential V(r) the matrix element is given by the volume integral:

$$M_{if} = \int \Psi_f^* V(r) \Psi_i d\tau$$

ψ_i = wave function of the incoming electron

ψ_f = wave function of the scattered electron

The *Born approximation* assumes the perturbation to be weak; we can represent ψ_i and ψ_f as plane waves

$$M_{if} = \int e^{i(\vec{k}_0 - \vec{k})\cdot\vec{r}} V(\vec{r}) d^3\vec{r}$$

where $\mathbf{k_0=p_0/\hbar}$ **e** $\mathbf{k=p/\hbar}$ are the initial and final propagation vectors, respectively.

$$\rho_f \to \frac{p^2 d\Omega}{h^3}\frac{dp}{dE_f}$$

E_f = total energy in the final state

$$dW = \frac{2\pi}{\hbar}\left|M_{if}\right|^2 \frac{p^2 d\Omega}{h^3}\frac{dp}{dE_f}$$

$$d\sigma = \frac{dW}{v}$$

v =velocity of the incident beam

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2\pi)^2\hbar^4}\frac{p^2}{v}\frac{dp}{dE_f}\left|M_{if}\right|^2$$

Let us now consider the nuclear recoil. We assume that both incident and scattered electrons are extreme relativistic ($v \approx 1$) and we set ħ=c=1.

$p_0 = k_0 = E_0 \qquad p = k = E$

From energy conservation:

$$\underset{\substack{\uparrow \\ \text{initial energy}}}{E_i} = p_0 + \underset{\substack{\uparrow \\ \text{nucleus mass}}}{M} = p + W = \underset{\substack{\uparrow \\ \text{final energy}}}{E_f}$$

From momentum conservation:

$$\vec{p}_0 = \vec{p}\,' + \vec{p}$$

$$E_f = p + W = p + \sqrt{\vec{p}'^2 + M^2}$$

$$= p + \sqrt{\left(\vec{p}_0 - \vec{p}\right)^2 + M^2}$$

$$= p + \sqrt{p_0^2 + p^2 - 2 p_0 p \cos\vartheta + M^2}$$

$$\left(\frac{dp}{dE_f}\right)_\vartheta = \frac{W}{E_f - p_0\cos\vartheta} = \frac{W}{M}\frac{p}{p_0}$$

$$\frac{p}{p_0} = \frac{1}{1 + \frac{p_0}{M}(1 - \cos\theta)} = \frac{1}{1 + \frac{2 p_0}{M}\sin^2\frac{\theta}{2}}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2} p^2 \frac{W}{M}\frac{p}{p_0}\left|\int e^{i\vec{q}\cdot\vec{r}} V(\vec{r}) d^3\vec{r}\right|^2}$$

$\vec{q} = \vec{p}_0 - \vec{p}$ momentum transfer

$$\left(\frac{dp}{dE_f}\right)_\vartheta = \frac{1}{\left(\frac{dE_f}{dp}\right)_\vartheta} = \frac{1}{1+\frac{2p-2p_0\cos\vartheta}{2\sqrt{p_0^2+p^2-2p_0p\cos\vartheta+M^2}}}$$

$$= \frac{W}{W+p-p_0\cos\vartheta}$$

$$= \frac{W}{E_f-p_0\cos\vartheta}$$

$$= \frac{W}{M}\frac{M}{p_0+M-p_0\cos\vartheta}$$

$$\boxed{= \frac{W}{M}\frac{p}{p_0}}$$

$$p_0 + M = p + \sqrt{p_0^2 + p^2 - 2p_0 p\cos\theta + M^2}$$

$$\left(p_0 + M - p\right)^2 = p_0^2 + p^2 - 2p_0 p\cos\theta + M^2$$

$$\cancel{p_0^2} + \cancel{M^2} + \cancel{p^2} + 2p_0 M - 2p_0 p - 2Mp = \cancel{p_0^2} + \cancel{p^2} - 2p_0 p\cos\theta + \cancel{M^2}$$

$$: 2p_0^2$$

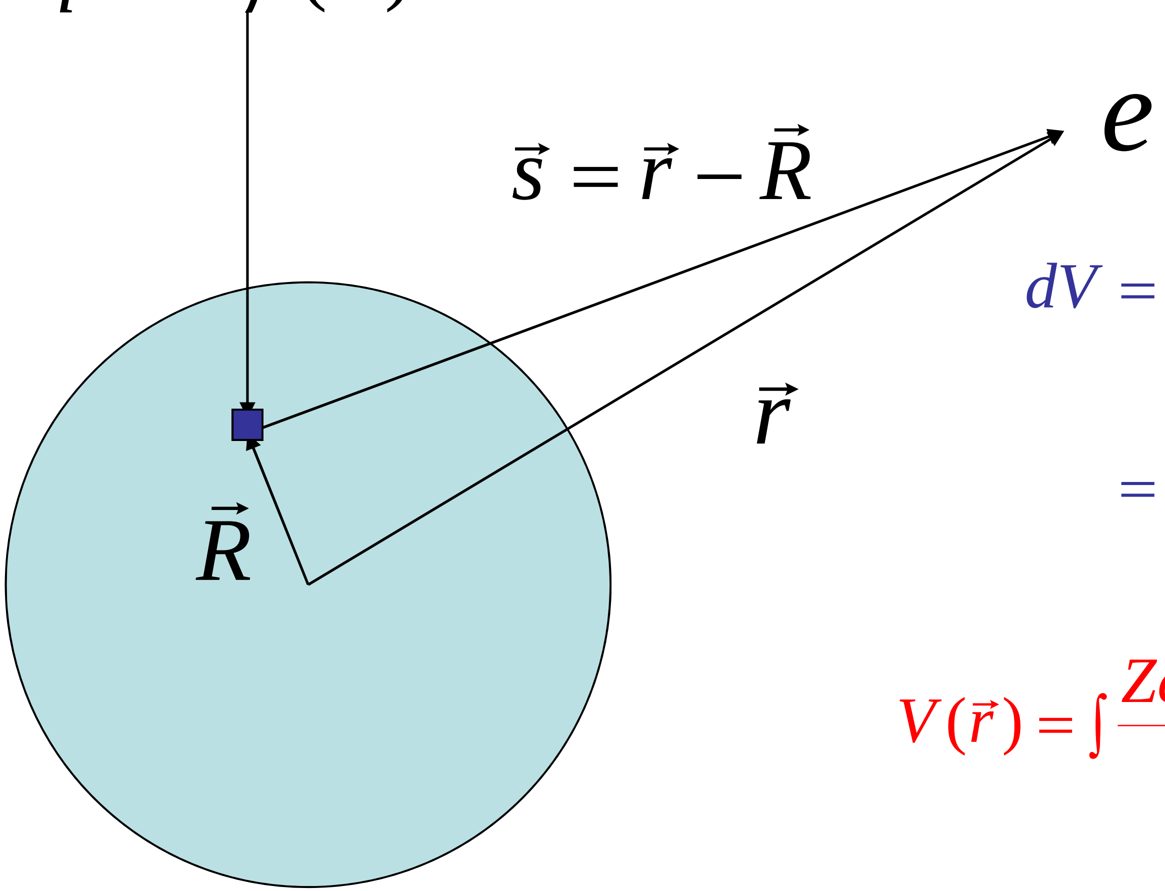
$$\frac{M}{p_0} - \frac{p}{p_0} - M\frac{p}{p_0^2} + \frac{p}{p_0}\cos\theta = 0$$

$$\frac{p}{p_0} = \frac{M}{p_0 + M - p_0\cos\theta}\boxed{= \frac{1}{1 + \frac{p_0}{M}(1 - \cos\theta)}}$$

Let us now calculate the matrix element, taking for V(r) the coulomb interaction.
We represent the nucleus by a sphere of charge density $Ze\rho(\vec{R})$

$$\int \rho(\vec{R})\, d^3\vec{R} = 1$$

$$dq = Ze\rho(\vec{R})\, d^3\vec{R}$$



$$dV = \frac{edq}{4\pi|\vec{s}|}$$

$$= \frac{Ze^2\rho(\vec{R})\, d^3\vec{R}}{4\pi|\vec{r} - \vec{R}|}$$

$$V(\vec{r}) = \int \frac{Ze^2\rho(\vec{R})\, d^3\vec{R}}{4\pi|\vec{r} - \vec{R}|}$$

$$M_{if} = \int d^3\vec{r} e^{i\vec{q}\cdot\vec{r}} \int d^3\vec{R} \frac{Ze^2 \rho(\vec{R})}{4\pi|\vec{r}-\vec{R}|}$$

$$= \frac{Ze^2}{4\pi} \int d^3\vec{r} \int d^3\vec{R} \frac{e^{i\vec{q}\cdot(\vec{r}-\vec{R})} e^{i\vec{q}\cdot\vec{R}} \rho(\vec{R})}{|\vec{r}-\vec{R}|}$$

$$= \frac{Ze^2}{4\pi} \int d^3\vec{R} \rho(\vec{R}) e^{i\vec{q}\cdot\vec{R}} \int \frac{e^{iqs\cos\alpha}}{s} 2\pi s^2 ds d(\cos\alpha)$$

α polar angle between $\vec{s}$ and $\vec{q}$

Let us define the Nuclear Form Factor $F(\vec{q}) = \int \rho(\vec{R}) e^{i\vec{q}\cdot\vec{R}} d^3\vec{R}$

If $\rho(\vec{R})$ is spherically symmetric $\rho(\vec{R}) = \rho(R)$

$$F(\vec{q}) = \int \left(1 + i\vec{q}\cdot\vec{R} - \frac{(\vec{q}\cdot\vec{R})^2}{2} + \cdots\right) \rho(R) d^3\vec{R}$$

$$q^2 = |\vec{q}|^2$$

$$= 1 - \frac{1}{6} q^2 \langle R \rangle^2 + \cdots \quad \Longrightarrow \quad F = F(q^2)$$

