The Structure of Hadrons

Introduction to Elementary Particle Physics

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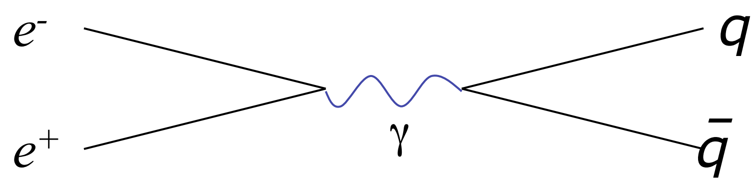
Outline

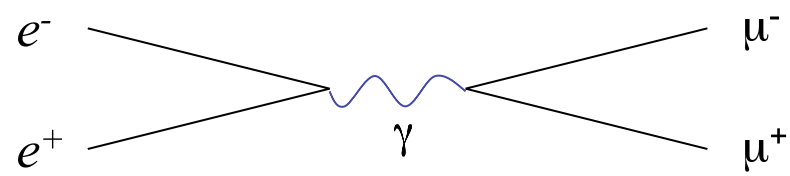
- The process $e^+e^- \rightarrow$ hadrons
- *Deep Inelastic Scattering* (DIS)
- Scale invariance
- The parton model
- The Drell-Yan process
- Jet formation
- Quantum Chromodynamics

The process $e^+e^- \rightarrow$ hadrons

Hadron production in e^+e^- annihilation occurs via the hadronization of the $q\bar{q}$ pairs produced in the process $e^+e^- \rightarrow q\bar{q}$.

The total cross section can be related to that of the process $e^+e^- \rightarrow \mu^+\mu^-$.





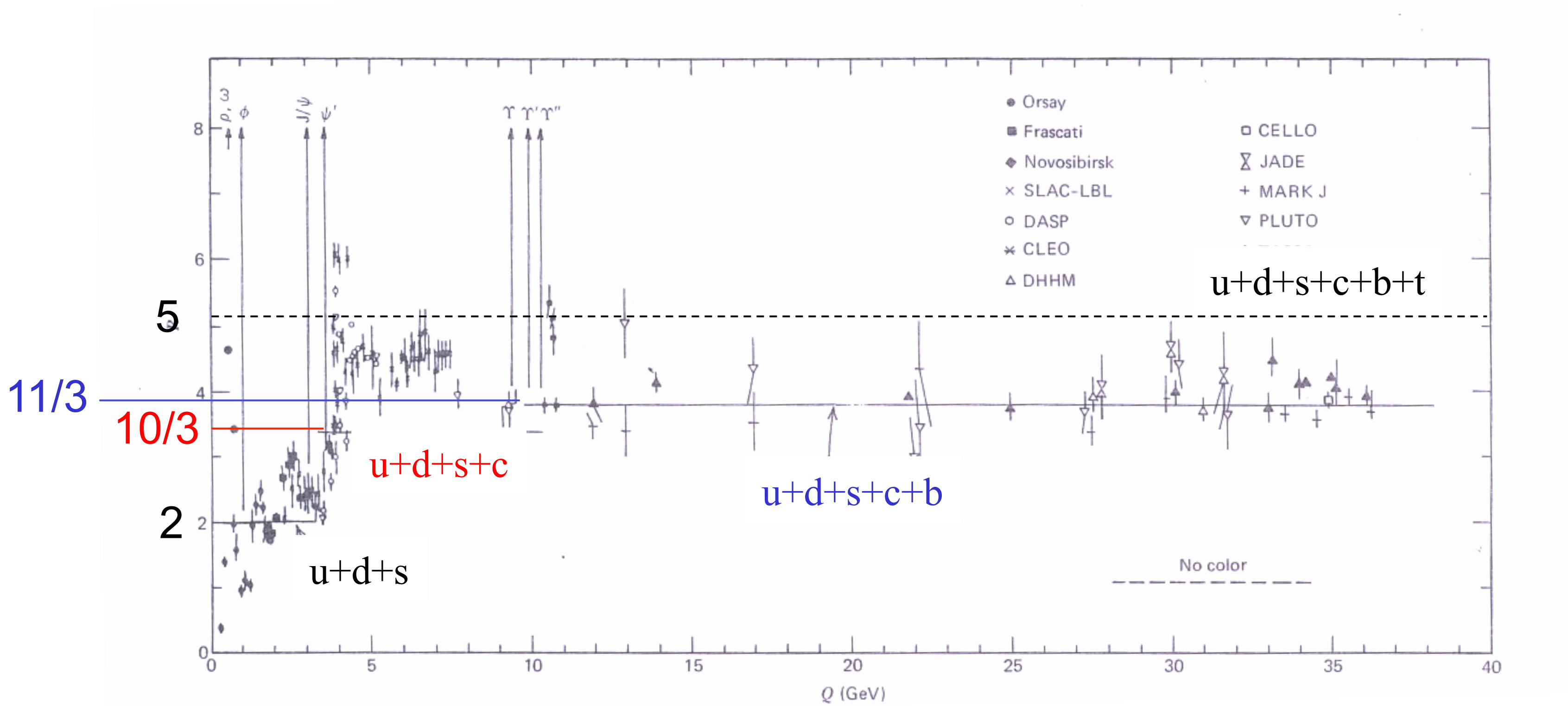
$$\sigma\left(e^+e^- \rightarrow \mu^+\mu^-\right) = \frac{4\pi}{3}\frac{\alpha^2}{s}$$

$$\sigma\left(e^+e^- \rightarrow q\bar{q}\right) = \underset{\text{color}}{3}\, e_q^2\,\sigma\left(e^+e^- \rightarrow \mu^+\mu^-\right)$$

$$e_q = \begin{cases} -\frac{1}{3} & q = d, s, b \\ +\frac{2}{3} & q = u, c, t \end{cases}$$

$$\boxed{\sigma\left(e^+e^- \rightarrow adroni\right) = \sum_q 3e_q^2\,\sigma\left(e^+e^- \rightarrow \mu^+\mu^-\right)}$$

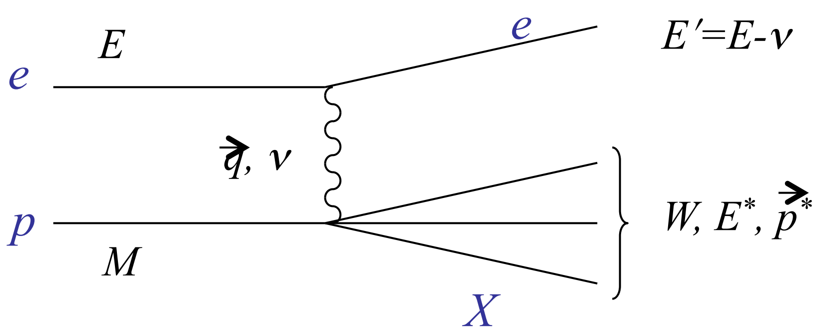
$$R \equiv \frac{\sigma\left(e^+e^- \to adroni\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} = 3\sum_q e_q^2$$



As the energy increases, the R ratio increases each time a threshold for the production of a new flavor is crossed. The experimental measurement of R is also a confirmation that quarks come in three colors.

with QCD corrections $$R = 3\sum_q e_q^2\left(1 + \frac{\alpha_s(Q^2)}{\pi}\right)$$

Deep Inelastic Scattering $e + p \rightarrow e + X$



$$q^2 = \left(E^* - M\right)^2 - \left(\vec{p}^* - 0\right)^2$$

$$Q^2 = -q^2$$

$$\nu = E^* - M$$

$$W^2 = E^{*2} - \vec{p}^{*2}$$

$$\boxed{Q^2 = 2M\nu - W^2 + M^2}$$

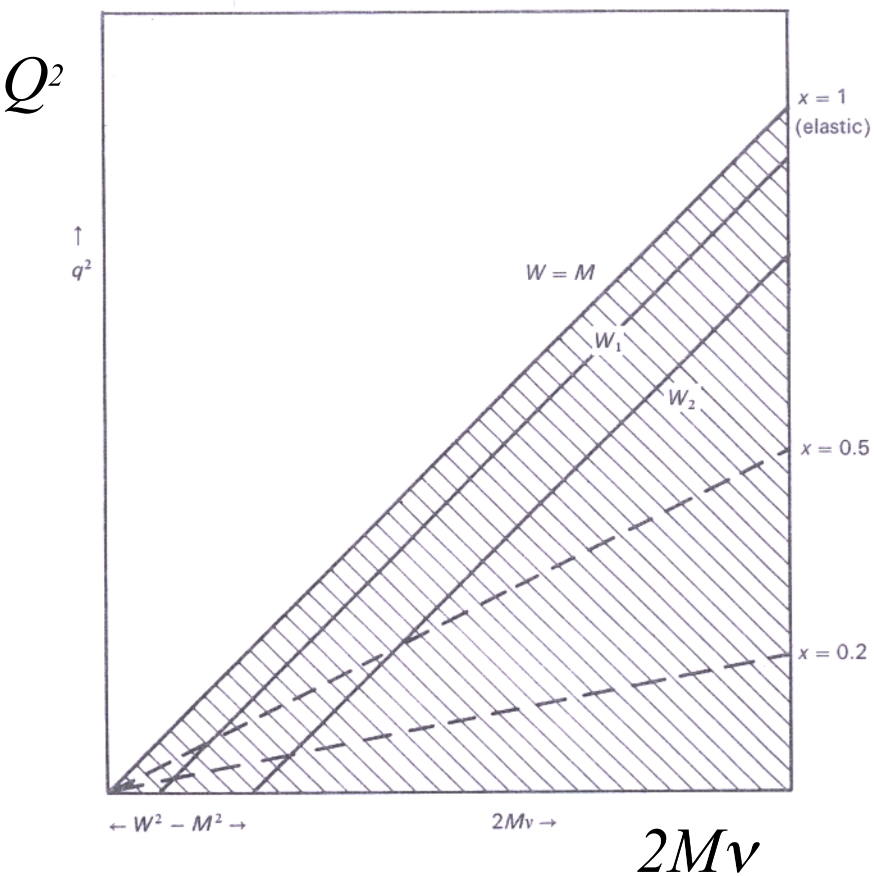
If $W=M$ we obtain elastic scattering

$$Q^2 = 2M\nu$$

For higher q^2 values we can excite the proton, for example:

$$e + p \rightarrow e + \Delta^+ \rightarrow e + p + \pi^0$$

When q^2 becomes very large the initial state proton loses its identity completely and a new formalism must be devised to extract information from the measurements.



