Weak Interactions

Introduction to Elementary Particle Physics

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Introduction

Weak interactions were first observed in the slow process of nuclear β decay. They take place in circumstances where the much faster strong or electromagnetic decays are forbidden by the conservation laws, or processes involving neutrinos, which can only have weak interaction.

Examples:

$n \rightarrow p + e^{-} + \overline{\nu}_e$	$\tau \approx 10^{3}\ s$	B, Q, L
$\overline{\nu}_e + p \rightarrow n + e^{+}$	$\sigma \approx 10^{-43}\ cm^2$	
$\Lambda \rightarrow p + \pi^{-}$	$\tau \approx 10^{-10}\ s$	ΔS=1
$\pi^{+} \rightarrow \mu^{+} + \nu_{\mu}$	$\tau \approx 10^{-8}\ s$	Energy

Weak processes are characterized by smaller cross sections and longer lifetimes than for strong and electromagnetic processes.

$$\left.\begin{array}{ll} \nu_{\mu} + N \rightarrow N + \pi + \mu^{-} & \sigma \approx 10^{-38}\ cm^2 \\ \pi + N \rightarrow \pi + N & \sigma \approx 10^{-26}\ cm^2 \end{array}\right\} \text{at 1 GeV}$$

The weak interactions can be classified as to whether they involve leptons only, leptons and hadrons or hadrons only:

Leptonic

$$\begin{cases} \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \\ \nu_e + e^- \to \nu_e + e^- \end{cases}$$

Semileptonic

$$\begin{cases} \left.\begin{array}{l} n \to p + e^- + \bar{\nu}_e \\ \bar{\nu}_e + p \to n + e^+ \end{array}\right\} \quad \Delta S = 0 \\ \left.\begin{array}{l} K^+ \to \pi^0 + e^+ + \nu_e \\ K^+ \to \mu^+ + \nu_\mu \end{array}\right\} \quad \Delta S = 1 \\ D^+ \to \bar{K}^0 + \mu^+ + \nu_\mu \quad \Delta S = 1;\ \Delta C = 1 \end{cases}$$

Nonleptonic

$$\begin{cases} \Lambda \to \pi^- + p \\ K^+ \to \pi^+ + \pi^0 \\ K^+ \to \pi^+ + \pi^- + \pi^+ \\ D^+ \to \pi^+ + \pi^+ + \pi^- + \pi^0 \end{cases}$$

Natural Radioactivity

Discovered in 1896 by H. Becquerel (one year before the electron was discovered and several years before nuclei were known to exist !!!)

Three types of radiation, classified by Rutherford:

- α-rays. Easy to absorbe. Bent slightly in the presence of magnetic fields (positive charge, "heavy"). α-rays emitted by a well defined isotope are monoenergetic.
- β -rays. Harder to absorb than α-rays. Bent significantly in the presence of magnetic fields (negative charge, "light").
- γ -rays. Not bent in the presence of magnetic fields (no charge), very hard to absorb.

Each type of radiation is due to a different interaction:

$\alpha \rightarrow$ strong, $\beta \rightarrow$ weak, $\gamma \rightarrow$ electromagnetic.

Nuclear β decay

Early studies of β-rays revealed two fundamental properties of this type of radiation:

- They were identical to cathode rays (electrons).
- Their energy spectrum was discrete (like the spectrum of α-rays)
  - studies of the absorption of β-rays seemed to indicated that these were monoenergetic.
  - the spectrum of a β-ray beam incident on a photographic plate in the presence of a constant magnetic field seemed to be discrete.
  - β-ray emission was a quantum phenomenon, to be associated to a discrete spectrum.
  - ${}^{A}Z \rightarrow {}^{A}(Z+1) + e^{-}$ energy conservation implies

$$E(e^{-}) = M({}^{A}Z) - M({}^{A}(Z+1))$$

Only in 1914 did Chadwick show that the observed β-ray spectrum is continuous and only 15 years later it was shown that the nuclear β-ray spectrum is continuous.

A Wrong Model for the Nucleus

In the 1920's it was postulated that nuclei were made up of protons and electrons, such that ^{A}Z contained A protons and A-Z electrons (e.g. $^{4}He = 4p$; $^{14}N = 14p + 7e^{-}$). In addition to energy conservation in β-ray emission there were several other problems:

- Magnetic moment of nuclei: $\mu_e >> \mu_p$, $\mu_e >> \mu_{nucleo}$. Impossible if the nucleus is made of protons and electrons.
- Spin-statistics: according to this model ^{14}N consists of an odd number of fermions ($14p+7e^{-}$), implying a half-integer spin, whereas experimentally it was found to be a boson.

Solution:

<u>electrons bound in nuclei behave differently from free electrons !!! (Bohr)</u>

<u>Gamow</u>: "*This would mean that the idea of energy and its conservation fails in dealing with processes involving the emission or capture of nuclear electrons. This does not sound improbable if we remember all that has been said about peculiar properties of electrons in the nucleus".*

Pauli and the "*neutron*"

In 1930 W. Pauli made the hypothesis that there was a third constituent inside the nucleus: the "*neutron*": a fermion with no electric charge, which interacted very weakly with matter and with a mass of less than 1% of the proton mass: $^{14}N = 14p + 7e^- + 7'\nu'$.

In this way:

- The spin-statistics problems was solved, since the number of fermions in the nucleus was now "correct".
- The apparent violation of energy conservation in β-decay was explained assuming that the correct physical process was:

$$^{A}Z \rightarrow {}^{A}(Z-1) + e^- + `\nu'$$

  i.e. a three-body decay, with a continuous energy spectrum for the electron.

Pauli (1930)

Dear Radioactive Ladies and Gentlemen,

I have come upon a desperate way out regarding the wrong statistics of the ^{14}N and ^{6}Li nuclei, as well as the continuous β spectrum, in order to save the “alternation law” of statistics and the energy law. Namely, the possibility that there could exist in the nucleus electrically neutral particles, which I shall call “neutrons”, which have spin ½ and satisfy the exclusion principle and which further differ from light quanta in that they do not travel with the speed of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any case not larger than 0.01 times the proton mass. The countinuous β spectrum would then become understandable from the assumption that in β decay a neutron is emitted along with the electron, in such a way that the sum of the energies of the neutron and the electron is constant. For the time being I dare not publish anything about this idea and address myself to you, dear radioactive ones, with the question how it would be with experimental proof of such a neutron, if it were to have the penetrating power equal to about ten times larger than a γ ray.

I admit that my way out may not seem very probable *a priori* since one would probably have seen the neutrons a long time ago if they exist. But only the one who dares wins, and the seriousness of the situation concerning the continuous β spectrum is illuminated by my honored predecessor, Mr Debye, who recently said to me in Brussels: “ Oh, it is best not to think about this at all, as with new taxes”. One must therefore discuss seriously every road to salvation. Thus, dear radioactive ones, examine and judge. Unfortunately I cannot appear personally in Tübingen since a ball in Zürich makes my presence here indispensible.

Your most humble servant, W. Pauli

From a letter written by W. Pauli , dated 4 december 1930, to the physicists attending a Nuclear Physics conference in Tübingen.

The Discovery of the Neutron

In 1932 Chadwick discovered a neutral nuclear constituent. By studying the properties of the neutral radiation n emitted in the process

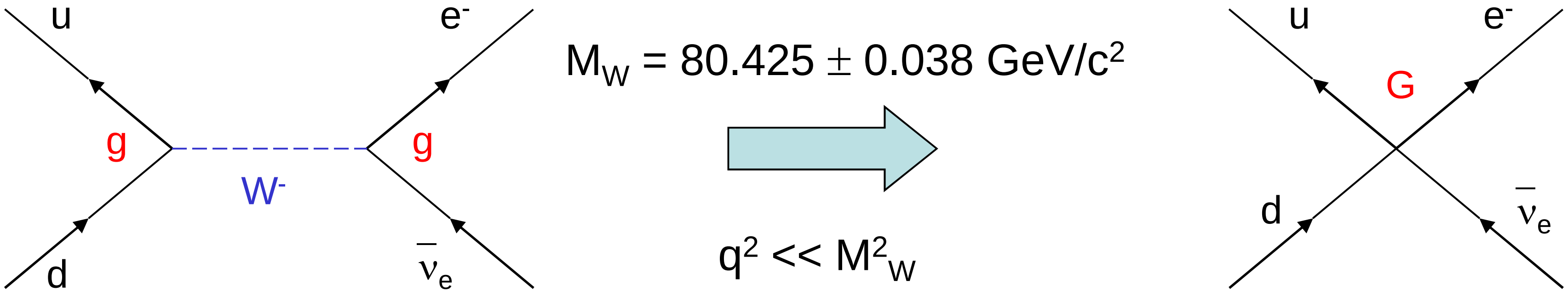
$$^{9}Be + \alpha \rightarrow {}^{12}C + n$$

he found out that the particle n, the **neutron**, was a deeply penetrating neutral particle slightly heavier than the proton, quite distinct from γ-rays, i.e. <u>a different particle from the neutron postulated by Pauli</u>. Given the fact that Chadwick's neutron was much heavier than Pauli's, Fermi renamed Pauli's neutron the **neutrino**.

Fermi Theory of β Decay

$$n \rightarrow p + e^- + \overline{\nu}_e \qquad \tau = 885.7 \pm 0.8\, s$$

$$\left(d \rightarrow u + e^- + \overline{\nu}_e\right)$$



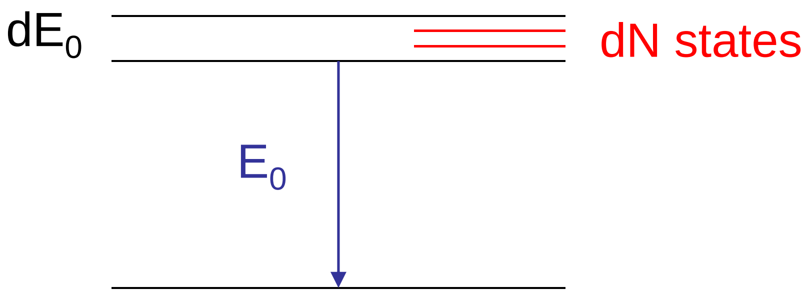
The interaction is practically pointlike, described by a 4 fermion coupling

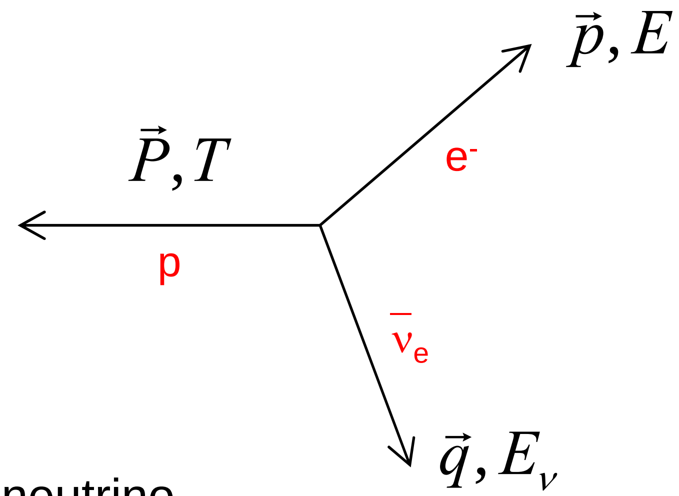
$$G = \frac{g^2}{M_W^2}$$

In Fermi theory the transition probability per unit time is given by:

$$W = \frac{2\pi}{\hbar} G^2 |M|^2 \frac{dn}{dE_0}$$

if	J(leptons) = 0	$\lvert M\rvert^2 = 1$	Fermi transition
if	J(leptons) = 1	$\lvert M\rvert^2 = 3$	Gamow-Teller transition