The matrix element becomes:

$$M_{if} = \frac{Ze^2}{2} F(q^2) \int_0^{\infty} s\,ds \int_{-1}^{1} e^{iqs\cos\alpha} d(\cos\alpha)$$

$$= \frac{Ze^2}{2} F(q^2) \int_0^{\infty} s \frac{e^{iqs} - e^{-iqs}}{iqs} ds$$

This integral diverges.
We modify V(r) by a factor $e^{-r/a}$ which takes into account the screening of the nucleus by atomic electrons.
Since a ≈ atomic dimensions >> R ≈ nuclear dimension, we can write:

$$e^{-r/a} \approx e^{-s/a} \qquad\qquad R << a$$

And the matrix element becomes:

$$M_{if} = \frac{Ze^2}{2iq} F(q^2) \int_0^{\infty} e^{-s/a} (e^{iqs} - e^{-iqs}) ds$$

$$\int e^{-s(\frac{1}{a}-iq)}ds - \int e^{-s(\frac{1}{a}+iq)}ds$$

$$= \frac{1}{\frac{1}{a}-iq} - \frac{1}{\frac{1}{a}+iq} = \frac{\frac{1}{a}+iq-\frac{1}{a}+iq}{q^2+\frac{1}{a^2}} = \frac{2iq}{q^2+\frac{1}{a^2}}$$

$$M_{if} = \frac{Ze^2 F(q^2)}{q^2+\left(\frac{1}{a}\right)^2}$$

$a \approx 10^{-8}$ cm = 10^{-10} m $\approx 5\times10^{5}$ GeV^{-1}
$1/a \approx 0.2\times10^{-5}$ GeV = 2 keV
$q^2 \gg (1/a)^2$

$\Rightarrow$

$$M_{if} = \frac{Ze^2}{q^2}F(q^2)$$

$$\boxed{\frac{d\sigma}{d\Omega} = 4Z^2\left(\frac{e^2}{4\pi}\right)^2 \frac{1}{q^4} p^2 \frac{W}{M}\frac{p}{p_0}\left[F(q^2)\right]^2}$$

Let us assume $\dfrac{W}{M} \approx 1$ (Non relativistic nuclear recoil)

$$p' = q << p_0 \quad \Rightarrow \quad p \approx p_0 \quad (\frac{p}{p_0} \approx 1)$$

$$q^2 = (\vec{p}_0 - \vec{p})^2 \approx 2p_0^2 - 2p_0^2 \cos\theta = 4p_0^2 \sin^2\frac{\theta}{2}$$

$$\frac{d\sigma}{d\Omega} = 4Z^2\left(\frac{e^2}{4\pi}\right)^2 \frac{1}{16p_0^4 \sin^4\frac{\theta}{2}} p_0^2 \left[F(q^2)\right]^2 = \frac{Z^2\alpha^2}{4p_0^2\sin^4\frac{\theta}{2}}\left[F(q^2)\right]^2$$

$$[d\Omega = 2\pi d(\cos\theta); \quad dq^2 = 2p_0^2 d(\cos\theta); \quad d\Omega = \frac{\pi}{p_0^2} dq^2]$$

For an effectively pointlike nucleus, i.e. low values of q^2, $F(q^2) \approx 1$

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2}{4p_0^2\sin^4\frac{\theta}{2}}$$

$$\frac{d\sigma}{dq^2} = \frac{4\pi Z^2\alpha^2}{q^4}$$

Rutherford cross section

Four-momentum Transfer

Initial state

$$\begin{cases} P_0 \equiv (E_0, \vec{p}_0) & \text{incident electron} \\ P_0' \equiv (M, \vec{0}) & \text{nucleus at rest in the laboratory} \end{cases}$$

Final state

$$\begin{cases} P \equiv (E, \vec{p}) & \text{scattered electron} \\ P' \equiv (W, \vec{p}\,') & \text{recoiling nucleus} \end{cases}$$

$$\begin{aligned} q^2 \equiv (P_0 - P)^2 &= (E_0 - E)^2 - (\vec{p}_0 - \vec{p})^2 \\ &= E_0^2 + E^2 - 2E_0E - p_0^2 - p^2 + 2\vec{p}_0 \cdot \vec{p} \\ &= 2m^2 - 2E_0E + 2p_0 p\cos\theta \qquad m << E \\ &\approx -2p_0 p(1-\cos\theta) = -4p_0 p \sin^2\frac{\theta}{2} \end{aligned}$$

$q^2 < 0$ spacelike (scattering processes)

$q^2 > 0$ timelike (annihilation, e.g. $e^+e^- \to \pi^+\pi^-$)

Considering the four-momentum transfer to the nucleus:

$$-q^2 = (P_0' - P')^2 = (M - W)^2 - \vec{p}'^2 = 2M^2 - 2MW = -2MK$$

where K = W-M = kinetic energy of the nucleus

Thus: $$\frac{W}{M} = 1 + \frac{q^2}{2M^2}$$

The nucleus recoils coherently for $q^2 << 2M^2$

$$\boxed{\frac{W}{M} \approx 1}$$

The nuclear form factor:

$$F(q^2) = \int \rho(R) e^{iqR\cos\theta} 2\pi R^2 d(\cos\theta) dR$$

$$= \int \rho(R) \frac{e^{iqR} - e^{-iqR}}{iqR} 2\pi R^2 dR$$

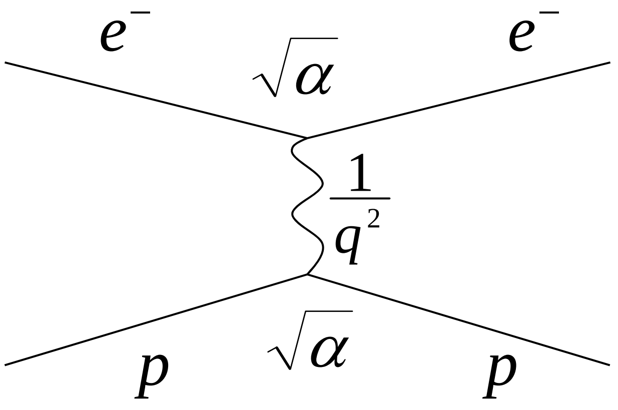
$$= \int \rho(R) \frac{\sin qR}{qR} 4\pi R^2 dR$$

Typical nuclear radius is R $\approx$ a few fm. For example, if R = 4 fm, qR = 1, q=1/R

$$qc = \frac{\hbar c}{R} = \frac{197 MeV \cdot fm}{4\ fm} \approx 50 MeV$$

Therefore if q << 50 MeV/c, qR $\rightarrow$ 0 and F(q^2) $\rightarrow$1

$$\frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{q^4}$$



Electron Spin

For a relativistic fermion the spin vector $\vec{\sigma}$ is aligned with the momentum vector $\vec{p}$

$$\langle \sigma_z \rangle = \pm 1 \qquad \langle \sigma_x \rangle = \langle \sigma_y \rangle = 0$$

if $\vec{p}$ defines the z axis.

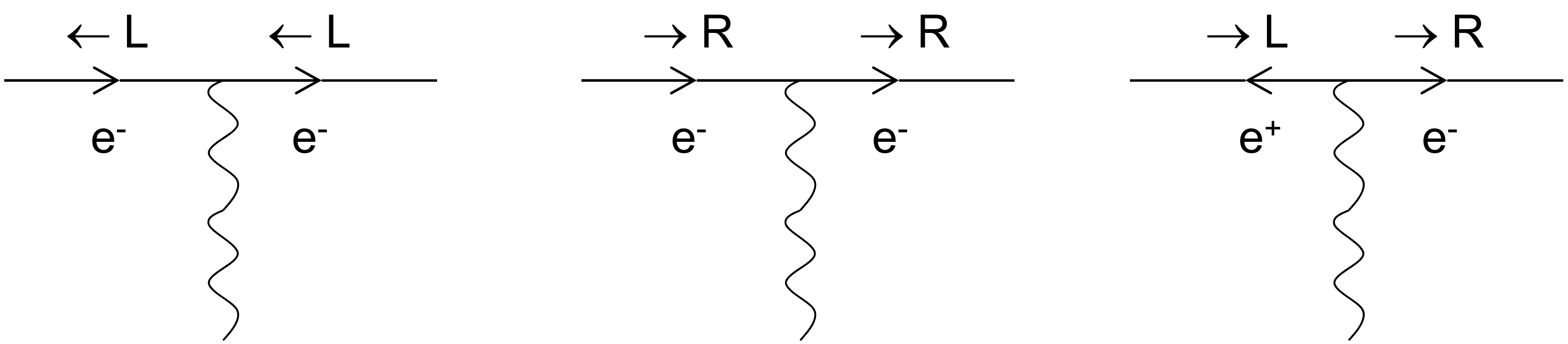
The helicity H is defines as:

$$H = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{p}|} = \pm 1$$

H=+1 right-handed ψ_R

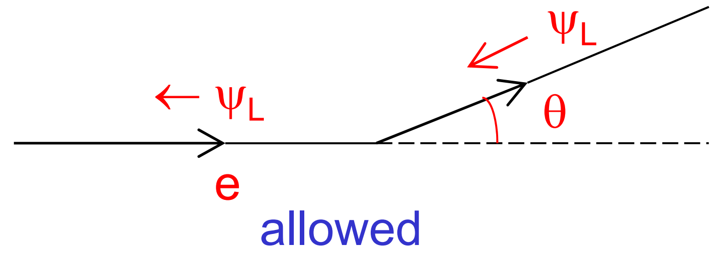
H= -1 left-handed ψ_L

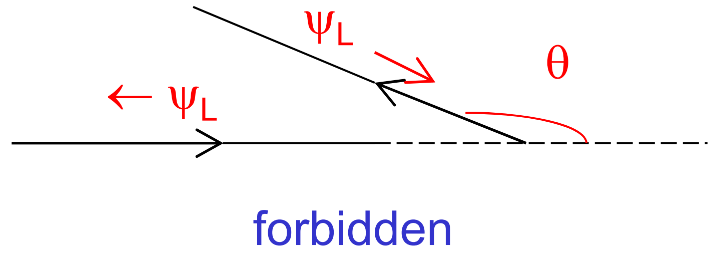
In electromagnetic interactions helicity is conserved.