The kinematically allowed region is

$$0 < x = \frac{Q^2}{2M\nu} < 1$$

Contours of fixed W are 45^0 lines.

$x = 1$ *Elastic*

$x < 1$ *Inelastic*

The cross section for the process:

$$e + p \rightarrow e + X$$

can be expressed in terms of two structure functions:

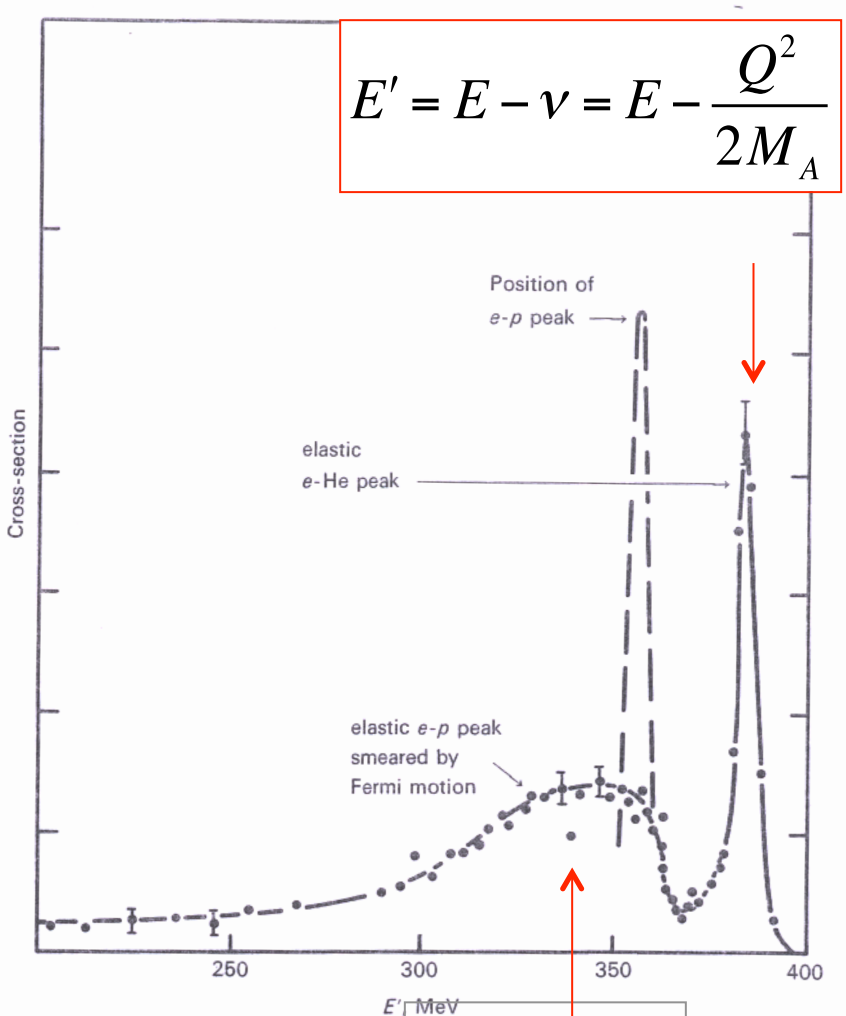
$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{4\pi\alpha^2}{q^4}\frac{E'}{EM}\left[W_2\left(q^2,\nu\right)\cos^2\frac{\theta}{2} + 2W_1\left(q^2,\nu\right)\sin^2\frac{\theta}{2}\right]$$

(where it is assumed that $E >> m$, $E' >> m$)

Let us consider how this cross section behaves as a function of q^2 in two cases:

1. electron-nucleus scattering
2. electron-nucleon scattering

$$e^- + A \to e^- + X$$



$$E' = E - \nu = E - \frac{Q^2}{2M_A}$$

$$\nu \approx \frac{Q^2}{2M_N}$$

Elastic peak at high values of E′
(low values of Q^2)

The broader peak at larger q^2 values corresponds to quasi-elastic scattering on single nucleons. If nucleons were free we would have a single narrow peak at:

$$\nu \approx \frac{Q^2}{2M_N}$$

Nucleons however are in a potential well of radius R ~ 1 fm, so that they have a Fermi momentum:

$$p_F \approx \frac{1}{R} \approx 200\ MeV$$

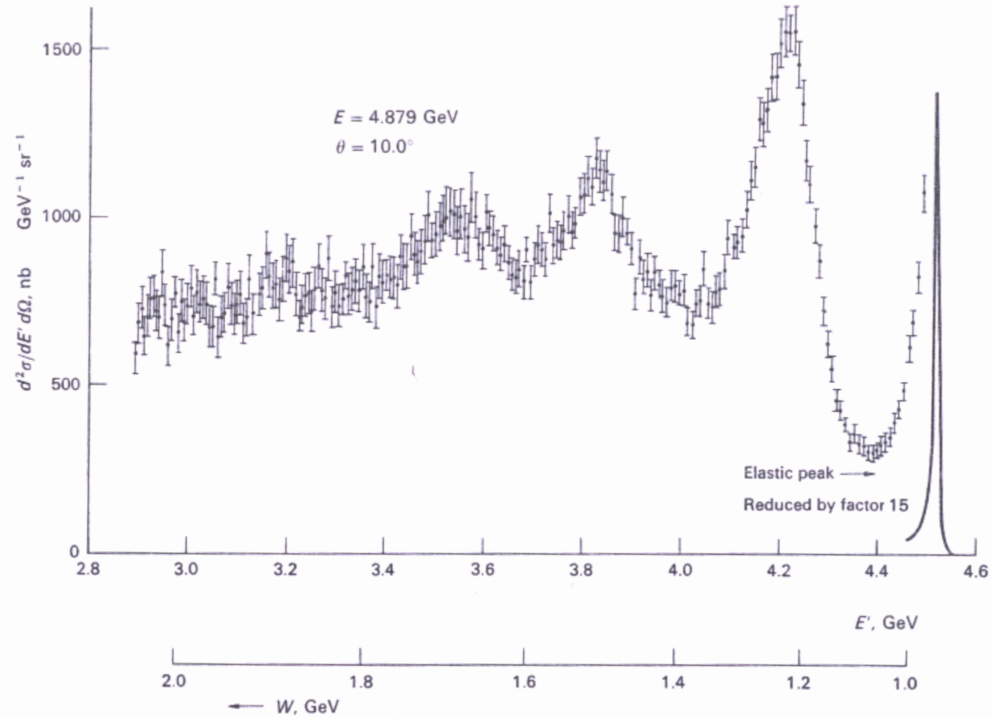
which broadens the elastic peak:

$$\frac{\Delta\nu}{\nu} = \pm\frac{p_F}{M} \approx 10\%$$

$$e^- + p \rightarrow e^- + X$$

The same phenomena, but at higher q^2 values, can be observed in $ep \rightarrow eX$:

- elastic peak at $x=1$ ($Q^2=2M\nu$)
- Peaks corresponding to resonances (Δ, N ..) at higher q^2 values.



If the pointlike constituents are quarks we can expect $m \sim M/3$, hence:

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2m\nu}\frac{m}{M} = \frac{1}{3}$$

The broadening of the elastic peak in this case is:

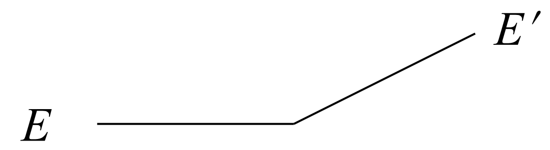
$$\frac{\Delta x}{x} \approx \frac{p_F}{mc} \approx 1$$

therefore we observe no sharp peak.

Scale Invariance (Bjorken Scaling)

The differential cross sections for the scattering of electrons (pointlike fermions) from various targets can all be written in the form:

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\alpha^2 E'^2}{q^4}\{\;\}$$



$$\{\;\}_{e\mu\to e\mu} = \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M}\sin^2\frac{\theta}{2}\right)\delta\left(\nu - \frac{Q^2}{2m}\right)$$

For a μ target of mass m (or quark target of mass m, $\alpha^2 \to \alpha^2 e_q^{\,2}$)

$$\{\;\}_{ep\to ep} = \left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\right)\delta\left(\nu - \frac{Q^2}{2M}\right)$$

