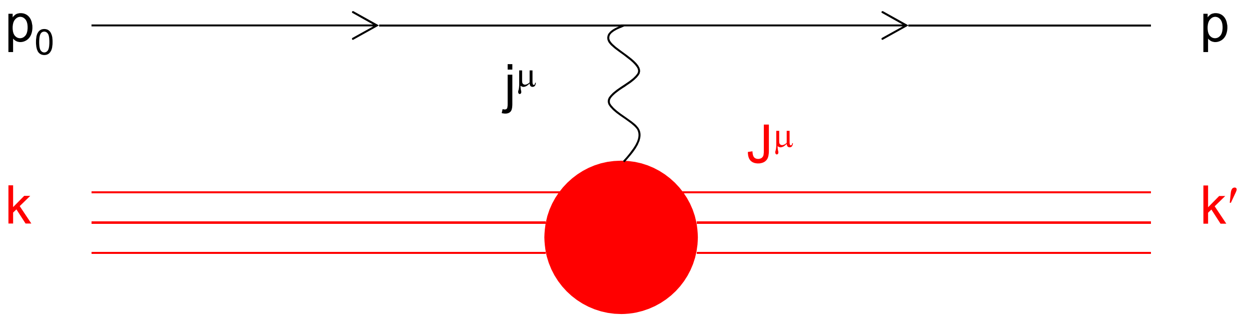
Form Factors

Diego Bettoni
Istituto Nazionale di Fisica Nucleare, Ferrara

Introduction



$$J^{\mu} = e\left[F_1\left(q^2\right)\gamma^{\mu} + \frac{\kappa}{2M} F_2\left(q^2\right) i\sigma^{\mu\nu} q_{\nu} \right]$$

$$F_1^p(0) = 1 \quad F_2^p(0) = 1$$

$$F_1^n(0) = 0 \quad F_2^n(0) = 1$$

Dirac and Pauli
Form Factors

Sachs Form Factors

$$G_E \equiv F_1 + \frac{\kappa q^2}{4M^2} F_2$$
$$G_M \equiv F_1 + \kappa F_2$$

- G_E and G_M are Fourier transforms of nucleon charge and magnetization density distributions (in the Breit Frame).

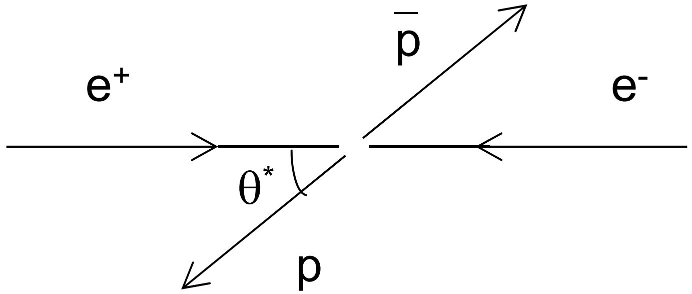
- Spacelike form factors are real, timelike are complex.

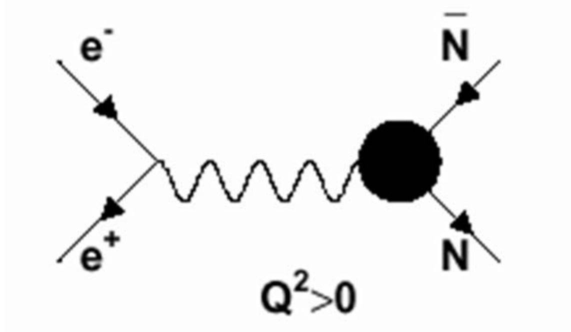
- The analytic structure of the timelike form factors is connected by dispersion relations to the spacelike regime.

- By definition they do not interfere in the expression of the cross section, therefore, in the timelike case, only polarization observables allow to get the relative phase.

Time-like Form Factors

$$e^+ + e^- \rightarrow \bar{N} + N$$





$s = Q^2 > 0$

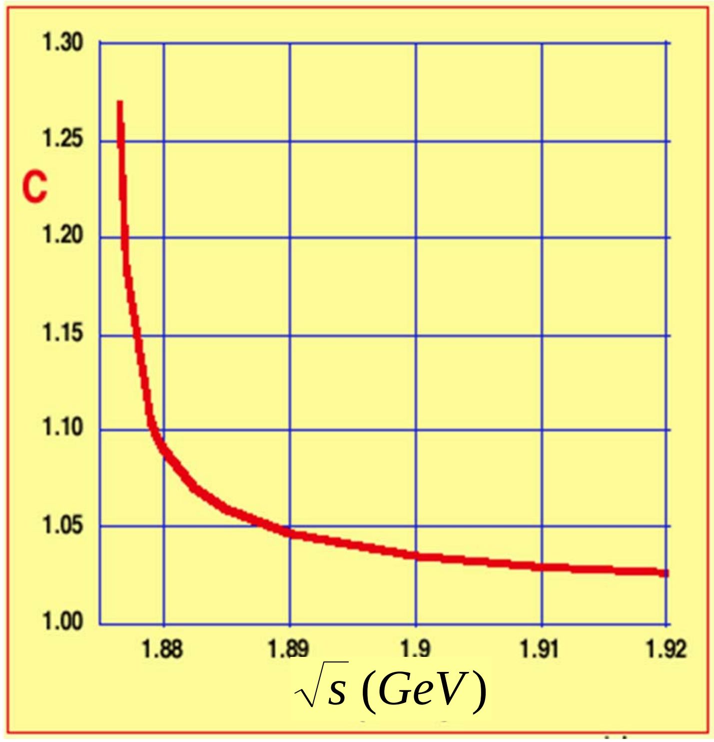
$$\sigma = \frac{4\alpha^2 \pi \beta C}{3s}\left[|G_M(s)|^2 + \frac{2m_N^2}{s}|G_E(s)|^2 \right]$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4s}\left[|G_M(s)|^2 (1+\cos^2\theta^*) + \frac{4m_N^2}{s}|G_E(s)|^2 \sin^2\theta^* \right]$$

C is the Coulomb correction factor, taking into account the QED coulomb interaction. Important at threshold.

$$\boxed{C = \frac{1}{1-e^{-y}}}$$

$$y = \frac{2\pi\alpha M_N}{\beta\sqrt{s}}$$



$$C \xrightarrow[s\to 4M_N^2]{} \frac{1}{\beta}$$ ⇨ σ finite

$$\boxed{\sigma\left(4M_N^2\right) = \frac{\pi^2\alpha^3}{4M_N^2}\left|G_E(4M_N^2)\right|^2 \approx 0.1\,nb}$$

There is no Coulomb correction in the neutron case.

Form Factor Properties

- At threshold $G_E=G_M$ by definition, if F_1 and F_2 are analytic functions with a continuous behaviour through threshold.

$$G_E(4m_p^2) = G_M(4m_p^2)$$

- Timelike G_E and G_M are the analytical continuation of non spin flip and, respectively, spin flip spacelike form factors. Since timelike form factors are complex functions, this continuity requirement imposes theoretical constraints.
- Two-photon contribution can be measured from asymmetry in angular distribution.

Form Factor Properties

- Perturbative QCD and analyticity relate timelike and spacelike form factors, predicting a continuous transition and spacelike-timelike equalitity at high Q^2.
- At high Q^2 PQCD predicts:

$$F_1(Q^2) \propto \frac{\alpha_s^2(Q^2)}{Q^4} \qquad F_2(Q^2) \propto \frac{\alpha_s^2(Q^2)}{Q^6}$$

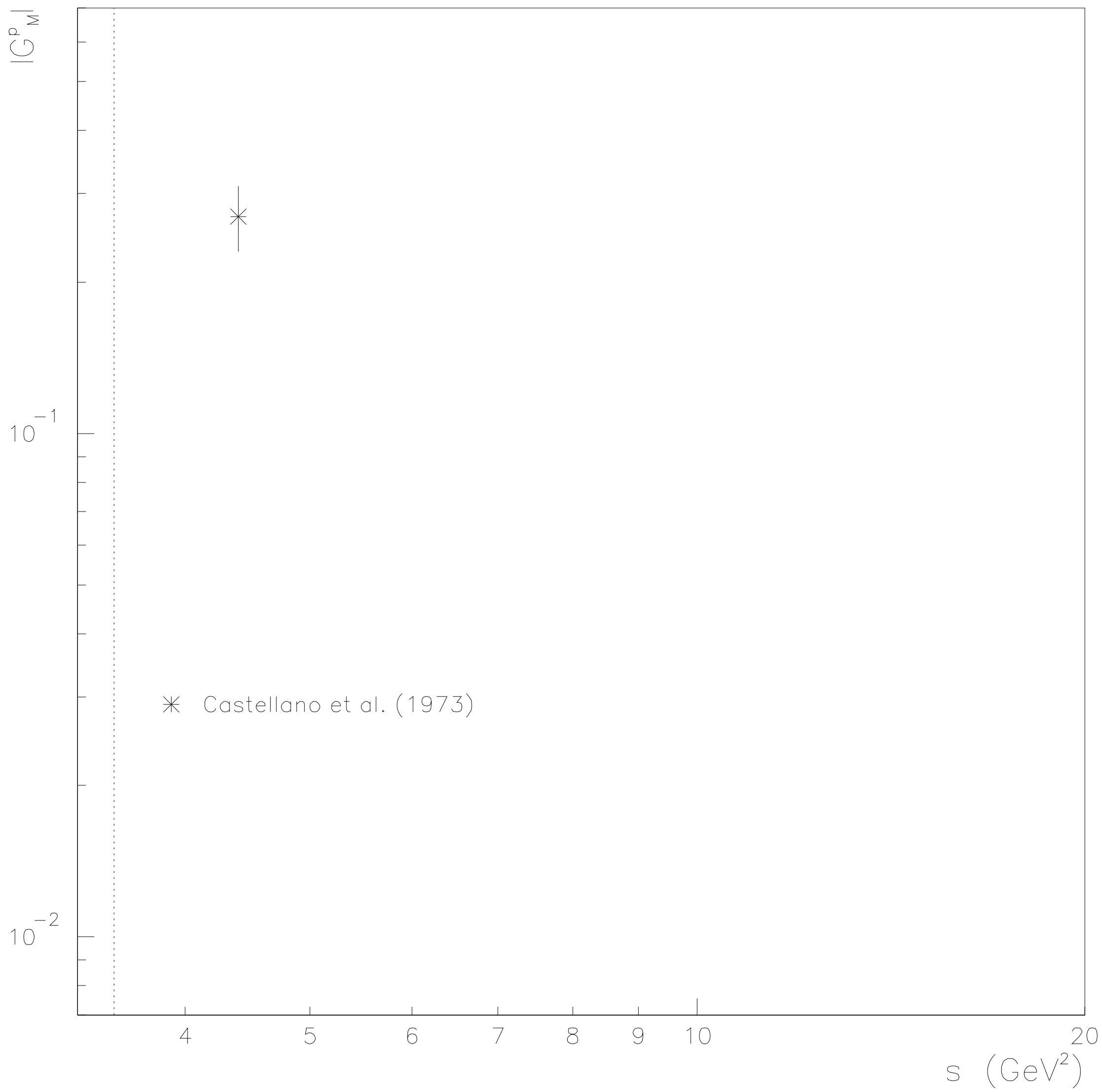
- Naïve prediction for the neutron:

$$\left|\frac{G_M^n}{G_M^p}\right|^2 \approx \left(\frac{q_d}{q_u}\right)^2 = 0.25$$

Proton Form Factors

- The moduli of the Form Factors can be derived from measurements of the cross sections for $e^+e^- \leftrightarrow \bar{p}p$
- Due to the low value of the cross sections and the consequent limited statistics, most experiments could not determine $|G_M|$ and $|G_E|$ separately from the analysis of the angular distributions, but extracted $|G_M|$ using the (arbitrary) assumption $|G_E| = |G_M|$.
- The magnetic form factor has been derived in this way by many e^+e^- and $\bar{p}p$ experiments. The timelike electric form factor is basically unknown.
- Recently BaBar has attempted to measure $|G_M|/ |G_E|$ by means of ISR, but the final result is quoted using $|G_E| = |G_M|$.

Proton Magnetic Form Factor $|G_M|$

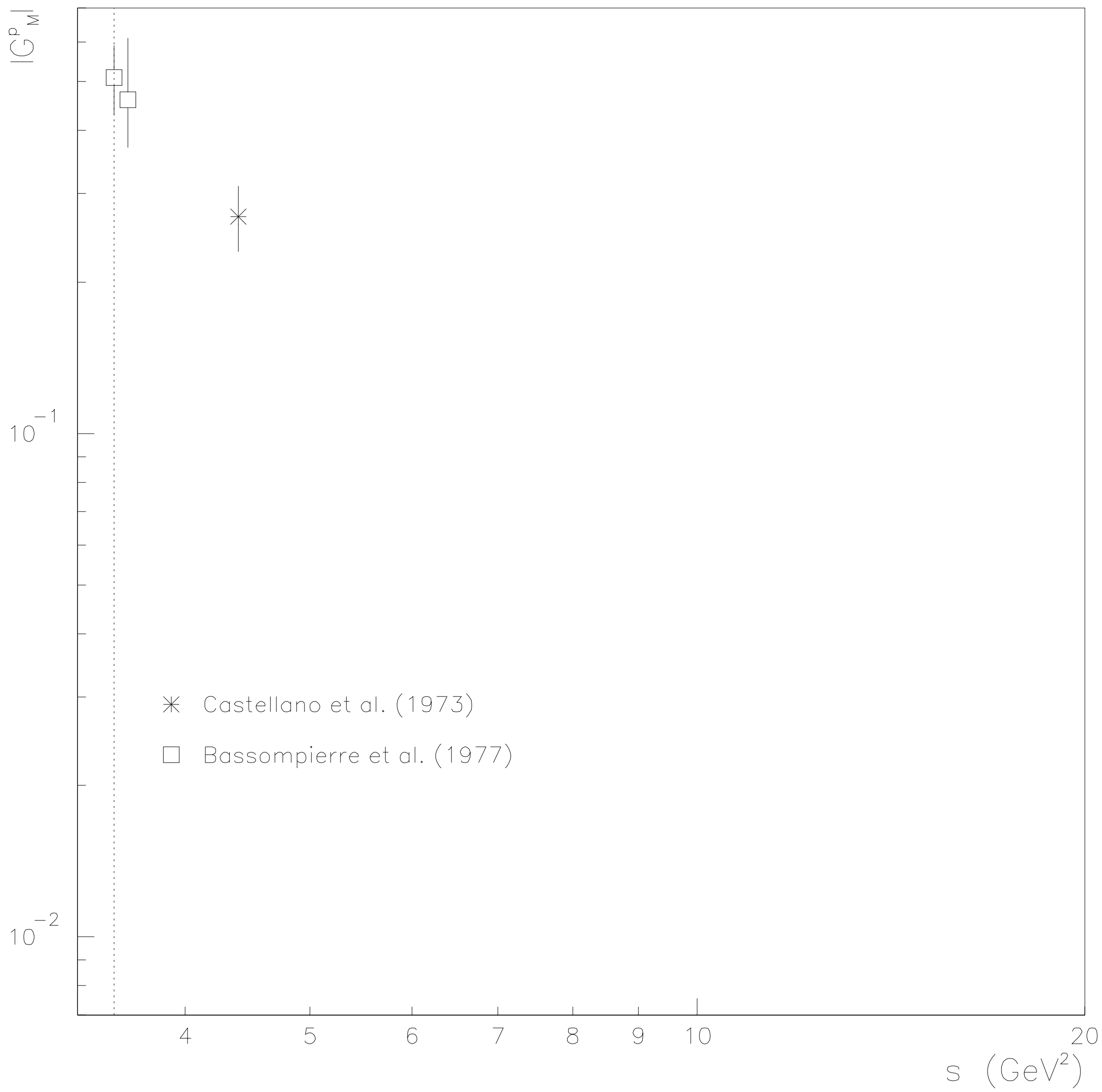


The first experiment to produce a positive result for the proton timelike form factor was carried out at ADONE in Frascati