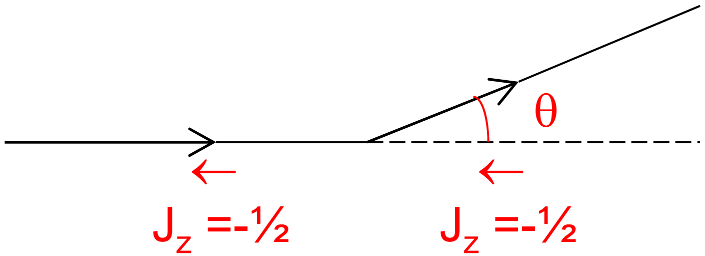


$J_z = \pm 1$ Transverse photon

In the relativistic limit fermion and antifermion have opposite helicities.







Helicity amplitudes

$$d^{j}_{mm'}(\theta)$$

$$\left\{\begin{array}{ll} J_z = -\frac{1}{2} \rightarrow J_z = -\frac{1}{2} & \quad d^{\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}} = \cos\frac{\theta}{2} \\ J_z = +\frac{1}{2} \rightarrow J_z = +\frac{1}{2} & \quad d^{\frac{1}{2}}_{\frac{1}{2},+\frac{1}{2}} = \cos\frac{\theta}{2} \\ J_z = \pm\frac{1}{2} \rightarrow J_z = \mp\frac{1}{2} & \quad d^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}} = \sin\frac{\theta}{2} \end{array}\right.$$

Mott cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{Z^2\alpha^2\cos^2\frac{\theta}{2}}{4p_0^2\sin^4\frac{\theta}{2}} \frac{1}{1+\frac{2p_0}{M}\sin^2\frac{\theta}{2}}$$

the factor p/p_0 takes into account the nuclear recoil.

e-N Scattering

Let us now consider also the spin of the target. In the scattering of electrons by hypothetical pointlike protons there will be a magnetic and an electrical interaction.

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \left(\cos^2\frac{\theta}{2} + \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{Z^2\alpha^2}{4p_0^2\sin^4\frac{\theta}{2}} \frac{1}{1+\frac{2p_0}{M}\sin^2\frac{\theta}{2}}$$

$\cos^2\frac{\theta}{2}$: electrical (non spin-flip)

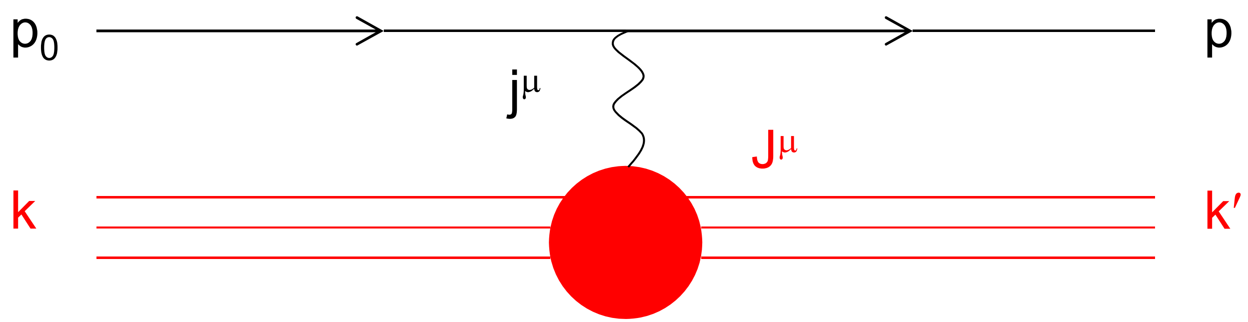
$\frac{q^2}{2M^2}\sin^2\frac{\theta}{2}$: magnetic (spin-flip)

If nucleons were pointlike this would be the cross section. However p and n are not pointlike, as shown by their anomalous magnetic moments:

	μ(Dirac)	μ(experimental)
p	eħ/2mc = 1 n.m.	+2.79 n.m.
n	0	-1.91 n.m.

Nucleons have an **extended structure**

Proton Structure



$$j^{\mu} = -e\bar{u}(p)\gamma^{\mu}u(p_0)e^{i(p-p_0)\cdot x}$$

$$J^{\mu} = e\bar{u}(k')[\cdots]u(k)e^{i(k'-k)\cdot x}$$

$[\cdots] =$ most general 4-vector which can be constructed from k, k′, q and the Dirac matrices γ^{μ}.

There are only two independent terms, γ^{μ} and $i\sigma_{\mu\nu}q^{\nu}$, and their coefficients are functions of q^2.

$$[\cdots] = F_1(q^2)\gamma^{\mu} + \frac{\kappa}{2M}F_2(q^2)i\sigma^{\mu\nu}q_{\nu}$$

F_1 and F_2 are the **Form Factors of the Proton**

As $q^2 \rightarrow 0$ we see a particle of charge e and magnetic moment $\frac{(1+\kappa)e\hbar}{2Mc}$

$F_1(0)=1$	$F_2(0)=1$	proton
$F_1(0)=0$	$F_2(0)=1$	neutron

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}\times\left[\left(F_1^2-\frac{\kappa^2 q^2}{4M^2}F_2^2\right)\cos^2\frac{\theta}{2}-\frac{q^2}{2M^2}(F_1+\kappa F_2)^2\sin^2\frac{\theta}{2}\right]$$

Sachs Form Factors

$$G_E \equiv F_1+\frac{\kappa q^2}{4M^2}F_2$$ electric Form Factor

$$G_M \equiv F_1+\kappa F_2$$ magnetic Form Factor

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}\times\left[\frac{G_E^2+\tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2}+2\tau G_M^2\sin^2\frac{\theta}{2}\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)=\left(\frac{d\sigma}{d\Omega}\right)_{Mott}\times\left[\frac{G_E^2+\tau G_M^2}{1+\tau}+2\tau G_M^2\tan^2\frac{\theta}{2}\right]$$

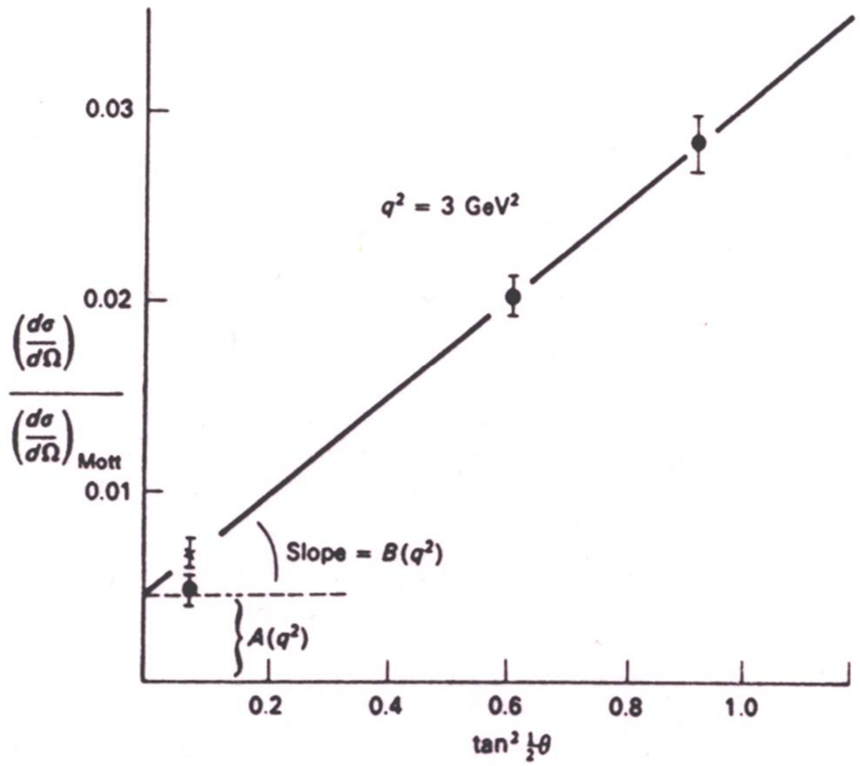
$$\tau=\frac{q^2}{4M^2}$$

Rosenbluth Formula

$$G_E = G_E(q^2) \qquad G_M = G_M(q^2) \qquad G_E^p(0) = 1 \qquad G_E^n(0) = 0$$

$$G_M^p(0) = +2.79 \qquad G_M^n(0) = -1.91$$

$$\left(\frac{d\sigma}{d\Omega}\right) \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{Mott} = A(q^2) + B(q^2)\tan^2\frac{\theta}{2}$$



Rosenbluth Plot

The experimental determination of the nucleon form factors in the spacelike region ($q^2 < 0$) is carried out by directing e^- beams of energy between 400 MeV and 16 GeV at a hydrogen target (for the proton) or deuterium (for the neutron).

$$e^- + p \to e^- + p$$

$$e^- + d \to e^- + d$$

For the neutron:

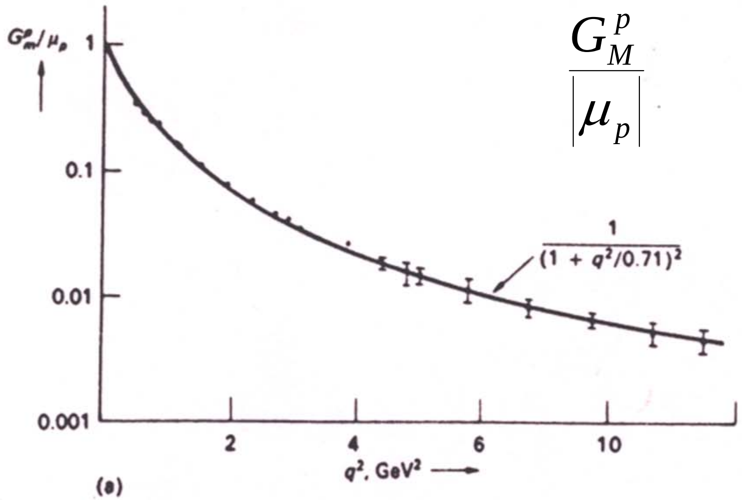
$$\frac{d\sigma}{d\Omega}(en) = \frac{d\sigma}{d\Omega}(ed) - \frac{d\sigma}{d\Omega}(ep) + \textit{fattori di correzione}$$

