Masses and the Higgs Mechanism

Elementary Particle Physics
Strong Interaction Fenomenology

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The assumption is made that the universe is filled with a spin-zero field, called a Higgs field, which is a doublet in $SU(2)$ and with a nonzero $U(1)$ hypercharge, but a singlet in color space. The gauge bosons and fermions can interact with this field, and in this interaction they acquire mass. The $SU(2)$ and $U(1)$ quantum numbers of the ground state (i.e. the vacuum are non-zero, so the $SU(2)$ and $U(1)$ symmetries are effectively broken (spontaneously brokedn symmetries). In this context the masses of the $W^\pm$ and $Z^0$ bosons can be calculated in terms of measurable parameters.
Spontaneous Symmetry breaking

\[ \mathcal{L} = T - V = \frac{1}{2} \partial_{\mu}\phi \partial^{\mu}\phi - \left( \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \right) \]

• We can require \( \lambda > 0 \) in order that the potential be bounded below as \( \phi \to \infty \).
• The theory is invariant under \( \phi \to -\phi \).
• To find the spectrum it is necessary:
  – to find the minimum of the potential (ground state \( \to \) vacuum)
  – to expand the fields around their value at the minimum and determine the excitations
• The \( \phi^4 \) term represents an interaction of strength \( \lambda \).
• Higher powers of \( \phi \) would lead to infinities in physical quantities and must therefore be excluded (the theory would not be renormalizable).
If $\mu^2 > 0$ the vacuum corresponds to $\phi = 0$, which minimizes the potential. In that case $\mu^2$ can be interpreted as $(\text{mass})^2$.

If, however, $\mu^2 < 0$ we find the minimum of the potential by setting:

$$\frac{\partial V}{\partial \phi} = 0 \quad \Rightarrow \quad \phi(\mu^2 + \lambda \phi^2) = 0$$

Minimum kinetic energy: $\phi = \text{constant}$.
The choice $\phi = 0$ is not a minimum of the potential.

$$\phi = \pm \sqrt{-\frac{\mu^2}{\lambda}} \equiv v$$

$v$ is called the vacuum expectation value of $\phi$. The field $\phi$ is called a Higgs field.
To determine the particle spectrum we study the theory in the region of the minimum.

\[ \phi(x) = v + \eta(x) \]

We expand around \( \eta = 0 \).

We could also have chosen \( \phi(x) = -v + \eta(x) \), but the physics conclusions would be the same, since the theory is symmetric under \( \phi \rightarrow -\phi \).

\[ \mathcal{L} = \frac{1}{2} \left( \partial_\mu \eta \partial^\mu \eta \right) - \left\{ \frac{1}{2} \mu^2 \left[ v^2 + 2\eta v + \eta^2 \right] + \frac{1}{4} \lambda \left[ v^4 + 4v^3 \eta + 6v^2 \eta^2 + 4v \eta^3 + \eta^4 \right] \right\} \]

\[ = \frac{1}{2} \left( \partial_\mu \eta \partial^\mu \eta \right) - \left\{ \frac{v^2}{2} \left( \mu^2 + \frac{1}{2} \lambda v^2 \right) + \eta v \left( \mu^2 + \lambda v^2 \right) + \frac{\eta^2}{2} \left( \mu^2 + 3\lambda v^2 \right) + \lambda v \eta^3 + \frac{1}{4} \lambda \eta^4 \right\} \]

\[ \mathcal{L} = \frac{1}{2} \left( \partial_\mu \eta \partial^\mu \eta \right) - \left\{ \lambda v^2 \eta^2 + \lambda v \eta^3 + \frac{1}{4} \lambda \eta^4 \right\} + \text{costante} \]

\[ m_\eta^2 = 2\lambda v^2 = -2\mu^2 \]
• The two descriptions of the theory in terms of $\phi$ or $\eta$ must be equivalent.
• It is essential to perturb around the minimum to have a convergent description.
• The scalar particle described by the theory with $\mu^2<0$ is a real scalar, with a mass obtained by its self-interaction with other scalar, because at the minimum of the potential there is a non-zero expectation value $\nu$.
• There is no trace of the original reflection symmetry. The symmetry was broken when a specific vacuum was chose ($+\nu$ rather than $-\nu$): the vacuum does not have the symmetry of the original Lagrangian, so the solutions do not.
• This is called a spontaneous symmetry breaking.
Complex Scalar Field