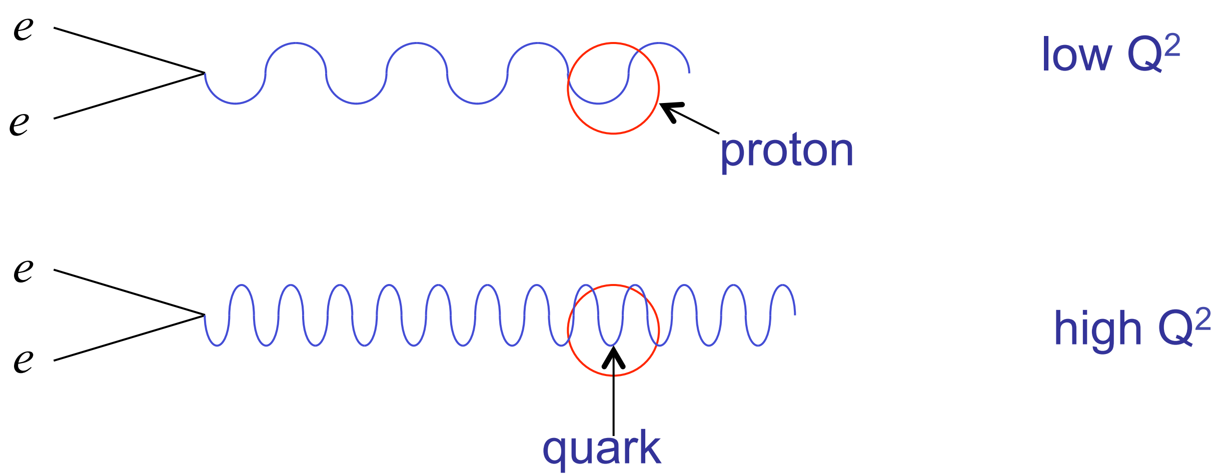
Elastic scattering from a proton target (mass M)

$$\left(\tau = Q^2/4M^2\right)$$

$$\{\;\}_{ep\to eX} = \left(W_2\left(Q^2,\nu\right)\cos^2\frac{\theta}{2} + 2W_1\left(Q^2,\nu\right)\sin^2\frac{\theta}{2}\right)$$

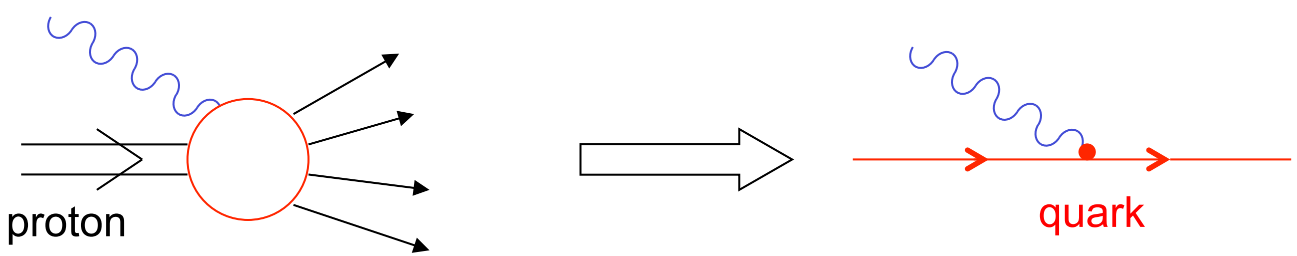
Inelastic scattering from a proton

For high q^2 values we probe the proton with high-frequency (virtual) photons



In this case we can view the inelastic $ep \to eX$ scattering as (quasi) elastic scattering on a pointlike free constituent within the proton. Correspondingly we expect the cross sections in this q^2 regime to behave like those for a pointlike target. Thus the structure functions become:

$$2W_1^{\text{point}} = \frac{q^2}{2m^2}\delta\left(\nu - \frac{Q^2}{2m}\right) \qquad W_2^{\text{point}} = \delta\left(\nu - \frac{Q^2}{2m}\right)$$



Using the identity $\delta(x/a) = a\delta(x)$ we can introduce dimensionless structure functions:

$$2mW_1^{\text{point}} = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right) \qquad \nu W_2^{\text{point}} = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

which are only functions of the ratio $\dfrac{Q^2}{2m\nu}$, and not of Q^2 and ν independently.

For *ep* elastic scattering this cannot be done. If for simplicity we set $G_E = G_M = G$ we obtain:

$$W_1^{\text{el}} = \frac{q^2}{4M^2}G^2(Q^2)\delta\left(\nu - \frac{Q^2}{2M}\right) \qquad W_2^{\text{el}} = G^2(Q^2)\delta\left(\nu - \frac{Q^2}{2M}\right)$$

in these expressions a mass scale is explicitly present, which reflects the inverse size of the proton. **The structure functions W_1^{el} e W_2^{el} cannot be rearranged to be functions of a single dimensionless variable.**

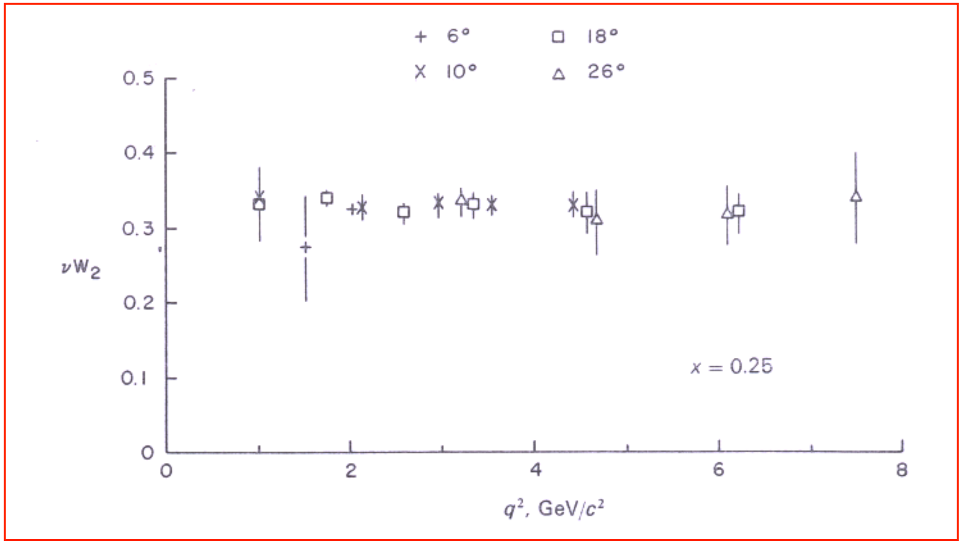
If large q^2 virtual photons are used to probe the structure of the proton then:

$$MW_1(\nu,Q^2) \xrightarrow[q^2\ alti]{} F_1\left(\frac{Q^2}{2M\nu}\right)$$

$$\nu W_2(\nu,Q^2) \xrightarrow[q^2\ alti]{} F_2\left(\frac{Q^2}{2M\nu}\right)$$

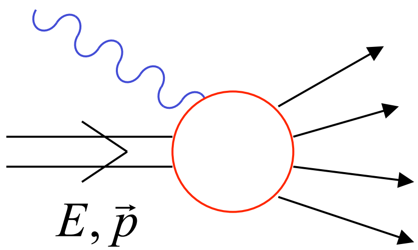
Bjorken Scaling

experimental data

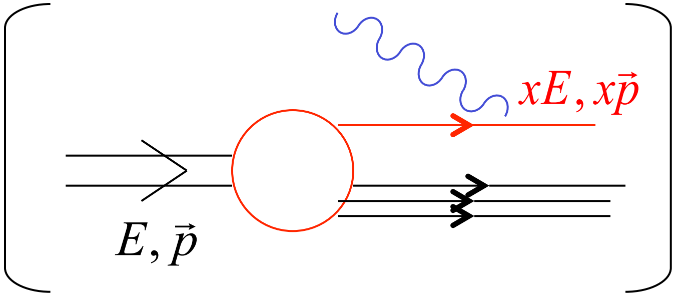


The absence of an explicit mass scale indicates that the scattering takes place on pointlike constituents (partons)

Bjorken Scaling and the Parton Model

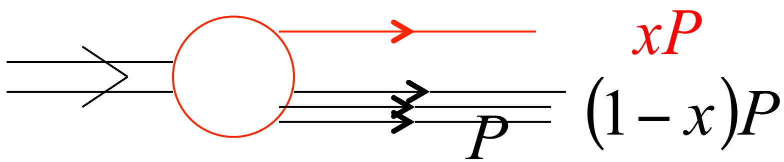


$$= \sum_i \int dx e_i^2$$



The sum is over partons of charge e_i, each carrying a fraction x of the proton energy and momentum. We introduce the parton momentum distribution:

$$f_i(x) = \frac{dP_i}{dx} =$$



$$\sum_i \int x f_i(x) dx = 1$$

Kinematics	proton	parton
Energy	E	xE
Momentum	p_L $p_T=0$	xp_L $p_T=0$
Mass	M	$m = \sqrt{x^2E^2 - x^2p^2} = xM$

This kinematics is valid only if $m=M=0$. Let us choose a reference frame in which:

$$|\vec{p}| >> m, M$$

Infinite Momentum Frame

In the final state we obviously observe hadrons, not partons: partons must somehow recombine to form colorless hadrons. This has to happen with probability 1 because of color confinement and due to the size of the hadron it requires a much longer time scale than that of the interaction between parton and virtual photon:

$$t_1 \approx \frac{\hbar}{\nu}$$

Interaction time

$$t_2 \approx \gamma \frac{\hbar}{W}$$

Hadronization time

$$\gamma = \frac{\nu}{W} \qquad W^2 \approx 2M\nu$$

$$t_2 > \frac{\hbar \nu}{W^2} = \frac{\hbar}{M} >> t_1$$ se $$\nu >> M$$

During the time t_1 in which it interacts with the virtual photon the parton behaves essentially as a free particle. Over a much longer time scale (given by t_2) partons recombine to form hadrons (color singlets).

This final state interaction does not alter the calculation of structure functions.