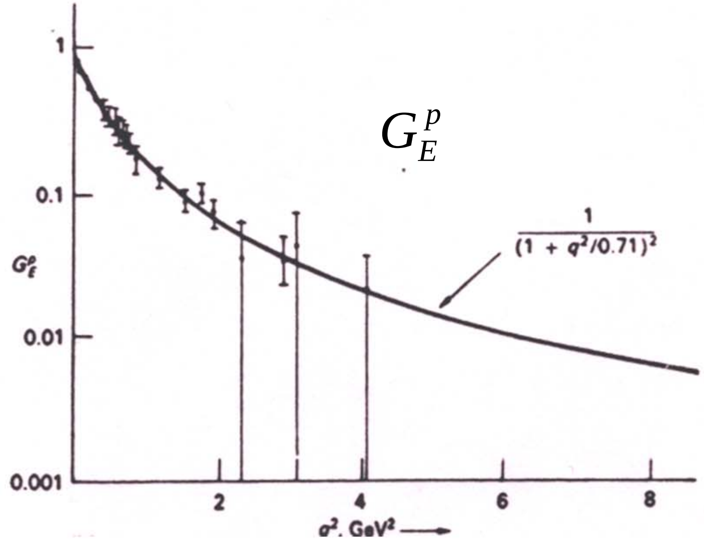
Scaling laws for the form factors:

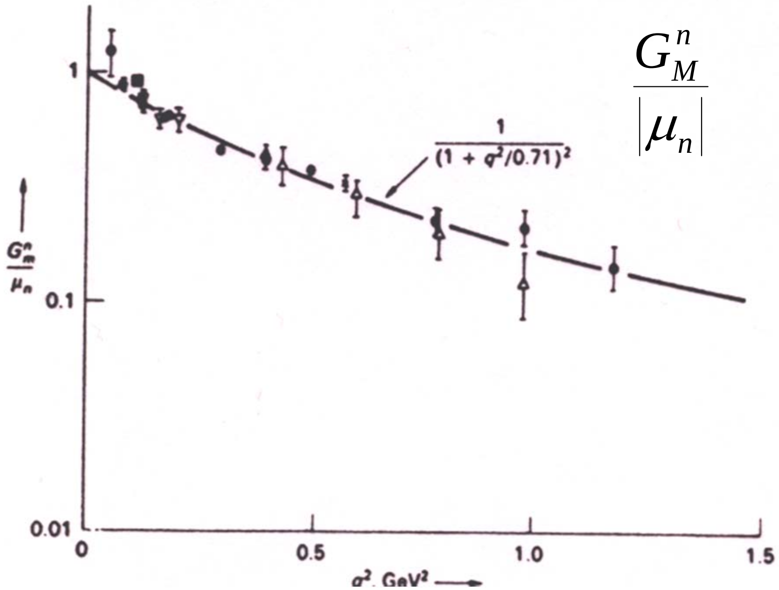
$$G_E^p\left(q^2\right) = \frac{G_M^p\left(q^2\right)}{\left|\mu_p\right|} = \frac{G_M^n\left(q^2\right)}{\left|\mu_n\right|} = G\left(q^2\right)$$

$$G_E^n\left(q^2\right) = 0$$

Nucleon Spacelike Form Factors







Proton Form Factors at High q^2

Dipole formula: $$G(q^2) = \frac{1}{\left(1 + \frac{q^2}{M_V^2}\right)^2}$$

$$M_V^2 = (0.84\ GeV)^2$$

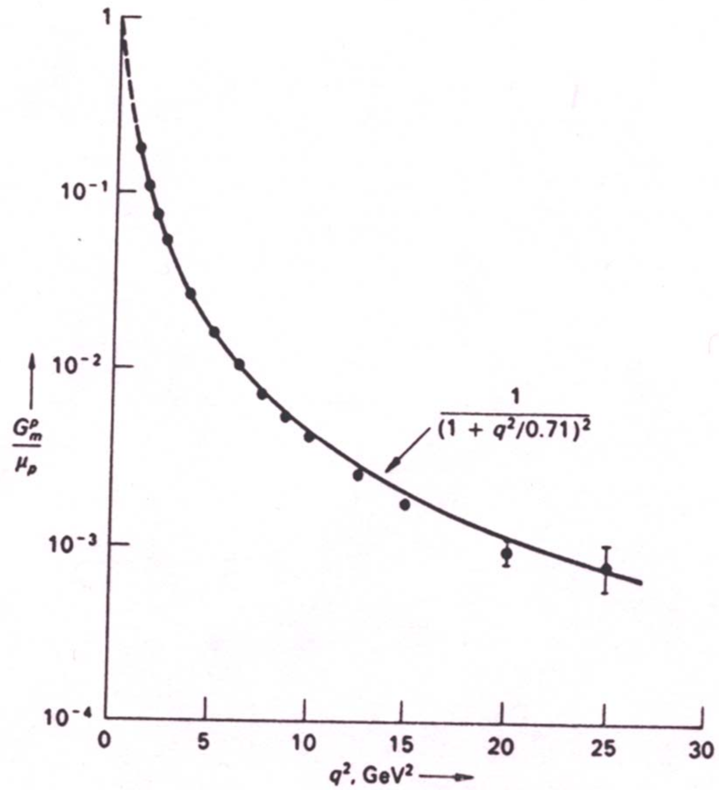
The dipole form corresponds to an exponential charge distribution

$$\rho(R) = \rho_0 e^{-M_V R}$$

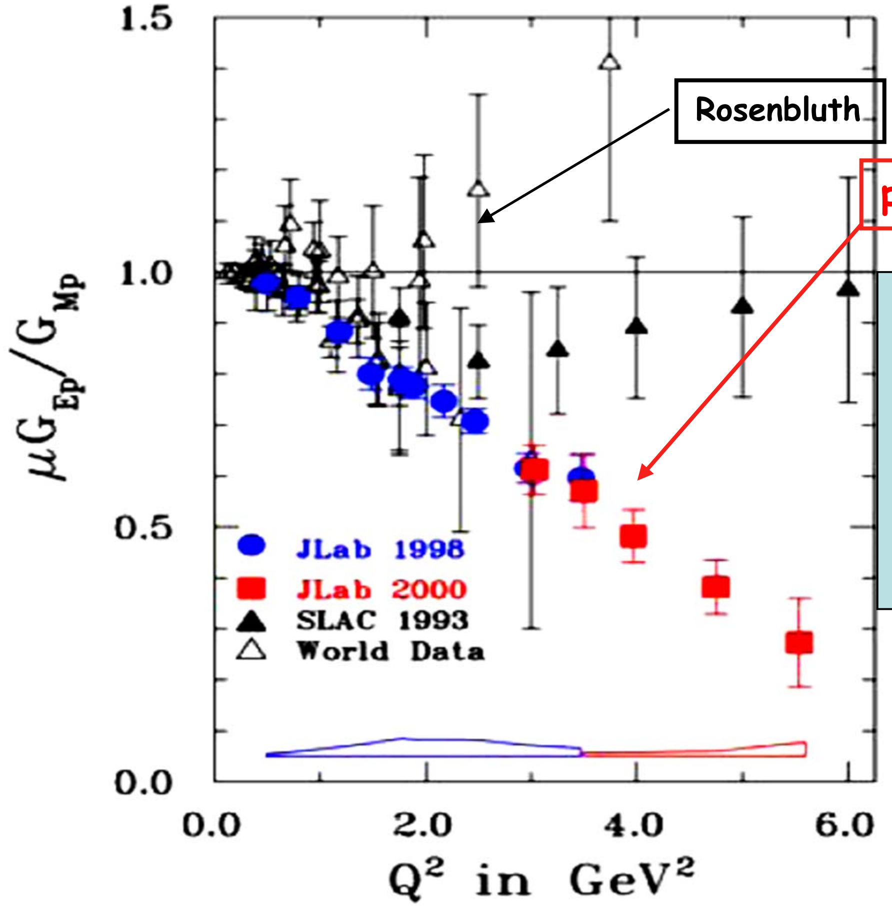
with an rms radius

$$\langle R^2 \rangle = \frac{\int_0^\infty \rho_0 e^{-M_V R} R^2 d^3R}{\int_0^\infty \rho_0 e^{-M_V R} d^3R} = \frac{12}{M_V^2}$$

For the proton: $$\sqrt{\langle R^2 \rangle} = \frac{\sqrt{12}}{M_V} = 0.80\ \ fm$$

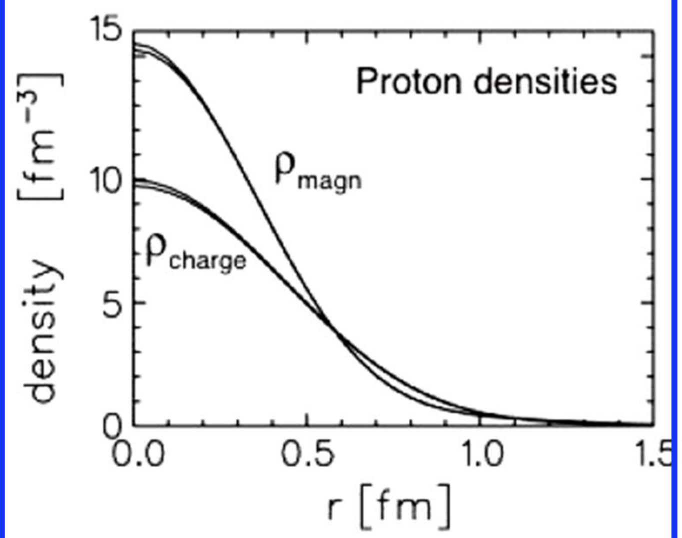


Form Factor Measurements Using Polarization



Linear deviation from dipole

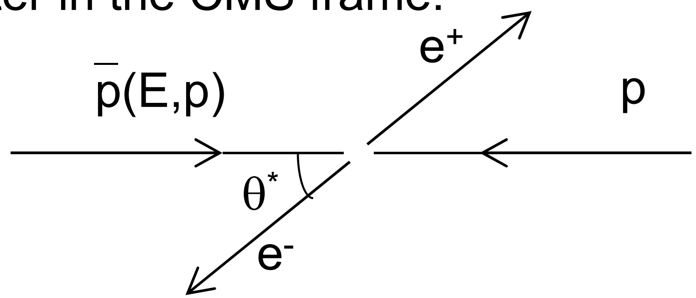
$G_E \neq \mu G_M$



- Different charge and magnetization distributions
- Quark angular momentum contribution?