$e^+e^- \rightarrow \bar{p}p$

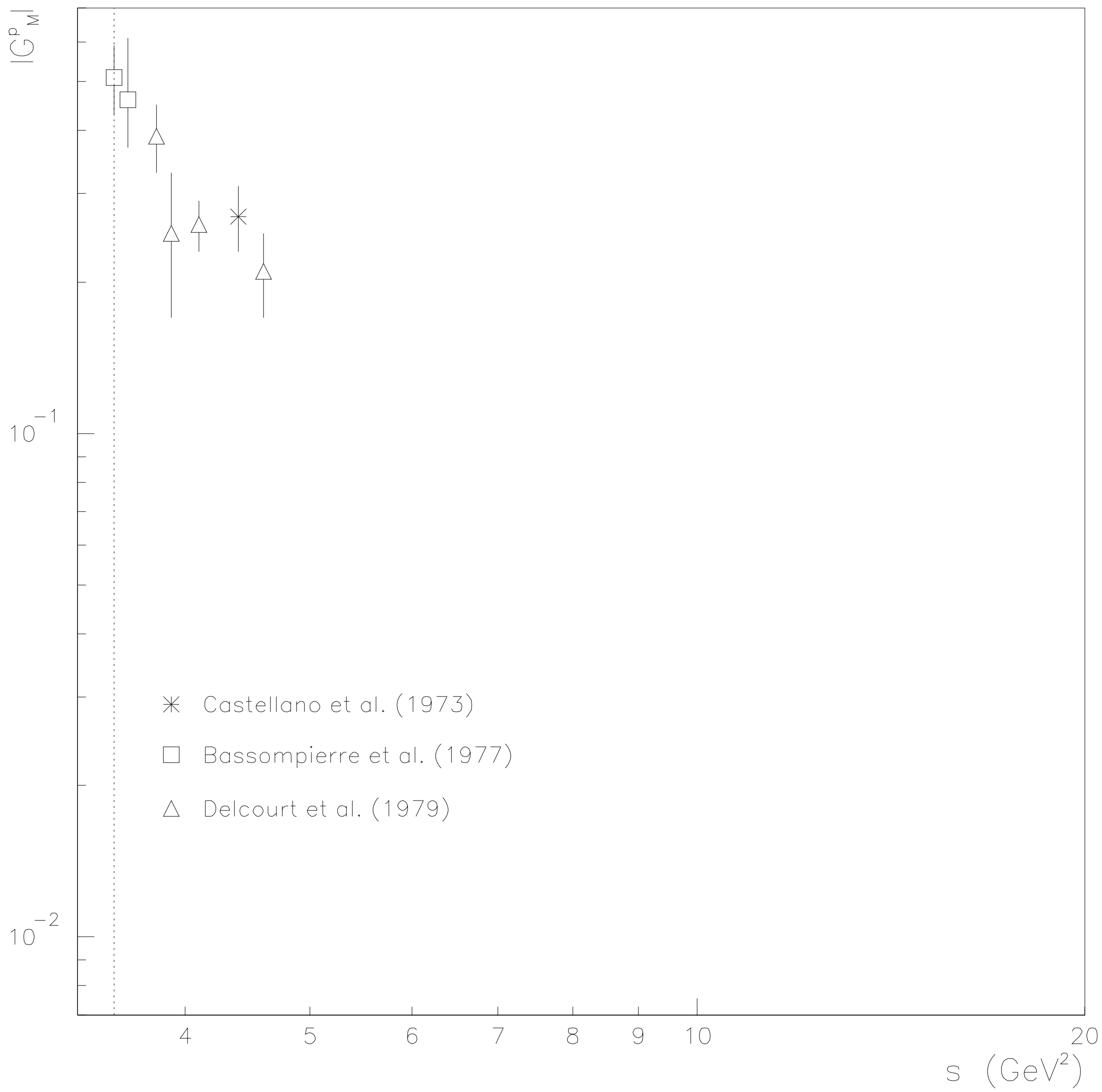
The measurement was based on 0.2 pb^{-1} of data at 4.4 GeV^2 yielding 25 events.

Proton Magnetic Form Factor $|G_M|$



The first measurement of the timelike form factors at threshold is due to the ELPAR experiment at CERN. They observed 34 events of $\bar{p}p$ annihilation at rest in a liquid H_2 target.
The measurement assumes $|G_E|=|G_M|$

Proton Magnetic Form Factor $|G_M|$

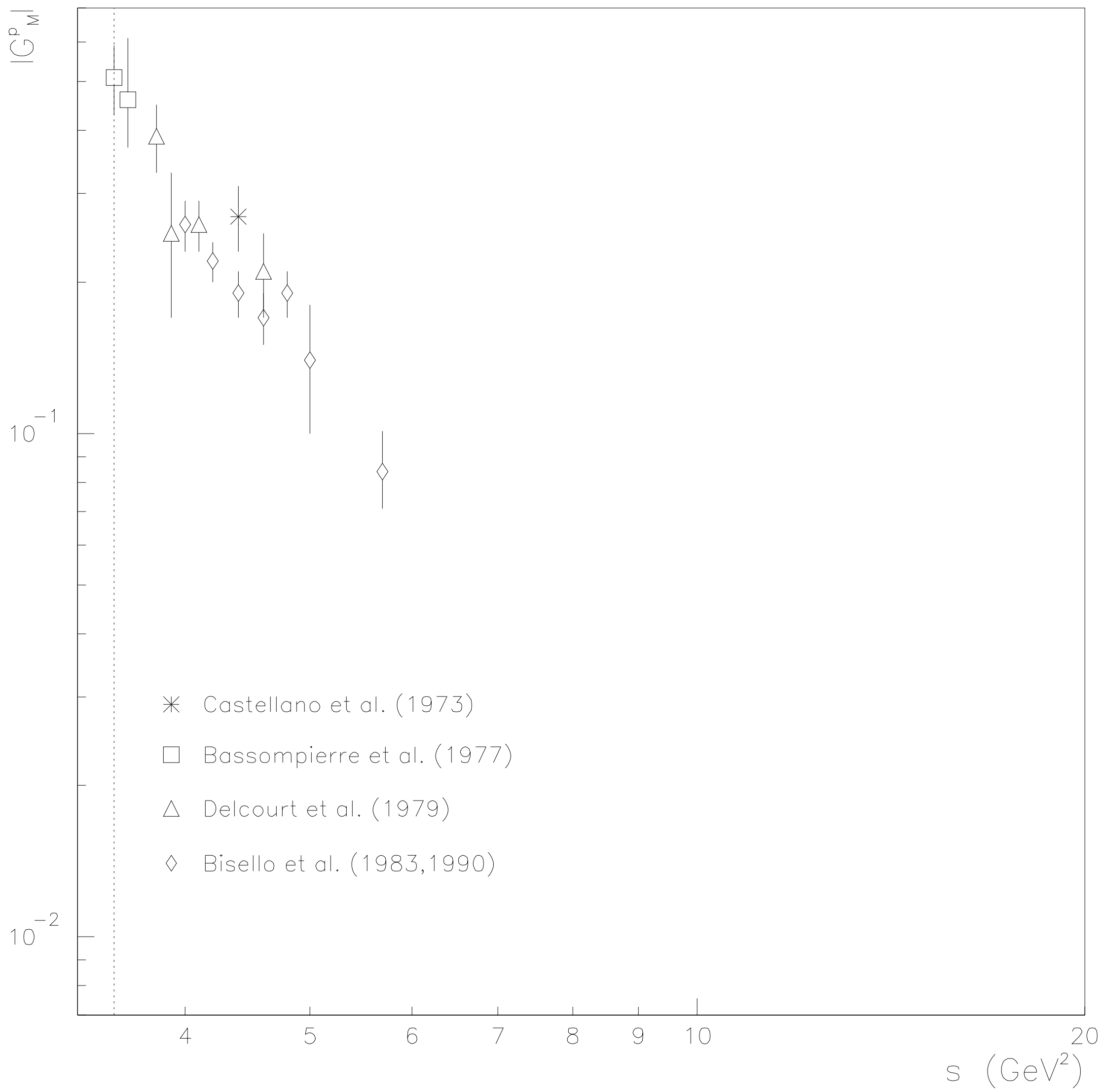


Various measurements of the proton form factors were carried out at DCI in Orsay using

$e^+e^- \rightarrow \bar{p}p$

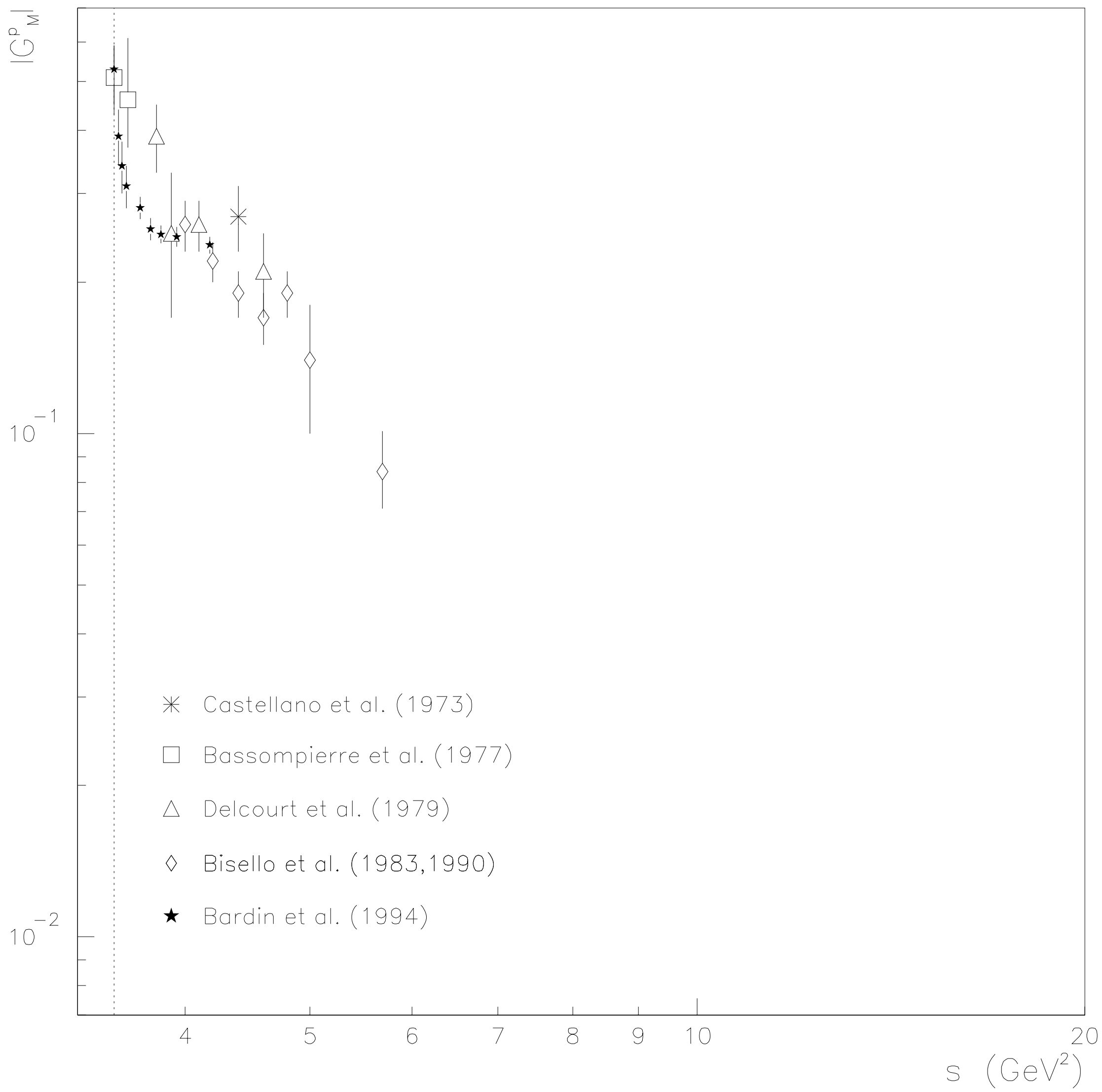
The first experiment was DM1 which recorded 63 events in 4 data points.

Proton Magnetic Form Factor $|G_M|$



At DCI in ORSAY
the DM2 collected data
in three data taking runs
for a total of 0.7 pb^{-1}.
With a total of 112 events
in 6 points they attempted
to measure the angular
distribution, from which they
could fit $|G_M|/|G_E|=0.34$,
but $|G_E|=|G_M|$ was still allowed.

Proton Magnetic Form Factor $|G_M|$



The first high-statistics measurement of the timelike form factors was carried out at LEAR by the PS 170 collaboration. They recorded a total of 3667

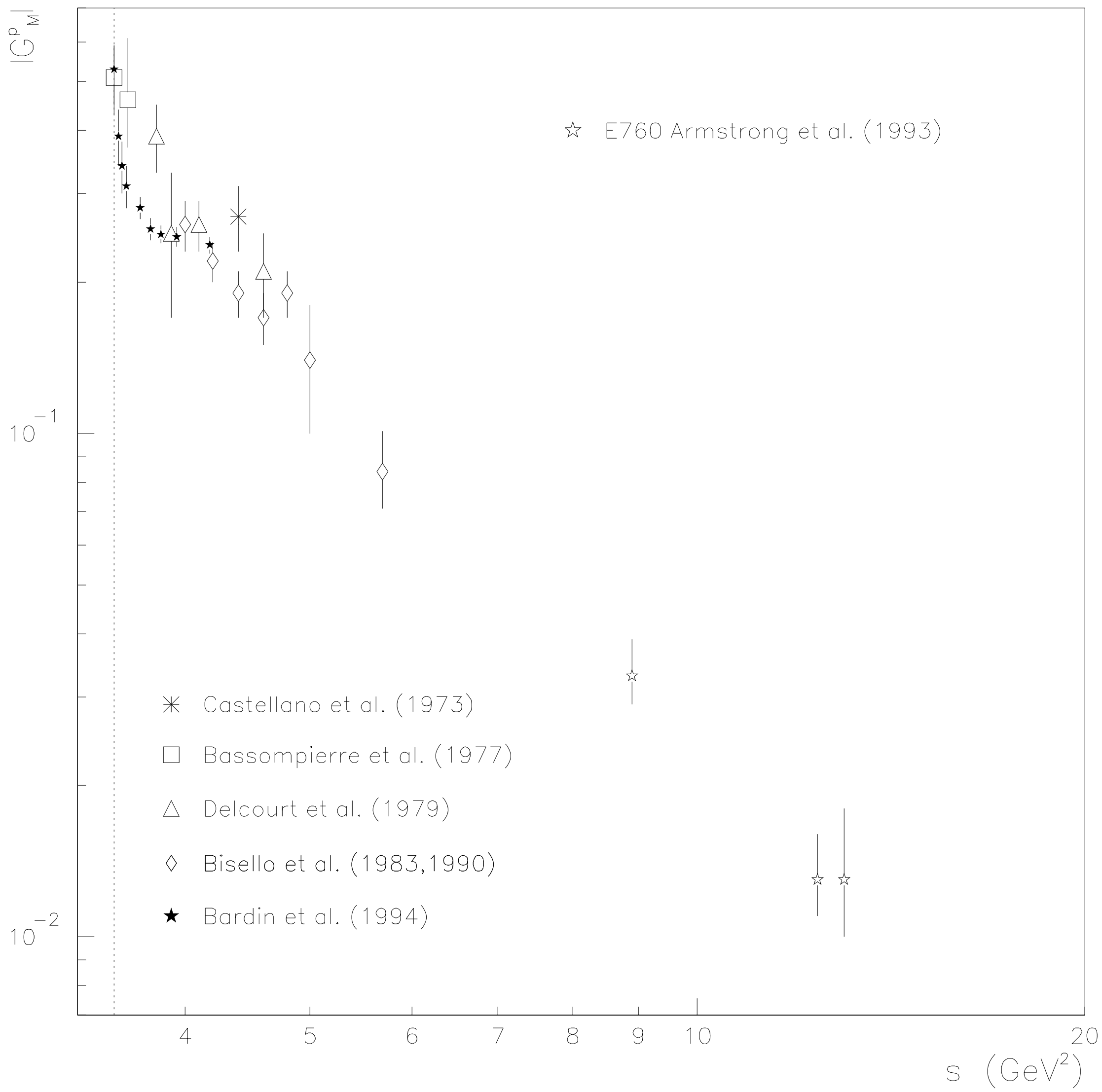
$\bar{p}p \rightarrow e^+e^-$

events in 9 data points.

The angular distribution is compatible with $|G_E|=|G_M|$.

First indication of steep rise near threshold.