T, E, E_ν kinetic energies of proton, electron, antineutrino

Energy and momentum conservation

$$\vec{P} + \vec{p} + \vec{q} = 0$$

$$T + E_\nu + E = E_0$$

$$E_0 = m_n - m_p - m_e \approx 0.8\ MeV$$

$$T = \frac{P^2}{2M} < \frac{(0.8\ MeV)^2}{2 \times 938.27\ MeV} < 10^{-3}\ MeV \Rightarrow T \approx 0$$

In the hypothesis $m_\nu = 0$ $\quad E_\nu = qc \approx E_0 - E \qquad q = \frac{E_0 - E}{c} \Rightarrow dq = \frac{dE_0}{c}$

For the electron the number of states with momentum between p and $p+dp$ is given by:

$$dn = \frac{1}{h^3} d^3p = \frac{p^2 dp d\Omega}{h^3} \xrightarrow{\text{integrating over } d\Omega} dn_e = \frac{4\pi p^2 dp}{h^3}$$

Similarly for the neutrino: $\quad dn_\nu = \frac{4\pi q^2 dq}{h^3}$

Hence considering $\vec{p}$ and $\vec{q}$ as uncorrelated:

$$d^2N = \frac{16\pi^2}{h^6} p^2 q^2 dp dq \qquad \frac{d^2N}{dE_0} = \frac{16\pi^2}{h^6 c^3} p^2 (E_0 - E)^2 dp$$

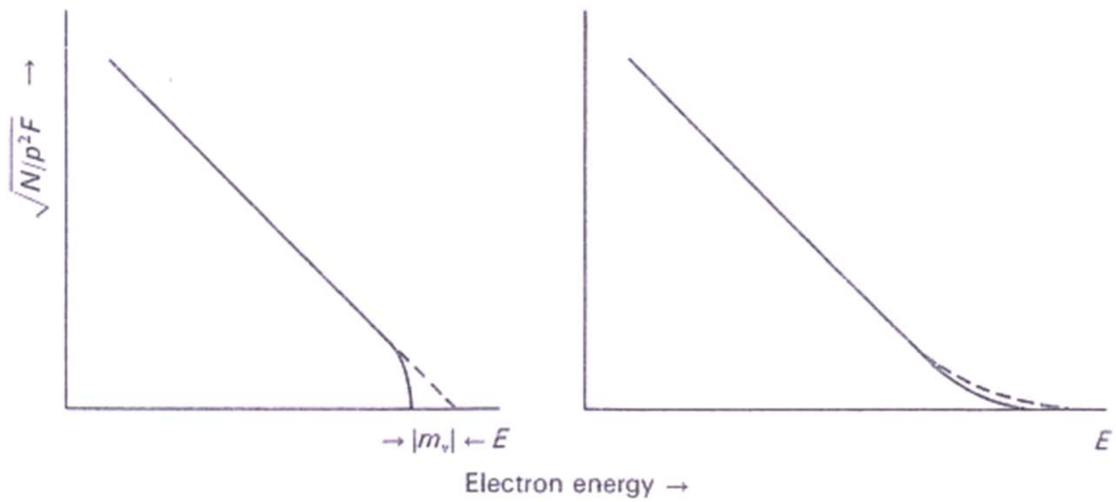
Hence if $|M|^2$ is constant the electron spectrum is given by:

$$\boxed{N(p)dp \propto p^2 (E_0 - E)^2 dp}$$

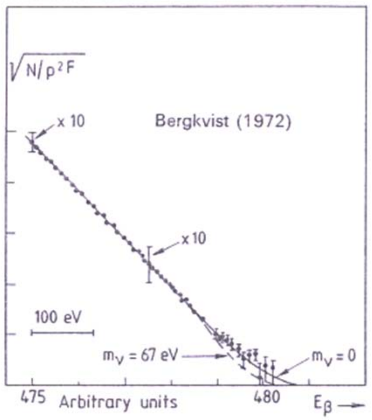
Kurie Plot

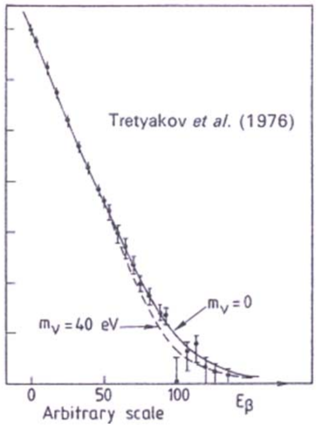
$$\frac{\sqrt{N(p)}}{p} \; vs \; E \qquad \propto (E_0 - E) \qquad m_\nu = 0$$

$$N(p) \propto p^2 (E_0 - E)^2 \sqrt{1 - \left(\frac{m_\nu c^2}{E_0 - E}\right)^2} \qquad m_\nu \neq 0$$



From Kurie plot one can measure the mass of the neutrino.





Some measurements from tritium β decay

$$H^3 \rightarrow He^3 + e^- + \bar{\nu}_e$$

Langer, Moffatt	1952	m_ν < 10 keV
Bergkvist	1972	m_ν < 65 eV
Tretyakov	1976	m_ν < 35 eV
Lyubimov	1980	m_ν < 30 eV
Fritschi	1986	m_ν < 18 eV
Robertson	1991	m_ν < 9.3 eV
Stoeffl	1995	m_ν < 7 eV
Weinheimer	1999	m_ν < 2.8 eV
Lobashev	1999	m_ν < 2.5 eV

present limit

$$\boxed{M(\nu_e) < 3\, eV}$$

Sargent Rule

The total decay rate is obtained integrating over the electron energy spectrum.
For extreme relativistic electrons $E \sim pc$ and we obtain:

$$N = \int_0^{E_0} N(p)dp \propto \int_0^{E_0} E^2 (E_0 - E)^2 dE = \boxed{\frac{E_0^5}{30}}$$

Sargent rule

From Fermi's rule, the value of G can be obtained from the observed decay rate.
For example:

$$O^{14} \to N^{14*} + e^+ + \nu_e$$

Fermi transition $J^P = 0^+ \to J^P = 0^+$

From $\tau = 3100\ s$ we obtain

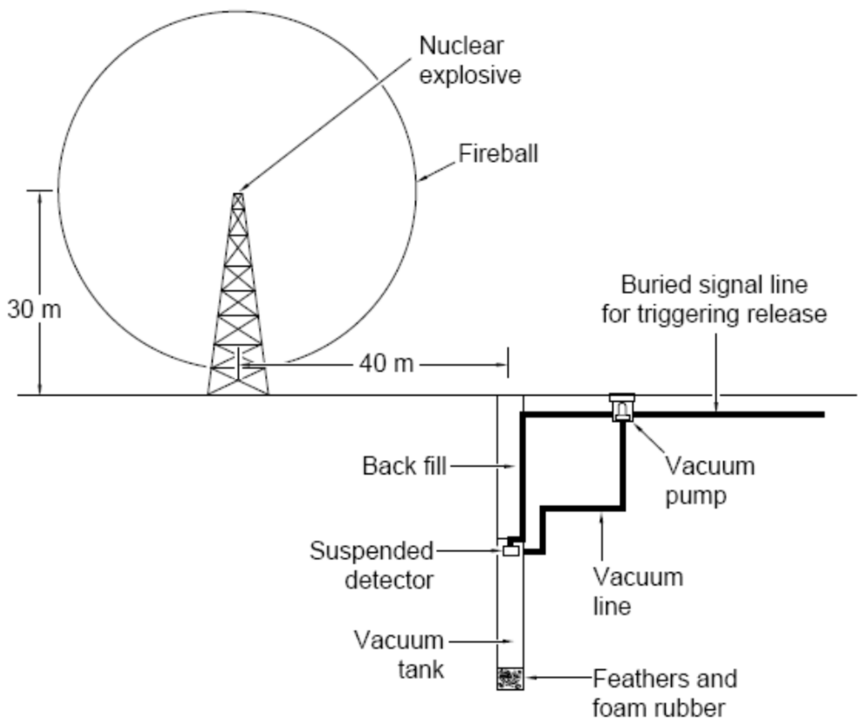
$$G = \frac{1.02 \times 10^{-5}}{M_p^2}$$

$$\hbar = c = 1$$

$$= 1.16 \times 10^{-5}\ GeV^{-2}$$

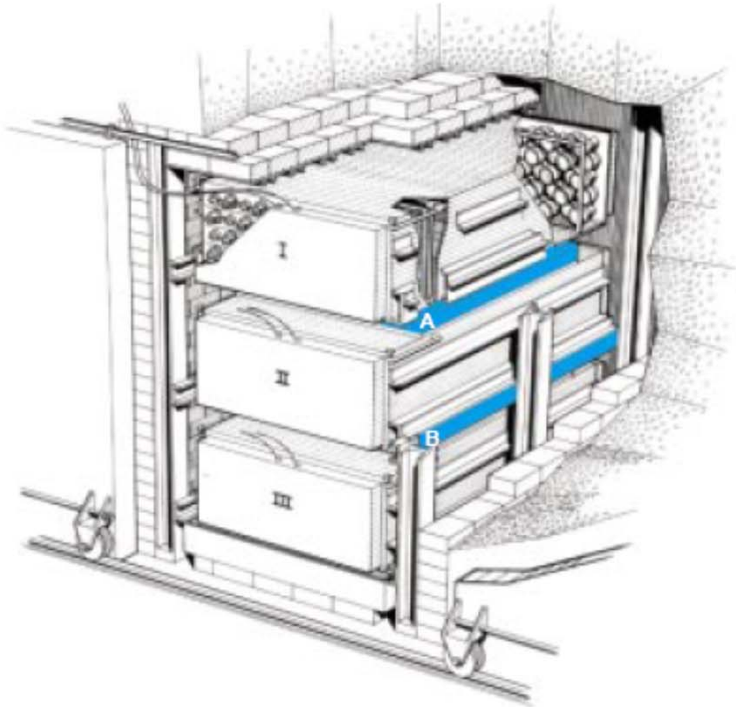
Project Poltergeist and the Discovery of the Neutrino

The goal of the project was the detection of (anti)neutrinos from the inverse β decay $\bar{\nu}_e + p \rightarrow e^+ + n$.
Original idea: detect antineutrinos from a nuclear explosion:

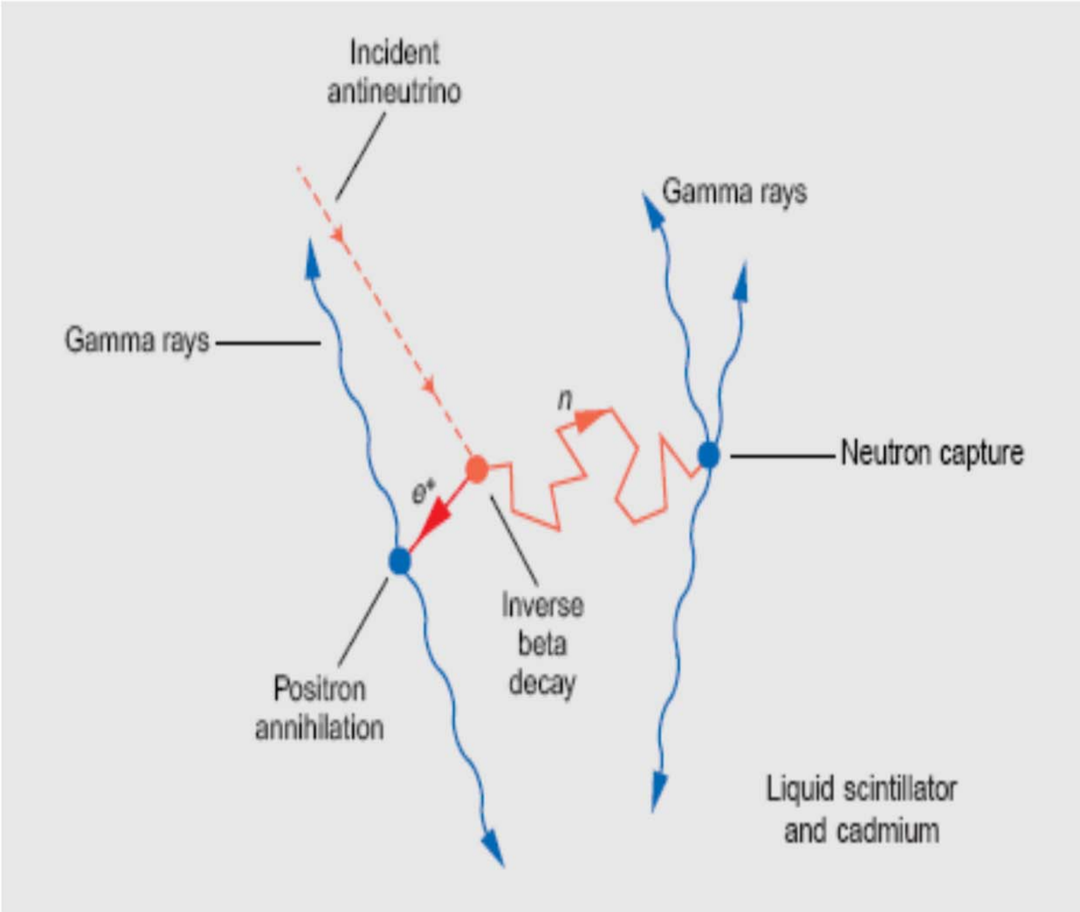


The antineutrinos from the nuclear explosion would have reached a liquid scintillator suspended in an underground cave at a distance of 40 m from the 30m high tower. In the original scheme of Reines and Cowan antineutrinos would have given rise to inverse β decay, whereas the detector would have registered the positrons produced in the process.

