Symmetries and Conservation Laws II
Isospin, Strangeness, G-parity

Introduction to Elementary Particle Physics

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Outline

- Isospin
  - Definition, conservation
  - Isospin in the πN system
- Strangeness
- G-parity

Isospin

$$m_p = 938.27\,MeV \quad m_n = 939.57\,MeV$$

$$m_p \approx m_n$$

Heisenberg (1932):

*Proton and neutron considered as different charge substates of one particle, the **Nucleon**.*

A nucleon is ascribed a quantum number, **isospin**, conserved in the strong interaction, not conserved in electromagnetic interactions.

Nucleon is assigned isospin $I = \frac{1}{2}$

$$I_3 = +\tfrac{1}{2} \quad p$$

$$I_3 = -\tfrac{1}{2} \quad n$$

$$\frac{Q}{e} = \frac{1}{2} + I_3$$

The nucleon has an internal degree of freedom with two allowed states (the proton and the neutron) which are not distinguished by the nuclear force.

Let us write the nucleon states as $|I, I_3\rangle$

$$|p\rangle = \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \qquad |n\rangle = \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

For a two-nucleon system we have therefore:

Triplet
(symmetric)

$$\begin{cases} \chi(1,1) = \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle \\ \chi(1,0) = \tfrac{1}{\sqrt{2}}\left(\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle + \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\right) \\ \chi(1,-1) = \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle \end{cases}$$

Singlet
(antisymmetric)

$$\left\{\chi(0,0) = \tfrac{1}{\sqrt{2}}\left(\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle - \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\right)\right.$$

Example: deuteron (S-wave *pn* bound state)

$$\psi = \phi(spazio) \times \alpha(spin) \times \chi(isospin)$$

$$(-1)^l = +1 \qquad (-1)^{S+1} = +1 \qquad (-1)^{I+1}$$

$$(l = 0) \qquad (S = 1)$$

$$(-1)^{I+1} = -1 \quad \Rightarrow \quad I = 0$$

ψ is the wave function for *two identical fermions* (two nucleons), hence it must be *globally antisymmetric*. This implies that the deuteron must have **zero isospin**:

$$I_d = 0$$

As an example let us consider the two reactions

$$p + p \to \pi^+ + d$$

$$p + n \to \pi^0 + d \qquad (I_\pi = 1)$$

Since I_d=0 in each case the final state has isospin 1.

Let us now consider the initial states:

$$pp = |1,1\rangle$$

$$np = \tfrac{1}{\sqrt{2}}\left(|1,0\rangle - |0,0\rangle\right)$$

The cross section

$$\sigma \propto |ampiezza|^2 \approx \sum_I \left|\langle I', I_3' | A | I, I_3 \rangle\right|^2$$

Isospin conservation implies

$$I = I' = 1 \quad I_3 = I_3'$$

The reaction $np \to \pi^0 d$ proceeds with probability $\left(\tfrac{1}{\sqrt{2}}\right)^2$ with respect to $pp \to \pi^+ d$ hence:

$$\frac{\sigma\left(pp \to \pi^+ d\right)}{\sigma\left(np \to \pi^0 d\right)} = 2$$

Isospin in the πN System

The π meson exists in three charge states of roughly the same mass:

$$m_{\pi^\pm} = 139.57\,MeV$$

$$m_{\pi^0} = 134.98\,MeV$$

Consequently it is assigned $I_\pi=1$, with the charge given by $Q/e=I_3$.

$$|\pi^+\rangle = |1,1\rangle \quad |\pi^0\rangle = |1,0\rangle \quad |\pi^-\rangle = |1,-1\rangle$$

For the π $B=0$:

$$\frac{Q}{e} = I_3 + \frac{B}{2}$$

For the πN system the total isospin can be either I=1/2 or I=3/2

$$\pi^+ p \rightarrow \pi^+ p$$
$$\pi^- n \rightarrow \pi^- n$$

pure
I=3/2

$$\pi^- p \rightarrow \pi^- p$$
$$\pi^- p \rightarrow \pi^0 n$$
$$\pi^+ n \rightarrow \pi^+ n$$
$$\pi^+ n \rightarrow \pi^0 p$$

combination of
I=1/2 and I=3/2

		$I=\frac{3}{2}$				$I=\frac{1}{2}$	
$1\times\frac{1}{2}$	I_3	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
$\pi^+ p$		1					
$\pi^+ n$			$\sqrt{\frac{1}{3}}$			$\sqrt{\frac{2}{3}}$	
$\pi^0 p$			$\sqrt{\frac{2}{3}}$			$-\sqrt{\frac{1}{3}}$	
$\pi^0 n$				$\sqrt{\frac{2}{3}}$			$\sqrt{\frac{1}{3}}$
$\pi^- p$				$\sqrt{\frac{1}{3}}$			$-\sqrt{\frac{2}{3}}$
$\pi^- n$					1		