Proton Magnetic Form Factor $|G_M|$

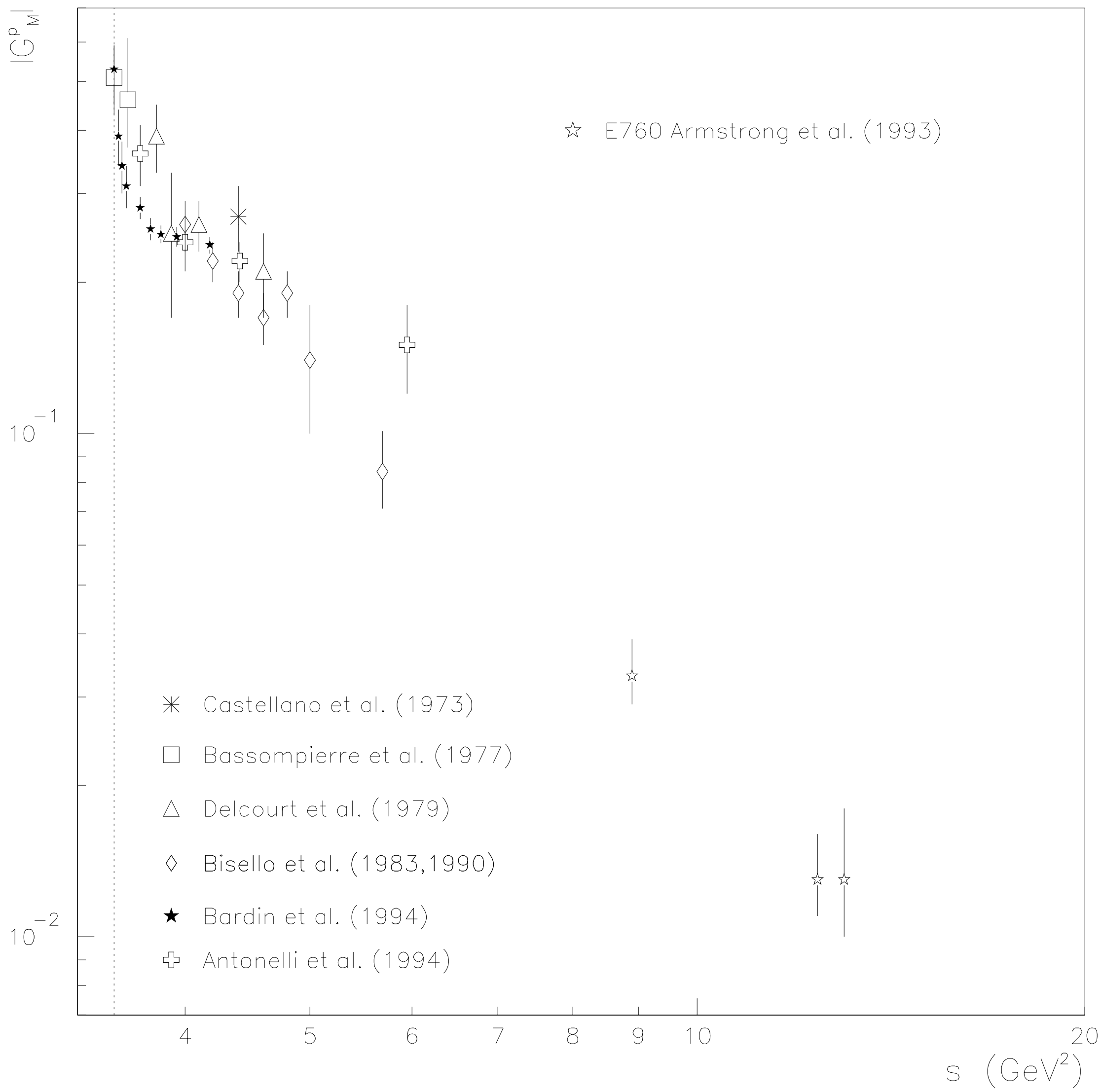


The E760 experiment at Fermilab produced the first measurement of the form factors at high Q^2

$\bar{p}p \rightarrow e^+e^-$

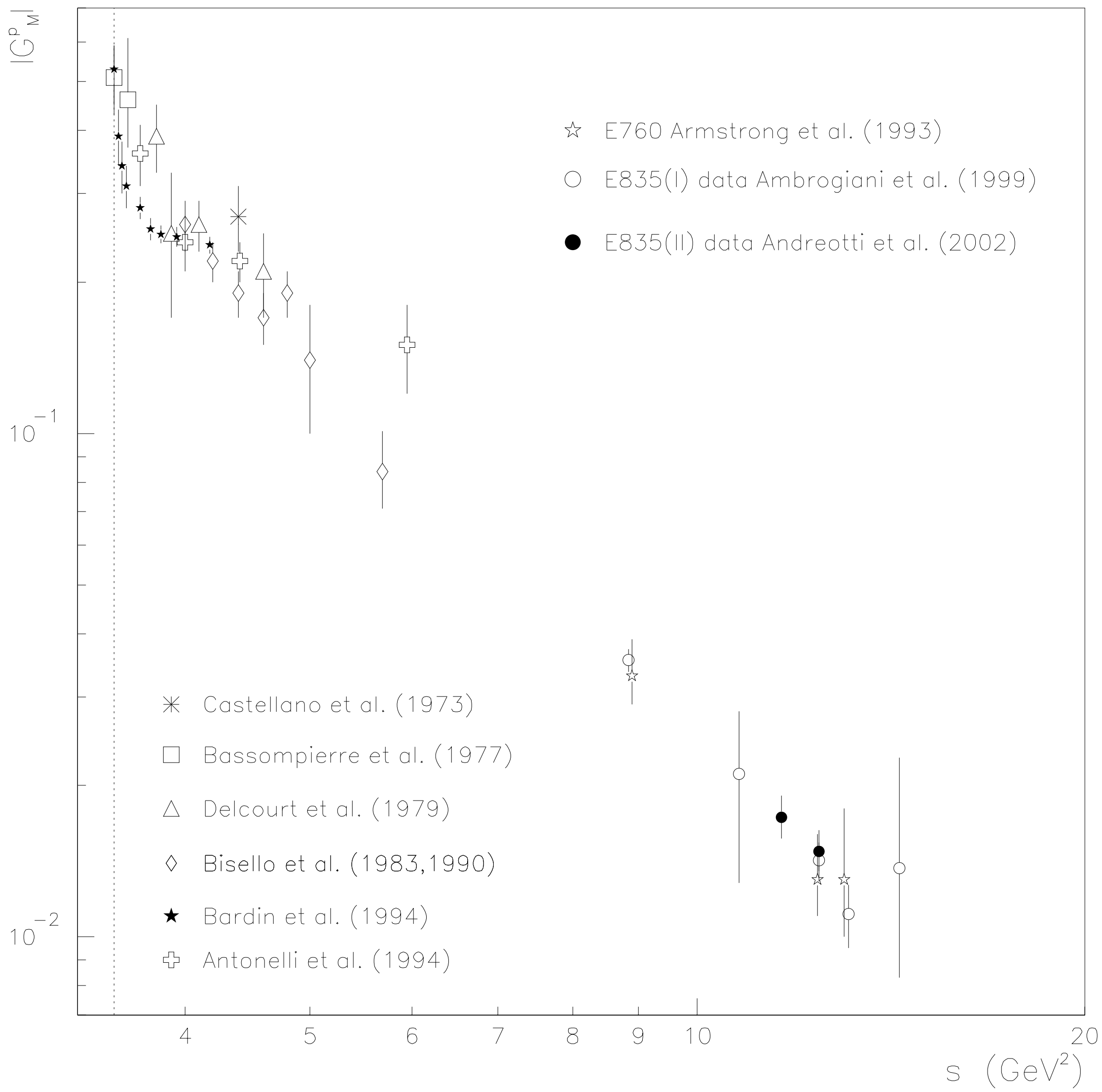
Very difficult measurement due to very small cross section. They recorded 29 events. The measurement assumes $|G_E|=|G_M|$.

Proton Magnetic Form Factor $|G_M|$



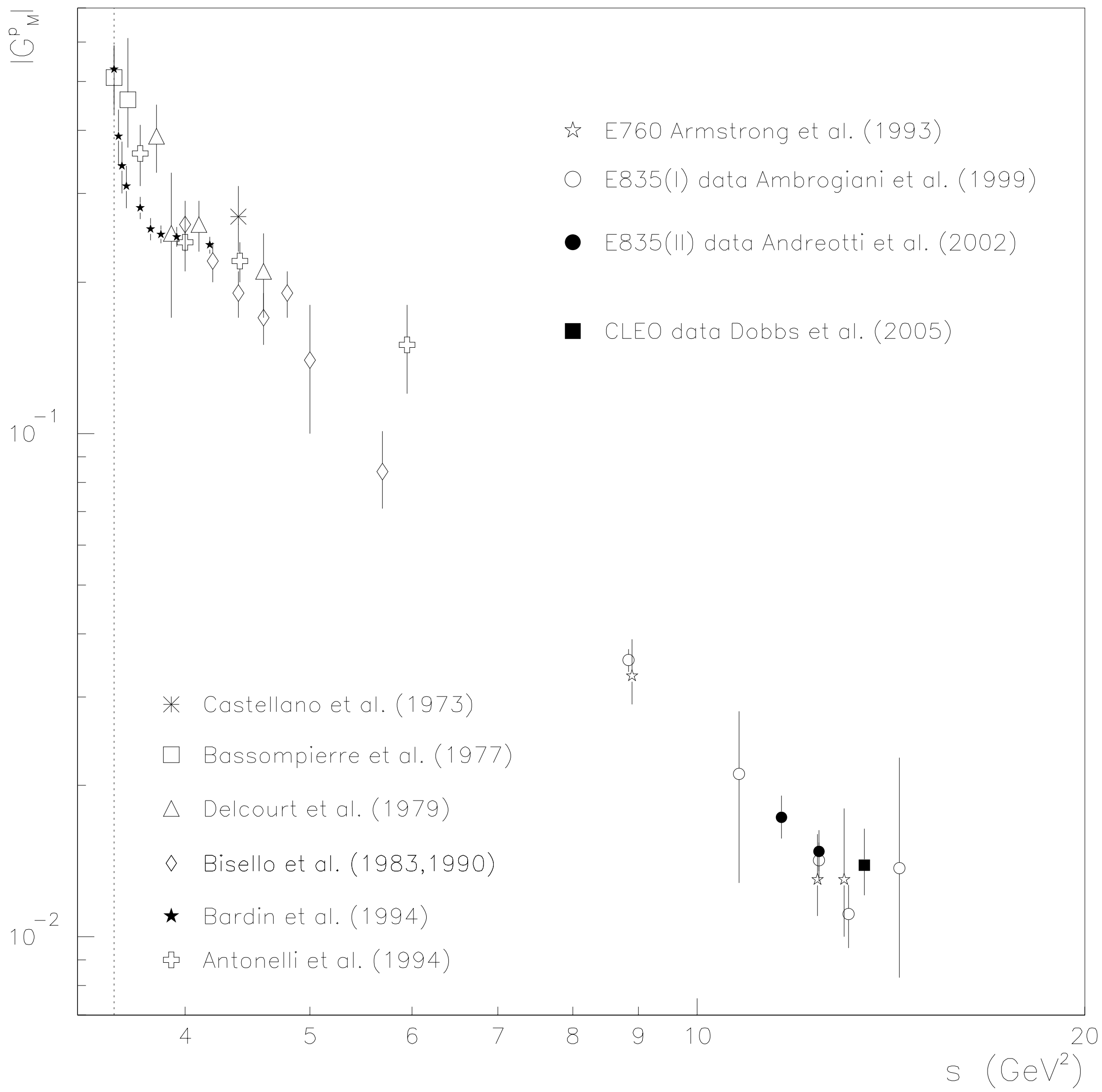
The FENICE experiment at ADONE, primarily devoted to the measurement of the neutron form factor, produced also a measurement of the proton magnetic form factor with 69 events in 4 points.

Proton Magnetic Form Factor $|G_M|$



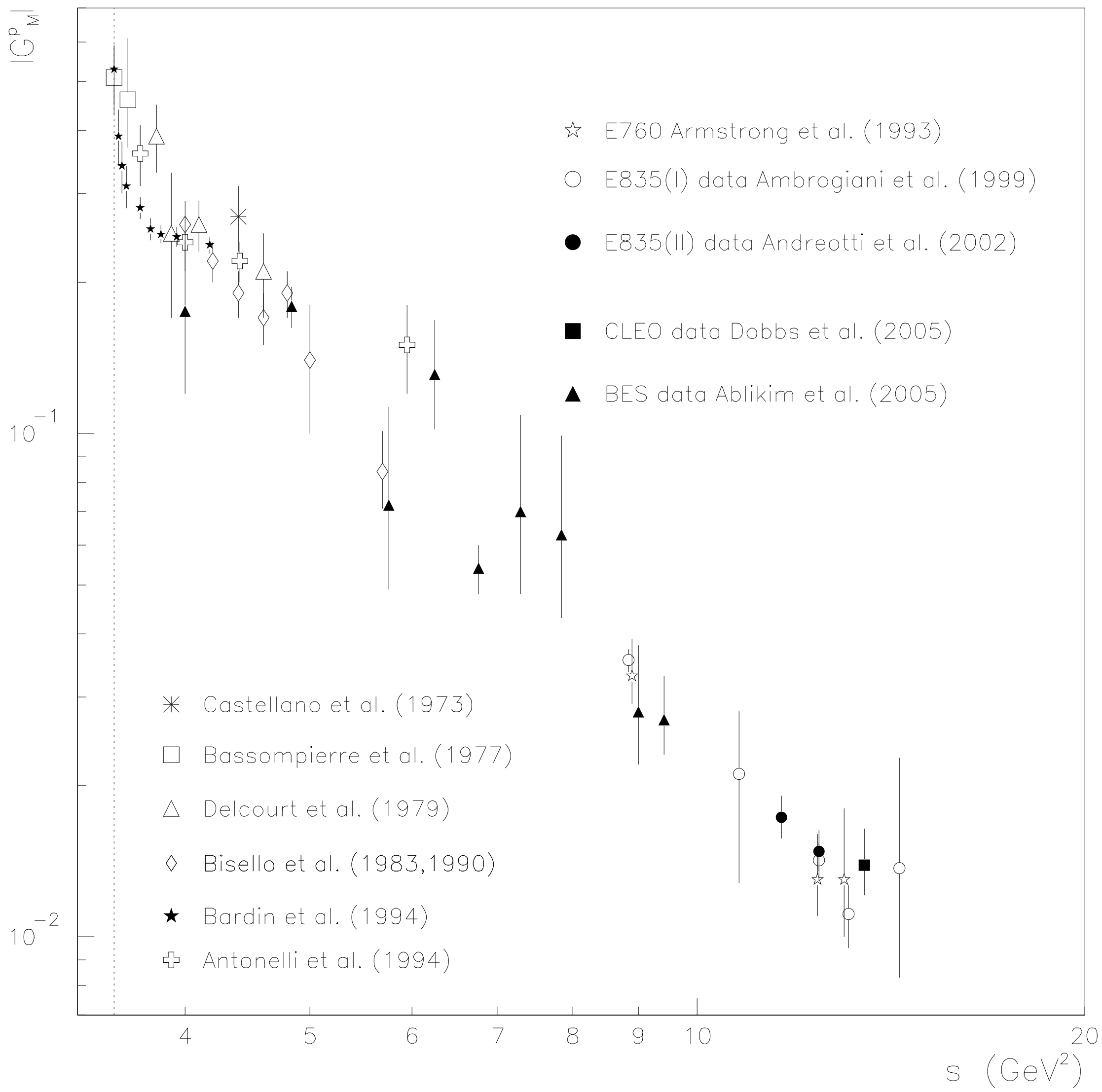
E835 at FNAL, continuation of E760, made further measurements at high Q^2 with a total of 206 events in 2 data taking runs.

Proton Magnetic Form Factor $|G_M|$



A new measurement at high Q^2 was recently made by the CLEO at CESR in $e^+e^- \rightarrow \bar{p}p$. It assumes $|G_E|=|G_M|$. The measurement is based on 14 events.

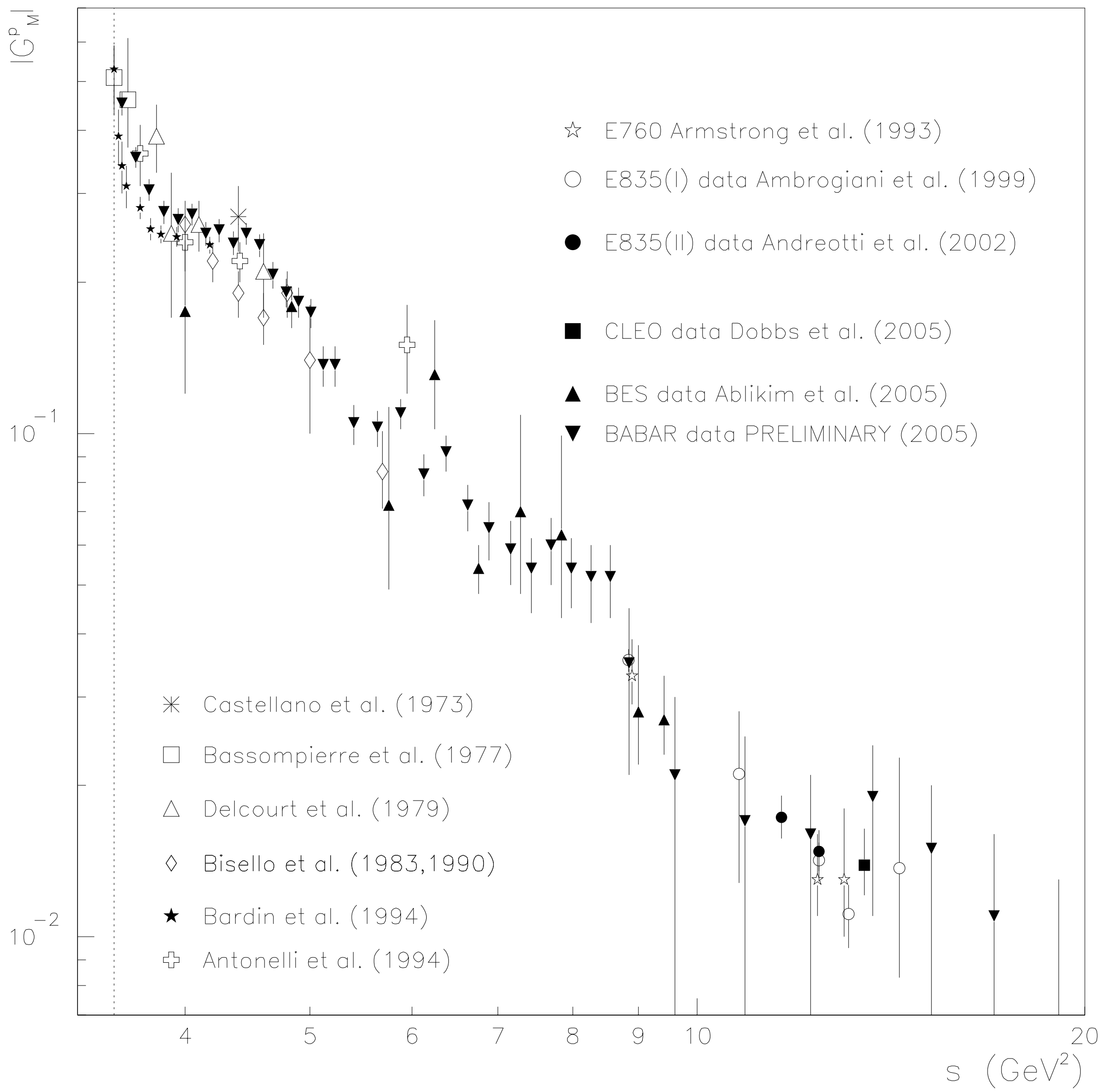
Proton Magnetic Form Factor $|G_M|$



Another measurement of the proton timelike form factors has been reported by BES.