The Savannah River Experiment (1956-1960)



A,B water tanks, whose protons acted as targets for the antineutrinos, whereas the function of the Cadmium Chloride was to capture the neutrons.
I, II, III scintillator detectors.



It is relevant to note that a different technique for observing antineutrinos was tried, without success, in parallel with project Poltergeist. In 1955 a radiochemical experiment led by Ray Davis located next to a nuclear reactor site failed to observe *inverse chlorine decay*, i.e. the reaction:

$$\bar{\nu}_e + {}^{37}\mathrm{Cl} \rightarrow e^- + {}^{37}\mathrm{Ar}$$

does not happen with a measurable rate. This null result can be interpreted as evidence that neutrinos and antineutrinos are distinct particles.

We currently interpret this null result as due to a conservation law, i.e. lepton number conservation.

A similar setup was eventually used to study solar neutrinos.

Parity Violation in β Decay

Parity violation in weak interactions had been postulated in 1956 by
Lee e Yang to explain the existence of the two decay modes:

$$K^+ \to \pi\pi \qquad K^+ \to \pi\pi\pi \qquad (\tau\text{-}\theta \textit{ paradox})$$

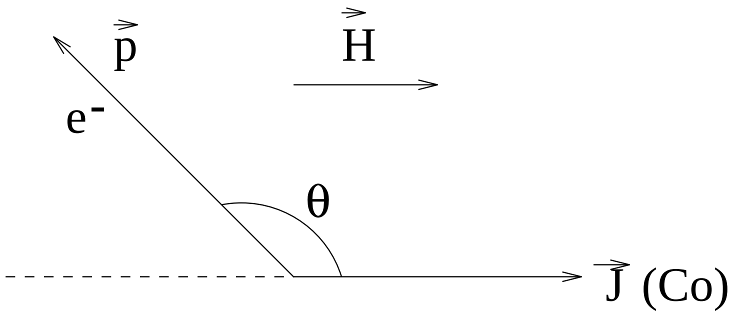
To test parity conservation an experiment was carried out by Wu et al (1957) who employed a sample of ^{60}Co at $T=0.01\ K$ inside a solenoid.

$$^{60}Co \to {}^{60}Ni^* + e^- + \bar{\nu}_e$$

$$J=5 \quad J=4$$

Gamow-Teller Transition

At a **temperature of *0.01 K*** a high proportion of ^{60}Co nuclei are aligned.
The relative electron intensities along and against the magnetic field direction were measured.



The results for electron intensities were consistent with a θ distribution of the form:

$$I(\theta) = 1 + \alpha \frac{\vec{\sigma} \cdot \vec{p}}{E}$$

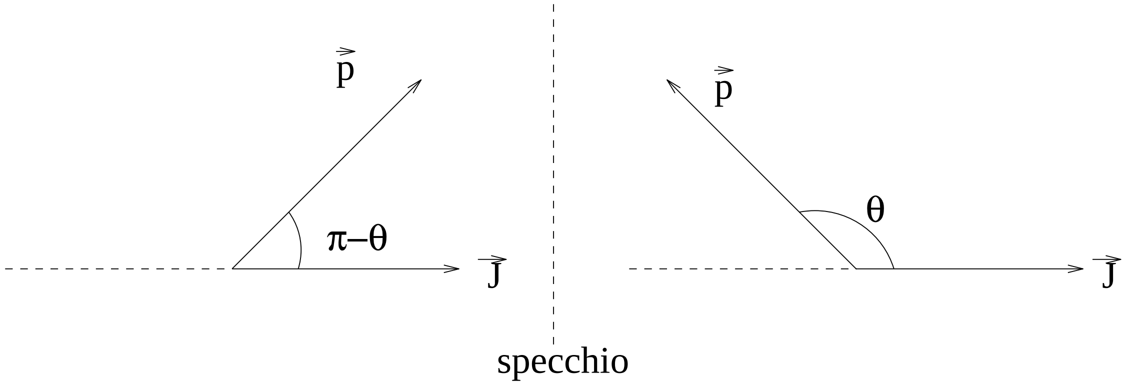
$$= 1 + \alpha \frac{v}{c} \cos\theta$$

$$\alpha = -1$$

$$\sigma = \frac{\vec{J}}{|\vec{J}|}$$

p,E electron momentum and energy

This form for $I(\theta)$ implies a fore-aft asymmetry which in turn implies that the **interaction violates parity conservation**.



Under parity:

$$\theta \rightarrow \pi - \theta$$

$$I(\theta) \rightarrow 1 - \alpha \frac{v}{c} \cos\theta$$

Let us now consider the helicity of the electrons emitted in ^{60}Co decay. The conservation of J_z implies that also the spin of the electron point in the direction $\vec{J}$. L:et $\vec{s}$ be a unit vector pointing in the electron spin direction:

$$I(\theta) = 1 + \alpha \frac{\vec{s} \cdot \vec{p}}{E}$$

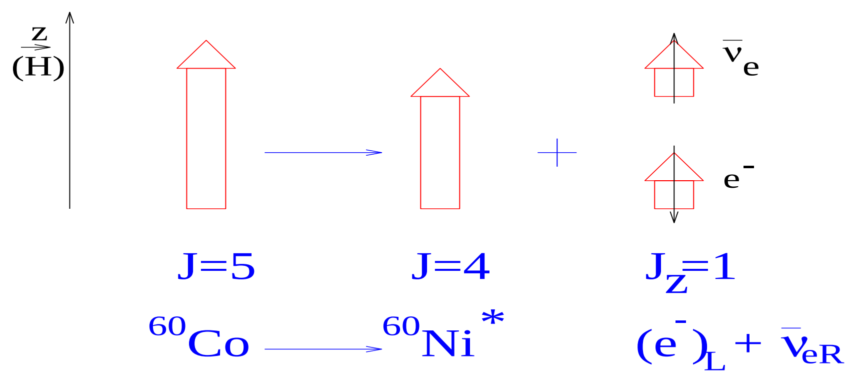
The *average longitudinal polarization* (or net helicity) can be defined as:

$$H = \frac{I_+ - I_-}{I_+ + I_-} \qquad I_+ = I(\theta = 0) \qquad I_- = I(\theta = \pi)$$

It turns out that: $H = \alpha \dfrac{v}{c}$

Experimentally:

$$H = \begin{cases} -\dfrac{v}{c} & e^- \ (\alpha = -1) \\ +\dfrac{v}{c} & e^+ \ (\alpha = +1) \end{cases}$$



Helicity of the Neutrino

From the previous discussion it follows that for a massless neutrino (*v*=*c*) helicity can assume the values *H=+1* o *H=-1*. This particle is therefere fully polarized.

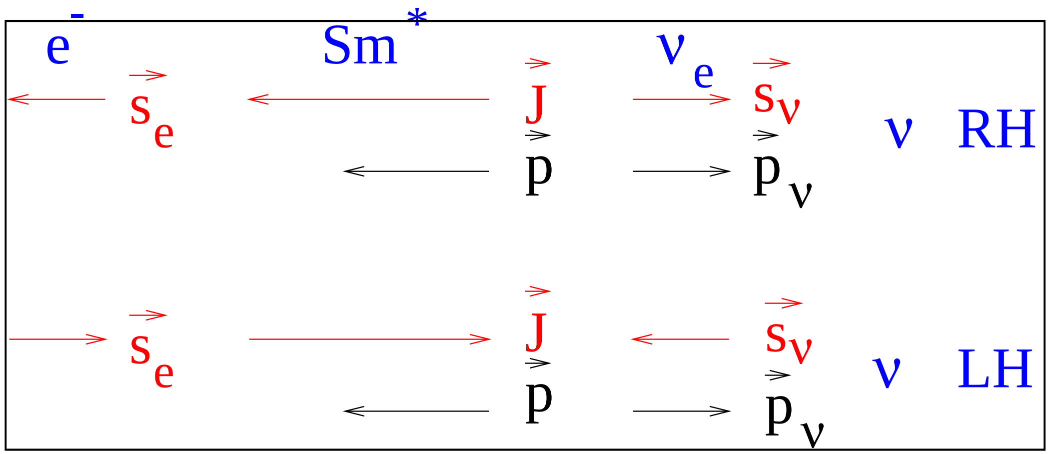
Goldhaber experiment (Phys Rev 109(1015)1958)

orbital electron of Europium $\rightarrow e^- + {}^{152}Eu \rightarrow {}^{152}Sm^* + \nu_e$

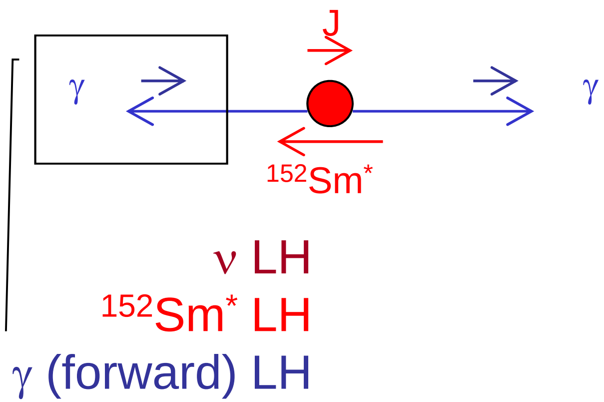
$$\underset{Z=63,\,J=0}{{}^{152}Eu} \qquad \underset{Z=62,\,J=1}{{}^{152}Sm^*}$$

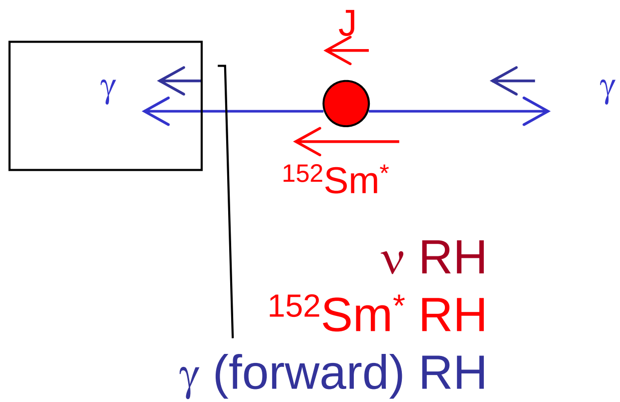
$$\underset{J=1}{{}^{152}Sm^*} \rightarrow \underset{J=0}{{}^{152}Sm} + \gamma(960KeV)$$

In this process the $^{152}Sm^*$ has the same polarization as the neutrino.