For an electron hitting a parton with momentum fraction x and unit charge the dimensionless structure functions are:

$$\tilde{F}_1^p(\omega) = \frac{Q^2}{4m\nu x}\delta\left(1 - \frac{Q^2}{2m\nu}\right) = \frac{1}{2x^2\omega}\delta\left(1 - \frac{1}{x\omega}\right)$$

$$\omega = \frac{2M\nu}{Q^2}$$

$$\tilde{F}_2^p(\omega) = \delta\left(1 - \frac{Q^2}{2m\nu}\right) = \delta\left(1 - \frac{1}{x\omega}\right)$$

Summing over the partons in the proton:

$$\tilde{F}_2(\omega) = \sum_i \int dx e_i^2 f_i(x) x \delta\left(x - \frac{1}{\omega}\right)$$

$$\tilde{F}_1(\omega) = \frac{\omega}{2}\tilde{F}_2(\omega)$$

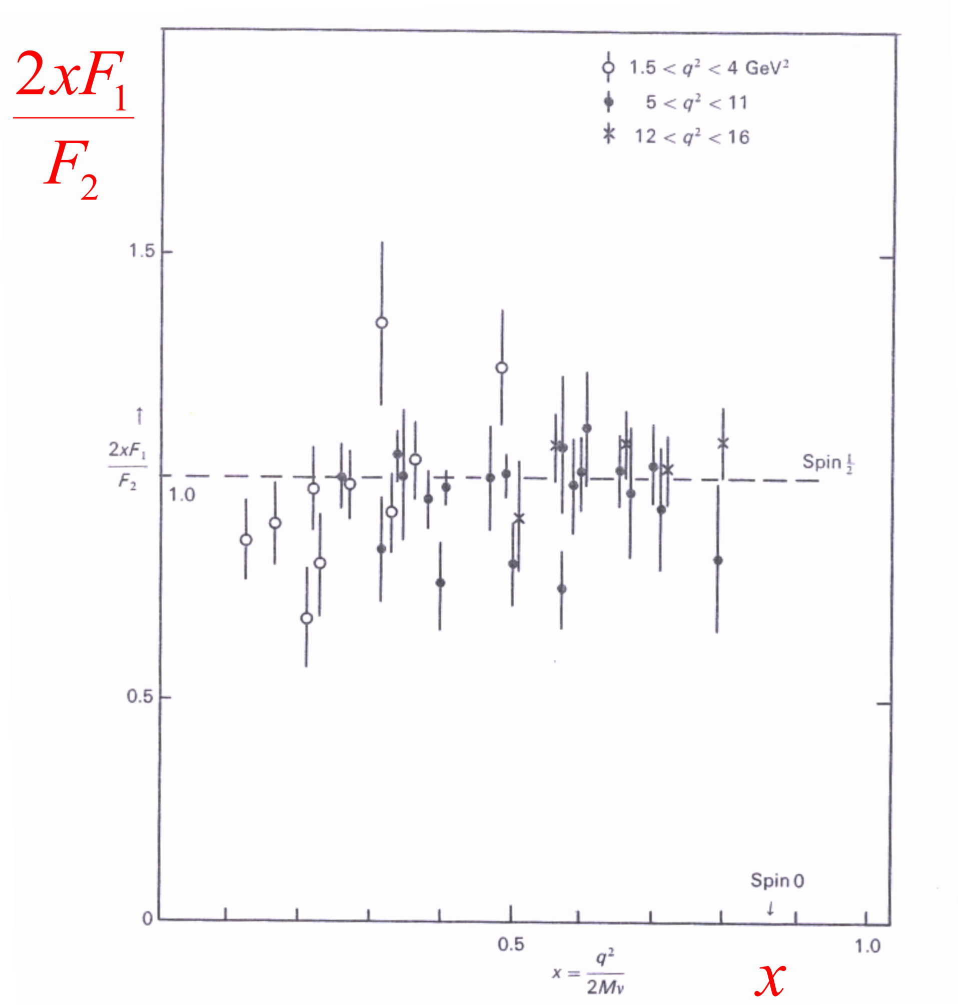
It is conventional to define F_1 and F_2 as functions of x:

$$\nu W_2(\nu, Q^2) \rightarrow F_2(x) = \sum_i e_i^2 x f_i(x)$$

$$MW_1(\nu, Q^2) \rightarrow F_1(x) = \sum_i \frac{e_i^2 f_i(x)}{2} = \frac{1}{2x}F_2(x)$$

The variable x introduced previously is found to represent the momentum fraction carried by the: **the virtual photon must have just the right value of x to be absorbed by a parton with momentum xp**.

Parton Spin



From the expressions for F_1 and F_2 we get:

$$2xF_1(x) = F_2(x)$$

known as the Callan-Gross relation.
It is a consequence of the quarks having spin ½. It implies:

$$\frac{\sigma_L}{\sigma_T} \xrightarrow[\substack{q^2\to\infty \\ \nu\to\infty}]{} 0 \quad \Big\} x \text{ fissato}$$

By conservation of J_z a spin-0 quark, cannot absorb a photon with helicity $\pm1 \Rightarrow \sigma_T=0$.
With spin ½ helicity conservation implies $J_z= \pm1$ (transverse photon) $\Rightarrow \sigma_L=0$.

The Quarks within the Proton

Assuming that the proton structure is dominated by the light quarks u,d,s we can write the structure function in the following way:

$$\frac{1}{x}F_2^{ep}(x) = \left(\frac{2}{3}\right)^2\left[u^p(x)+\bar{u}^p(x)\right]+\left(\frac{1}{3}\right)^2\left[d^p(x)+\bar{d}^p(x)\right]+\left(\frac{1}{3}\right)^2\left[s^p(x)+\bar{s}^p(x)\right]$$

similarly for the neutron:

$$\frac{1}{x}F_2^{en}(x) = \left(\frac{2}{3}\right)^2\left[u^n(x)+\bar{u}^n(x)\right]+\left(\frac{1}{3}\right)^2\left[d^n(x)+\bar{d}^n(x)\right]+\left(\frac{1}{3}\right)^2\left[s^n(x)+\bar{s}^n(x)\right]$$

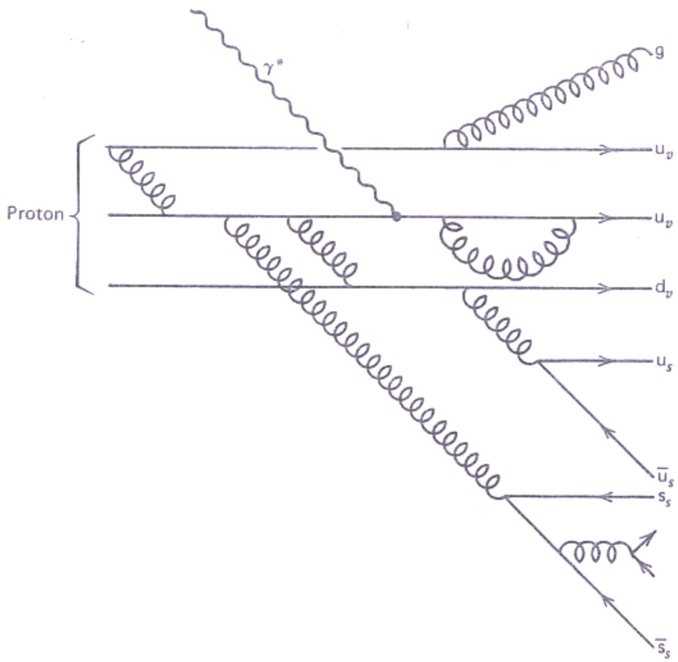
$$u^p(x) = d^n(x) \equiv u(x)$$

$$d^p(x) = u^n(x) \equiv d(x)$$

$$s^p(x) = s^n(x) \equiv s(x)$$

As proton and neutron are members of an isospin doublet their quark content is related. There are as many u quarks in a proton as d quarks in a neutron etc.

$$\underbrace{\frac{1}{4}}_{\text{only } u} \le \frac{F_2^{en}(x)}{F_2^{ep}(x)} \le \underbrace{4}_{\text{only } d}$$



We distinguish between valence and sea quarks:

$$u_s(x) = \bar{u}_s(x) = d_s(x) = \bar{d}_s(x) = s_s(x) = \bar{s}_s(x) = S(x)$$

$$u(x) = u_v(x) + u_s(x)$$

$$d(x) = d_v(x) + d_s(x)$$

$S(x)$ = sea quark distribution common to all quark flavors.

By summing over all contributing partons we must recover the quantum numbers of the proton: B=1, Q=1, S=0.

$$\int_0^1 \left[u(x) - \bar{u}(x)\right] dx = 2$$

$$\int_0^1 \left[d(x) - \bar{d}(x)\right] dx = 1 \qquad \Longleftrightarrow \qquad \int_0^1 \left[u_v(x) + d_v(x)\right] dx = 3$$

Gross Llewellyn-Smith

$$\int_0^1 \left[s(x) - \bar{s}(x)\right] dx = 0$$

where:

$$u - \bar{u} = u - \bar{u}_s = u - u_s = u_v$$

$$d - \bar{d} = d - \bar{d}_s = d - d_s = d_v$$

$$s - \bar{s} = s_s - \bar{s}_s = 0$$

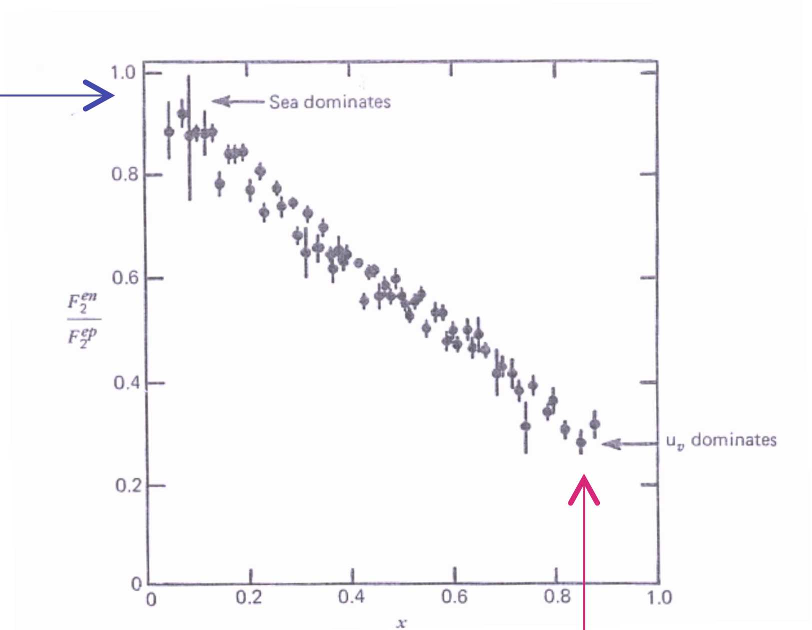
Combining these relations we obtain:

$$\frac{1}{x} F_2^{ep} = \frac{1}{9}\left[4u_v + d_v\right] + \frac{4}{3} S$$

$$\frac{1}{x} F_2^{en} = \frac{1}{9}\left[u_v + 4d_v\right] + \frac{4}{3} S$$

At small x values we expect that the sea dominates:

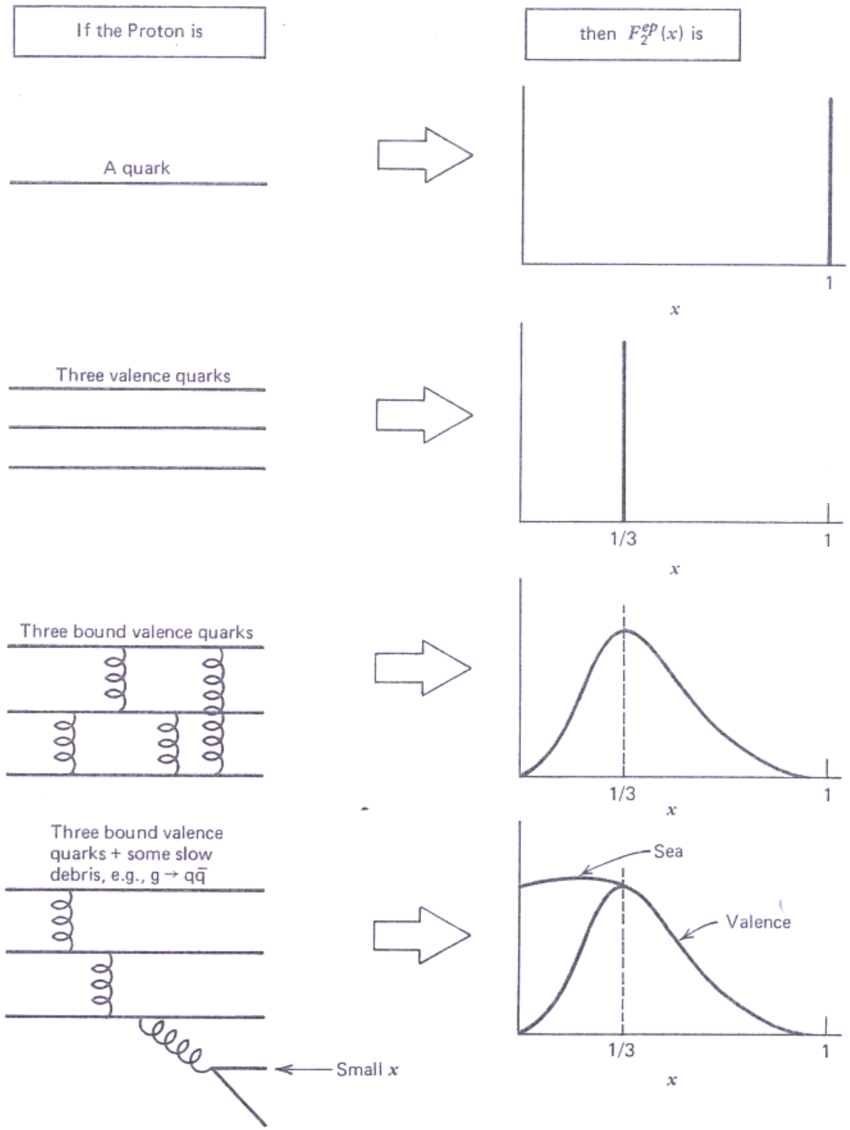
$$\boxed{\frac{F_2^{en}(x)}{F_2^{ep}(x)} \xrightarrow[x\to 0]{} 1}$$



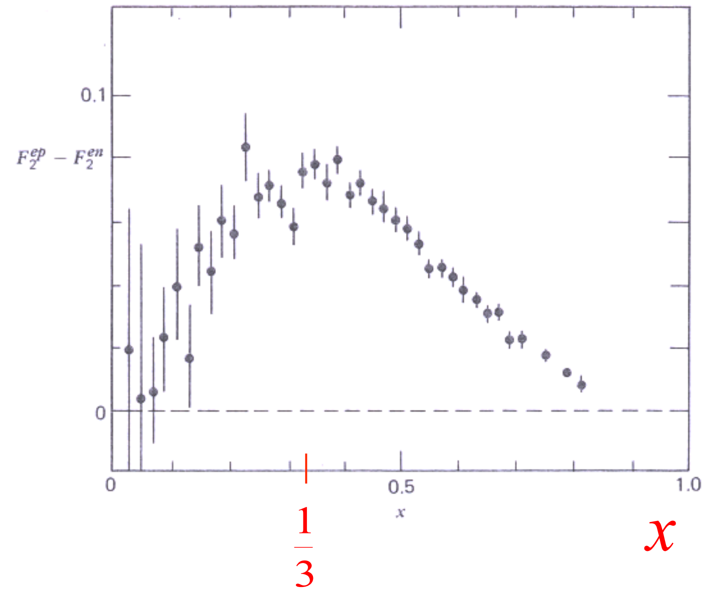
At high x values we expect that valence quarks dominate:

$$\boxed{\frac{F_2^{en}(x)}{F_2^{ep}(x)} \xrightarrow[x\to 1]{} \frac{u_v + 4d_v}{4u_v + d_v}}$$

The experimental data are consistent with this prediction, indicating $u_v >> d_v$ at high x so that the ratio $\rightarrow$ 0.25



$$\frac{1}{x}\left[F_2^{ep}(x) - F_2^{en}(x)\right] = \frac{1}{3}\left[u_v(x) - d_v(x)\right]$$



The Role of Gluons

By summing over all parton momenta we must recover the proton momentum

$$\int_0^1 dx(xp)\left[u+\bar{u}+d+\bar{d}+s+\bar{s}\right]=p-p_g$$

gluon momentum

Dividing by p:

$$\int_0^1 dxx\left[u+\bar{u}+d+\bar{d}+s+\bar{s}\right]=1-\varepsilon_g \qquad \varepsilon_g \equiv \frac{p_g}{p}$$

From experimental data:

$$\int_0^1 dxF_2^{ep}(x)=\frac{4}{9}\varepsilon_u+\frac{1}{9}\varepsilon_d=0.18 \qquad \varepsilon_u \equiv \int_0^1 dxx(u+\bar{u})$$

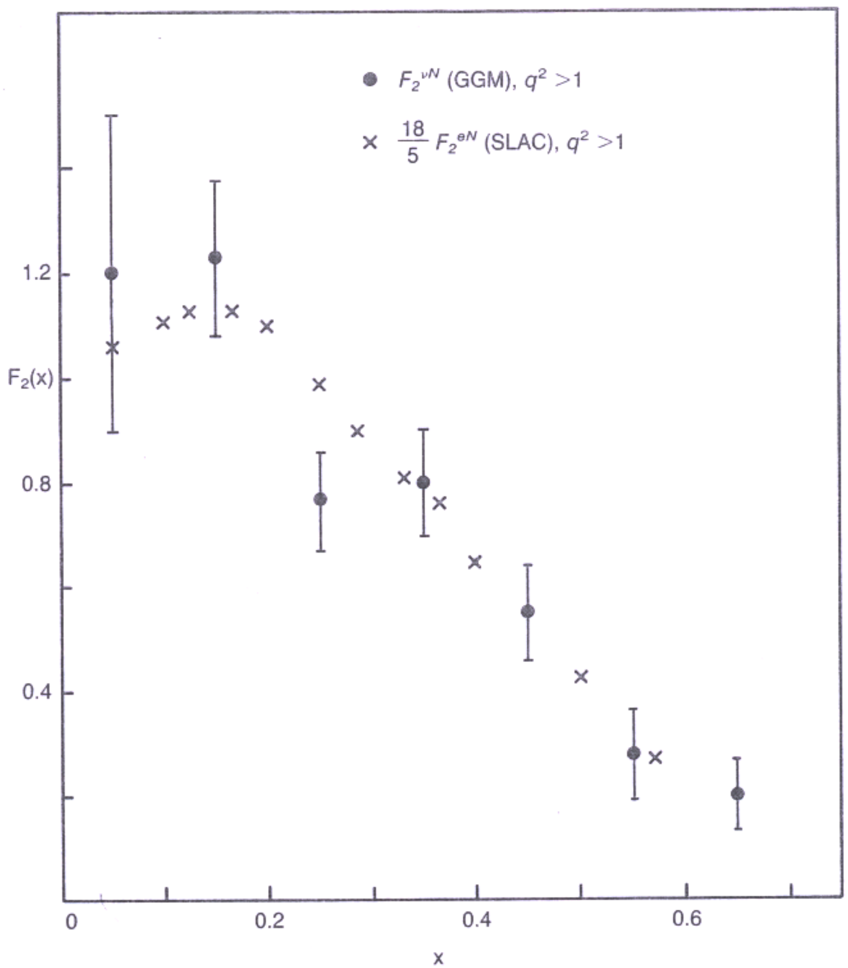
$$\int_0^1 dxF_2^{en}(x)=\frac{1}{9}\varepsilon_u+\frac{4}{9}\varepsilon_d=0.12 \qquad \varepsilon_d \equiv \int_0^1 dxx(d+\bar{d})$$

furthermore $\varepsilon_g=1-\varepsilon_u-\varepsilon_d$

$$\varepsilon_u=0.36 \qquad \varepsilon_d=0.18 \qquad \varepsilon_g=0.46$$

Gluons carry approximately 50 % of the proton momentum

νN Inelasting Scattering



$$\frac{4\pi\alpha^2}{q^4} \rightarrow \frac{G^2}{2\pi}$$

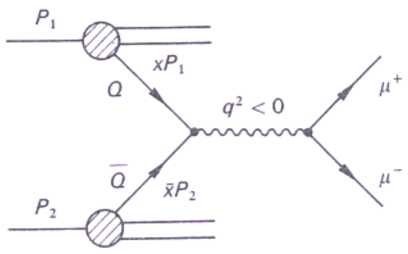
We need to introduce a third structure function $F_3(x)$, to account for parity non-conservation in the weak interaction.

$$F_2^{\nu N}(x) \leq \frac{18}{5} F_2^{eN}(x)$$

Experimental data confirm that neutrinos and electrons see the sam structure in the nucleon and that partons have the same fractional charges as quarks.