The measurement covers 9 data points from $(2.0\ \mathrm{GeV})^2$ to $(3.07\ \mathrm{GeV})^2$ using the hypothesis $|G_E|=|G_M|$.

Proton Magnetic Form Factor $|G_M|$



BaBar measurement using Initial State Radiation (ISR)

$e^+e^- \to \gamma\, \bar{p}p$

Advantages:

- All energies at the same time

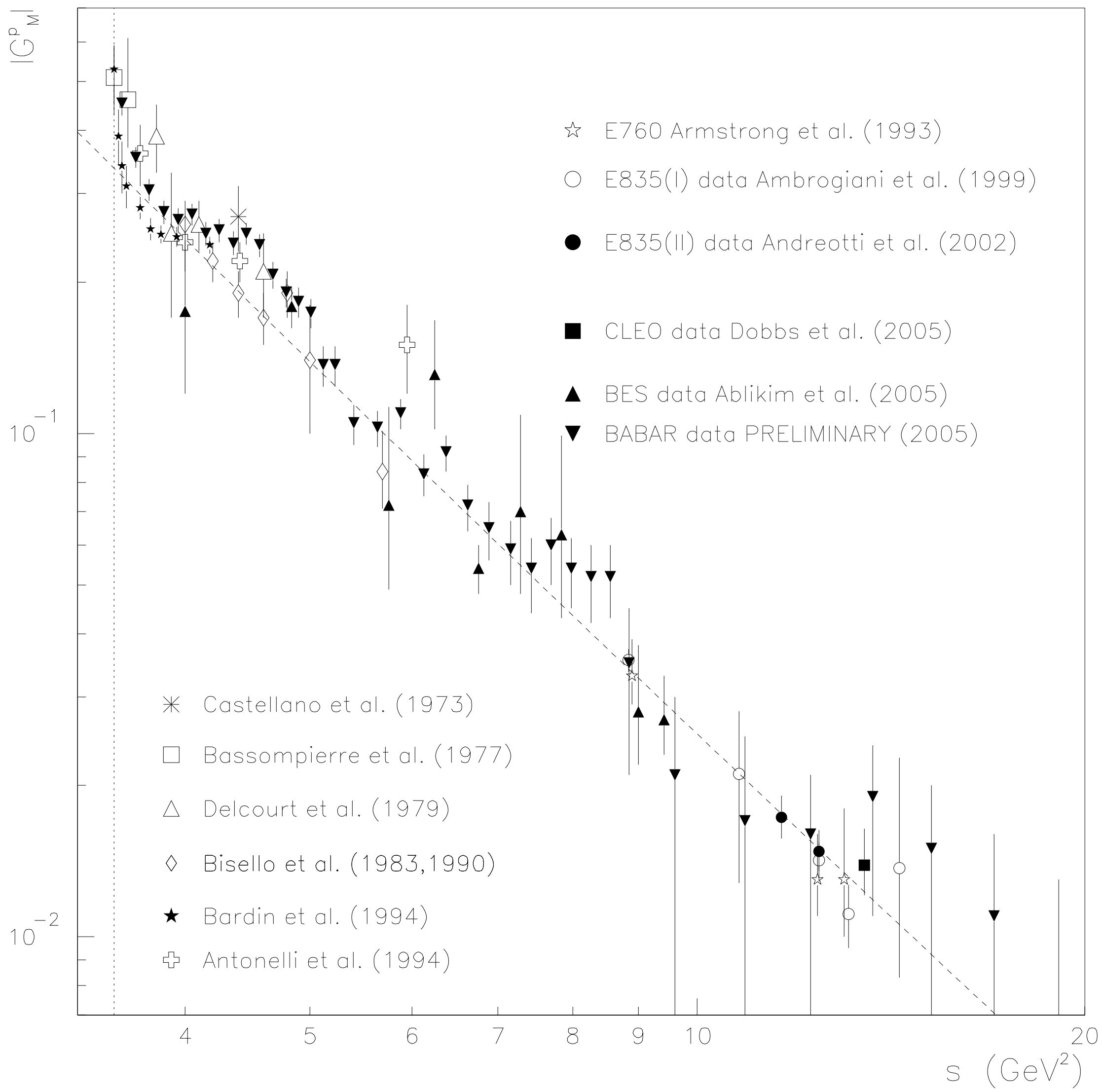
  $\Rightarrow$ fewer systematics
- CMS boost

  $\Rightarrow$ easier measurement at threshold

Disadvantages

- Luminosity proportional to invariant mass bin $L \propto \Delta\sqrt{s}$
- More background

Asymptotic Behavior

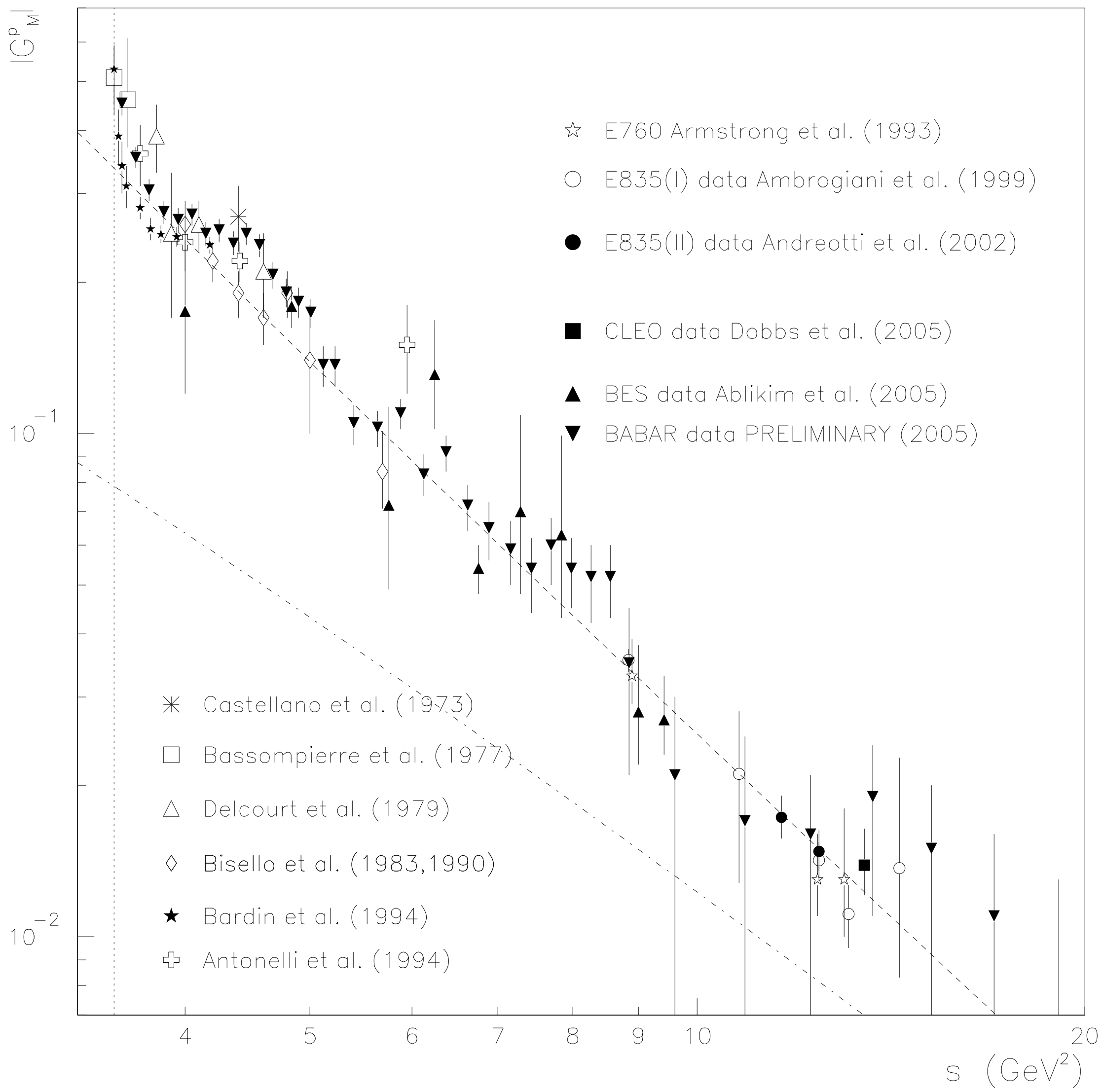


The dashed line is a fit to the PQCD prediction

$$\frac{|G_M|}{\mu_p} = \frac{C}{s^2 \ln^2\left(\frac{s}{\Lambda^2}\right)}$$

The expected Q^2 behaviour is reached quite early, however ...

Asymptotic Behavior



The dashed line is a fit to the PQCD prediction

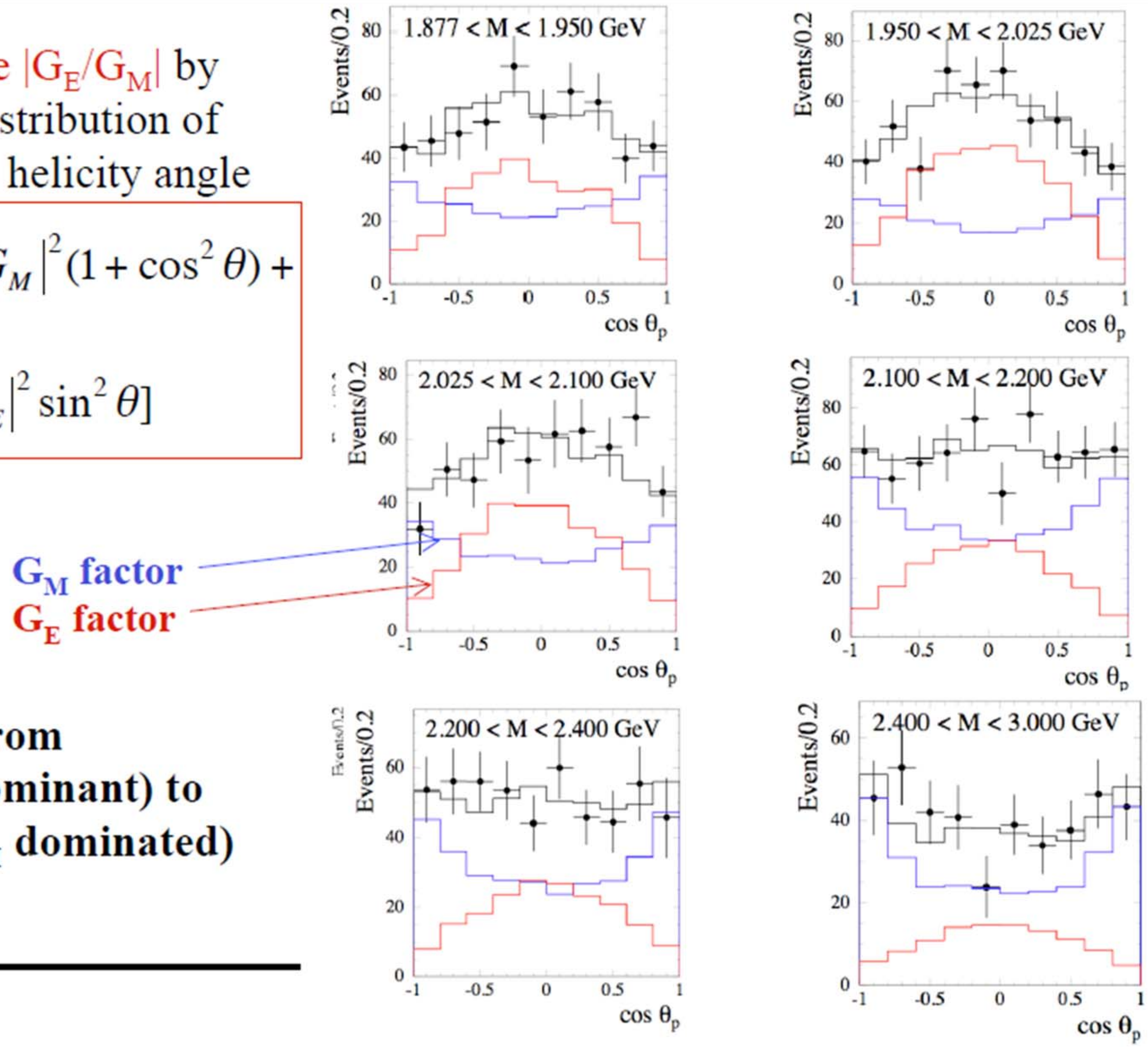
$$\frac{|G_M|}{\mu_p} = \frac{C}{s^2 \ln^2\left(\frac{s}{\Lambda^2}\right)}$$

The expected Q^2 behaviour is reached quite early, however ...
... there is still a factor of 2 between timelike and spacelike.

G_E and G_M angular distributions

Can measure $|G_E/G_M|$ by fitting the distribution of the cosine of helicity angle

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{2s\beta}[|G_M|^2(1+\cos^2\theta) + \frac{4m^2}{s}|G_E|^2\sin^2\theta]$$

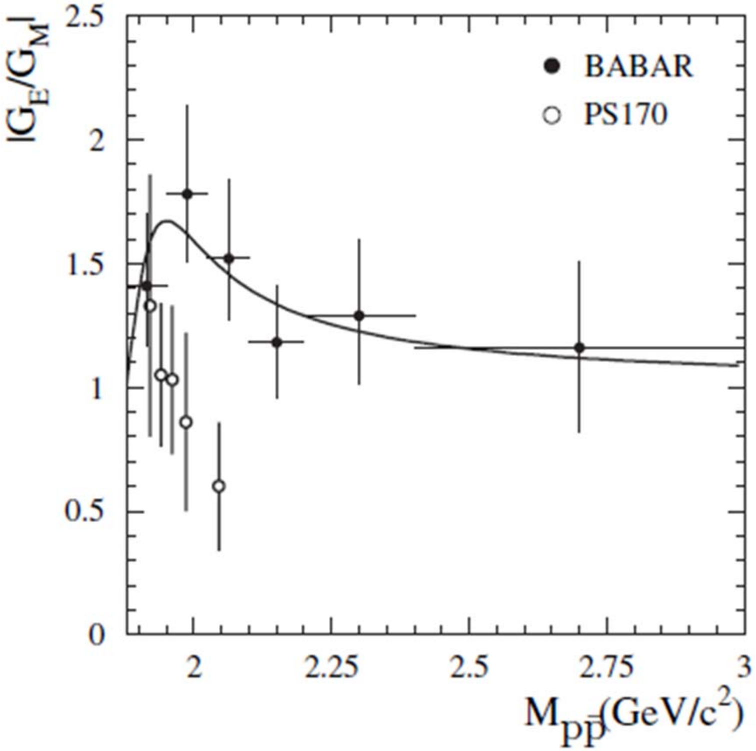


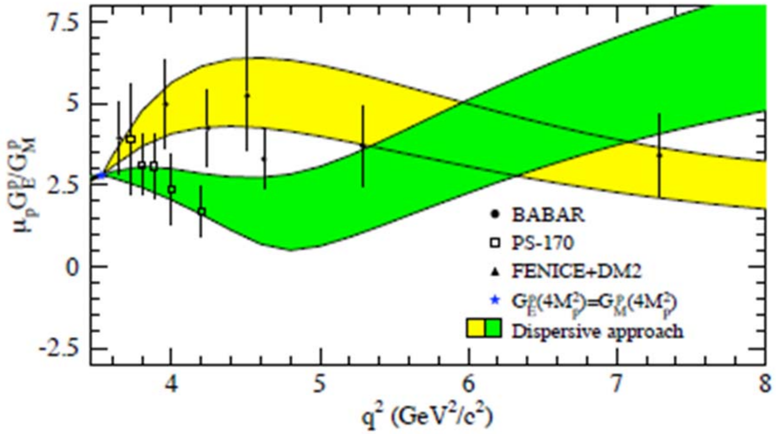
Transition from $\sin^2\theta$ (G_E dominant) to $1+\cos^2\theta$ (G_M dominated)