Timelike Proton Form Factors

They can be measured via the processes $e^+e^- \rightarrow p\ \bar{p}$, or $p\ \bar{p} \rightarrow e^+e^-$.
For the latter in the CMS frame:



$$\frac{d\sigma}{d\left(\cos\theta^*\right)} = \frac{\pi\alpha^2\hbar^2c^2}{2xs}\left[\left|G_M\right|^2\left(1+\cos^2\theta^*\right) + \frac{4m_p^2}{s}\left|G_E\right|^2\left(1-\cos^2\theta^*\right)\right]$$

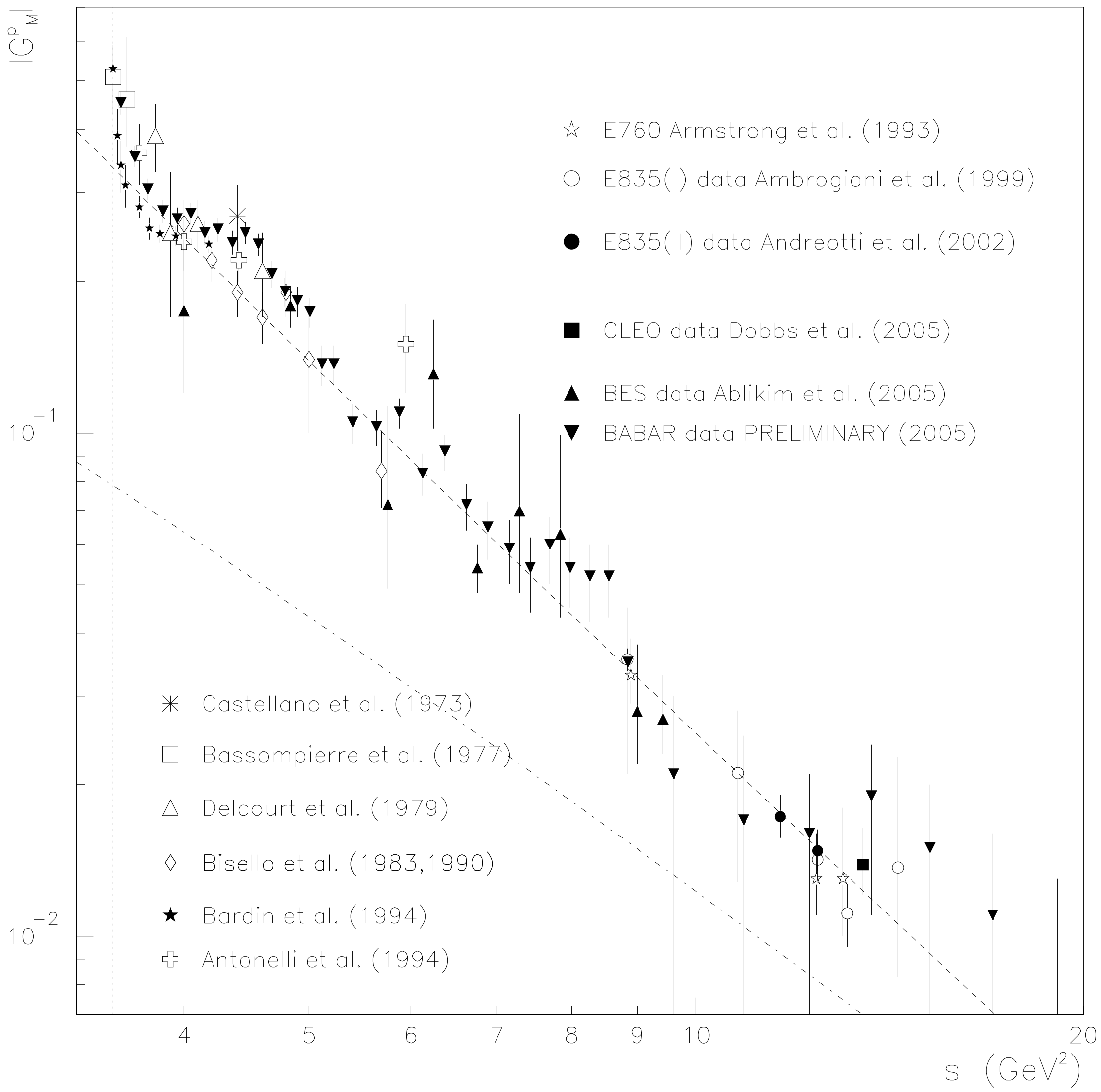
$\sqrt{s}$ = CM Energy

One measures the
total cross section σ.

$$\sigma = \int_0^{2\pi} d\phi \int_{-\left|\cos\theta^*\right|_{\max}}^{+\left|\cos\theta^*\right|_{\max}} \frac{d\sigma}{d\Omega} d(\cos\theta^*)$$

$$= \frac{\pi\alpha^2}{2\beta_p s}\left[A\left|G_M\right|^2 + \frac{4m_p^2}{s} B\left|G_E\right|^2\right]$$

Proton Magnetic Form Factor



The dashed line is a fit to the PQCD prediction

$$\frac{|G_M|}{\mu_p} = \frac{C}{s^2 \ln^2\left(\frac{s}{\Lambda^2}\right)}$$

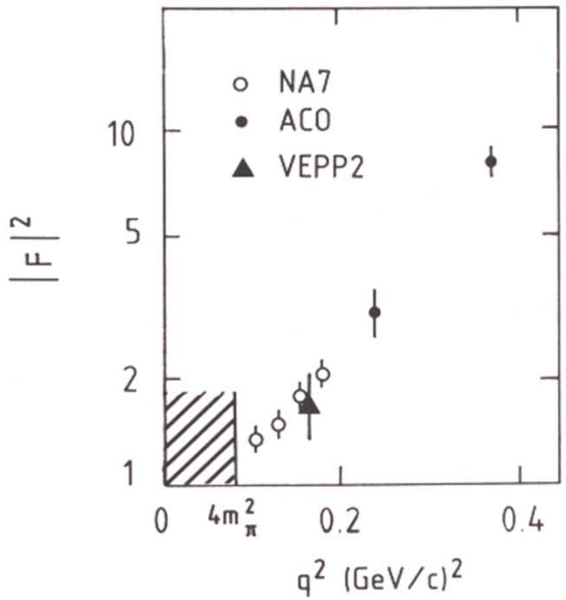
The expected Q^2 behaviour is reached quite early, however ...

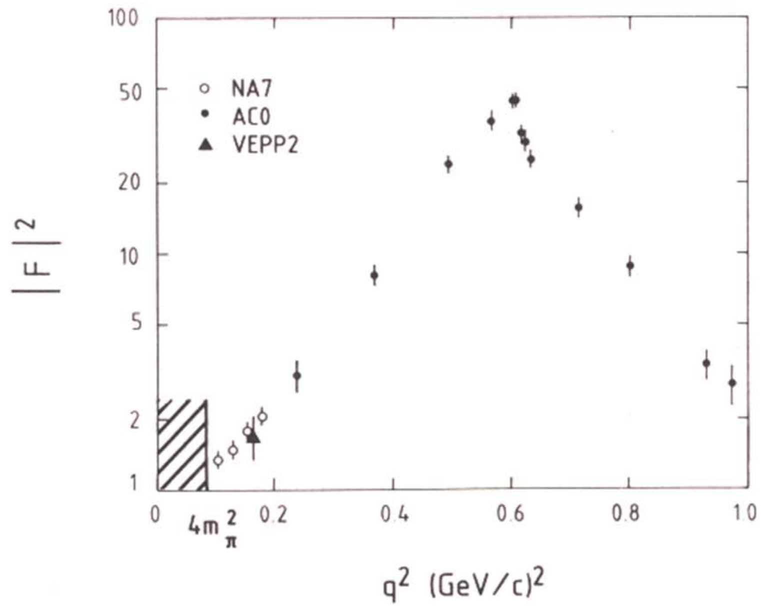
... there is still a factor of 2 between timelike and spacelike.

Timelike Form Factor of the π

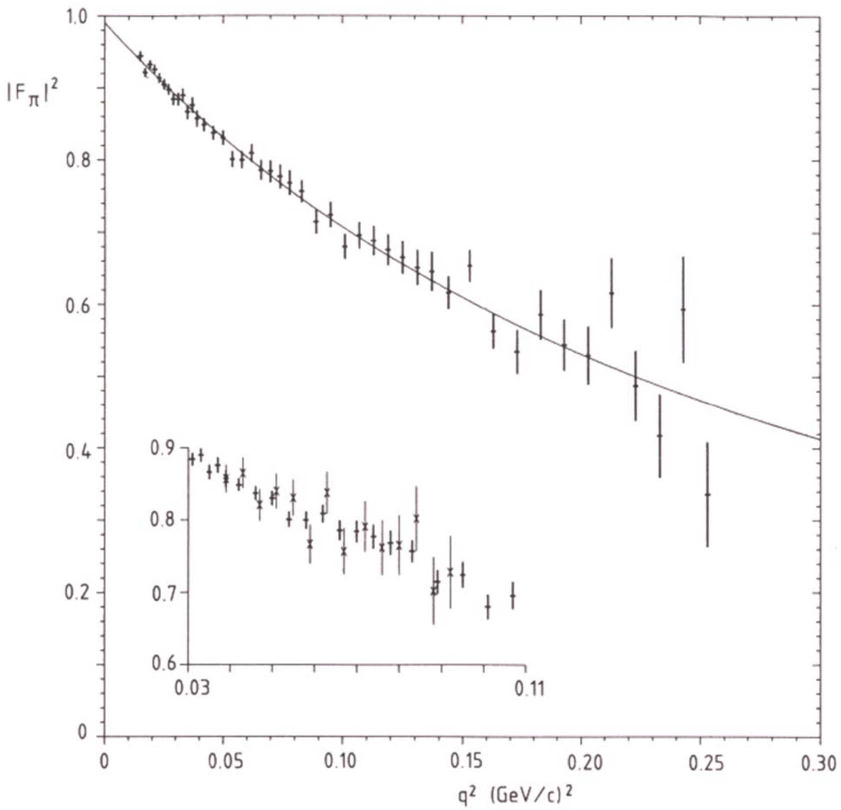
$$e^{+} + e^{-} \rightarrow \pi^{+} + \pi^{-}$$