\[ \phi = \frac{\phi_1 + i \phi_2}{\sqrt{2}} \]

\[ \mathcal{L} = (\partial_{\mu} \phi)^* (\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \]

\[ \phi \rightarrow \phi' = e^{i \chi} \phi \quad \text{Global } U(1) \text{ symmetry} \]

\[ \mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi_1)^2 + \frac{1}{2} (\partial_{\mu} \phi_2)^2 - \frac{1}{2} \mu^2 (\phi_1^2 + \phi_2^2) - \frac{\lambda}{4} (\phi_1^2 + \phi_2^2)^2 \]

For \( \mu^2 > 0 \) the minimum is at the origin

For \( \mu^2 < 0 \) the minimum is along a circle of radius \( v \).

\[ \phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda} \]

We choose \( \phi_1 = v, \phi_2 = 0 \)

\[ \phi(x) = \frac{v + \eta(x) + i \rho(x)}{\sqrt{2}} \]

Mexican Hat Potential
\[ \mathcal{L} = \frac{1}{2} (\partial_\mu \rho)^2 + \frac{1}{2} (\partial_\mu \eta)^2 + \mu^2 \eta^2 - \lambda \nu (\eta \rho^2 + \eta^3) - \frac{\lambda}{2} \eta^2 \rho^2 - \frac{\lambda}{4} \eta^4 - \frac{\lambda}{4} \rho^4 + \text{costante} \]

\[ m_\eta^2 = 2|\mu^2| \]

- the \( \eta \) field corresponds to a massive particle
- The terms \( \rho^2 \) vanish, thus \( \rho \) has zero mass: Goldstone boson!
- Goldstone theorem: whenever a continuous global symmetry is spontaneously broken the spectrum contains a massless, spin-zero boson.
- In this case the global \( U(1) \) symmetry is broken because we had to choose a particular point on the circle to expand around.
- The potential is a minimum along a circle: radial excitations correspond to massive particles (curvature of the potential), whereas along the circle the potential is flat, hence the massless excitation.
The Abelian Higgs Mechanism

Let us require local gauge invariance:

$$\phi(x) \rightarrow \phi'(x) = e^{ix(x)} \phi(x) \quad \partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu - igA_\mu$$

$$A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{g} \partial_\mu \chi(x)$$

$$\mathcal{L} = (\mathcal{D}_\mu \phi)^* (\mathcal{D}^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

• For \( \mu^2 > 0 \) this describes the interaction of a scalar particle of mass \( \mu \) with the electromagnetic field \( A_\mu \).

• There is no mass term for \( A_\mu \).

• This Lagrangian contains four independent fields: the two real scalars \( \phi_1 \) and \( \phi_2 \) and the two transverse polarization states of the massless vector boson \( A_\mu \).

• Also in this case let us look for solutions for \( \mu^2 < 0 \).

$$\phi(x) = \eta(x) e^{-i\rho(x)}$$

$$\phi(x) = \frac{v + h(x)}{\sqrt{2}}$$

\( \eta, \rho \) and \( h \) real, using a suitable \( \chi \).
There is now a mass term for the gauge boson!!!

\[ M_A = gv \]

The Higgs boson $h$ with mass $\sqrt{2\lambda v^2}$

- La Lagrangian is gauge invariant, but the vacuum is not.
- The number of independent fields is still 4: the Higgs boson and the three polarization states of the massive gauge boson.
- The Goldstone boson of the previous case (global $U(1)$ symmetry) has become the longitudinal polarization state of the gauge boson.
- **Higgs Mechanism.**
- The Higgs boson should exist as a physical particle.
- The gauge boson mass is fixed if $g^2$ and $v$ are known, but the mass of the Higgs boson $h$ depends on the unknown parameter $\lambda$. 