Achim Denig

The ratio $|G_E|/|G_M|$

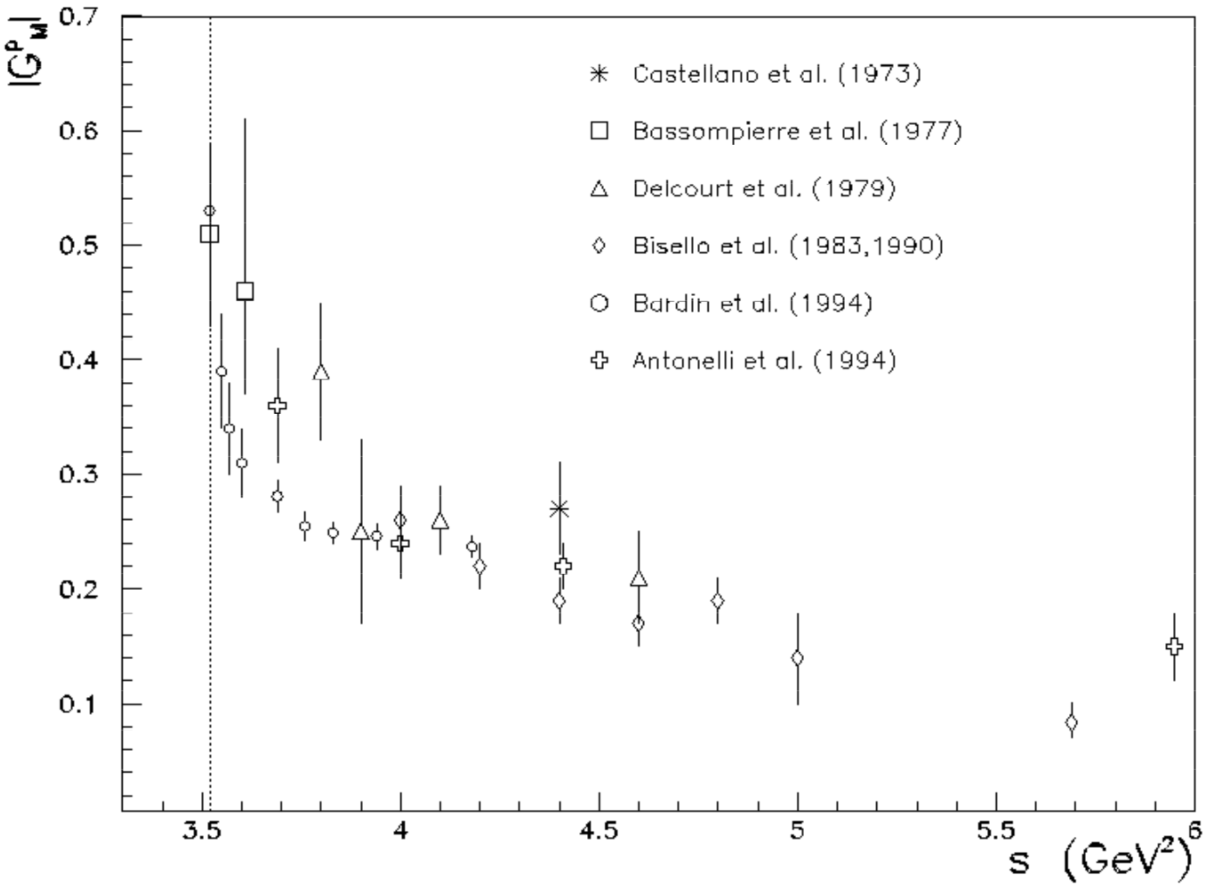
So far only two experiments have collected enough statistics to analyze the angular distribution and attempt to extract $|G_E|$ and $|G_M|$ independently.





The present accuracy in the ratio $|G_E|$ and $|G_M|$ is of the order of 50 %.

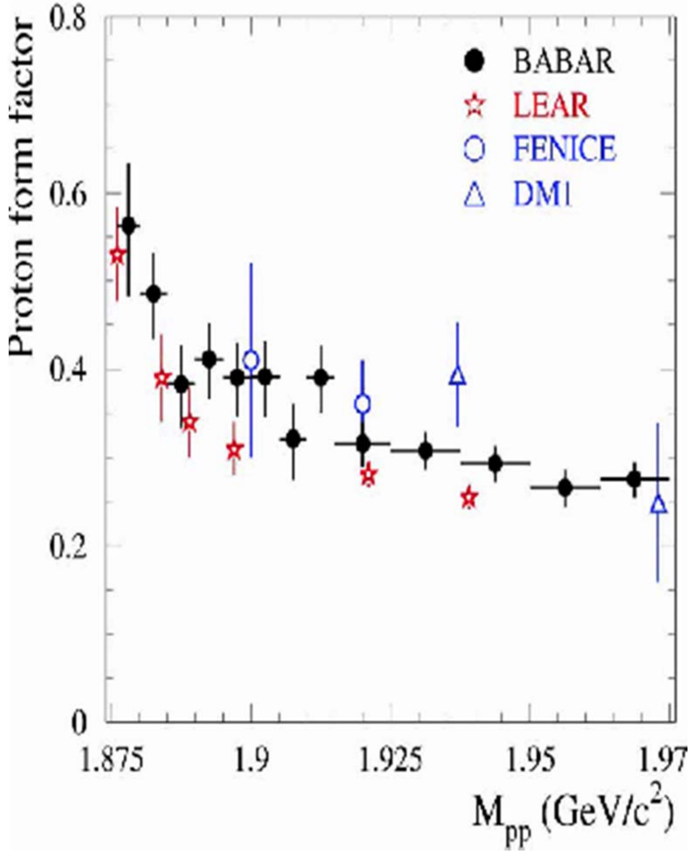
Threshold Q^2 Dependence



Steep behavior near threshold observed by PS 170 at LEAR (2000 events).

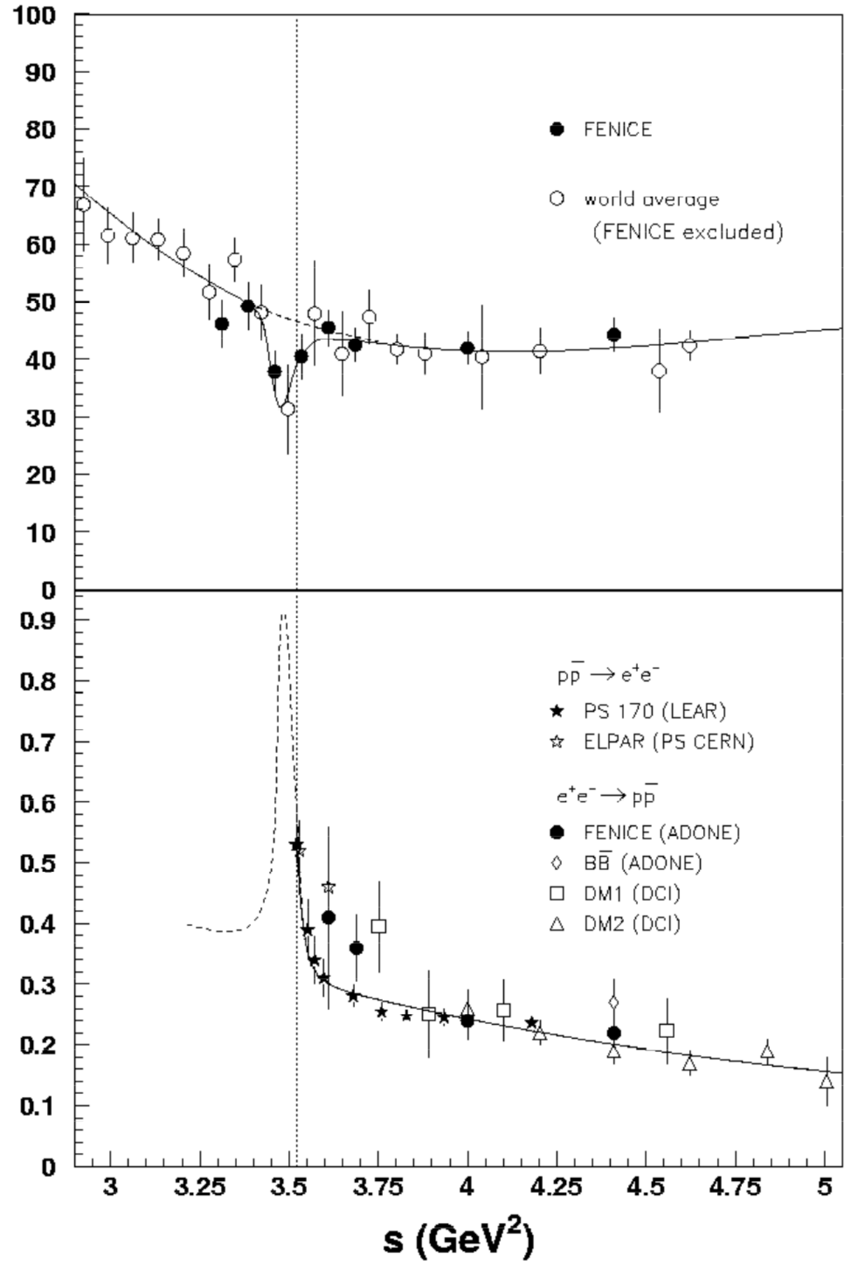
BaBar Measurement using ISR

BaBar measurement very near threshold confirms steep rise of Form Factor

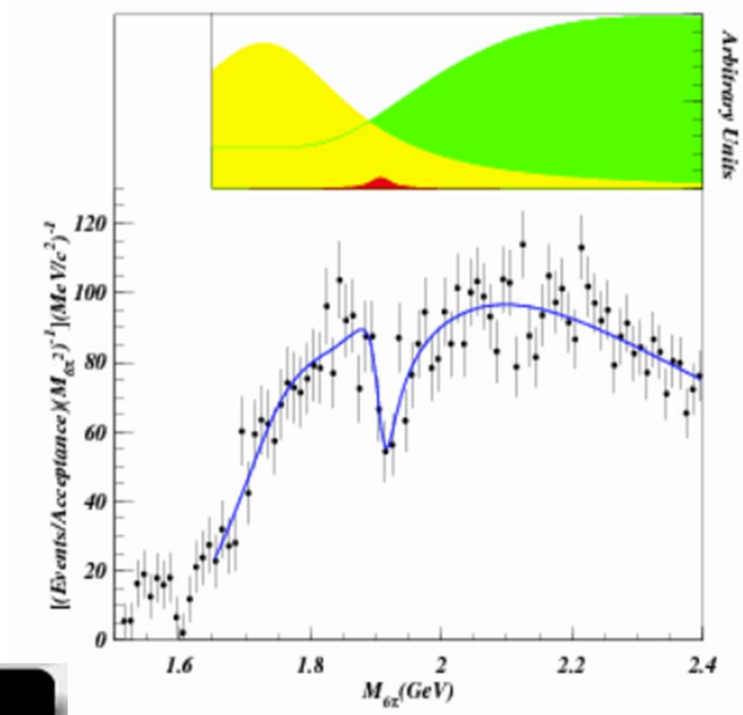


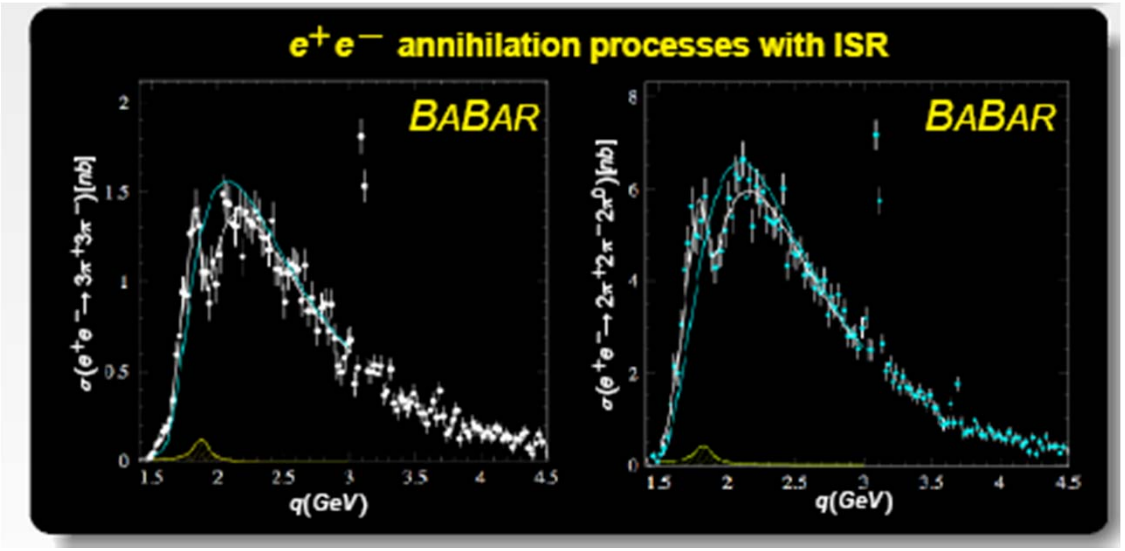
Resonant Structures

The dip in the total multihadronic cross section and the steep variation of the proton form factor near threshold may be fitted with a narrow vector meson resonance, with a mass M ~1.87 GeV and a width Γ ~ 10-20 MeV, consistent with an N $\bar{\mathrm{N}}$ bound state.



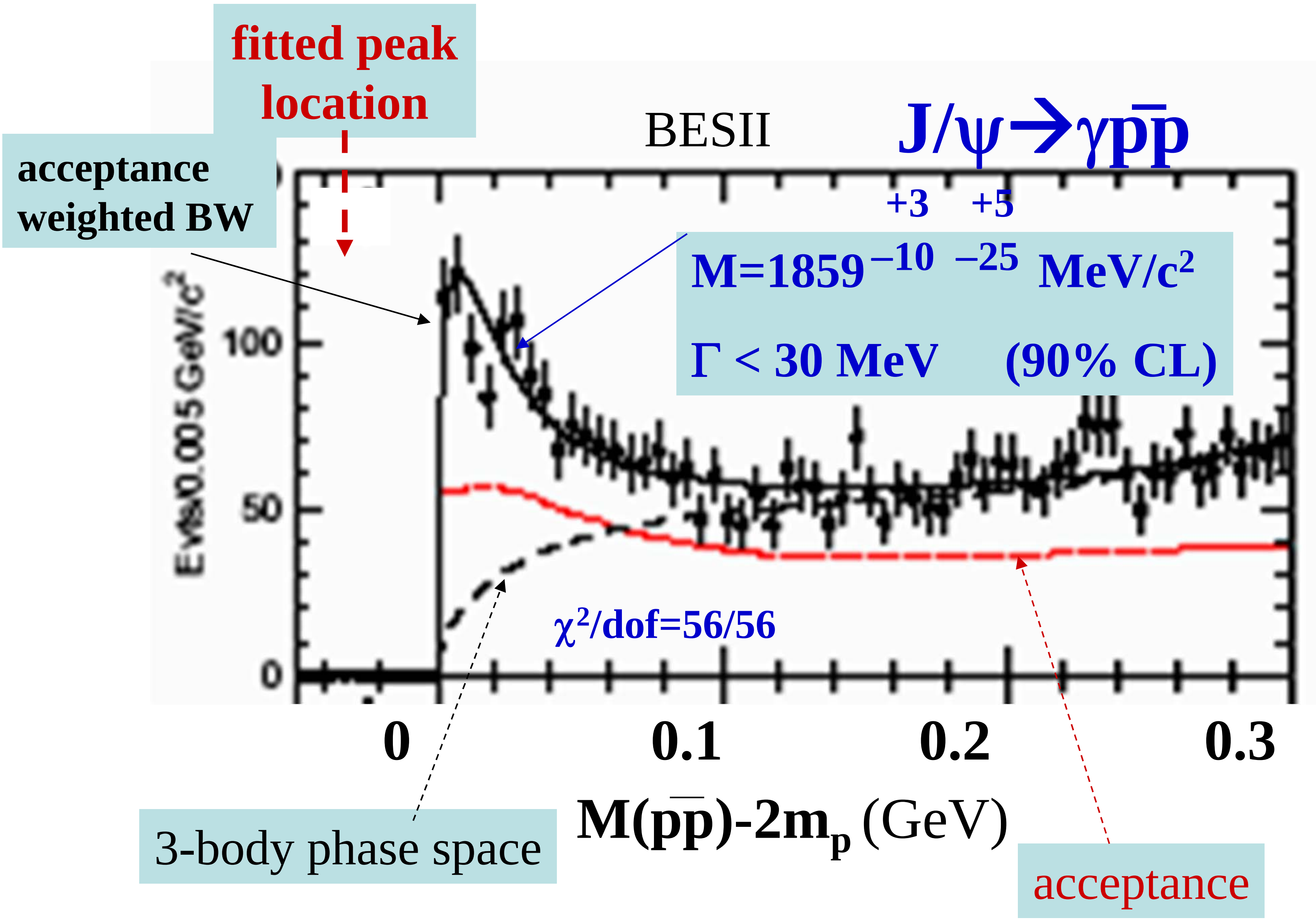
- Dip observed in 6 π diffractive photoproduction by E687 at Fermilab
- New results from Babar in e^+e^- annihilation to 6 π with ISR