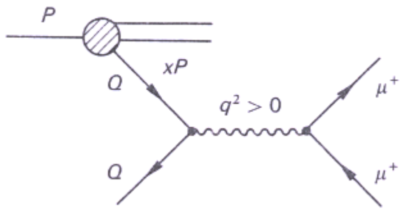
Drell-Yan

$$p + p \rightarrow \mu^+ + \mu^- + X$$



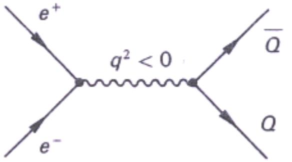
(a)

$$\mu + N \rightarrow \mu + X$$



(b)

$$e^+ + e^- \rightarrow Q + \overline{Q}$$



(c)

The cross section for the Drell-Yan process can be calculated starting from $q\bar{q} \to \mu^+\mu^-$

$$\hat{\sigma}\left(q\bar{q} \to \mu^+\mu^-\right) = \frac{4\pi\alpha^2}{3q^2} e_q^2$$

$$\frac{d\hat{\sigma}}{dq^2}\left(q\bar{q} \to \mu^+\mu^-\right) = \frac{4\pi\alpha^2}{3q^2} e_q^2 \delta\left(q^2 - m^2\right)$$

m^2 is the invariant mass of $\mu^+\mu^-$ (or $q\bar{q}$)

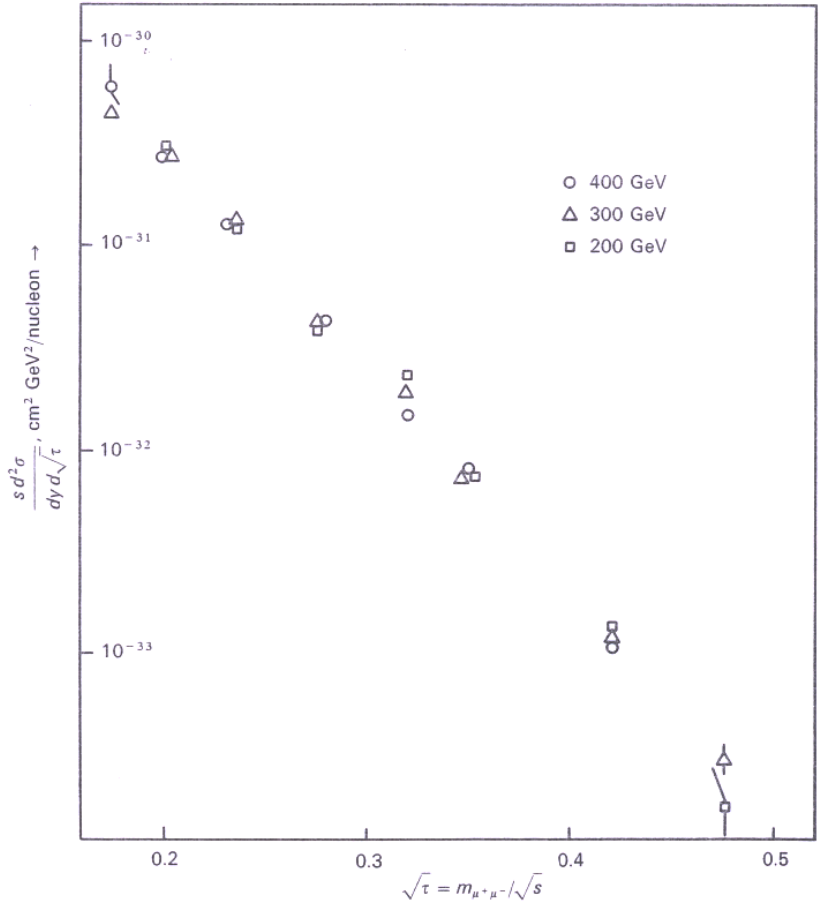
$$m^2 = \left(xP_1 + yP_2\right)^2 = x^2P_1^2 + y^2P_2^2 + 2xyP_1P_2 = \left(x^2 + y^2\right)M^2 + 2xyP_1P_2 \approx 2xyP_1P_2$$

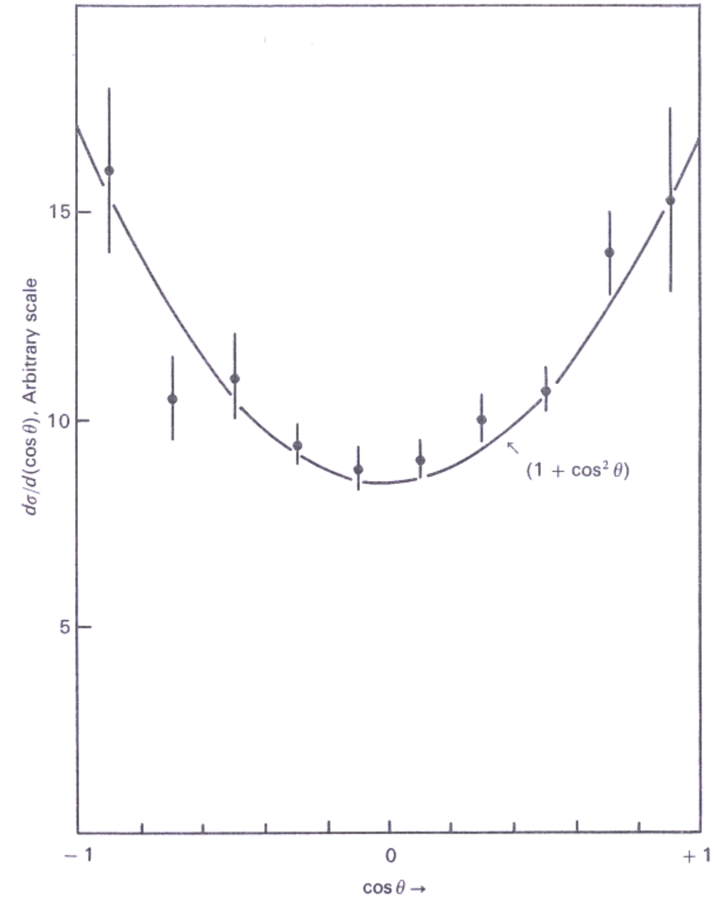
$$s = \left(P_1 + P_2\right)^2 = P_1^2 + P_2^2 + 2P_1P_2 \approx 2P_1P_2 \qquad \boxed{m^2 = xys}$$

Summing over the quarks in the proton:

$$\frac{d\sigma}{dq^2}\left(pp \to \mu^+\mu^- X\right) = 3\sum_q e_q^2 \int_0^1 dx \int_0^1 dy f_q(x) f_{\bar{q}}(y) \frac{d\hat{\sigma}}{dq^2}$$

$$\frac{d\sigma}{dq^2}\left(pp \to \mu^+\mu^- X\right) = \frac{4\pi\alpha^2}{q^4} \sum_q e_q^2 \int_0^1 dx \int_0^1 dy f_q(x) f_{\bar{q}}(y) \delta\left(1 - xy\frac{s}{q^2}\right)$$





The Drell-Yan cross section depends both on q^2 and on s, but the quantity:

$$q^4 \frac{d\sigma}{dq^2} = F(\tau) \qquad \tau = \frac{q^2}{s}$$

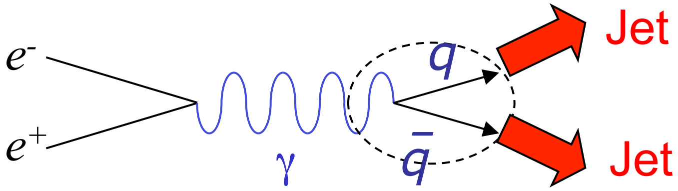
depends only on the ratio q^2/s. Experimental measurements are consistent with this scaling law.