The ${}^{152}Sm^*$ is excited (because it lacks one internal electron) decays to its fundamental state emitting a 960 keV γ ($\tau \sim 3\times10^{-14}$ s). In this process the γ travelling in the direction of ${}^{152}Sm^*$ have the same polarization as the ν.



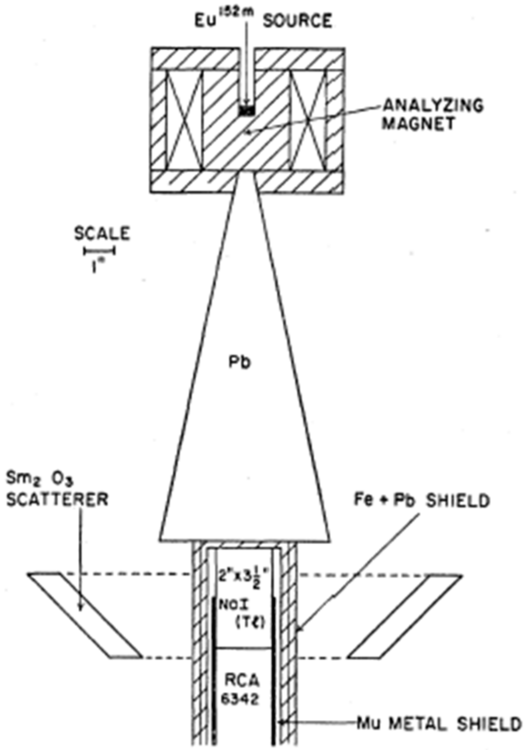


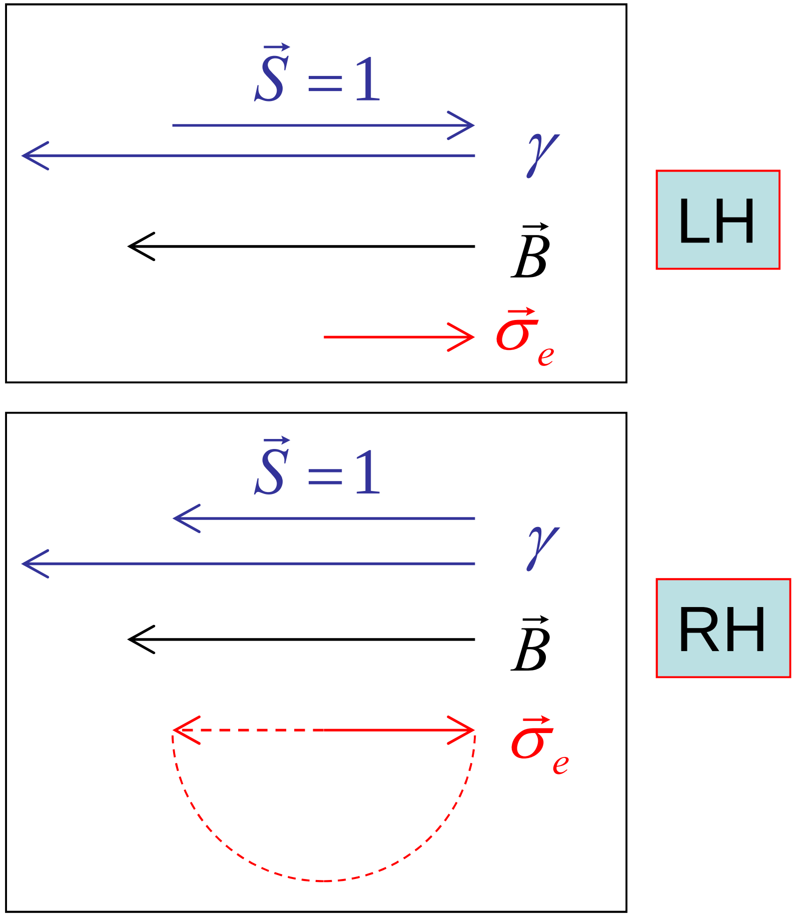
To determine the polarization of the foward γ rays (and thus of the neutrino) the resonant scattering process was used:

$$\gamma + {}^{152}Sm \rightarrow {}^{152}Sm^* \rightarrow \gamma + {}^{152}Sm$$

For which the forward emitted γ-rays carry the right energy.

To determine the polarization of the γ they were made to pass through magnetized iron.





The transmission in the iron is bigger for the LH γ than for RH γ; the γ polarization can be determined comparing the counts with $\vec{B}$ “up” and $\vec{B}$ “down.

I risultati danno **elicità negativa per i neutrini**.

Particle	e^-	e^+	ν_e	$\bar{\nu}_e$
Helicity	-v/c	+v/c	-1	+1

Parity Violation in Λ Decay

$\pi^- + p \rightarrow \Lambda + K^0$

$\Lambda \rightarrow \pi^- + p \qquad B.R. = (63.9 \pm 0.5)\%$

$\Lambda \rightarrow \pi^0 + n \quad (35.8\%)$

$\Lambda \rightarrow p + e^- + \bar{\nu}_e \quad (8.32 \times 10^{-4})$

In the production process the spin of the Λ must be orthogonal to the production plane, in order to conserve parity.

$$\vec{\sigma}_\Lambda \propto -\vec{p}_K \wedge \vec{p}_\Lambda$$

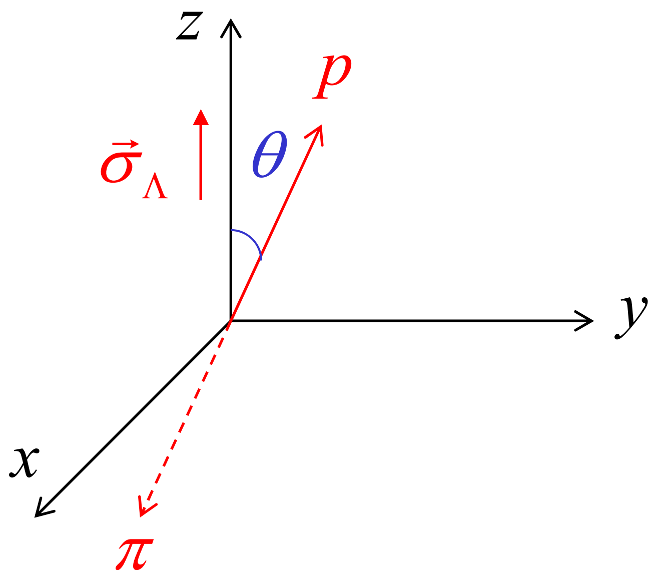
A polarization in the production plane generally changes sign under P and is not allowed. Experimentally the mean transverse polarization is found to be 70 %

$$P_\Lambda = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

$N_\uparrow$ Number of counts with spin up

$N_\downarrow$ Number of counts with spin down

Decay process: $\Lambda \rightarrow \pi^- + p$

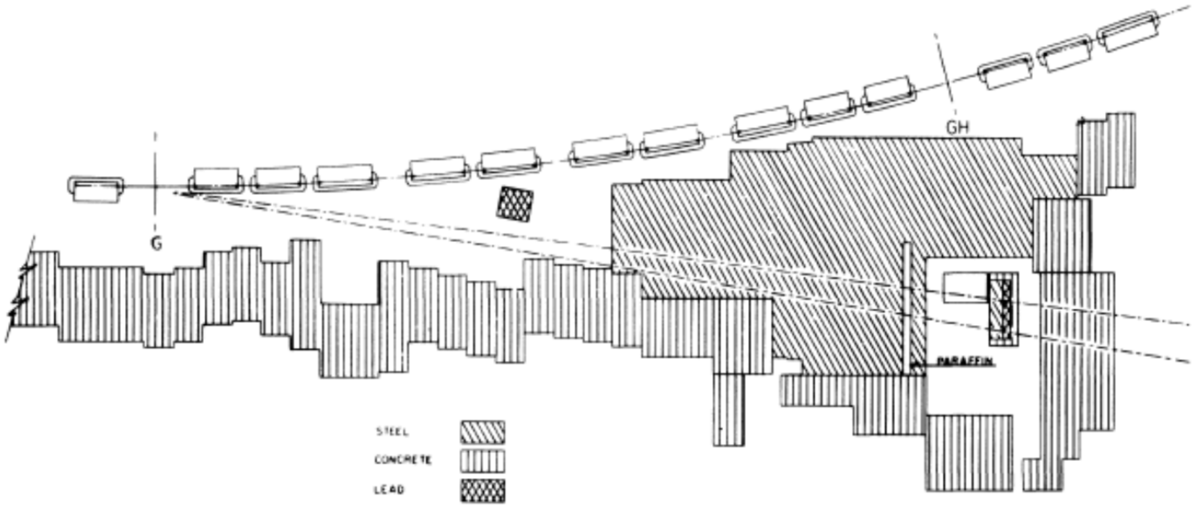


The angular distribution is of the

$$I(\theta) = 1 - \alpha P \cos\theta$$

<u>This up-down asymmetry is a manifestation of parity violation in Λ decay.</u>

The Discovery of ν_μ



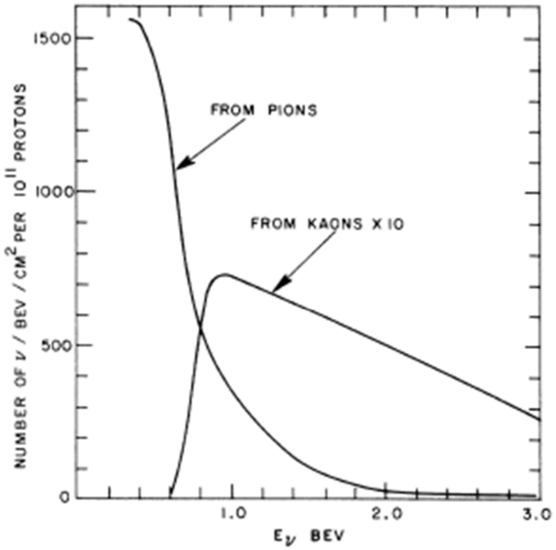


FIG. 2. Energy spectrum of neutrinos expected in the arrangement of Fig. 1 for 15-BeV protons on Be.

A neutrino beam coming from the decay of charged pions $\pi^\pm \to \mu^\pm + (\nu/\ \bar{\nu})$ was sent on a spark chamber: only muons were produced, but no electrons demonstrating the existence of two types of neutrino:

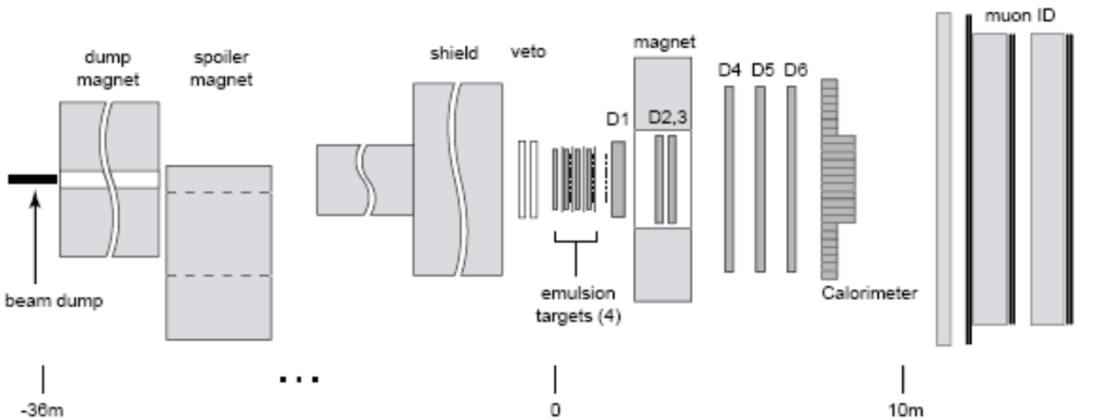
$$\boxed{\nu_\mu + X \to \mu + Y}$$

$$\boxed{\nu_\mu + X \not\to e + Y}$$

The Third Lepton Family and the ν_τ

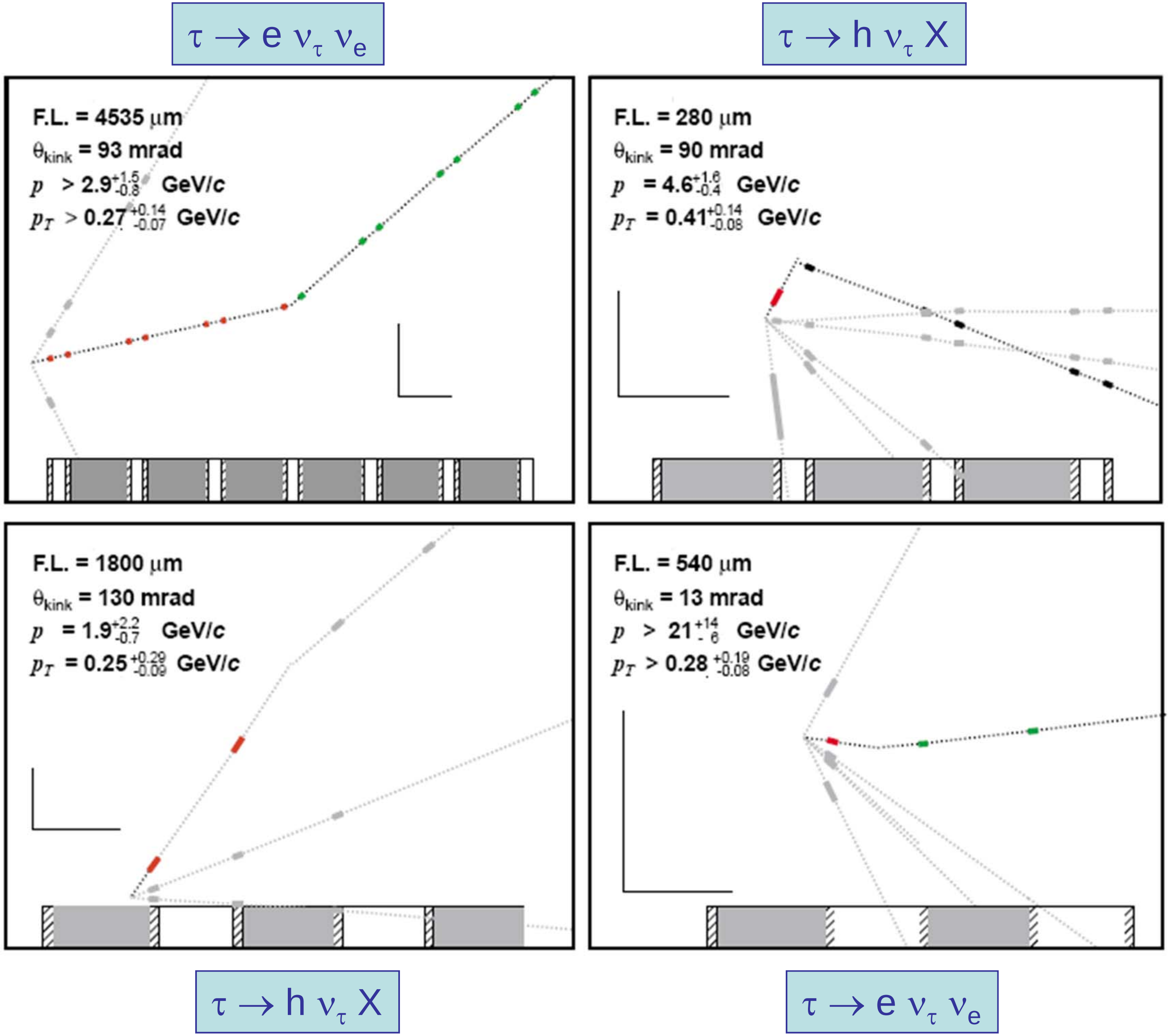
- In 1975 an experiment led by M. Perl unveiled the existence of the third lepton, the τ. This implied the existence of a third neutrino flavour.
- A first indirect evidence for the existence of a third neutrino came from the LEP experiments, where the measurement of the invisible width of the Z^0 was consistent with the predictions of the Standard Model with three neutrinos.
- Cosmological calculations based on the quantity of ^4He in the universe indicate the existence of three neutrino species at the time of the Big Bang.
- The direct observation of the ν_τ was made in 2001 thanks to the DONUT experiment (Direct Observation of NU Tau) at Fermilab, which detected four events due to ν_τ interactions over a background of 0.34 events, consistent consistent with the predictions of the Standard Model.

The DONUT Experiment



The experiment was designed to detect the CC interactions of the ν_τ, identifying the τ as the only charged lepton in the event. At the energies of DONUT the τ decays within 2 mm into a final state with a single charged particle (BR 86 %), therefore the signature is a prong with a "kink". The detector consists in an emulsion target followed by a magnetic spectrometer. The neutrino beam is made starting from the 800 GeV protons from the Fermilab Tevatron hitting a "beam dump" made of tungsten, 1 m in length and located 36 m upstream of the emulsions.

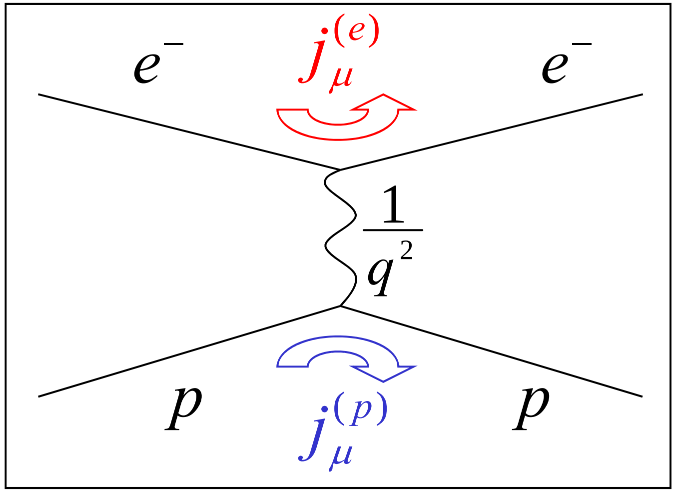
The primary source of ν_τ is the decay $D_S \rightarrow \tau\ \bar{\nu}_\tau$ and the subsequent decay of the τ into ν_τ.


τ → e ν_τ ν_e
τ → h ν_τ X
F.L. = 4535 μm
θ_kink = 93 mrad
p > 2.9 +1.5 -0.8 GeV/c
p_T > 0.27 +0.14 -0.07 GeV/c
F.L. = 280 μm
θ_kink = 90 mrad
p = 4.6 +1.6 -0.4 GeV/c
p_T = 0.41 +0.14 -0.08 GeV/c
F.L. = 1800 μm
θ_kink = 130 mrad
p = 1.9 +2.2 -0.7 GeV/c
p_T = 0.25 +0.29 -0.09 GeV/c
F.L. = 540 μm
θ_kink = 13 mrad
p > 21 +14 -6 GeV/c
p_T > 0.28 +0.19 -0.08 GeV/c
τ → h ν_τ X
τ → e ν_τ ν_e

The V-A Interaction

Fermi developed his theory of β decay in analogy with electromagnetic interactions.

$$e^- + p \rightarrow e^- + p$$

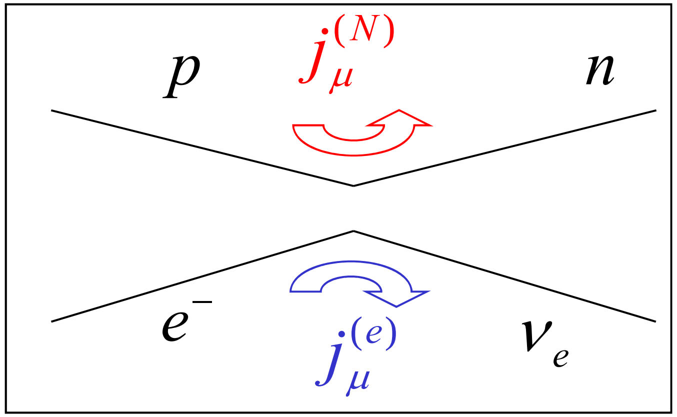


$$M_{em} \propto \underbrace{\left(e\overline{\psi}_p\gamma_\mu\psi_p\right)}_{ej_\mu^{(p)}}\frac{1}{q^2}\underbrace{\left(-e\overline{\psi}_e\gamma_\mu\psi_e\right)}_{ej_\mu^{(e)}}$$

$$M \propto \frac{e^2}{q^2}\, j_\mu^{(p)}\, j_\mu^{(e)}$$

$$n \rightarrow p + e^- + \overline{\nu}_e$$

$$p + e^- \rightarrow n + \nu_e$$



$$M_W \propto G\underbrace{\left(\overline{\psi}_n\gamma_\mu\psi_p\right)}_{J_\mu^{(N)}}\underbrace{\left(\overline{\psi}_{\nu_e}\gamma_\mu\psi_e\right)}_{J_\mu^{(e)}}$$

- G is the weak coupling constant
- The weak current J_μ changes the sign of the electric charge: charged weak current.
- In the matrix element there is no propagator: the interaction is pointlike.

The matrix element written in this way is a scalar quantity and it implies parity conservation. The violation of parity in the weak interaction requires requires the inclusion of a parity-violationg term $\gamma^5\gamma^\mu$. The weak current turns out to be a combination of a Lorentz vector (γ^μ) and of a pseudovector (or axial vector, $\gamma^5\gamma^\mu$). Hence the name V-A.

The matrix element is written as:

$$M(p+e^- \rightarrow n+\nu_e) = \frac{G}{\sqrt{2}}\left[\overline{\psi}_n \gamma^\mu (1-\gamma_5)\psi_p\right]\left[\overline{\psi}_{\nu_e}\gamma_\mu(1-\gamma_5)\psi_e\right]$$

A similar expression can be written for muon decay.

The charged weak current

$$J_\mu = \overline{\psi}_{\nu_e}\gamma_\mu \frac{(1-\gamma_5)}{2}\psi_e$$

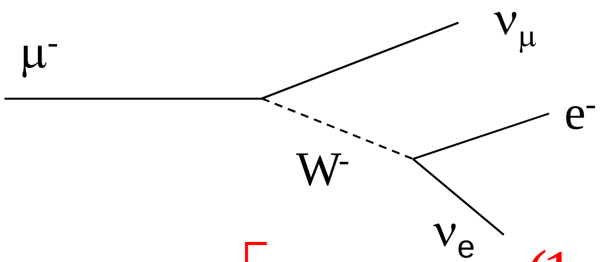
couples an incoming LH electron to an outgoing LH neutrino.

The amplitudes for weak interaction are of the form:

$$M = \frac{4G}{\sqrt{2}} J^\mu J^+_\mu$$

Interpretation of G

If we compare electromagnetic and weak amplitudes we see that G replaces e^2/q^2. Hence G is not dimensionless: [G] = [GeV]$^{-2}$. We can try to extend the between em and weak interaction by postulating that the latter are also mediated by vector boson, for instance:



$$\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$$

$$M = \left[\frac{g}{\sqrt{2}} \overline{\psi}_{\nu_\mu} \gamma^\sigma \frac{(1-\gamma_5)}{2} \psi_\mu \right] \frac{1}{M_W^2 + q^2} \left[\frac{g}{\sqrt{2}} \overline{\psi}_e \gamma_\sigma \frac{(1-\gamma_5)}{2} \psi_{\nu_e} \right]$$

If $q^2 << M_W^2$ (as is the case for β and μ decays)

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

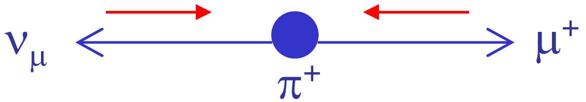
and the weak interaction becomes pointlike. We see that the interaction is weak not because g << e, but because the W mass is big. If g~e than for energies >> M_W electromagnetic and weak interactions are of comparable strength.

g ~ e unification of electromagnetic and weak interactions.