V_0	M(MeV)	Γ(MeV)
hadrons	~1870	10÷20
E687	1910 ± 10	37 ± 13
BaBar	1880 ± 50	130 ± 30
BaBar(π^0)	1860 ± 20	160 ± 20

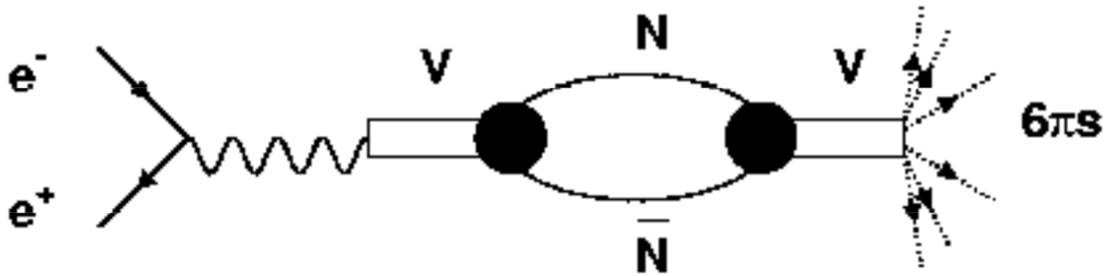
Threshold Enhancement observed by BES



Possible Explanations

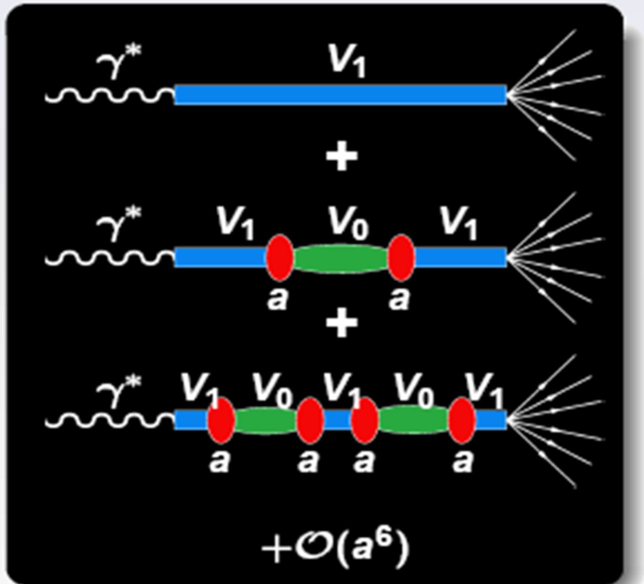
- Tail of a narrow resonance below threshold (baryonium ?).
- Dominance of π exchange in $\bar{p}p$ final state interaction.
- Underestimation of the Coulomb correction factor.

Possible test for baryonium: a vector meson with very small coupling to e^+e^- (and relatively small hadronic width), lying on top of a ρ/ω recurrence, should show up as a dip in some hadronic cross section.



"Baryonium" → dip in multihadronic processes

P.J. Franzini and F.J. Gilman, 1985



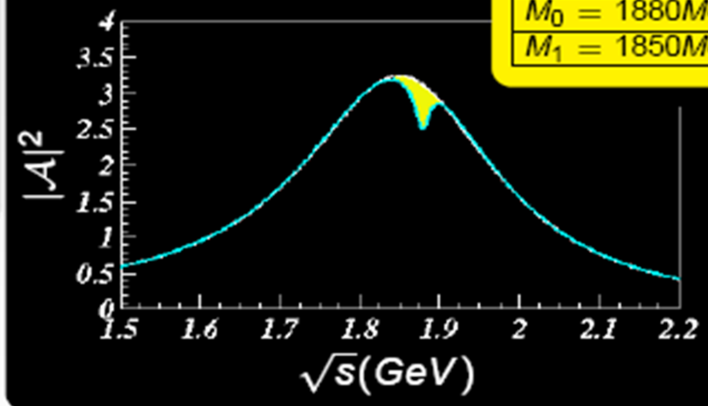
A vector meson V_0 ($J^{PC} = 1^{--}$), with vanishing e^+e^- coupling, which decays through an intermediate broad vector meson V_1

$$\mathcal{A} \propto \frac{1}{s - M_1^2}\left(1 + a\frac{1}{s - M_0^2}a\frac{1}{s - M_1^2} + \cdots\right)$$

$$\mathcal{A} = \frac{s - M_0^2}{(s - M_1^2)(s - M_0^2) - a^2}$$

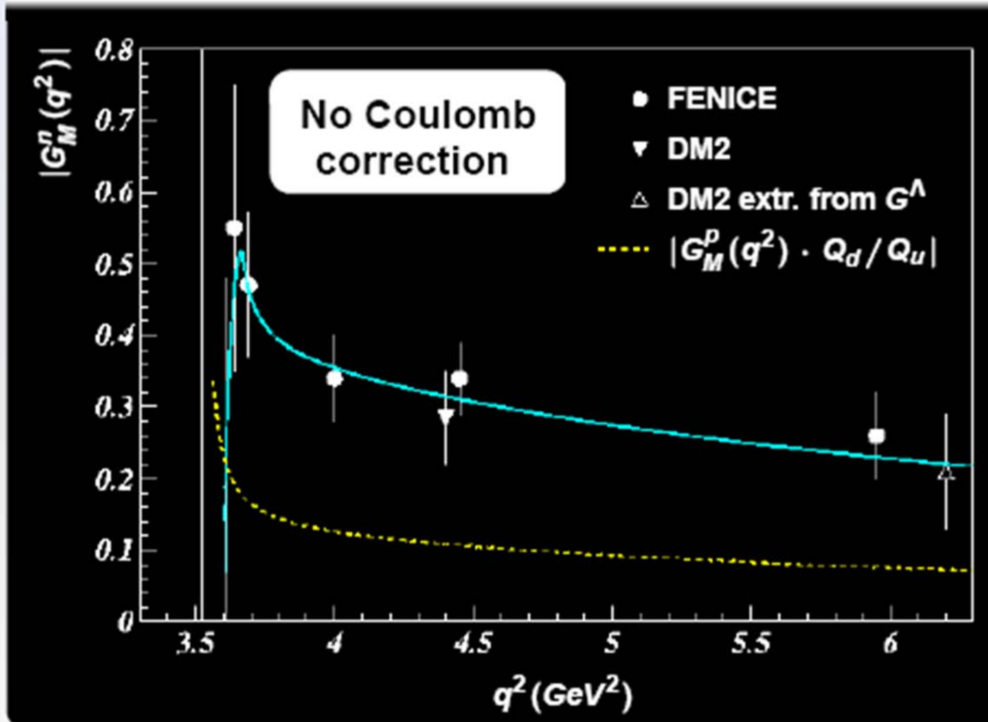
For instance...

$M_0 = 1880 MeV$	$\Gamma_0 = 20 MeV$
$M_1 = 1850 MeV$	$\Gamma_1 = 300 MeV$



Neutron Timelike Form Factor

Only two measurements by FENICE and DM2

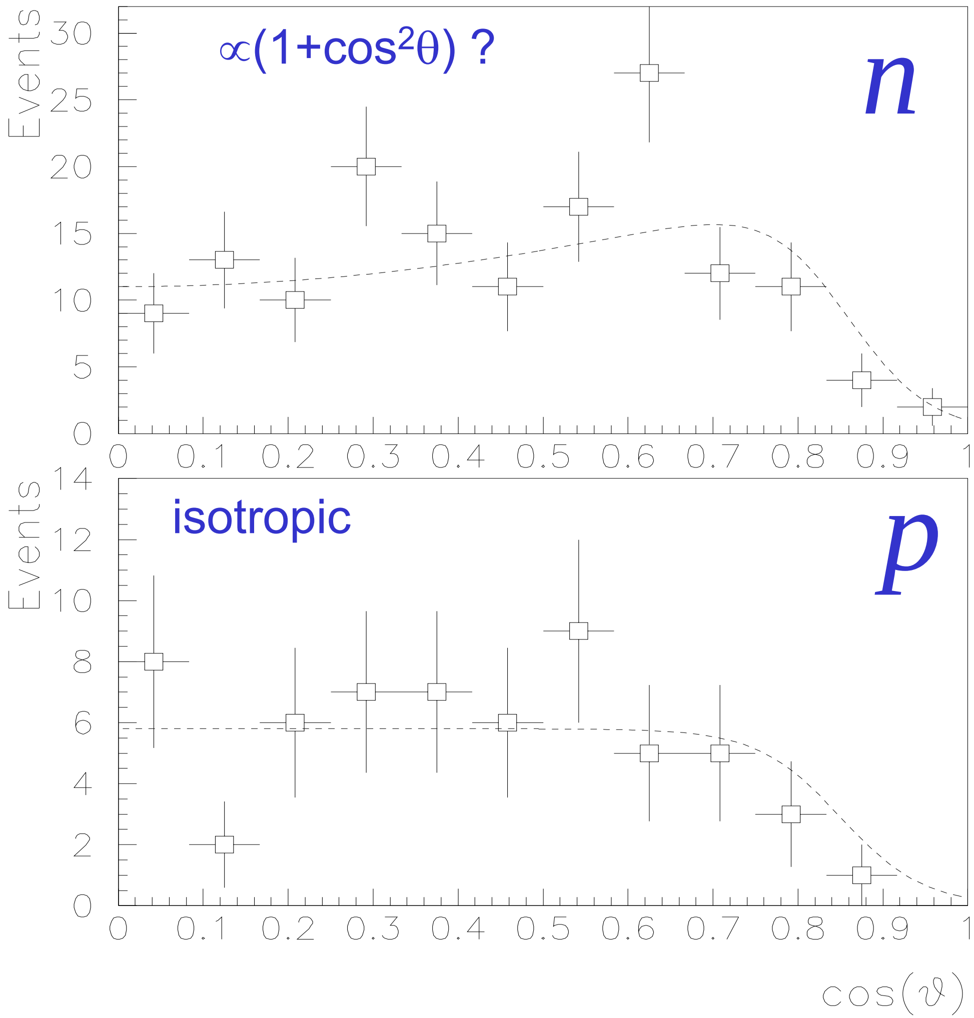


	$\lvert G_M^n/G_M^p \rvert$
Data	~ 1.5
Naively	$\sim \lvert Q_d/Q_u \rvert$
pQCD	< 1
Soliton models	~ 1
VMD	$\gg 1$

$1.9 < \sqrt{s} < 2.55 \quad GeV$ $\qquad \int L dt = 0.4\ pb^{-1}$ $\qquad$ 80 events

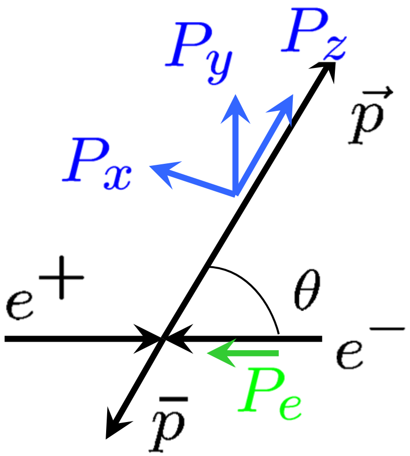
The neutron form factor is bigger than that of the proton !!!

Neutron Angular Distribution



$$\left|G_E^n\right| << \left|G_M^n\right|$$

Measuring the Phase between G_E and G_M



The relative phase δ_{ME} between G_M and G_E can only be measured by means of single- or double-polarization experiments.

$$P_x \propto P_e \cos\delta_{ME}$$

$$P_z \propto P_e$$

$$P_y = \frac{\sin 2\theta^*}{\left[|G_M(s)|^2(1+\cos^2\theta^*) + \frac{4m_N^2}{s}|G_E(s)|^2 \sin^2\theta^*\right]} \frac{4m_N^2}{s} |G_E||G_M| \sin\delta_{ME}$$

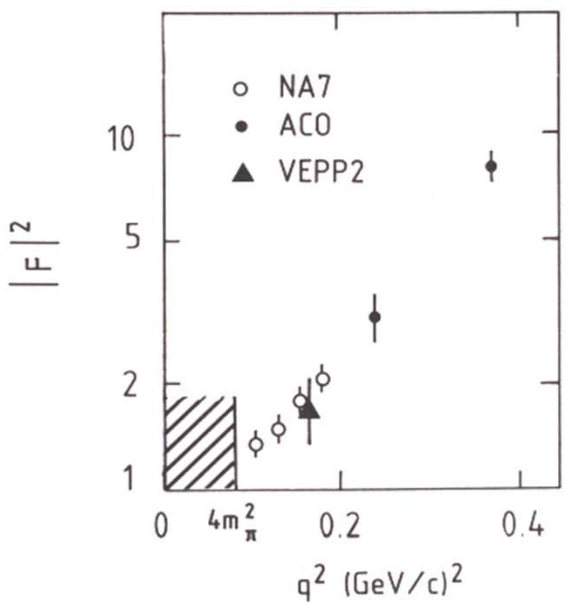
It takes the maximum value near scattering angles of 45^0 and 135^0 and vanishes at 90^0. Once this phase is known, by measuring the ratio of the two components of the nucleon polarizations in the scattering plane with longitudinally polarized beams, the ratio $|G_M|/|G_E|$ can be obtained with small systematic uncertainties.

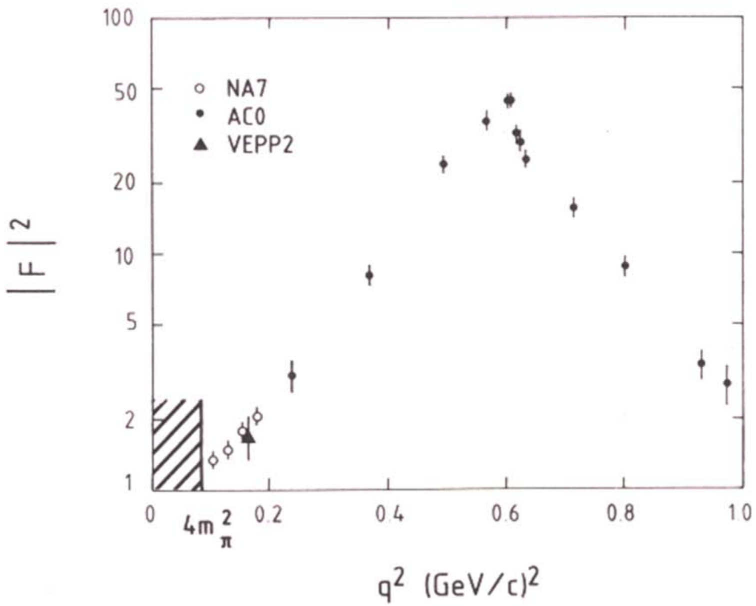
Pion Timelike Form Factor

Early Measurements near Threshold (NA7 Experiment at CERN)