The coefficients in the linear combinations, i.e. the relative weights of the 1/2 and 3/2 amplitudes, are given by *Clebsch-Gordan coefficients*

$$\left|\pi^+ n\right\rangle = \left|1,1\right\rangle \times \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$$

$$\left|\tfrac{3}{2},\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{3}}\left|\pi^+ n\right\rangle + \sqrt{\tfrac{2}{3}}\left|\pi^0 p\right\rangle = \sqrt{\tfrac{1}{3}}\left|1,1\right\rangle \times \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\left|1,0\right\rangle \times \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$$

$$(1)\quad \pi^+ p \to \pi^+ p$$
$$(2)\quad \pi^- p \to \pi^- p$$
Elastic scattering

$$(3)\quad \pi^- p \to \pi^0 n$$
Charge exchange

$$\sigma \propto \left|\langle f|H|i\rangle\right|^2 = \left|M_{if}\right|^2$$

H = { H_1 if it acts between states of *I=1/2*; H_3 if it acts between states of *I=3/2* }

let $M_1 = \langle I=\tfrac{1}{2}|H_1|I=\tfrac{1}{2}\rangle$

$M_3 = \langle I=\tfrac{3}{2}|H_3|I=\tfrac{3}{2}\rangle$

(1) $\sigma_1 = K\left|M_3\right|^2$

$$|i\rangle = |f\rangle = \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

(2) $\sigma_2 = K\left|\langle f|(H_1+H_3)|i\rangle\right|^2$

$$\sigma_2 = K\left|\tfrac{1}{3}M_3 + \tfrac{2}{3}M_1\right|^2$$

(3) $|i\rangle = \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$

$$|f\rangle = \sqrt{\tfrac{2}{3}}\left|\tfrac{3}{2},-\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

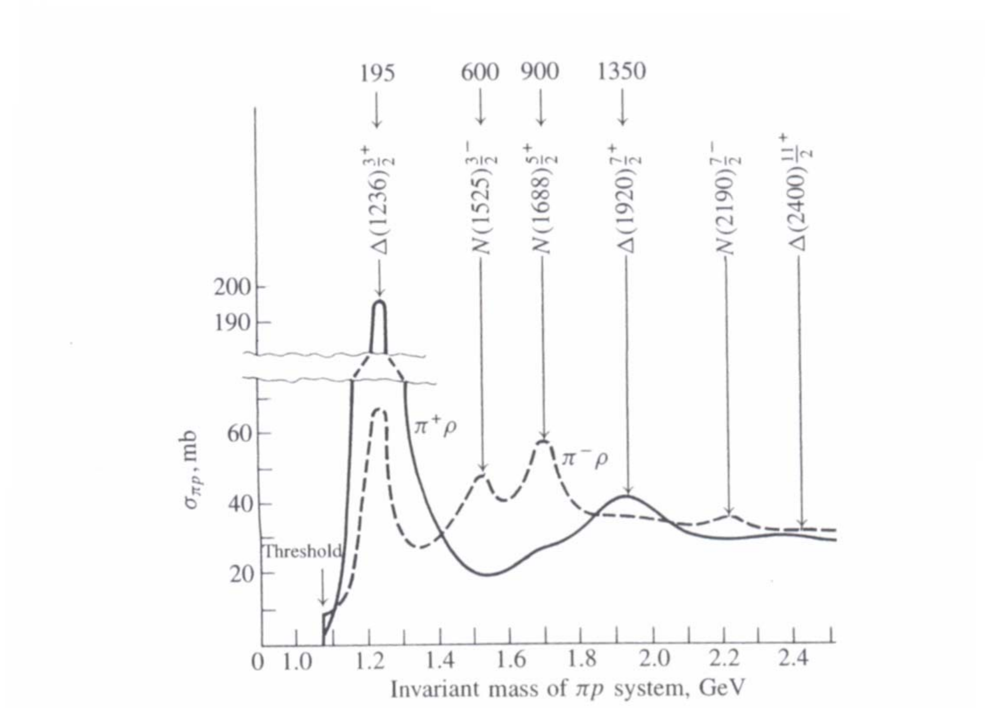
$$\sigma_3 = K\left|\sqrt{\tfrac{2}{9}}M_3 - \sqrt{\tfrac{2}{9}}M_1\right|^2$$

$$\sigma_1 : \sigma_2 : \sigma_3 = \left|M_3\right|^2 : \tfrac{1}{9}\left|M_3 + 2M_1\right|^2 : \tfrac{2}{9}\left|M_3 - M_1\right|^2$$

$$M_3 >> M_1 \quad \sigma_1 : \sigma_2 : \sigma_3 = 9:1:2$$

$$M_1 >> M_3 \quad \sigma_1 : \sigma_2 : \sigma_3 = 0:2:1$$

$\pi^{\pm}p$ Total Cross Section



$\Delta(1236) \quad \Gamma = 120\,MeV$

$J^P = \frac{3}{2}^+ \quad I = \frac{3}{2} \quad (3,3)$

$$\sigma(E) = \frac{2J+1}{(2s_1+1)(2s_2+1)} \frac{\pi}{k^2} \frac{\Gamma_{ab}\Gamma_{cd}}{(E-M_R)^2 + \frac{\Gamma^2}{4}}$$

a+b $\rightarrow$ R $\rightarrow$ c+d

Strangeness S

Strange particles are copiously produced in strong interactions

They have a long lifetime, typical of a weak decay.