π and μ Decay

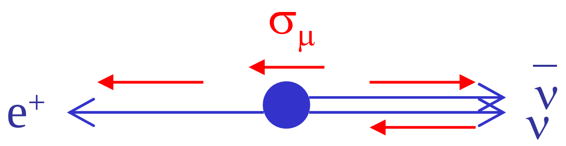
$$\pi^+ \rightarrow \mu^+ + \nu_\mu \qquad\qquad \mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$$

In the π^+ rest system



The π has spin 0, the ν is LH, thus also the μ must be LH.

In the subsequent muon decay the positron energy spectrum is peaked in the region of the maximum energy, so the most likely configuration is the following:



e^+ is RH

The angular distribution is of the form

$$\frac{dN}{d\Omega} \propto 1 - \frac{\alpha}{3}\cos\theta$$

in agreement with V-A. The muon lifetime is given by:

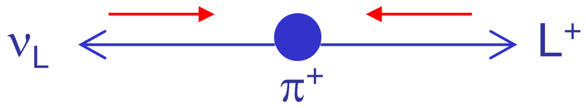
$$\tau_\mu^{-1} = \frac{G^2 m_\mu^5}{192\pi^3}$$

Using the values m_μ = 105.6593 MeV/c^2 and τ = 2.19709 μs we obtain:

$$G = 1.16637 \times 10^{-5}\ GeV^{-2}$$

The Decay of the π

Angular momentum conservation requires that the leptons have the same helicity.



The leptons are emitted with helicities

$$\begin{cases} +\dfrac{v}{c} & e^+, \mu^+ \\ -\dfrac{v}{c} & e^-, \mu^- \end{cases}$$

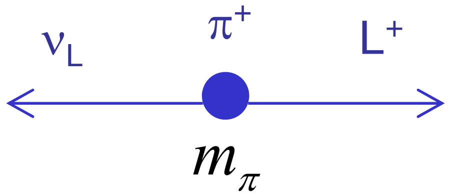
The probability that an e^+ or a μ^+ are emitted with velocity v and helicity $-v/c$ is proportional to $(1-v/c)$

$$\Gamma \propto \left(1 - \frac{v}{c}\right)$$

In order to obtain the transition probability we must take into account the phase space:

$$\frac{dn}{dE_0} \propto p^2 \frac{dp}{dE_0}$$

$-\vec{p}, m=0, c$



$\vec{p}, m, v$

In the π rest system

The total energy is thus $E_0 = m_\pi = p + \sqrt{p^2 + m^2}$

$$\frac{dp}{dE_0} = \frac{\sqrt{p^2+m^2}}{m_\pi} \qquad p^2 \frac{dp}{dE_0} = \frac{(m_\pi^2 + m^2)(m_\pi^2 - m^2)^2}{8m_\pi^4}$$

$$1 - \frac{v}{c} = 1 - \frac{p}{\sqrt{p^2+m^2}} = \frac{2m^2}{m_\pi^2 + m^2} \qquad M \propto p^2 \frac{dp}{dE_0}\left(1 - \frac{v}{c}\right) = \frac{m^2}{4}\left(1 - \frac{m^2}{m_\pi^2}\right)^2$$

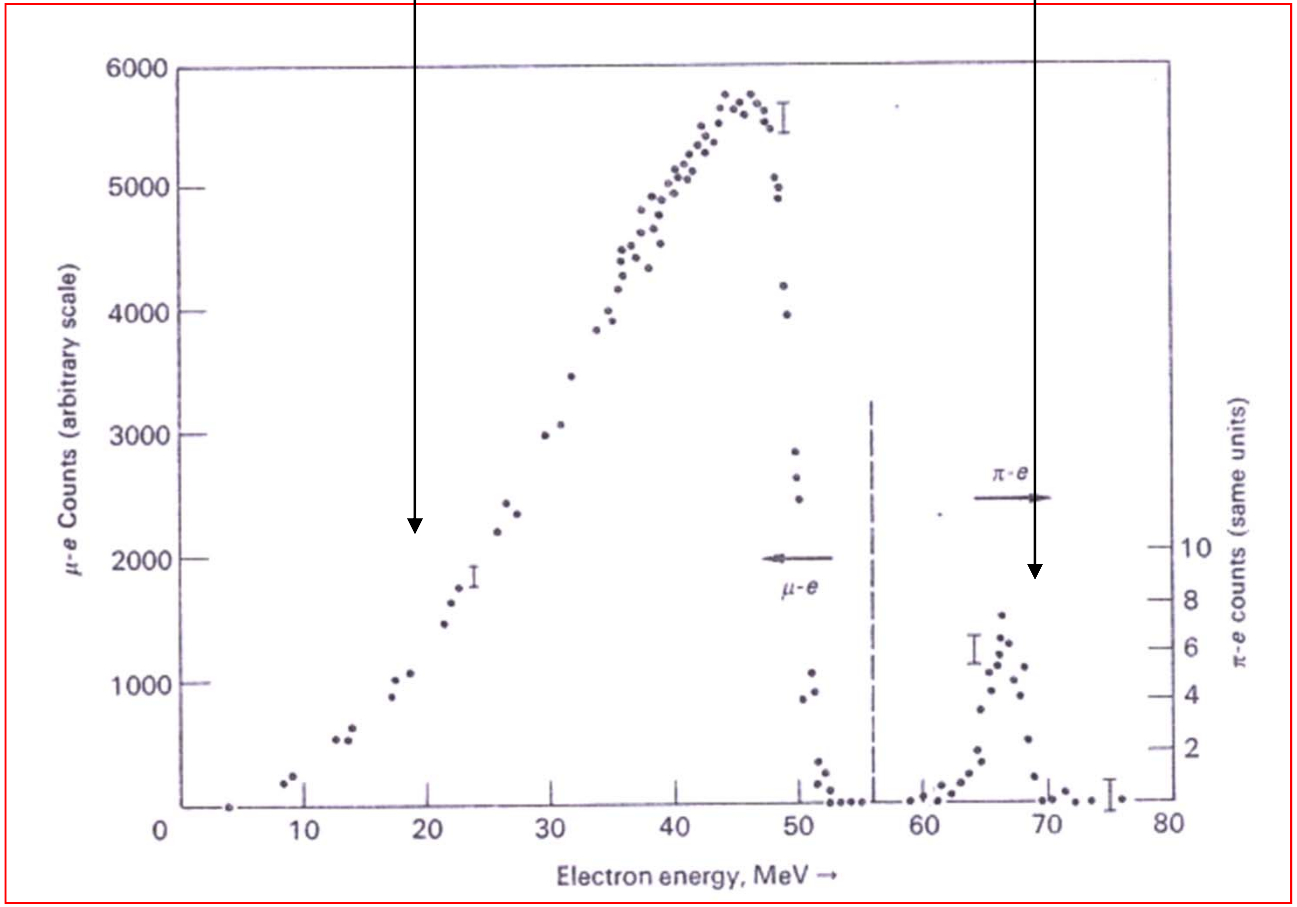
$$\boxed{\frac{\pi \to e\nu}{\pi \to \mu\nu} = \frac{m_e^2}{m_\mu^2} \frac{1}{\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2} = 1.275 \times 10^{-4}}$$

Experimentally: $\boxed{(1.267 \pm 0.023) \times 10^{-4}}$

Positron energy spectrum for the decays:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$$

$$\pi^+ \rightarrow e^+ + \nu_e$$



Some comments on the result:

- The decay $\pi \to e\nu$ would be enourmously favored by phase space with respect to $\pi \to \mu\nu$.

  However angular momentum conservation forces the charged lepton to have the “wrong” helicity. This is more easily realized for the μ, whose mass is approximately 200 times bigger than the electron’s.
- In the calculations we used the same value of G for both processes.

  Universality in the coupling of leptons to to weak interaction.
- Following the same scheme the V-A theory predicts for the K mesons:

$$\boxed{\frac{K \to e\nu}{K \to \mu\nu} \approx 2.5 \times 10^{-5}}$$

to be compared with the measured value:

$$\boxed{(2.43 \pm 0.14) \times 10^{-5}}$$

K^0 Decay

$S \backslash I_3$	$+1/2$	$-1/2$
+1	K^+	K^0
-1	$\overline{K}^0$	K^-

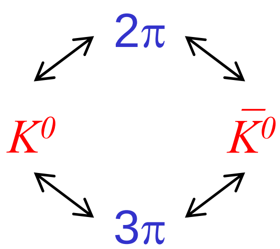
Production mechanisms:

K^0 $\qquad\qquad\qquad$ $\overline{K}^0$

$$\pi^- + p \rightarrow \Lambda + K^0 \qquad \pi^+ + p \rightarrow K^+ + \overline{K}^0 + p \qquad E_\pi > 1.5\, GeV$$

$$E_\pi > 0.91\, GeV \qquad \pi^- + p \rightarrow \overline{\Lambda} + \overline{K}^0 + n + n \qquad E_\pi > 6.0\, GeV$$

Therefore with a π beam of suitable energy it is possible to produce a pure beam of K^0.
K^0 e $\overline{K}^0$ are the kaon eigenstates as far as the strong interaction is concerned.
The kaons can decay by the weak interaction ($\Delta S = \pm 1$) into 2 or 3 π.



$$|K(t)\rangle = \alpha(t)|K^0\rangle + \beta(t)|\overline{K}^0\rangle$$

$$CP\left|K^0\right\rangle \rightarrow \eta\left|\overline{K}^0\right\rangle \qquad CP\left|\overline{K}^0\right\rangle \rightarrow \eta'\left|K^0\right\rangle \qquad \eta=\eta'=1$$

We can form two CP eigenstates:

$$\left|K_1\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\overline{K}^0\right\rangle\right) \qquad CP=+1$$

$$\left|K_2\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\overline{K}^0\right\rangle\right) \qquad CP=-1$$

K_1 and K_2 are distinguished by their mode of decay

• $\pi^0\pi^0$,$\pi^+\pi^-$	Bose Symmetry	CP=+1
• $\pi^+\pi^-\pi^0$	L=0; CP($\pi^+\pi^-$)=+1, CP(π^0)=-1	CP= -1
	(CP=±1 if L>0)	
• $\pi^0\pi^0\pi^0$	P=-1, C=+1	CP= -1

$$\boxed{\begin{array}{lll} K_1 \rightarrow 2\pi & CP=+1 & \tau_1 = 0.9\times10^{-10}\,s \\ K_2 \rightarrow 3\pi & CP=-1 & \tau_2 = 0.5\times10^{-7}\,s \end{array}}$$

Strangeness Oscillations

K_1 and K_2, are not particle and antiparticle, therefore they have different mass, because of their different weak couplings.