The angular distribution of the leptons (in their center-of-mass frame) has the form expected for $q\bar{q} \to \mu^+\mu^-$

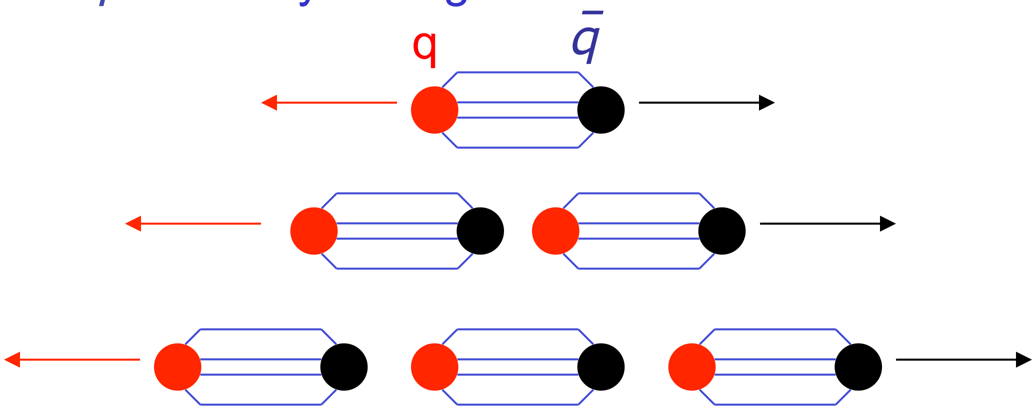
$$\frac{d\sigma}{d\Omega} \propto 1 + \cos^2\theta$$

Jet Formation in e^+e^-

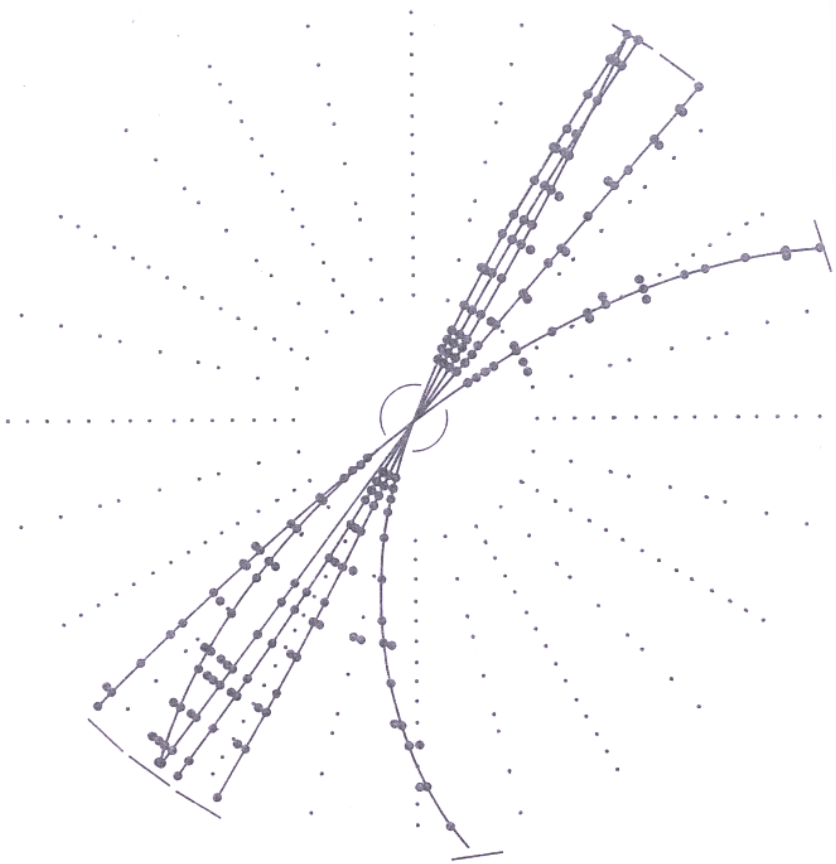
In the process $e^+e^- \rightarrow$ hadrons, final state hadrons are not distributed uniformly and isotropically, but they are collimated into hadron *streams* called **jets**.

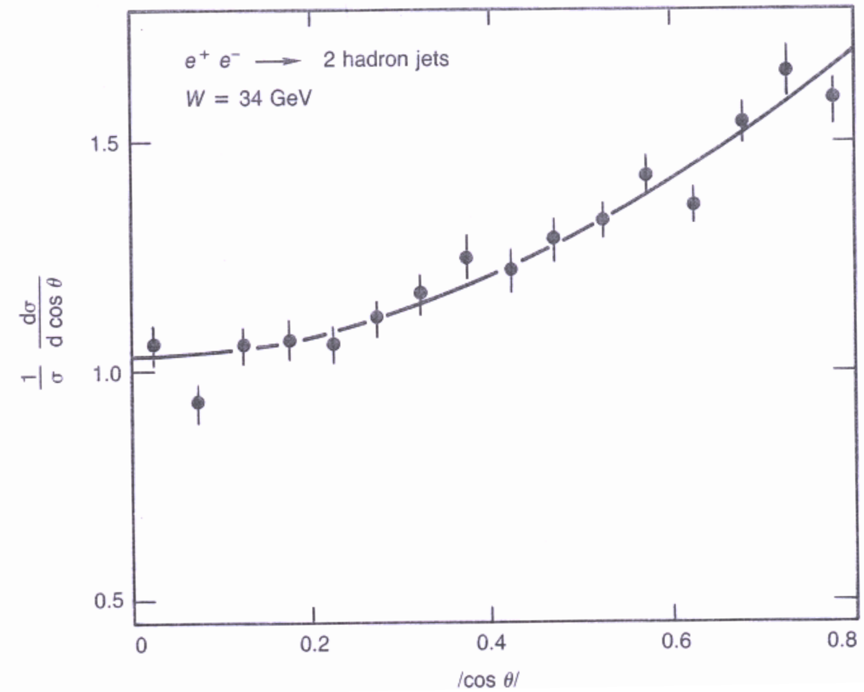


When the $q\bar{q}$ separates the color interaction becomes extremely strong (confinament) so that the quark and the antiquark get violently decelerated and radiate hadrons (Bremsstrahlung). These hadron jets are emitted in the direction of the q and of the $\bar{q}$ and they emerge "back-to-back".



Jet formation in e^+e^-
TASSO experiment at PETRA

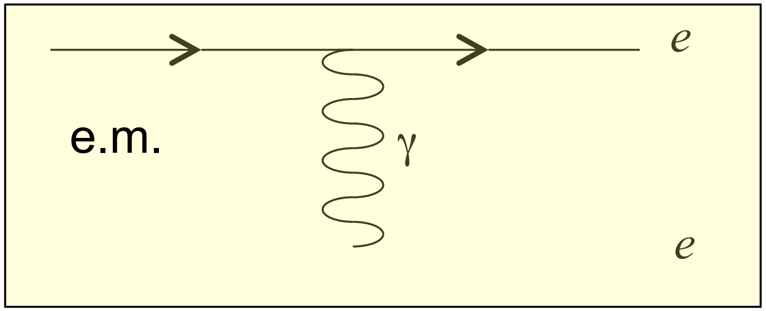


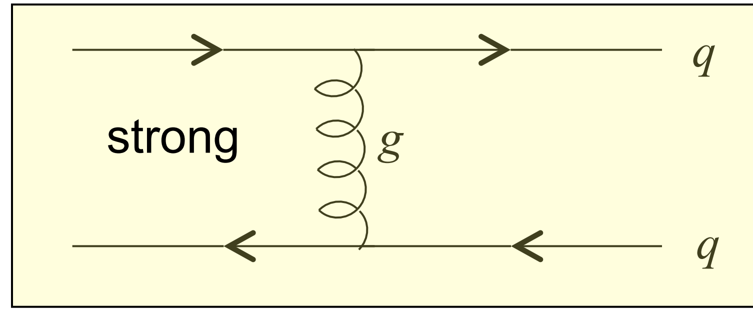


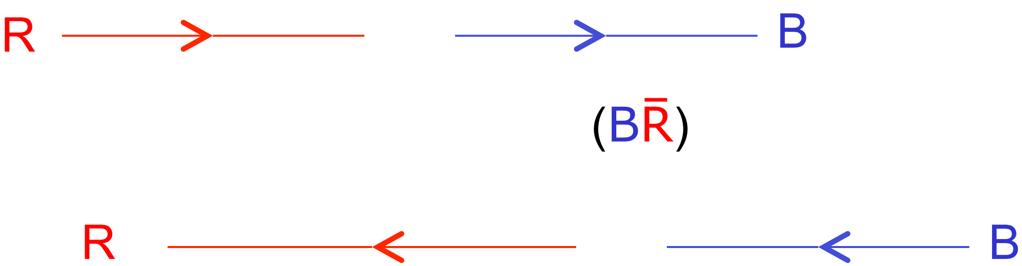
Angular distribution of two hadron jets, consistent with the form $(1+cos^2\theta)$ expected for the process $e^+e^- \rightarrow q\bar{q}$

Quantum ChromoDynamics (QCD)

The charge of the strong interaction is called color. The quanta of the strong field are called gluons.







Quarks interact strongly by color exchange. The gluon itself has a color charge, it is a bi-colored object (e.g.: $B\bar{R}$)

In analogy to QED the quantum field theory of the strong interaction is called Cromodinamica Quantistica (QCD).

Out of nine possible combinations, $\bar{R}R, \bar{G}R, \bar{B}R, \bar{R}G, \bar{G}G, \bar{B}G, \bar{R}B, \bar{G}B, \bar{B}B$, one is a color singlet ($R\bar{R}+B\bar{B}+G\bar{G}$). We are left with eight gluons, representing the quanta of the color field.

The color charge of gluons implies the existence of a direct coupling between gluons, which has no analogy in QED.