S quantum number: strangeness *conserved in strong and electromagnetic interactions*, *not conserved in weak interactions*.

Example: $\pi^- + p \rightarrow \Lambda + K^0$

$$\Lambda \rightarrow p + \pi^- \qquad \tau = 2.6 \times 10^{-10}\, s$$

$$\Lambda \rightarrow p + \pi^-$$

	Λ	p	π^-
I	0	$\frac{1}{2}$	1
I_3	0	$\frac{1}{2}$	-1

I = 0, because the Λ has no charged counterparts

$$\pi^- + p \rightarrow \Lambda + K^0$$

	π^-	p	Λ	K^0
I	1	$\frac{1}{2}$	0	$\frac{1}{2}$
I_3	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$

$$K^0, K^+ \quad \frac{Q}{e} = I_3 + \frac{1}{2}$$

$$\overline{K}^0, K^- \quad \frac{Q}{e} = I_3 - \frac{1}{2}$$

$$\frac{Q}{e} = I_3 + \frac{B+S}{2}$$ (Gell-Mann Nishijima)

Y = B+S hypercharge

Using the Gell-Mann Nishijima formula strangeness is assigned together with isospin.

Example.:

n, p	$S = 0$	$I = \frac{1}{2}$
Λ	$S = -1$	$I = 0$
K^0, K^+	$S = 1$	$I = \frac{1}{2}$
$K^-, \overline{K}^0$	$S = -1$	$I = \frac{1}{2}$

Example of strangeness conservation:

	K^-	$+\, p$	$\rightarrow \Lambda$	$+\, \pi^0$
S	-1	0	-1	0
I_3	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0

	$\pi^\pm$	$+\, p$	$\rightarrow \Sigma^\pm$	$+\, K^+$
S	0	0	-1	$+1$
I	1	$\frac{1}{2}$	1	$\frac{1}{2}$
I_3	± 1	$\frac{1}{2}$	± 1	$\frac{1}{2}$

	Σ^0	$\rightarrow \Lambda$	$+\, \gamma$	*e.m.*
S	-1	-1	0	

	Σ^+	$\rightarrow n$	$+\, \pi^+$	*weak*
S	-1	0	0	

	Ξ^-	$\rightarrow \Lambda$	$+\, \pi^-$	*weak*
S	-2	-1	0	

G-parity G

$$G = Ce^{i\pi I_2}$$

Rotation of π around the 2 axis in isospin space followed by charge conjugation.

$$I_3 \xrightarrow{e^{i\pi I_2}} -I_3 \xrightarrow{C} I_3$$

Consider an isospin state $\chi(I,I_3=0)$: under isospin rotations this state behaves like $Y^0_l(\theta,\varphi)$ (under rotations in ordinary space)

The rotation around the 2 axis implies:

$$\vartheta \to \pi - \vartheta \quad \varphi \to \pi - \varphi$$

$$Y_l^0 \to (-1)^l Y_l^0$$

therefore

$$\chi(I,0) \to (-1)^I \chi(I,0)$$

Example: for a nucleon-antinucleon state the effect of C is to give a factor $(-1)^{l+s}$ (just is in the case of positronium). Therefore:

$$G\left|\psi(N\overline{N})\right\rangle = (-1)^{l+s+I}\left|\psi(N\overline{N})\right\rangle$$

This formula has **general validity**, not limited to the $I_3=0$ case.

For the π

$$G\left|\pi^{+}\right\rangle = \pm\left|\pi^{+}\right\rangle$$

$$G\left|\pi^{-}\right\rangle = \pm\left|\pi^{-}\right\rangle$$

$$G\left|\pi^{0}\right\rangle = \pm\left|\pi^{0}\right\rangle$$

For the π^0 C=+1 ($\pi^0 \rightarrow \gamma\gamma$), the rotation gives $(-1)^I=-1$ (I=1) so that G= -1.

$$G_{\pi^0} = -1$$

It is the practice to assign the phases so that *all members of an isospin triplet have the same G-parity as the neutral member*.

$$G\left|\pi^{\pm}\right\rangle = -\left|\pi^{\mp}\right\rangle \qquad \text{with} \quad C\left|\pi^{\pm}\right\rangle = -\left|\pi^{\mp}\right\rangle$$

Since the C operation reverses the sign of the baryon number B, **the eigenstates of G-parity must have baryon number zero B=0**.

G is a multiplicative quantum number, so for a system of *n* π

$$G=(-1)^n$$

$$\rho \to \pi\pi \qquad G_\rho = +1$$

$$\omega \to \pi\pi\pi \qquad G_\omega = -1 \qquad B.R. = 89\%$$

$$\omega \to \pi\pi \qquad G_f = +1 \qquad B.R. = 2.2\%$$

$\eta \to \gamma\gamma$ C=+1 which, with I=0, yields G=+1.

$$\eta \not\to \pi\pi \qquad viola\ P$$

$$\eta \to \pi\pi\pi \quad viola\ G \Rightarrow e.m.$$