Let us write the amplitude for the K_1 as a function of time:

$$a_1(t) = a_1(0)e^{-iE_1 t}e^{-\frac{\Gamma_1 t}{2}}$$

E_1 energy

$\tau_1 = 1/\Gamma_1$ lifetime

$$I(t) = a_1(t)a_1^*(t) = I(0)e^{-t/\tau_1}$$

In the K_1 rest system $E_1 = m_1$ (rest mass), τ_1= proper lifetime

$$a_1(t) = a_1(0)e^{-\left(\frac{\Gamma_1 t}{2}+im_1\right)} \qquad a_2(t) = a_2(0)e^{-\left(\frac{\Gamma_2 t}{2}+im_2\right)}$$

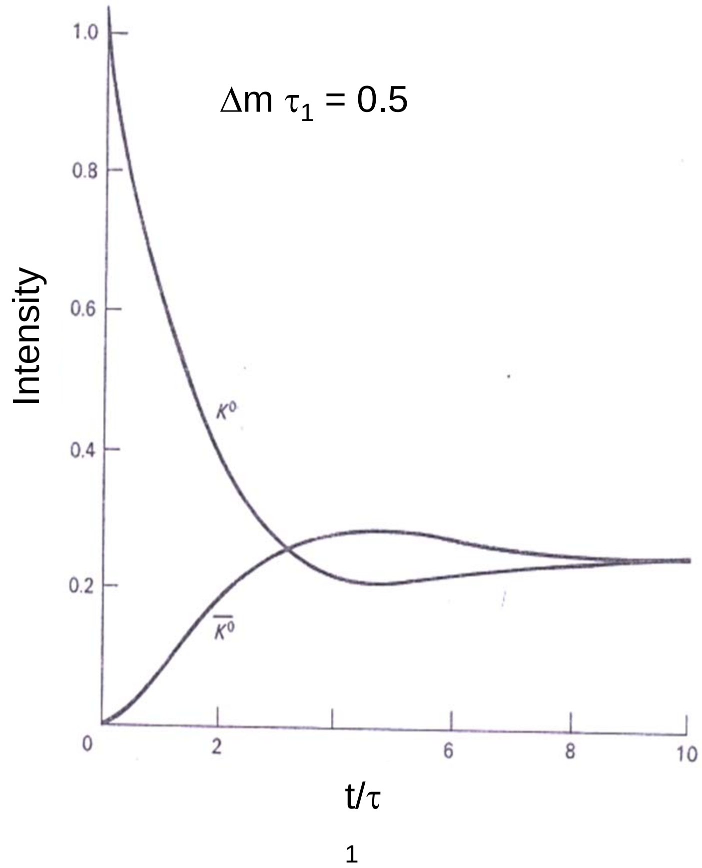
If we start from a pure K^0 beam:

$$a_1(0) = a_2(0) = \sqrt{\frac{1}{2}}$$

After a time t:

$$I(K^0) = \frac{a_1(t)+a_2(t)}{2} \cdot \frac{a_1^*(t)+a_2^*(t)}{2}$$

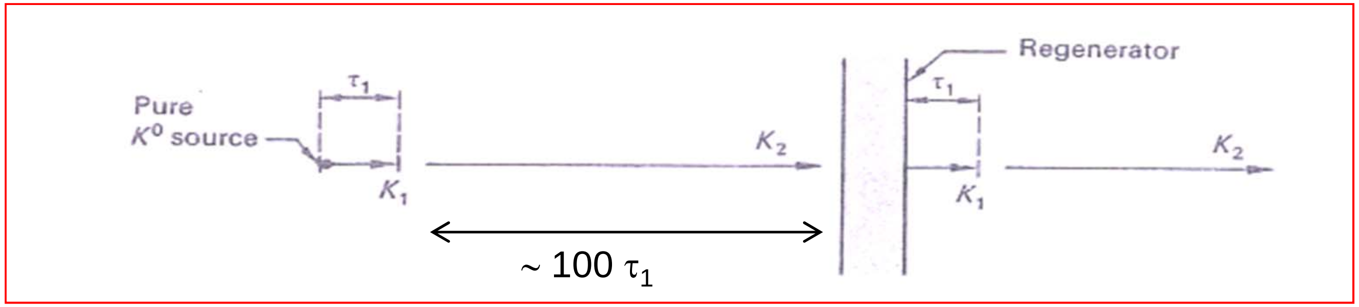
$$= \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-\frac{\Gamma_1+\Gamma_2}{2}t} \cos(\Delta m \cdot t) \right]$$



$$\Delta m = m_2 - m_1$$

$$\Delta m = (3.491 \pm 0.009) \times 10^{-12}\ MeV$$

$$\frac{\Delta m}{m} = 7 \times 10^{-15}$$



Starting from a pure K^0 beam after a high number of τ_1 only the K_2 survive.
In the target the strong interaction regenerates the components of S=+1, S=-1.

$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

K^0 e $\bar{K}^0$ are absorbed differently, since the K^0 only undergoes elastic scattering and charge exchange, whereas the $\bar{K}^0$ can also give rise to hyperons. Thus after the regenerator we have components $f|K^0>$ and $\bar{f}|\bar{K}^0>$, with $\bar{f}<f<1$.
Therefore after the regenerator:

$$\frac{1}{\sqrt{2}}\left(f|K^0\rangle - \bar{f}|\bar{K}^0\rangle\right) = \frac{f+\bar{f}}{2\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) + \frac{f-\bar{f}}{2\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$= \frac{f+\bar{f}}{2}|K_2\rangle + \frac{f-\bar{f}}{2}|K_1\rangle$$

Since $f \neq \bar{f}$ the K_1 component has been regenerated.

CP Violation in K^0 Decay

In 1964 it was discovered that the eigenstate of CP=-1 (K_2) can also decay to 2π with a branching ratio of the order of 10^{-3}.

$$K_1 \to K_S \qquad |K_S\rangle = \frac{|K_1\rangle - \varepsilon |K_2\rangle}{\sqrt{1+|\varepsilon|^2}}$$

Indirect CP violation

$$K_2 \to K_L \qquad |K_L\rangle = \frac{\varepsilon |K_1\rangle + |K_2\rangle}{\sqrt{1+|\varepsilon|^2}}$$

where ε is a parameter which quantifies CP violation.

$$|\eta_{+-}| = \frac{K_L \to \pi^+\pi^-}{K_S \to \pi^+\pi^-} = (2.286 \pm 0.014) \times 10^{-3}$$

$$|\eta_{00}| = \frac{K_L \to \pi^0\pi^0}{K_S \to \pi^0\pi^0} = (2.276 \pm 0.014) \times 10^{-3}$$

There exists also a direct CP violation, which originates in the decay.

Weak Decays of Strange Particles

Selection rule:

$$\Delta S = 1 \quad \Delta I = \frac{1}{2} \qquad s \to d \quad \underbrace{S = -1 \to S = 0}_{\Delta S = 1} \quad \underbrace{I = 0 \to I = \tfrac{1}{2}}_{\Delta I = \frac{1}{2}}$$

Example: $\Lambda \to p + \pi^-$ $\qquad \Lambda \to n + \pi^0$

The nucleon and the pion in the final state must be in an I=1/2 isospin state

$$\left| p\pi^- \right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle \qquad \left| n\pi^0 \right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$$\frac{\Gamma(\Lambda \to n\pi^0)}{\Gamma(\Lambda \to n\pi^0) + \Gamma(\Lambda \to p\pi^-)} = \underbrace{\left(\sqrt{\frac{1}{3}}\right)^2}_{\text{isospin}} \times \underbrace{1.036}_{\text{phase space}} = 0.345$$

Experimental value: 0.358 ± 0.005

isospin phase space

Semileptonic Decays

The semileptonic decays obey the selection rule $\Delta Q = \Delta S$.

ΔQ and ΔS are the changes in charge and strangeness of the hadrons.

$\Delta Q=\Delta S=1$ follows from $Q=I_3+(B+S)/2$ if $\Delta I_3=1/2$.

Examples: $\boxed{\Sigma^- \rightarrow \quad n + e^- + \bar{\nu}_e}$ $\qquad$ $\boxed{\Sigma^+ \rightarrow \quad n + e^+ + \nu_e}$

$$\begin{array}{llll} dds & udd & & \\ S=-1 & S=0 & \Delta S=1 & \\ Q=-1 & Q=0 & \Delta Q=1 & \end{array} \qquad \begin{array}{lll} uus & udd & \\ S=-1 & S=0 & \Delta S=1 \\ Q=+1 & Q=0 & \Delta Q=-1 \end{array}$$

$$\Delta Q = \Delta S = 1 \qquad\qquad \Delta Q \neq \Delta S$$

$$BR = 1.08\times10^{-3} \qquad\qquad BR < 5\times10^{-6}$$

Deviations from the $\Delta I = ½$ rule
$$\frac{\Gamma\left(\Omega^- \rightarrow \Xi^0 + \pi^-\right)}{\Gamma\left(\Omega^- \rightarrow \Xi^- + \pi^0\right)} = 2.94 \pm 0.35$$

whereas the selection rule predicts a ratio of 2.0. Thus the amplitude I=3/2 is also present, although heavily suppressed.

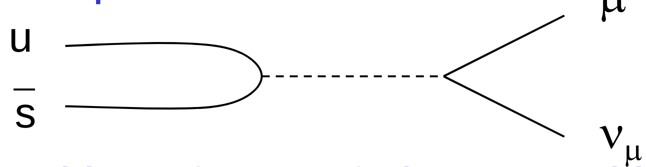
Cabibbo Theory

Leptons and quarks interact weakly through V-A currents which are built starting from the following leptonic doublets (LH):

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \longleftarrow ?$$

In addition we also have u-s couplings, for example:

$$K^+ \to \mu^+ + \nu_\mu$$



Furthermore decays with ΔS=1 are suppressed by a factor of about 20 with respect to those with ΔS=0.

Cabibbo(1963): the d and s quark states participating in the weak interaction are rotated by a mixing angle θ_C (Cabibbo angle)

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}$$

For either of these sets of doublets the weak coupling constant remains G.

For $\Delta S=0$ ($d \rightarrow u$) transitions the coupling will thus be proportional to $\cos\theta_C$, whereas for $\Delta S=1$ ($s \rightarrow u$) it will be proportional to $\sin\theta_C$.

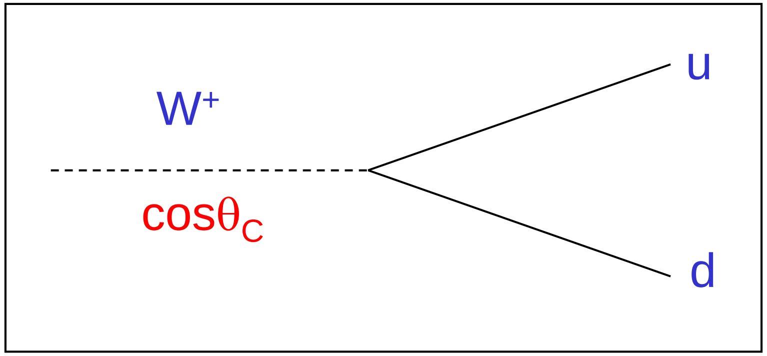
Thus for example:

$$\frac{\Gamma(K^+ \rightarrow \mu^+ + \nu_\mu)}{\Gamma(\pi^+ \rightarrow \mu^+ + \nu_\mu)} \propto \sin^2\theta_C$$

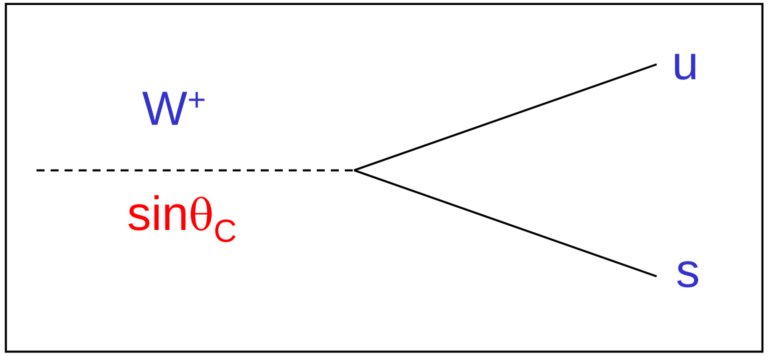
which yields $\theta_C \sim 15^o$.

We therefore have favored ($\propto \cos\theta_C$) and suppressed ($\propto \sin\theta_C$) transitions.