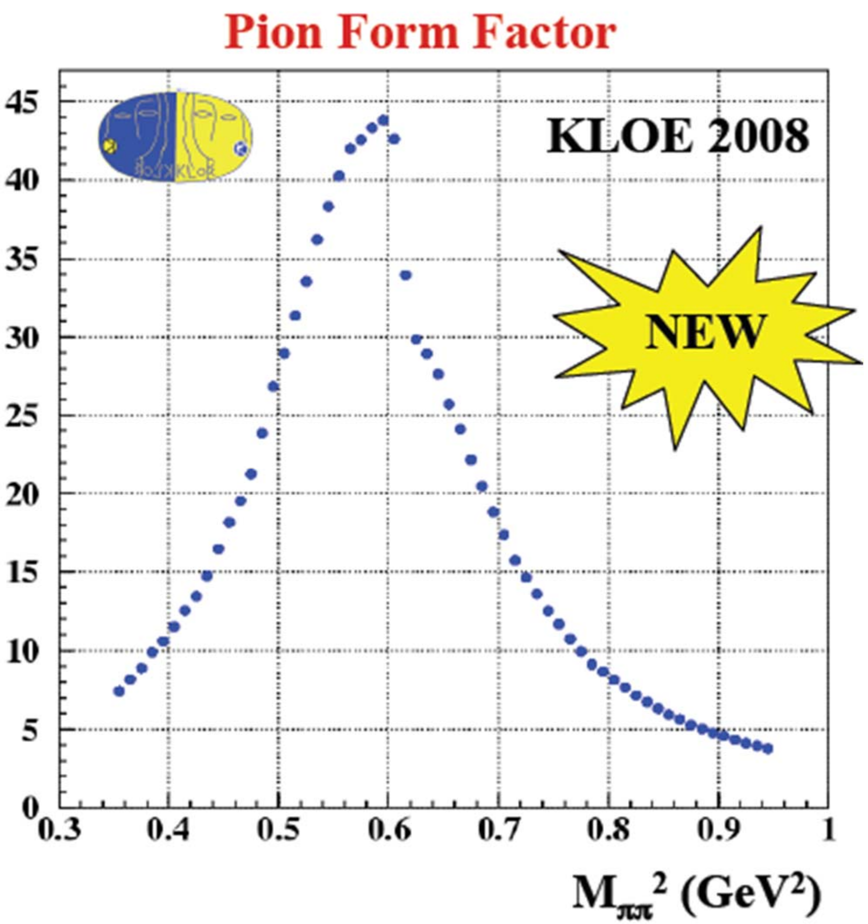
$$e^+ + e^- \rightarrow \pi^+ + \pi^-$$

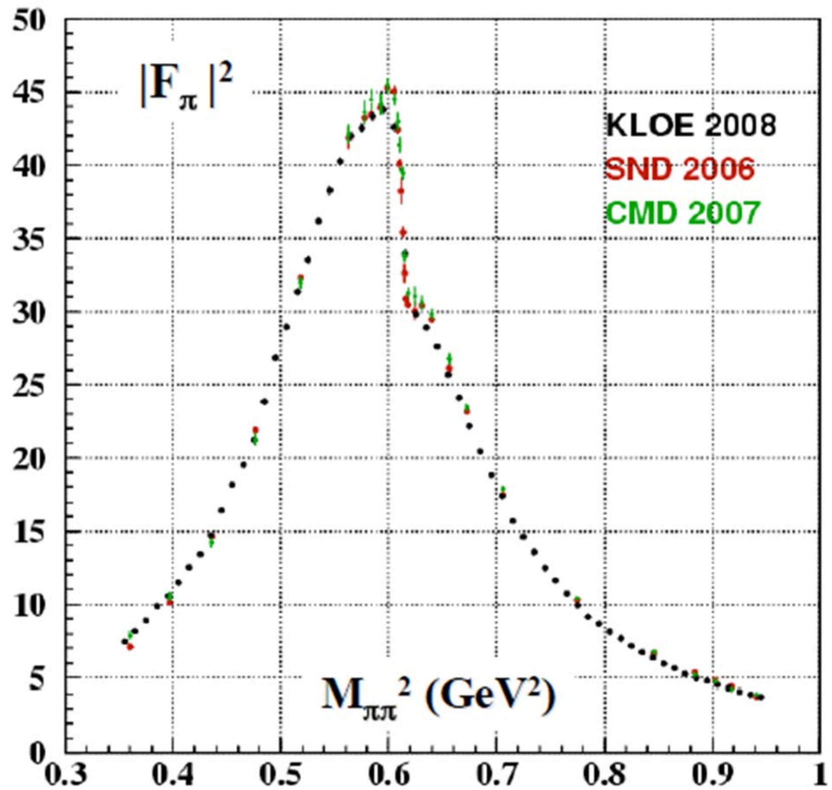
e^+ beam on H_2 target
100, 125, 150, 175 GeV



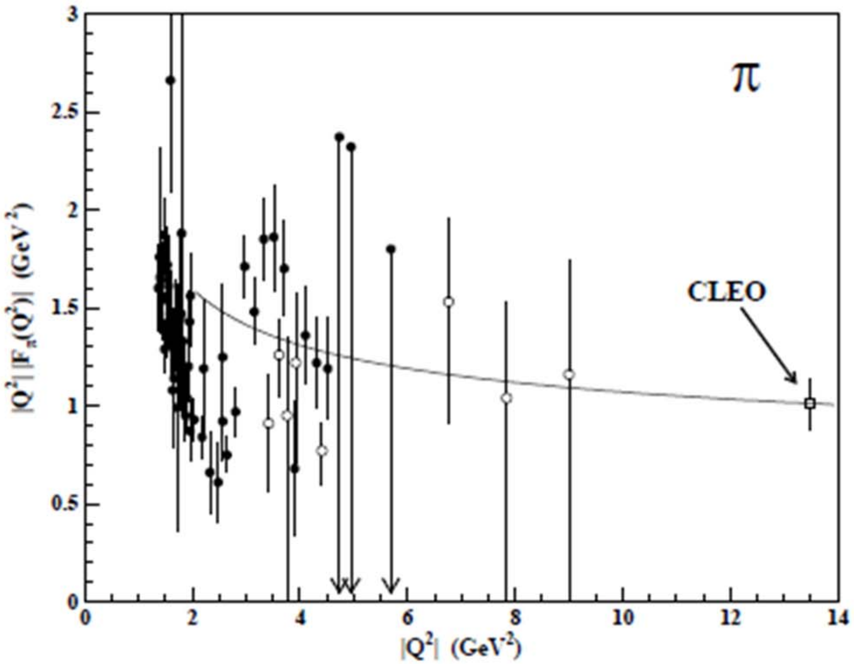


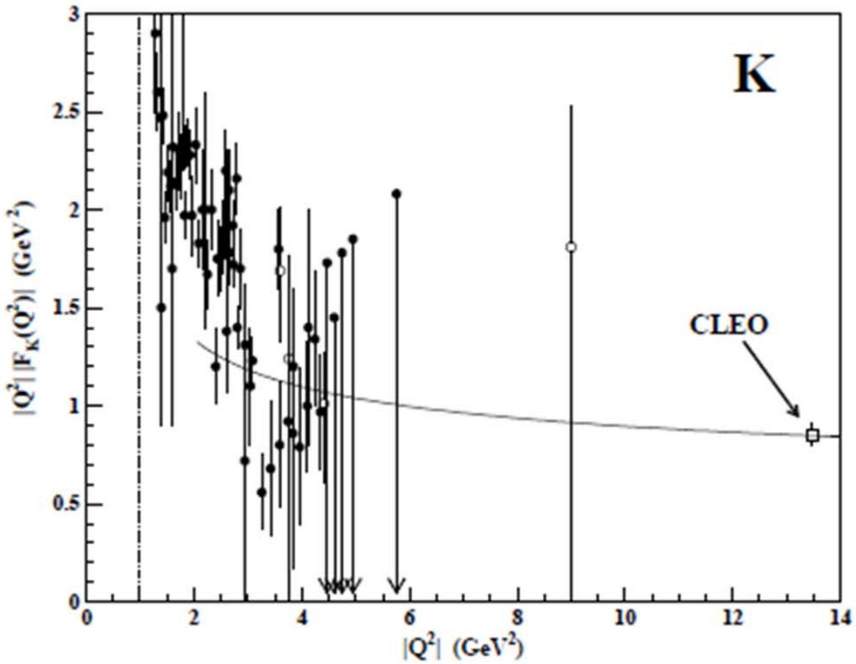
Pion Form Factor at KLOE





Pion and Kaon Form Factors at CLEO





Proton FF Measurement in $\overline{\text{P}}$ANDA at FAIR

The High-Energy Storage Ring (HESR)

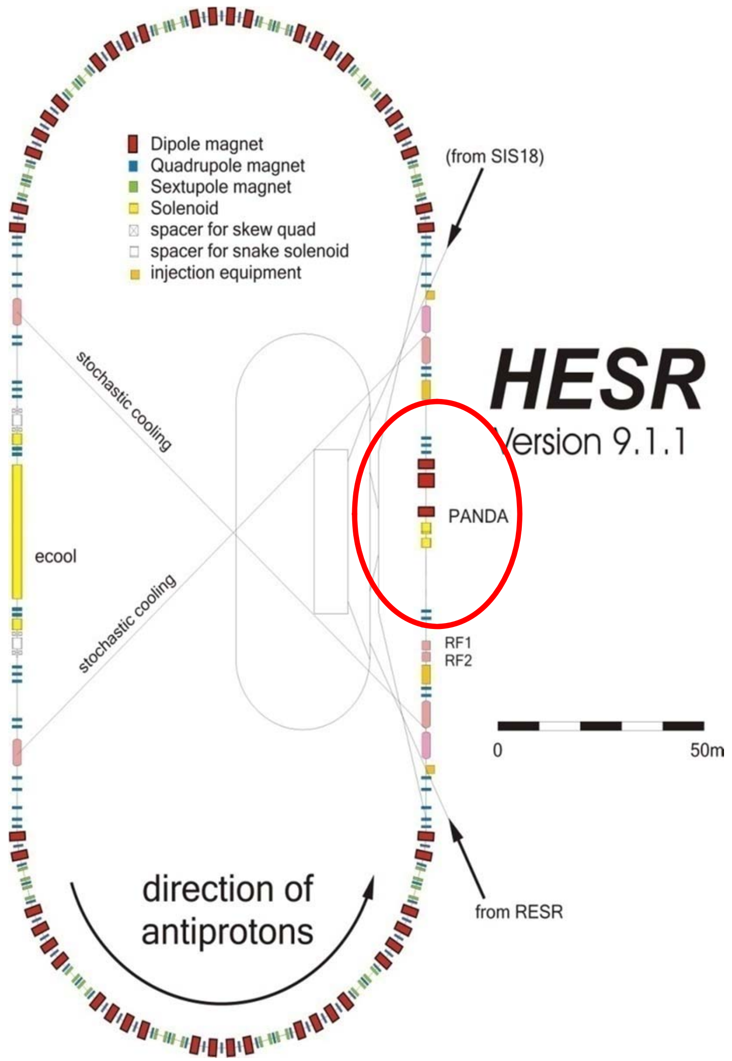
- Production rate 2×10^7/sec
- P_{beam} = 1 - 15 GeV/c
- N_{stored} = 5×10^{10} $\bar{p}$
- Internal Target

High resolution mode

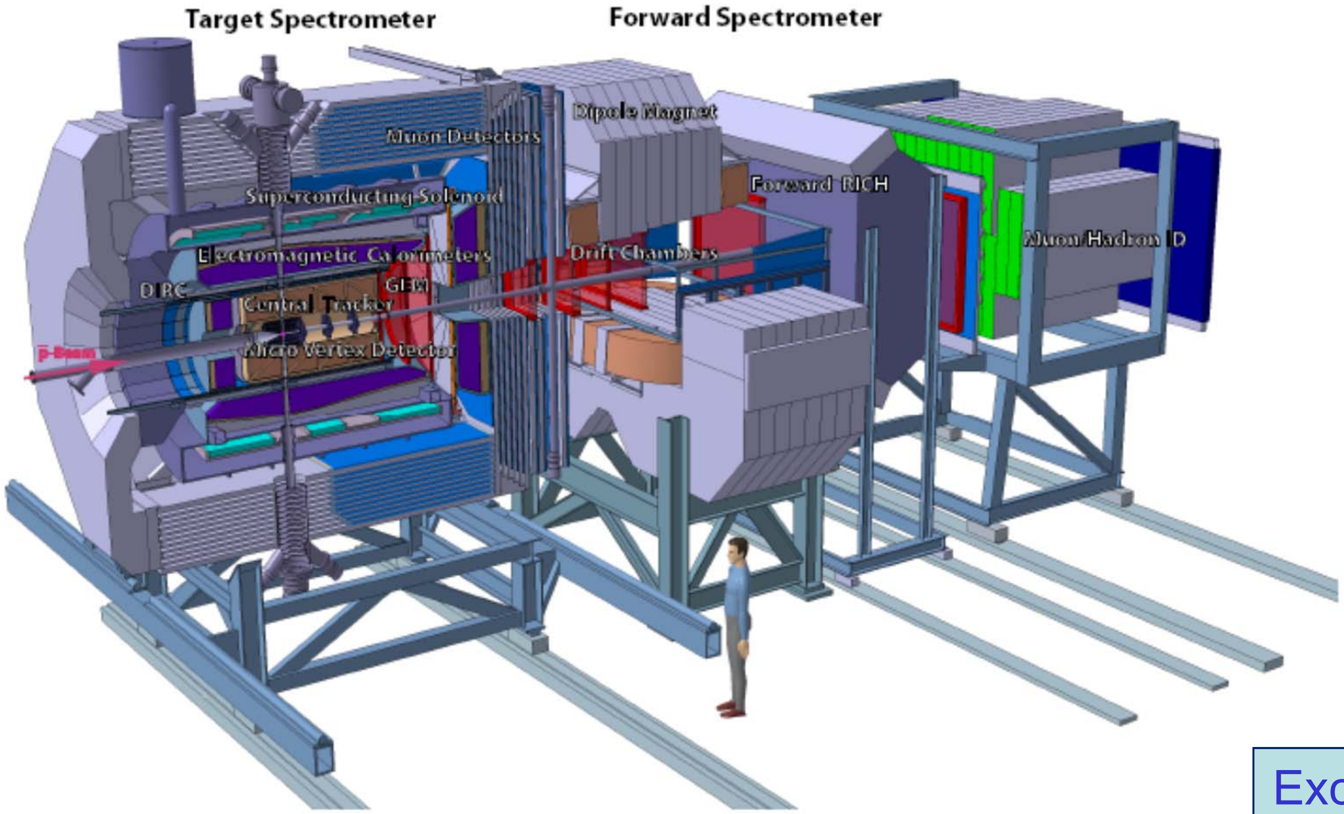
- $\delta p/p \sim 10^{-5}$ (electron cooling)
- Lumin. = 10^{31} cm^{-2} s^{-1}

High luminosity mode

- Lumin. = 2×10^{32} cm^{-2} s^{-1}
- $\delta p/p \sim 10^{-4}$ (stochastic cooling)



The $\overline{\text{P}}$ANDA Detector



High luminosity

Excellent Particle ID

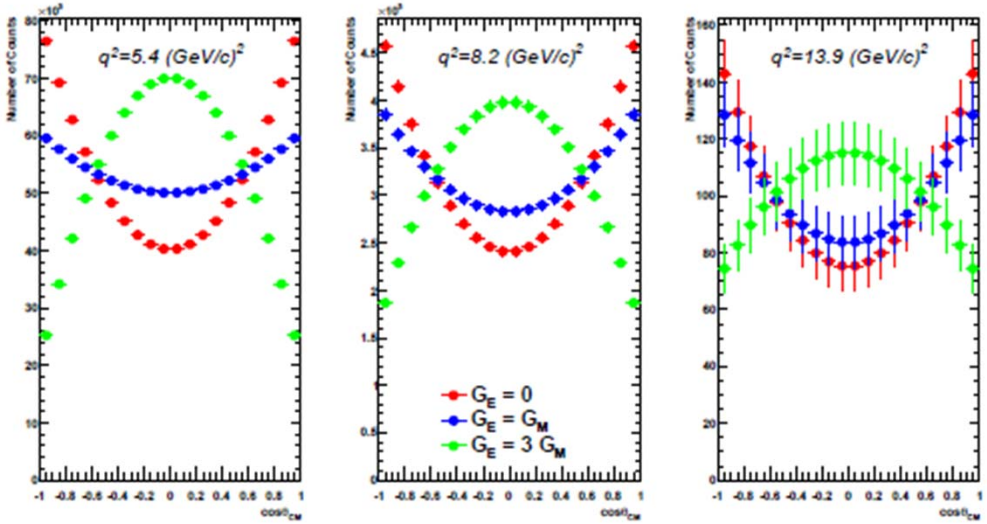
Excellent background rejection.

Form Factors in $\bar{P}$ANDA

The PANDA experiment will determine the moduli of the proton form factors in the time-like region by measuring the angular distribution of the process

$$\bar{p}p \rightarrow e^+e^-$$

in a q^2 range from 5 $(GeV/c)^2$ up tp 14 $(GeV/c)^2$. A determination of the form factor up to a q^2 of 22 $(GeV/c)^2$ will be possible by measuring the total cross section.