\[ \mathcal{L} = \frac{1}{2} \left[ \left( \partial_{\mu} + igA_{\mu} \right)(v + h) \right] \left[ \left( \partial_{\mu} - igA_{\mu} \right)(v + h) \right] \]

\[ -\frac{v^2}{2} (v + h)^2 - \frac{\lambda}{4} (v + h)^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]

\[ = \frac{1}{2} \left( \partial_{\mu} h \right) \left( \partial_{\mu} h \right) + \frac{1}{2} g^2 v^2 A_{\mu} A^{\mu} - \lambda v^2 h^2 - \lambda h h^3 \]

\[ - \frac{\lambda}{4} h^4 + g^2 v h A_{\mu} A^{\mu} + \frac{1}{2} g^2 v h^2 A_{\mu} A^{\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]
The Higgs Mechanism in the Standard Model

In the standard model the Higgs field is a doublet in $SU(2)$

$$\phi = \left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array}\right)$$

$$\phi^+ = \frac{\phi_1 + i\phi_2}{\sqrt{2}} \quad \phi^0 = \frac{\phi_3 + i\phi_4}{\sqrt{2}}$$

Complex fields

$$L = \left(\partial_\mu \phi^\dagger\right)\left(\partial^\mu \phi\right) - \mu^2 \phi^+ \phi - \lambda \left(\phi^+ \phi\right)^2$$

$$\phi^\dagger \phi = \left(\begin{array}{c} \phi^{0*} \\ \phi^0 \end{array}\right)\left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array}\right) = \phi^{0*} \phi^+ + \phi^0 \phi^0$$

$$\phi^\dagger \phi = \frac{\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2}{2}$$
Let us study the potential: \( V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2 \)

\( V(\phi) \) is invariant under the local gauge transformation:

\[
\phi(x) \rightarrow \phi'(x) = e^{i \vec{\alpha}(x) \cdot \vec{\tau}/2} \phi(x)
\]

For \( \mu^2 < 0 \) \( V(\phi) \) has a minimum at:

\[
\phi^* \phi = -\frac{\mu^2}{2\lambda} = \frac{v^2}{2}
\]

Let us choose a direction: \( \phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \phi_3 = v, \phi_1 = \phi_2 = \phi_4 = 0 \)

\[
\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}
\]

We look for equations satisfied by \( H(x) \).

This choice is always possible thanks to the local gauge invariance.

The original (\( O(4) \)) symmetry was:

\[
\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = \text{invariante}
\]

By choosing a direction we have three broken global symmetries so three massless Goldstone bosons, which will become the longitudinal parts of \( W^\pm \) and \( Z^0 \).
The electric charge, the weak isospin eigenvalue $T_3$ and the $U(1)$ hypercharge $Y_H$ of the Higgs field are related by:

$$Q = T_3 + \frac{Y_H}{2}$$

We have $Y_H = 1$.

The choice of $\phi^0$ as the component which gets a vacuum expectation value ensures the conservation of electric charge.

If the vacuum $\phi_0$ is invariant under some subgroups of the original $SU(2) \times U(1)$ any gauge bosons associated with that subgroup will still be massless.

Since only one component of the Higgs doublet gets a vacuum expectation value the $SU(2)$ symmetry is broken. Since $Y_H \neq 0$ then also the $U(1)$ symmetry is broken. However if we operate with the electric charge $Q$:

$$Q \phi_0 = \left( T_3 + \frac{Y}{2} \right) \phi_0$$

the vacuum is invariant under a transformation $\phi_0 \rightarrow \phi_0' = e^{i\alpha(x)Q} \phi_0 = \phi_0$

$U'(1)$ transformation corresponding to the electromagnetic interaction: the photon remains massless.
\[ D_\mu = \partial_\mu - ig_1 \frac{Y}{2} B_\mu - ig_2 \frac{\bar{\tau}}{2} \cdot \bar{W}_\mu \]

When $\phi$ gets a vacuum expectation value, proceeding as before the Lagrangian contains extra terms:

\[ \phi^* \left( ig_1 \frac{Y}{2} B_\mu + ig_2 \frac{\bar{\tau}}{2} \cdot \bar{W}_\mu \right)^* \left( ig_1 \frac{Y}{2} B_\mu + ig_2 \frac{\bar{\tau}}{2} \cdot \bar{W}_\mu \right) \phi \]