e^{+} beam with energies
100, 125, 150, 175 GeV





Spacelike Form Factor of the π



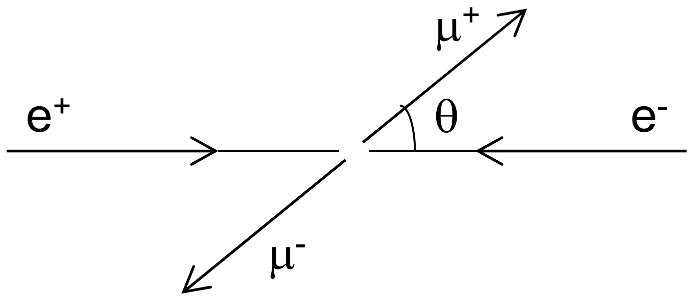
$$\pi^- + e^- \rightarrow \pi^- + e^-$$

300 GeV π^- beam

$$\langle r_\pi^2 \rangle = 0.439 \pm 0.008 \quad fm^2$$

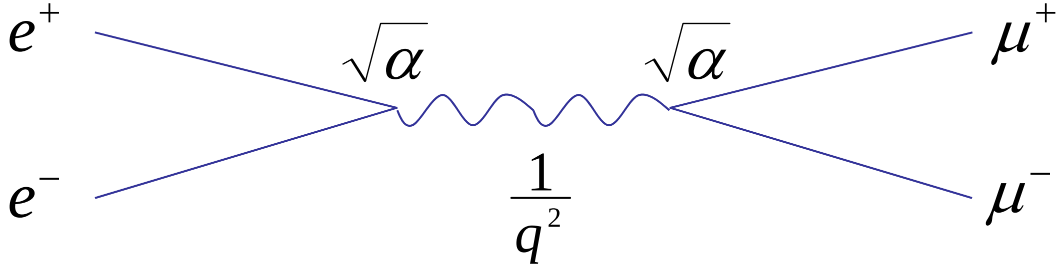
$$r_\pi \approx 0.66 \quad fm$$

$e^+e^- \rightarrow \mu^+\mu^-$



$$\sigma\left(e^+e^- \rightarrow \mu^+\mu^-\right) = \frac{4\pi\alpha^2}{3s}$$

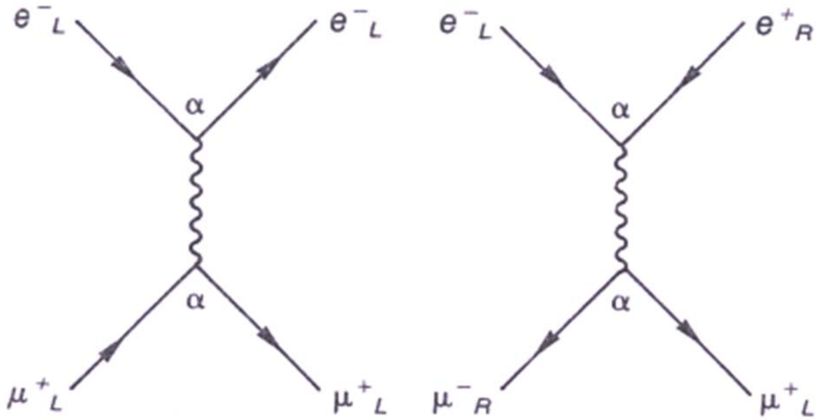
$s = 4E_1E_2$ if $s >> m_e^2, m_\mu^2$



- Each vertex gives a contribution $\sqrt{(\alpha)}$ to the matrix element. The cross section is therefore proportional to α^2. $\sigma \propto \alpha^2$.
- For a timelike process $q^2=s$, hence the propagator $\frac{1}{q^2} = \frac{1}{s}$

- σ has dimensions of $(\text{length})^2$, i.e. $(\text{energy})^{-2}$. If $s >> m_e^2, m_\mu^2$ s is the only energy scale in the process: $\sigma \propto \frac{1}{s}$
- The factor $(4/3)\pi$ comes from integration over solid angle and averaging over spins.

Angular Distribution for $e^+e^- \rightarrow \mu^+\mu^-$



$$\frac{d\sigma}{d\Omega} \propto \left(1+\cos^2\theta\right)$$

$$J=1 \qquad J_z=\pm 1$$

Because electromagnetic interactions conserve parity, both J_z=+1 and J_z=-1 occur with equal probability. The amplitude for emitting the μ^+ at angle θ to the e^+ starting from a J_Z=+1 state is given by:

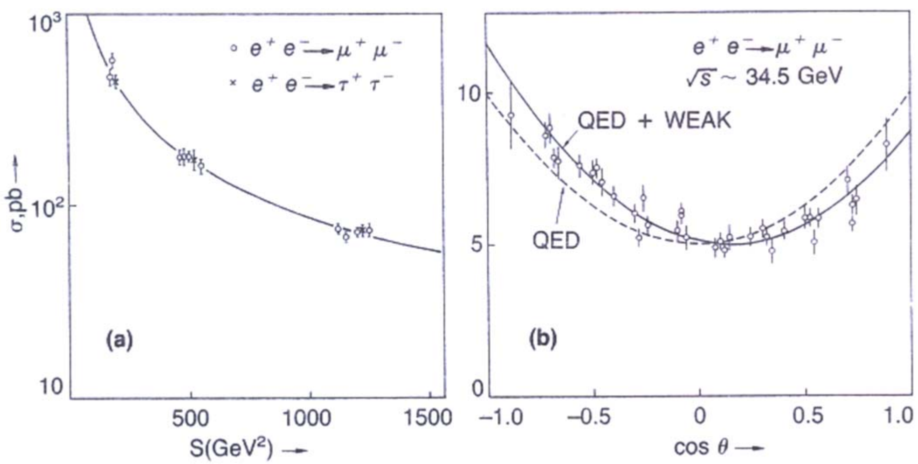
$$d^1_{1,1} = \tfrac{1}{2}\left(1+\cos\theta\right)$$

If the initial state has J_Z=-1 we have to replace θ by π-θ hence the amplitude becomes:

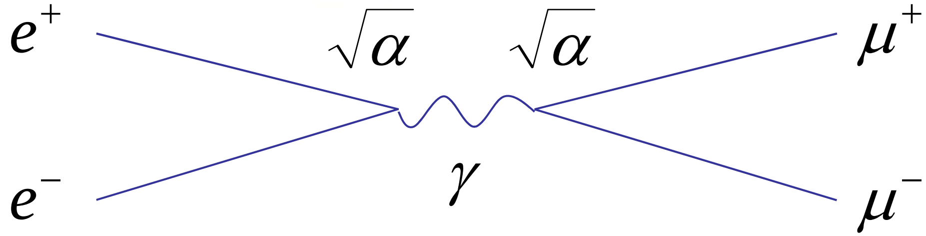
$$\tfrac{1}{2}\left(1-\cos\theta\right)$$

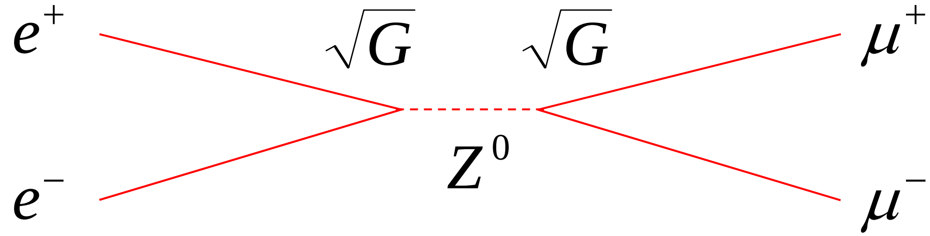
Squaring and adding the amplitudes of these two orthogonal states we obtain:

$$\frac{d\sigma}{d\Omega} \propto \left(1+\cos\theta\right)^2 + \left(1-\cos\theta\right)^2 \propto \left(1+\cos^2\theta\right)$$



The process $e^+e^- \to \mu^+\mu^-$ (or $e^+e^- \to \tau^+\tau^-$) is not purely electromagnetic: there is a weak contribution, due to Z^0 exchange.





$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}(QED) + \frac{d\sigma}{d\Omega}(weak) + \frac{d\sigma}{d\Omega}(interference)$$

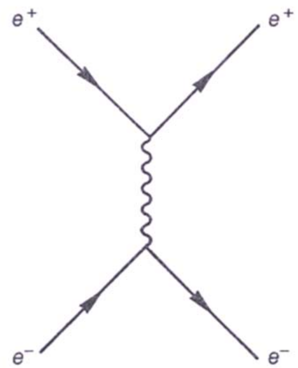
$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

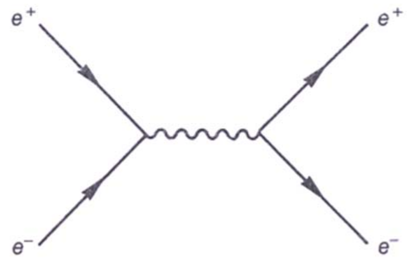
$$\frac{\alpha^2}{s} \qquad\qquad G^2 s \qquad\qquad \alpha G$$

The asymmetry arises from the interference term, the effect is of the order of 10 % for s = 1000 GeV^2.

$e^+e^- \rightarrow e^+e^-$
Bhabha Scattering

The dimensional arguments used for $e^+e^- \rightarrow \mu^+\mu^-$ apply equally well to *BhaBha scattering* to predict a 1/s dependence for the total cross section.
The angular distribution is however more complex, because two diagrams contribute:





The first diagram dominates at small angles. In this region the cross section is large and is used to monitor the luminosity in e^+e^- colliders.