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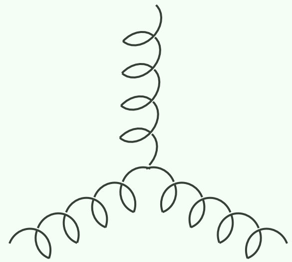
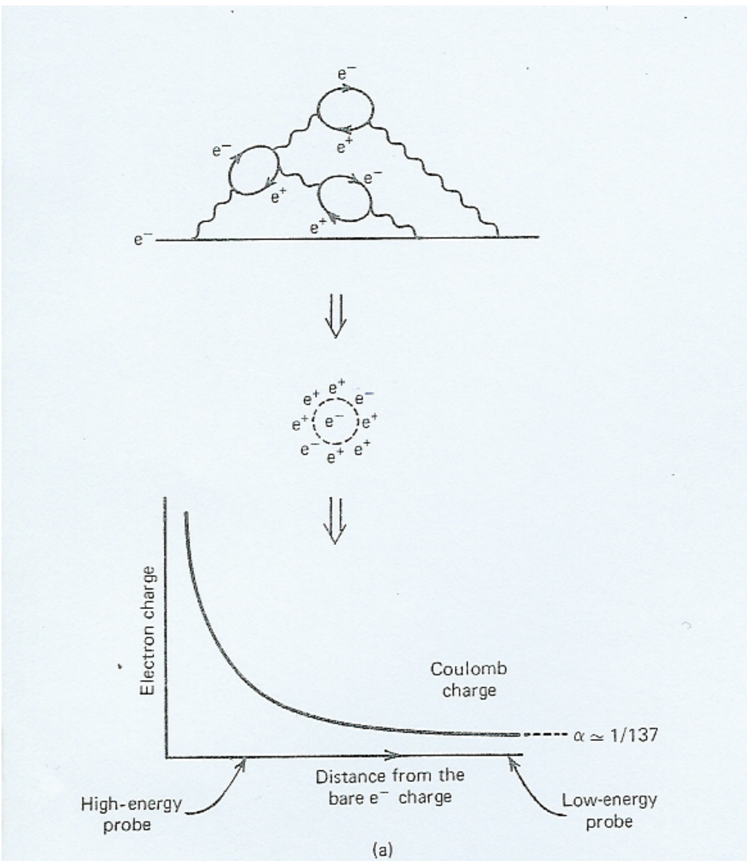
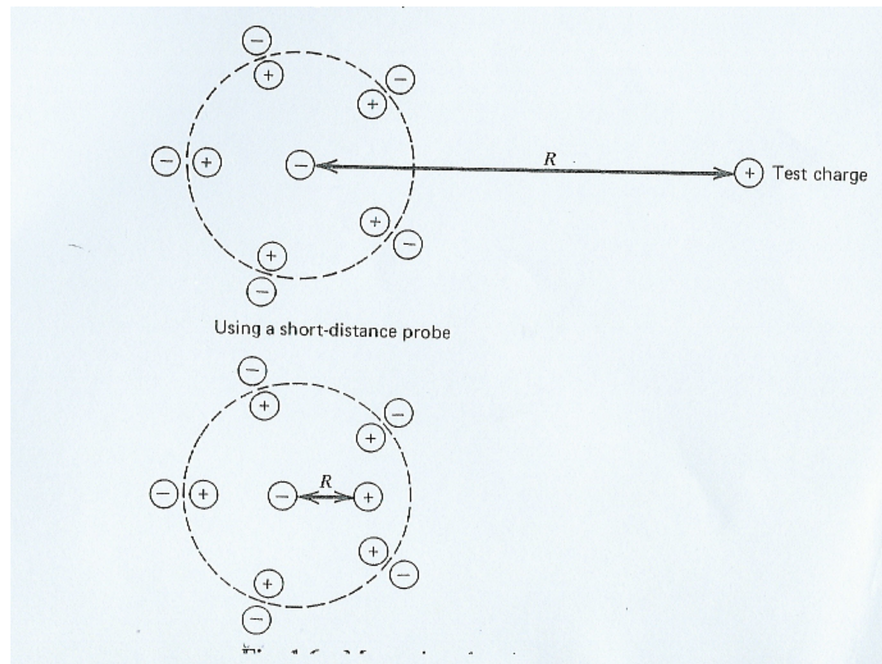


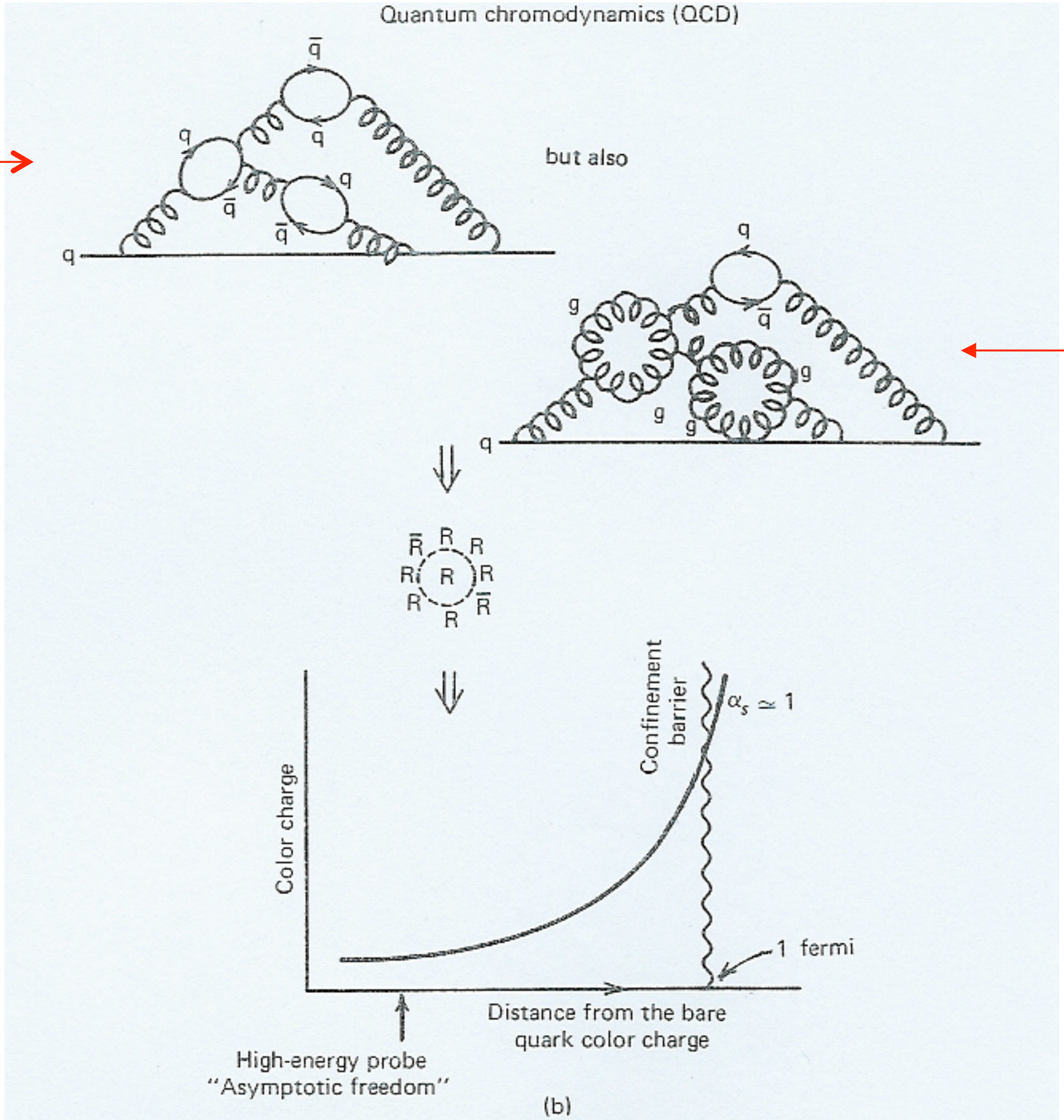
This fact has very important implications on charge screening in QED and QCD and on the asymptotic behavior of the two interactions.





Charge screening in QCD and running of α_S

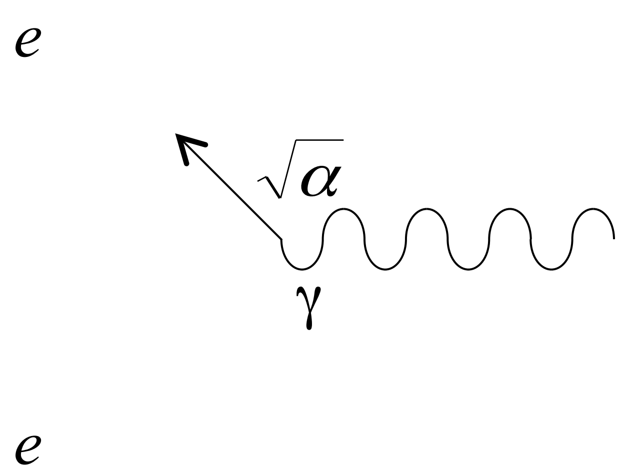
QED analog

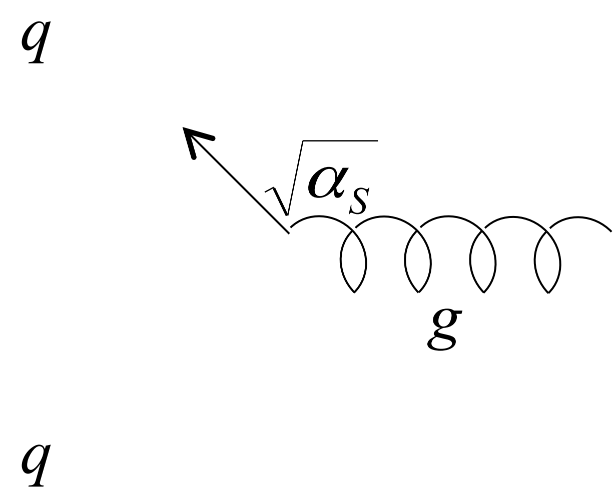


gluon self coupling in QCD

The fundamental properties of QCD can be summarized in the following points:

- quarks have a color charge (in addition to electric charge) which can take on 6 values: R, G, B, $\bar{R}$, $\bar{B}$, $\bar{G}$.
- color is exchanged by 8 bicolored gluons.
- quark-quark interactions can be computed using rules similar to QED, with the introduction of a color factor and the replacemente $\sqrt{\alpha} \rightarrow \sqrt{\alpha_s}$ at each vertex.





- gluons have a color charge and they can interact directly with one another.
- At small distances α_S is sufficiently small to make the perturbative approach possible (asymptotic freedom).