Using $\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ we obtain:

\[
\frac{1}{8} \left| \begin{pmatrix} g_1 B_\mu + g_2 W^3_\mu \\ g_2 \left( W^1_\mu - i W^2_\mu \right) \\ g_2 \left( W^1_\mu + i W^2_\mu \right) \end{pmatrix} \begin{pmatrix} 0 \\ v \\ v \end{pmatrix} \right|^2 = \frac{1}{8} v^2 g_2^2 \left( W^1_\mu \right)^2 + \left( W^2_\mu \right)^2 + \frac{1}{8} v^2 \left( g_1 B_\mu - g_2 W^3_\mu \right)^2
\]

\[
\left( \frac{1}{2} v g_2 \right)^2 W^+_\mu W^-_\mu
\]

\[
\frac{1}{2} \left( \frac{1}{2} v \sqrt{g_1^2 + g_2^2} \right)^2 Z_\mu Z^\mu
\]
For a charged boson the mass term would be of the form \( m^2 W^+ W^- \).
Comparing with:
\[
\left( \frac{1}{2} v g_2 \right)^2 W_\mu^+ W^-\mu
\]
We obtain:
\[
M_W = \frac{v g_2}{2}
\]
For a neutral boson the mass term would be of the form \( m^2 Z Z/2 \).
\[
M_Z = \frac{1}{2} \sqrt{g_1^2 + g_2^2}
\]
\[
M_\gamma = 0
\]
\[
\frac{M_W}{M_Z} = \cos \theta_W
\]
\[
\rho = \frac{M_W}{M_Z \cos \theta_W}
\]
Fermion Masses

Interaction Lagrangian of the leptons with the Higgs field:

\[ \mathcal{L}_{\text{int}} = g_e \left( \bar{L}_R \phi e_R^- + \phi^\dagger e^-_R L \right) \]

Invariant in \( SU(2) \). \( g_e \) arbitrary constant.

\[ \phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix} \]

\[ \mathcal{L}_{\text{int}} = \frac{g_e}{\sqrt{2}} \left( e^-_L e^-_R + e^-_e e^-_L \right) + \frac{g_e}{\sqrt{2}} \left( e^-_L e^-_R + e^-_e e^-_L \right) H \]

Mass term for \( e \)

\[ m_e = \frac{g_e \nu}{\sqrt{2}} \]

\[ \frac{g_e}{\sqrt{2}} = \frac{m_e}{\nu} \]

\[ \mathcal{L}_{\text{int}} = m_e \bar{e} e + \frac{m_e}{\nu} \bar{e} e H \]

For the neutrino there is no mass term, due to the absence of \( \nu_R \). This implies that the neutrino does not interact with the Higgs. If there were a \( \nu_R \) it would have \( T_3=0, Q=0 \) and it would not couple to \( W^\pm, Z^0, \gamma \), therefore it would be very difficult to observe.
For quark masses we must take into account also the existence of $u_R$.

$$\psi = \begin{pmatrix} a \\ b \end{pmatrix} \text{ doublet in } SU(2) \quad \iff \quad \psi_c = -i \tau_2 \psi^* = \begin{pmatrix} -b^* \\ a^* \end{pmatrix} \text{ doublet in } SU(2)$$

$$\phi_c = \begin{pmatrix} -\phi^0^* \\ \phi^- \end{pmatrix} \quad \phi_c \rightarrow \begin{pmatrix} -\frac{\nu + H}{2} \\ 0 \end{pmatrix}$$

$$\mathcal{L}_{\text{int}} = g_d \bar{Q}_L \phi_R d + g_u \bar{Q}_L \phi_c u_R + \text{h.c.}$$

$$\mathcal{L}_{\text{int}} = m_d \bar{d}d + m_u \bar{u}u + \frac{m_d}{\nu} \bar{d}dH + \frac{m_u}{\nu} \bar{u}uH$$

Also in this case the parameters $g_d$ and $g_u$ are arbitrary. Thus masses are included in the Standard Model, but they are not predicted: they must be measured.

This procedure can be repeated for the second and third families.
The Higgs interacts with fermions with a strength proportional to their mass $m_f$, therefore it couples more strongly to the heaviest fermions.