Lepton Magnetic Moments

According to the Dirac theory a pointlike fermion possesses a magnetic moment equal to the *Bohr magneton* . If *e* and *m* are the lepton charge and mass:

$$\mu_B = \frac{e}{2m}$$

In general the magnetic moment is related to the spin vector $\vec{s}$ by:

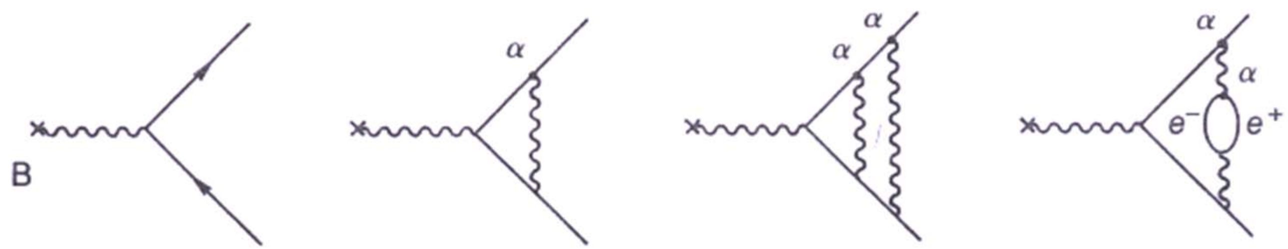
$$\vec{\mu} = g\mu_B\vec{s}$$

Where g is called the Landé factor and $g\mu_B$ is the gyromagnetic ratio.
For electron and muon $|\vec{s}|=½$ and the Dirac theory predicts g=2.
The actual g-values have been measured experimentally with great precision and have been found to differ by a small amount (0.2 %) from the value 2.
The Dirac picture of a structureless, point particle is not exact for the electron and muon.

The magnetic moment of a charged particle depends on the spatial distributions of charge and mass (e/m ratio). For a spin ½ a value g≠2 argues that processes are taking place which distort the relative charge and mass distributions. For example, for the proton g=5.59, due to its internal structure.

The electron, the μ, the τ consist of a bare, pointlike object surrounded by a cloud of virtual γ which are continually being emitted and reabsorbed. These γ carry part of the mass energy of the lepton, and hence the e/m ratio (and thus the magnetic moment) changes.

In terms of QED:



The Landé can be written as a perturbation series in (α/π).

To lowest order:

$$g = 2 \qquad \vec{\mu} = -\frac{e}{2m}\vec{\sigma}$$

At the next order:

$$g = 2 + \frac{\alpha}{\pi} \qquad \vec{\mu} = -\frac{e}{2m}\left(1 + \frac{\alpha}{\pi}\right)\vec{\sigma}$$

We can define the anomaly: $a \equiv \frac{g-2}{2}$

$$a_e \equiv \left(\frac{g-2}{2}\right)_e^{QED} = 0.5\left(\frac{\alpha}{\pi}\right) - 0.32848\left(\frac{\alpha}{\pi}\right)^2 + 1.19\left(\frac{\alpha}{\pi}\right)^3 + \cdots$$

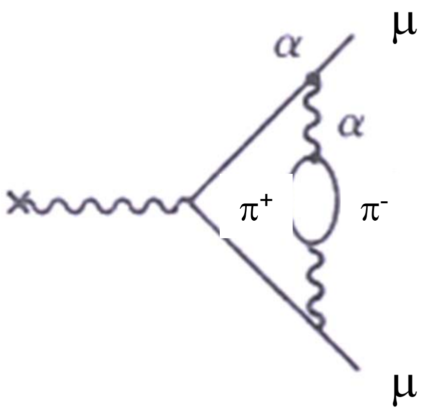
$$= (1159652140 \pm 28) \times 10^{-12}$$

$$a_\mu \equiv \left(\frac{g-2}{2}\right)_\mu^{QED} = 0.5\left(\frac{\alpha}{\pi}\right) - 0.76578\left(\frac{\alpha}{\pi}\right)^2 + 24.45\left(\frac{\alpha}{\pi}\right)^3 + \cdots$$

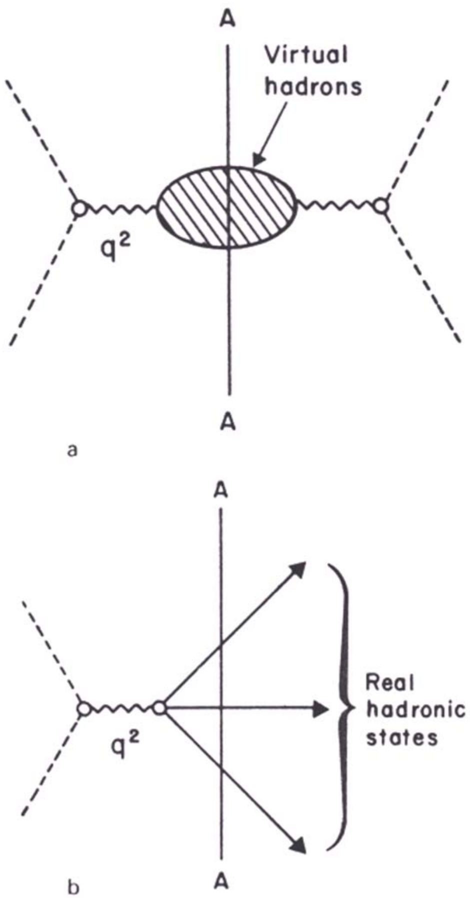
$$= (1165847008 \pm 18 \pm 28) \times 10^{-12}$$

For the μ the measured value differs by 9 standard deviations from the QED calculation. This arises from the fact that for the μ there are further corrections to a_μ due to the strong and weak interactions.

Hadrons do not couple directly to the μ, but they can couple to the virtual photon. We therefore expect hadronic contributions to vacuum polarization of the kind:



This contribution, which would be small due to the high π mass, gets amplified by resonances in the $\pi\pi$ system (vector resonances).



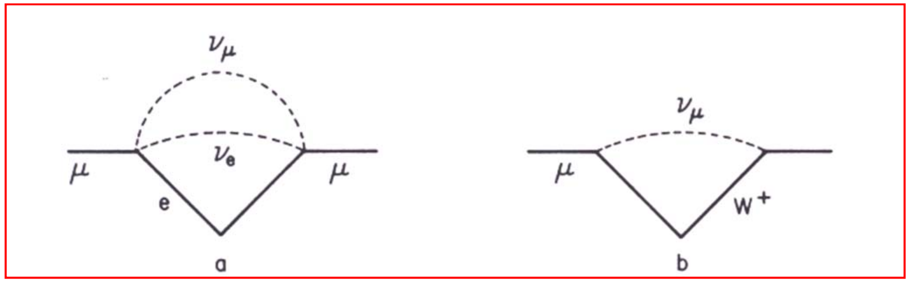
The strong contribution to the anomaly can be computed starting from the measurable cross section

$e^+e^- \to$ hadrons

via dispersion relations the two diagrams can be related one to the other.

$$\Delta a_\mu(\,forte) = \frac{m_\mu^2}{12\pi^3}\int_0^\infty \frac{\sigma\left(e^+e^- \to adroni\right)}{s}ds$$

Weak contributions



Measurement of the Muon (g-2)

Consider a longitudinally polarized charged particle moving in a static magnetic field $\vec{B}$. The particle momentum rotates at the cyclotron frequency:

$$\omega_c = \frac{eB}{mc}$$

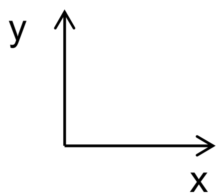
The spin precesses at the frequency:

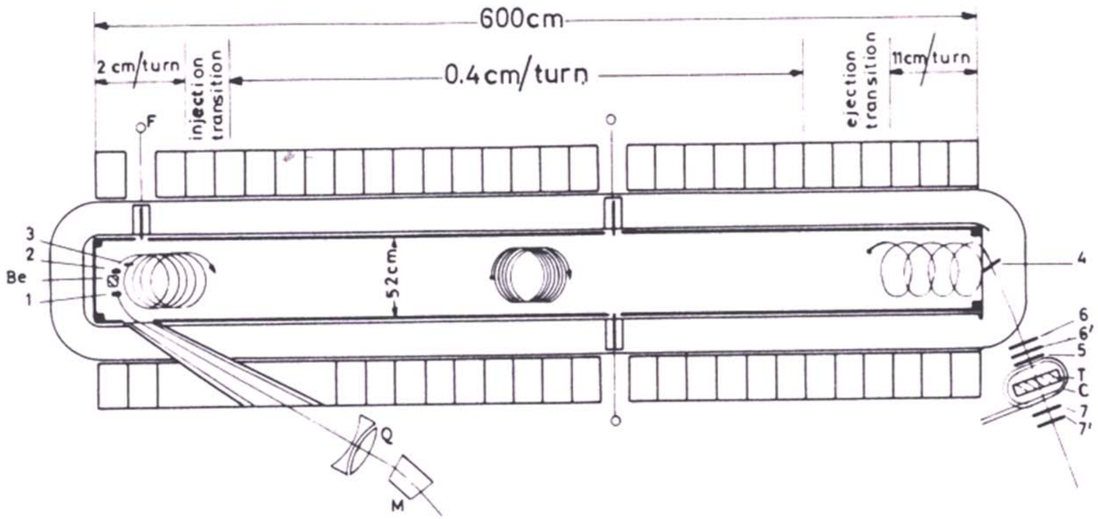
$$\omega_s = g\frac{eB}{2mc} = (1 + a_\mu)\frac{eB}{mc}$$

If g=2, i.e. a_μ=0, ω_s=ω_c and the particle will maintain its longitudinal polarization. If however g>2 (a_μ>0), ω_s>ω_c, spin precesses faster than momentum. The rotation frequency ω_a of the spin with respect to the momentum is given by:

$$\omega_a = \omega_s - \omega_c = a_\mu \frac{eB}{mc}$$

The measurement principle of a_μ is the following: muons are kept turning in a known magnetic field B, the angle between the spin and the direction of motion is measured as a function of time and from this the value of a_μ can be determined.
Since $a_\mu \approx 1/800$, the muon must make roughly 800 turns in the field for the spin to make 801 and the polarization to change gradually through 2π.