Examples:



Cabibbo favored



Cabibbo suppressed

Weak Neutral Currents

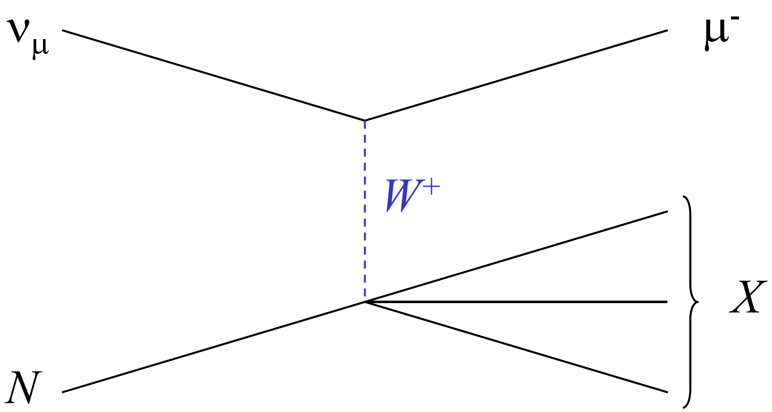
In 1973 the existence of muon neutrino interactions without a charged lepton in the final state was first demonstrated in a bubble chamber experiment at CERN.

$$\nu_\mu + N \to \nu_\mu + X$$

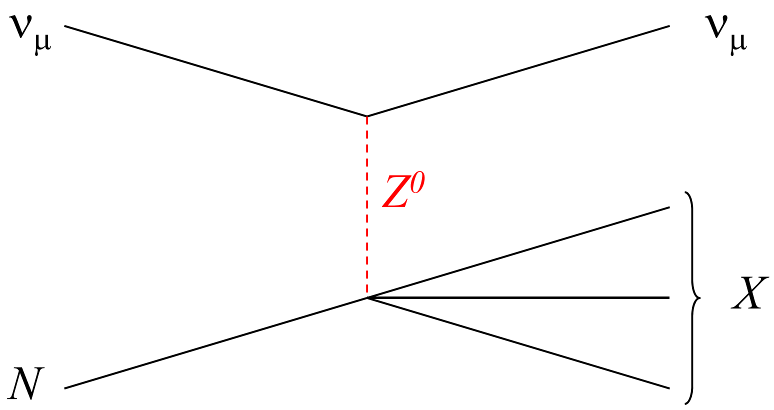
$$\bar{\nu}_\mu + N \to \bar{\nu}_\mu + X$$

Weak Neutral Currents, with rates comparable to charged currents

$$\frac{\sigma(\nu_\mu + N \to \nu_\mu + X)}{\sigma(\nu_\mu + N \to \mu^- + X)} \approx 0.25 \qquad \frac{\sigma(\bar{\nu}_\mu + N \to \bar{\nu}_\mu + X)}{\sigma(\bar{\nu}_\mu + N \to \mu^+ + X)} \approx 0.45$$



Charged current



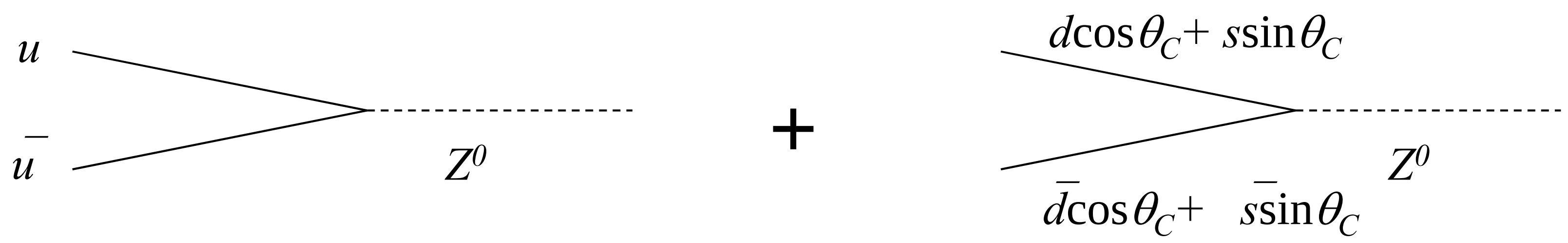
Neutral current

The GIM Model and Charm

All neutral current processes observed are characterized by the selection rule $\Delta S = 0$. Indeed the idea of weak neutral currents had been discarded because they had never been observed in decay processes:

$$\frac{K^+ \to \pi^+ \nu \bar{\nu}}{K^+ \to \pi^0 \mu^+ \nu_\mu} < 10^{-5} \qquad \Delta S = 1$$

Weak neutral currents are given by the diagrams:



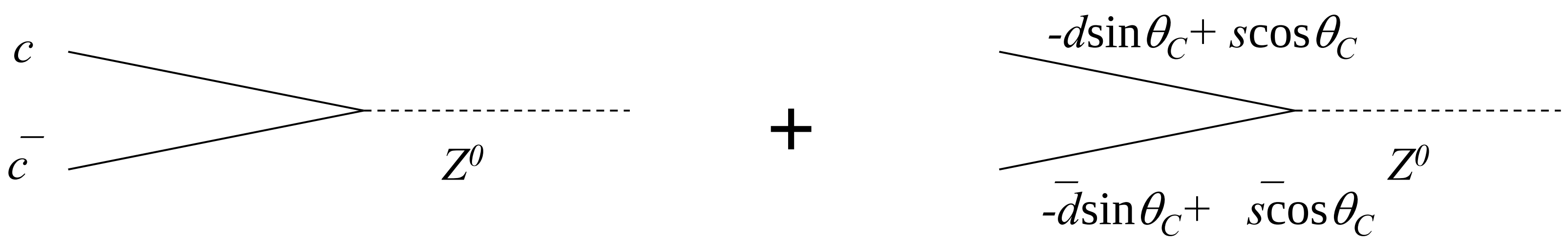
$$\underbrace{u\bar{u} + d\bar{d}\cos^2\theta_C + s\bar{s}\sin^2\theta_C}_{\Delta S = 0} + \underbrace{\left(s\bar{d} + \bar{s}d\right)\sin\theta_C\cos\theta_C}_{\Delta S = 1}$$

Therefore neutral currents with $\Delta S = 1$ should be possible (Strangeness Changing Neutral Currents, SCNC)

In order to explain the absence of SCNC Glashow, Iliopoulos e Maiani proposed in 1970 the introduction of a fourth quark, the charm (c), with charge 2/3, which allowed to introduce a second quark doublet for weak interactions:

$$\begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix} \qquad \begin{pmatrix} c \\ -d\sin\theta_C + s\cos\theta_C \end{pmatrix}$$

We have therefore two new diagrams contributing to the weak neutral current:



Therefore the weak neutral current becomes:

$$\underbrace{u\bar{u} + c\bar{c} + \left(d\bar{d} + s\bar{s}\right)\cos^2\theta_C + \left(s\bar{s} + d\bar{d}\right)\sin^2\theta_C}_{\Delta S = 0} +$$

$$\underbrace{\left(\overbrace{s\bar{d} + \bar{s}d - \bar{s}d - s\bar{d}}^{=0}\right)\sin\theta_C\cos\theta_C}_{\Delta S = 1}$$

The introduction of the fourth flavor cancels exactly the SCNC.
The charm quark was discovered experimentally in 1974.

Weak Mixing with 6 Quarks and the CKM Matrix

With four flavors the weak current has the form:

$$J_\mu = \begin{pmatrix}\bar{u} & \bar{c}\end{pmatrix} \frac{\gamma_\mu (1-\gamma_5)}{2} U \begin{pmatrix} d \\ s \end{pmatrix} \qquad U = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

With the introduction of two more flavors (b, charge -1/3 and t, charge 2/3):

$$J_\mu = \begin{pmatrix}\bar{u} & \bar{c} & \bar{t}\end{pmatrix} \frac{\gamma_\mu (1-\gamma_5)}{2} M \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$M = \begin{pmatrix} c_1 & c_3 s_1 & s_1 s_3 \\ -c_2 s_1 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}$$

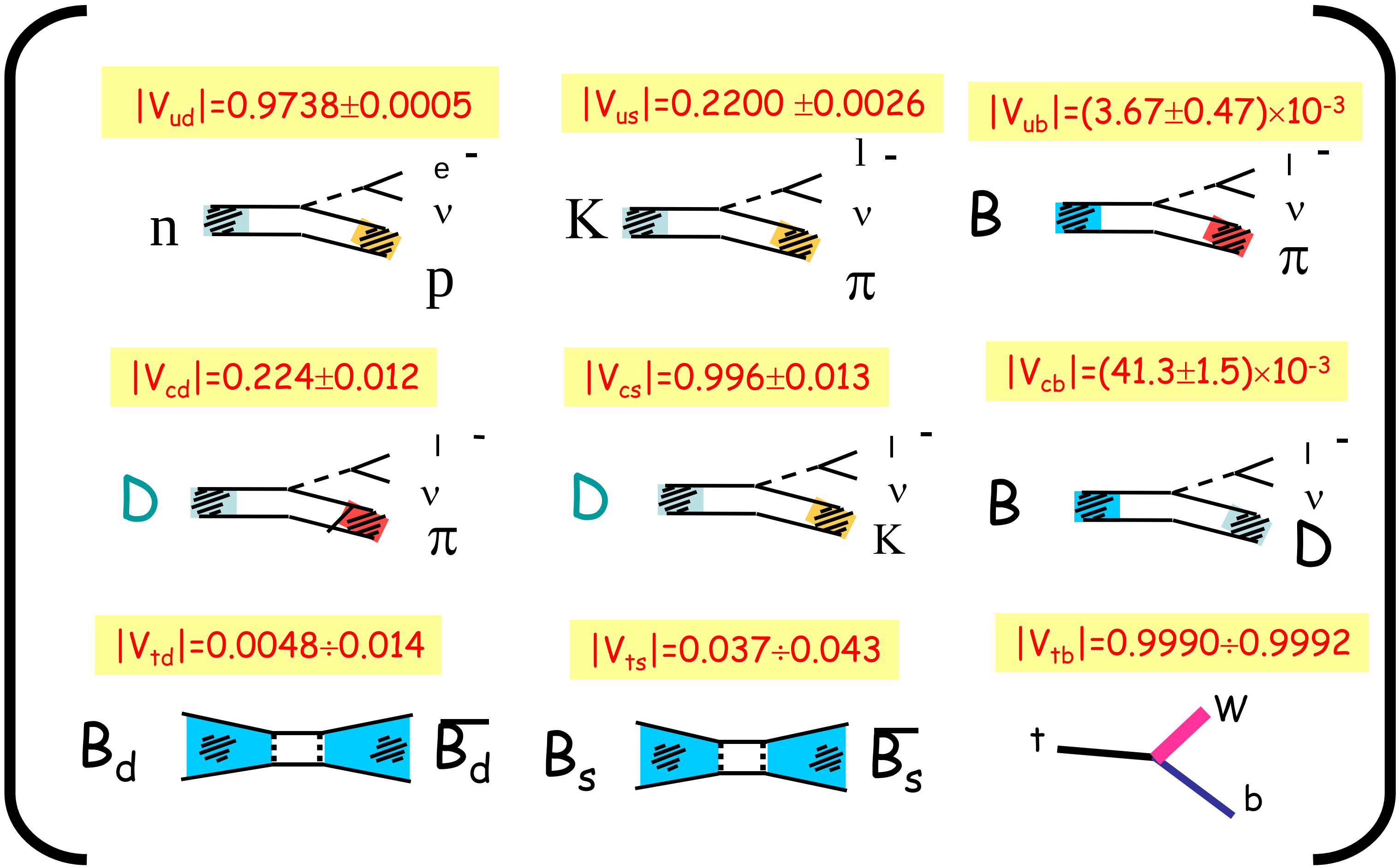
CKM matrix (Cabibbo-Kobayashi Maskawa)

$$c_i = \cos\theta_i \qquad s_i = \sin\theta_i$$

θ_1, θ_2, θ_3 mixing angles,

δ phase

CKM Matrix


$|V_{ud}|=0.9738\pm0.0005$
n
e^-
ν
p
$|V_{us}|=0.2200 \pm0.0026$
K
l^-
ν
π
$|V_{ub}|=(3.67\pm0.47)\times10^{-3}$
B
l^-
ν
π
$|V_{cd}|=0.224\pm0.012$
D
l^-
ν
π
$|V_{cs}|=0.996\pm0.013$
D
l^-
ν
K
$|V_{cb}|=(41.3\pm1.5)\times10^{-3}$
B
l^-
ν
D
$|V_{td}|=0.0048\div0.014$
B_d
$\bar{B}_d$
$|V_{ts}|=0.037\div0.043$
B_s
$\bar{B}_s$
$|V_{tb}|=0.9990\div0.9992$
t
W
b