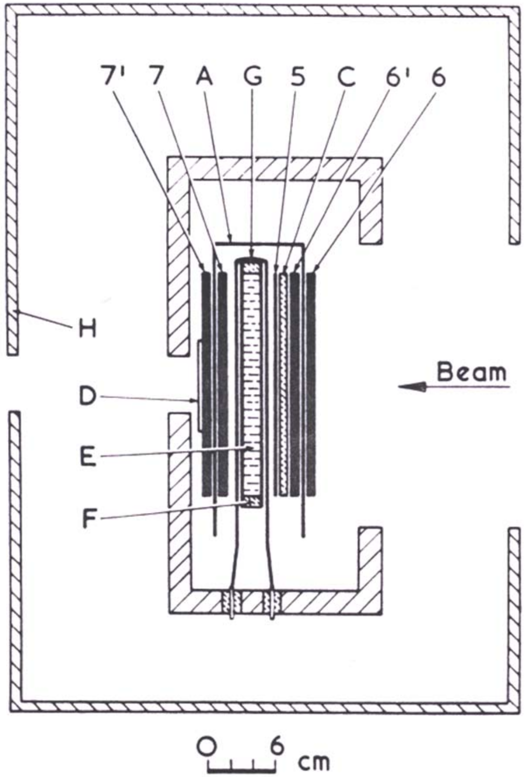




The muons take roughtly 2000 turns in the field. The field gradient displaces the orbit to the right. At the end a very large gradient is used to eject the muons, which are then stopped in the polarization analyzer.

$$B_z = B_0\left(1 + ay + by^2\right)$$

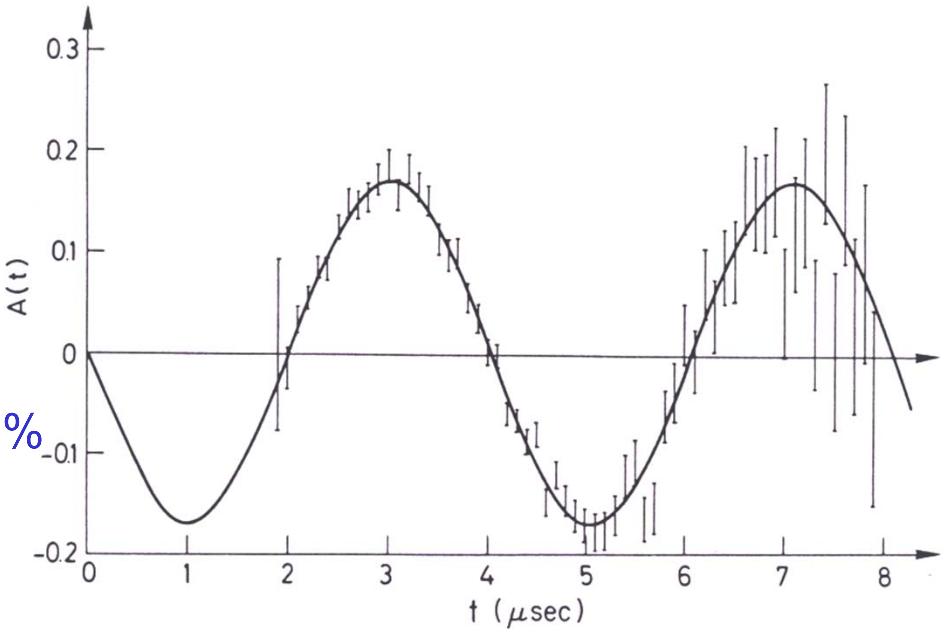
$$B = 1.6\,T \qquad p_\mu \approx 90\ MeV/c \qquad t \approx \tau_\mu = 2.2\ \mu s$$



Polarization analyzer. When a muon stops in the liquid methylene iodide (E) a pulse of current in coil G is used to flip the spin through $\pm 90^0$. Backward or forward decay electrons are detected in the counter telescopes 66′ and 77′. The asymmetry in counting rates as a function of time is given by:

$$A \equiv \frac{c_+ - c_-}{c_+ + c_-} = A_0 \sin \theta_s$$

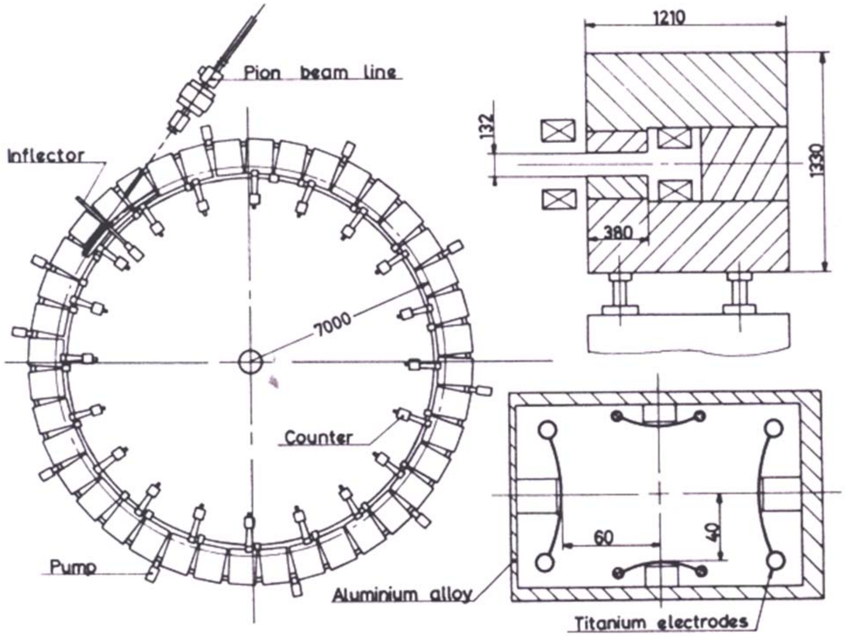
$$A = A_0 \sin\left\{ a_\mu \left(\frac{e}{mc} \right) Bt + \phi \right\}$$



Using this method the anomaly a_μ was measured with an accuracy of 0.4 %

$$a_\mu = (1162 \pm 5) \times 10^{-6}$$

Muon Storage Ring



In order to improve the measurement accuracy it was necessary to increase the number of (g-2) cycles, either by increasing the field B or by lengthening the storage time.
The usage of an electric field for the vertical focussing allowed the use of a uniform magnetic field.
For the precession of spin in combined electric and magnetic fields:

$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$

For γ=29.3 (*magic γ*) the coefficient of the second term vanishes and the precession is again $\propto a_\mu$.

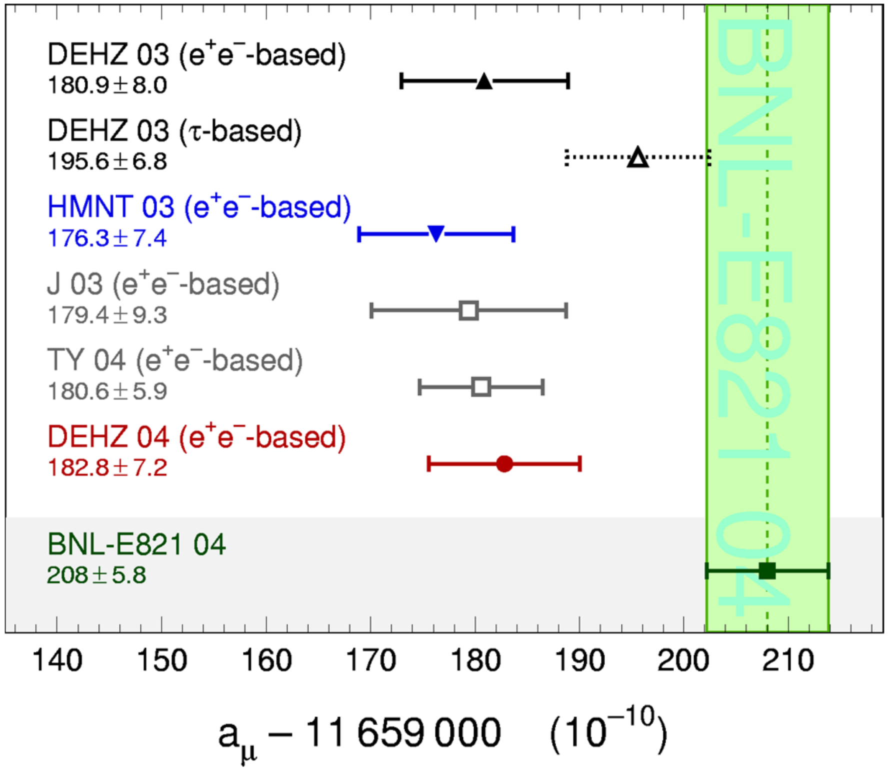
This method allowed to considerably increase the accuracy in the measurement of a_μ.

$$a_\mu = (1165924 \pm 9)\times 10^{-9}$$

Present Situation for a_μ

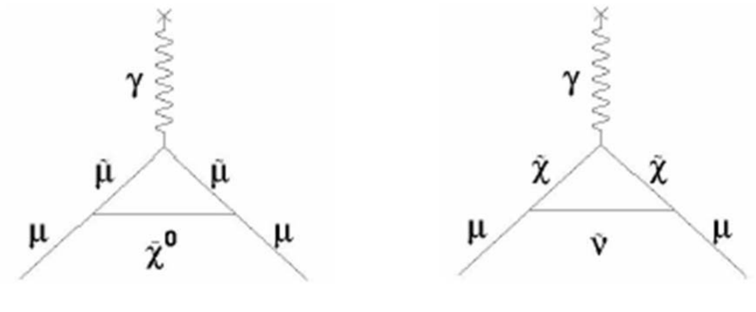
$$a_\mu^{SM}[e^+e^-] = (11\,659\,182.8 \pm 6.3_{had} \pm 3.5_{LBL} \pm 0.3_{QED+EW}) \times 10^{-10}$$

BNL E821 (2004) : $a_\mu^{exp} = (11\,659\,208.0 \pm 5.8)\ 10^{-10}$



$$\boxed{a_\mu^{exp} - a_\mu^{SM} = (25.2 \pm 9.2) \times 10^{-10}}$$

The discrepancy between theory and experiment is 2.7 standard deviations.



$$\Delta a_\mu^{SUSY} \approx 1.1\,\text{ppm} \times \left(\frac{100\ GeV}{\tilde{m}}\right)^2 \tan\beta$$