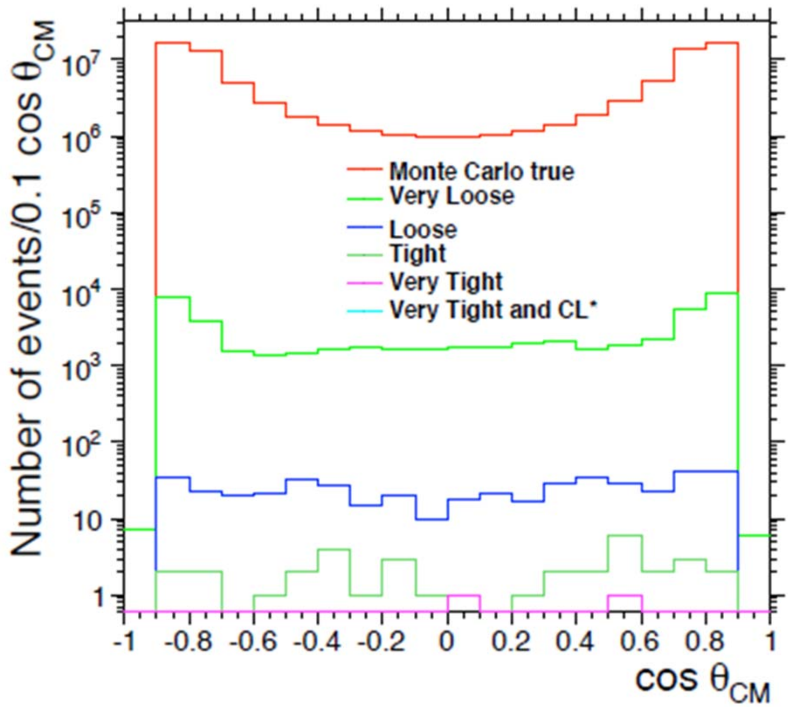


Background Rejection

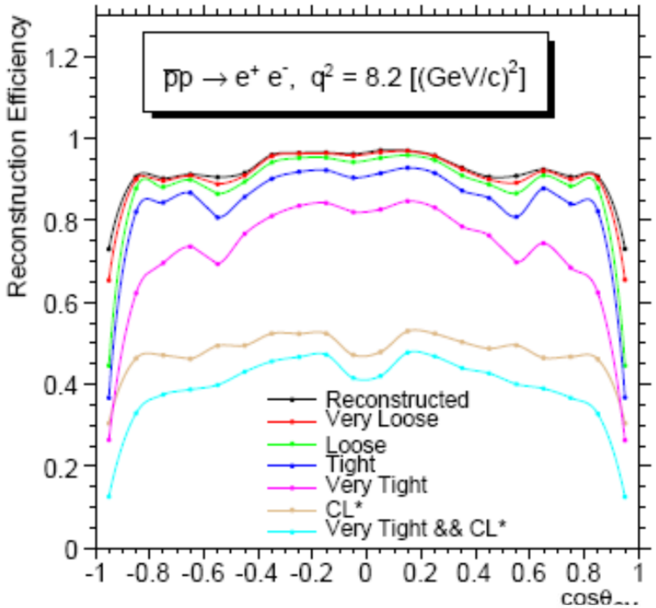
Very large background coming mainly from two-body hadronic final states, like $\bar{p}p \rightarrow \pi^+\pi^-$, with a cross section up to 10^6 times larger.

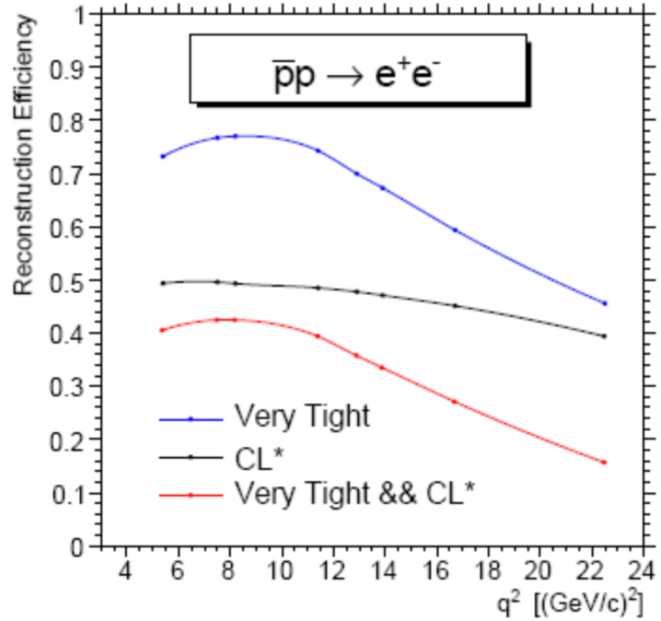
Background rejection done with particle identification (using information from all subdetectors) and kinematic fitting.

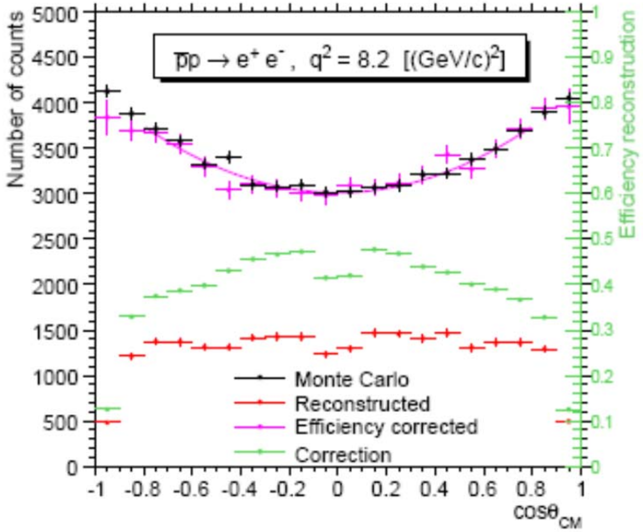
Monte Carlo simulations show that a total rejection factor of the order of 10^{10} is achieved.

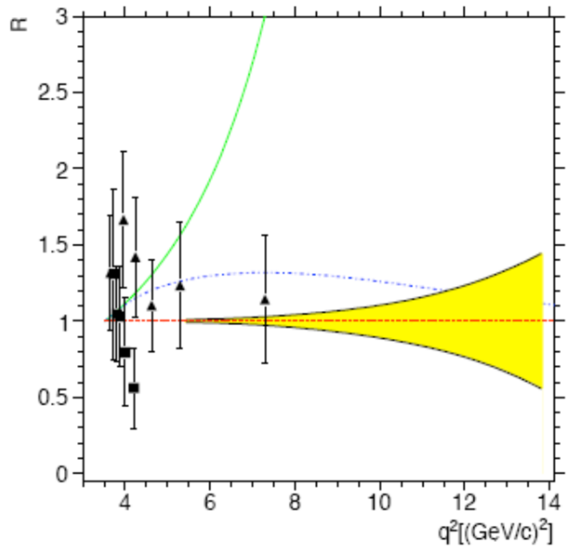


Signal Efficiency

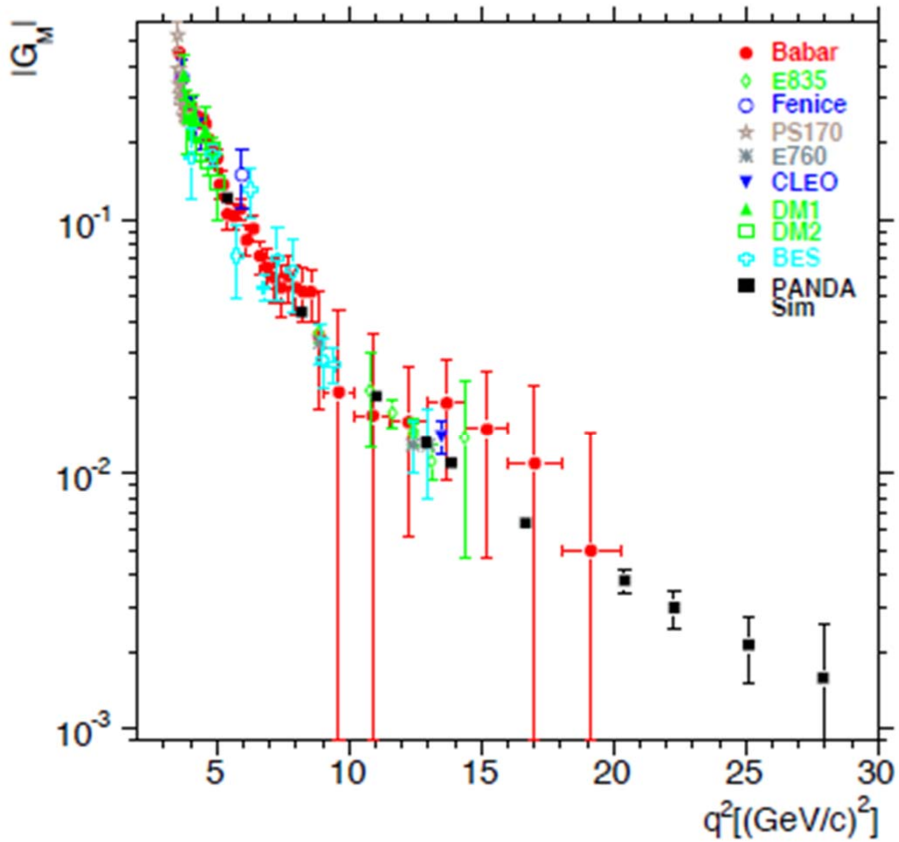








Projected $\bar{\mathrm{P}}$ANDA $|G_M|$ Measurement



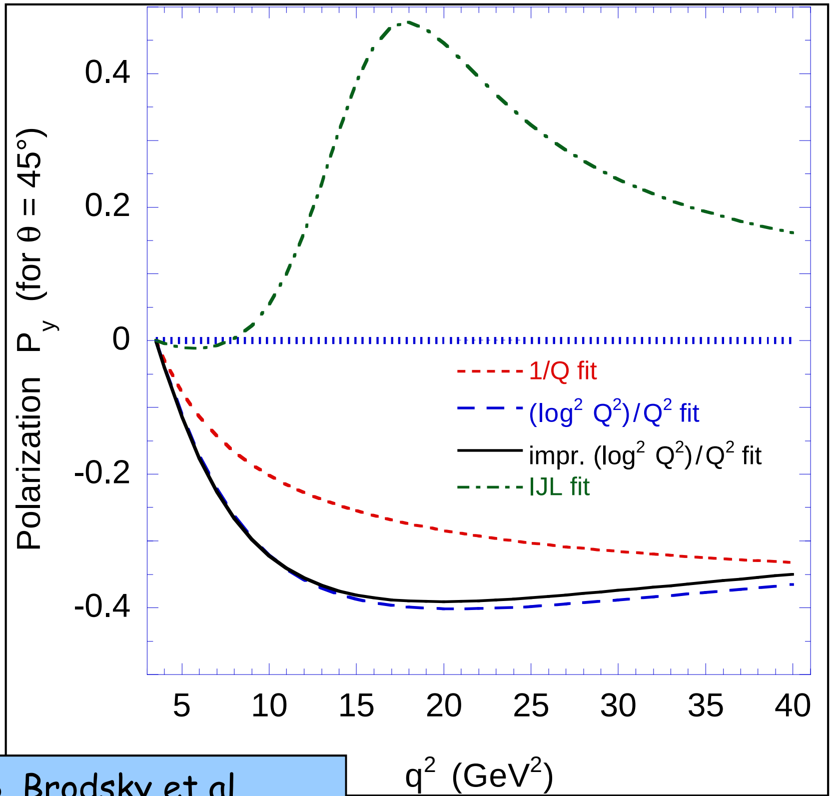
Proton FF in single- and double-polarisation experiments ($\bar{\text{P}}$ANDA and PAX)

- Double-spin asymmetry in pp → e^+e^-
  - independent G_E-G_m separation
  - test of Rosenbluth separation in the time-like region

- Single-spin asymmetry in pp → e^+e^-
  Measurement of relative phases of magnetic and electric FF in the time-like region

$$A_y = \frac{\sin(2\theta)\cdot \mathrm{Im}(G_E^* \cdot G_M)}{\left[\left(1+\cos^2(\theta)\right)|G_M|^2 + \sin^2(\theta)|G_E|^2/\tau\right]\sqrt{\tau}}$$

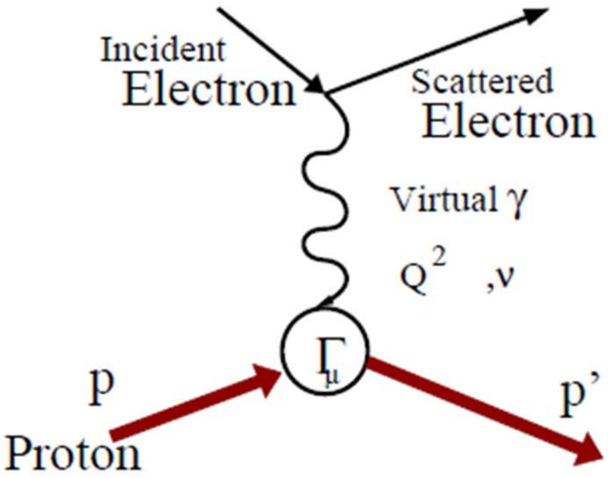
$$\tau = q^2/4m_p^{\,2}$$



S. Brodsky et al.,
Phys. Rev. D69 (2004)

Space-like Form Factors

Elastic Electron-Nucleon Scattering



Nucleon vertex:

$$\Gamma_\mu(p',p) = \underbrace{F_1(Q^2)}_{Dirac}\gamma_\mu + \frac{i\kappa_p}{2M_p}\underbrace{F_2(Q^2)}_{Pauli}\sigma_{\mu\nu}q^\nu$$

$$G_E(Q^2) = F_1(Q^2) - \kappa_N\tau\, F_2(Q^2)$$

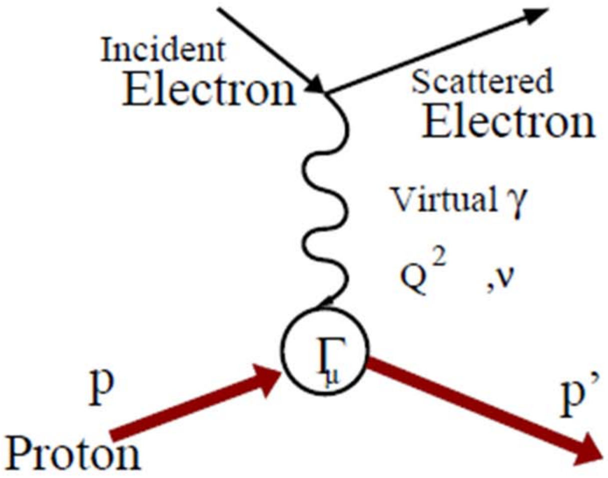
$$G_M(Q^2) = F_1(Q^2) + \kappa_N\, F_2(Q^2)\ ,\tau = \frac{Q^2}{4M_N}$$

At $Q^2 = 0 \quad G_{Mp} = 2.79 \quad G_{Mn} = -1.91$

$G_{Ep} = 1 \qquad G_{En} = 0$

$$\epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

Elastic Electron-Nucleon Scattering



Nucleon vertex:

$$\Gamma_\mu(p',p) = \underbrace{F_1(Q^2)}_{Dirac}\gamma_\mu + \frac{i\kappa_p}{2M_p}\underbrace{F_2(Q^2)}_{Pauli}\sigma_{\mu\nu}q^\nu$$

$$G_E(Q^2) = F_1(Q^2) - \kappa_N\tau\, F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + \kappa_N\, F_2(Q^2)\,, \tau = \frac{Q^2}{4M_N}$$

At $Q^2 = 0$ $\quad G_{Mp} = 2.79 \quad G_{Mn} = -1.91$

$G_{Ep} = 1 \quad G_{En} = 0$

$$\epsilon = \left[1 + 2(1+\tau)\tan^2\frac{\theta}{2}\right]^{-1}$$

Extract G_E^2 and G_M^2 from:

- Cross-section measurements $\quad N(e,e')$

Extract G_E/G_M from:

- Beam-target Asymmetries $\quad \vec{N}(\vec{e},e')N$
- Recoil polarization $\quad N(\vec{e},e')\vec{N}$

Rosenbluth separation technique

ep elastic cross-section:

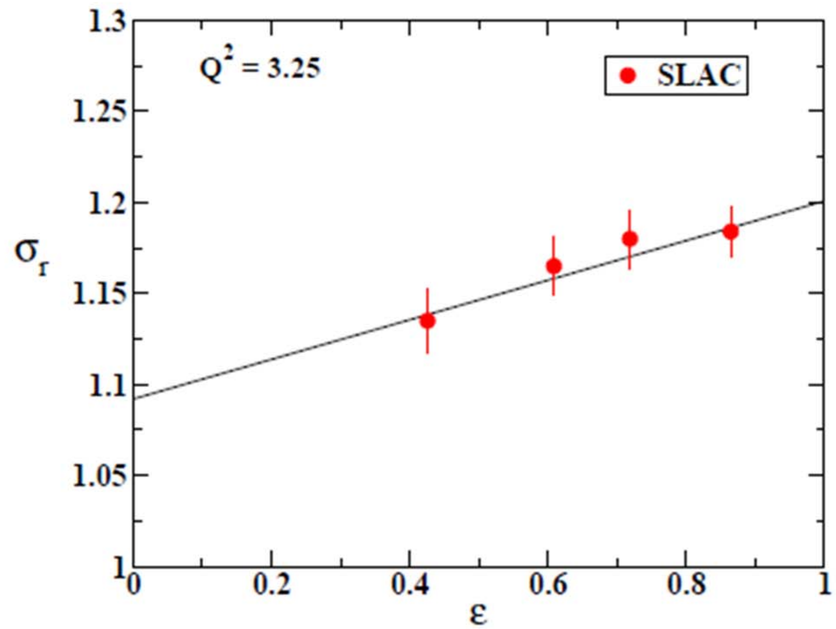
$$\sigma_r \propto \frac{\epsilon}{\tau}\left(\frac{G_E}{G_D}\right)^2 + \left(\frac{G_M}{G_D}\right)^2$$

$G_D = (1 + Q^2/.71)^{-2} \qquad \tau = Q^2/4M^2$

At fixed Q^2: Vary ϵ

G_E is slope

G_M is intercept



Rosenbluth separation technique

ep elastic cross-section:

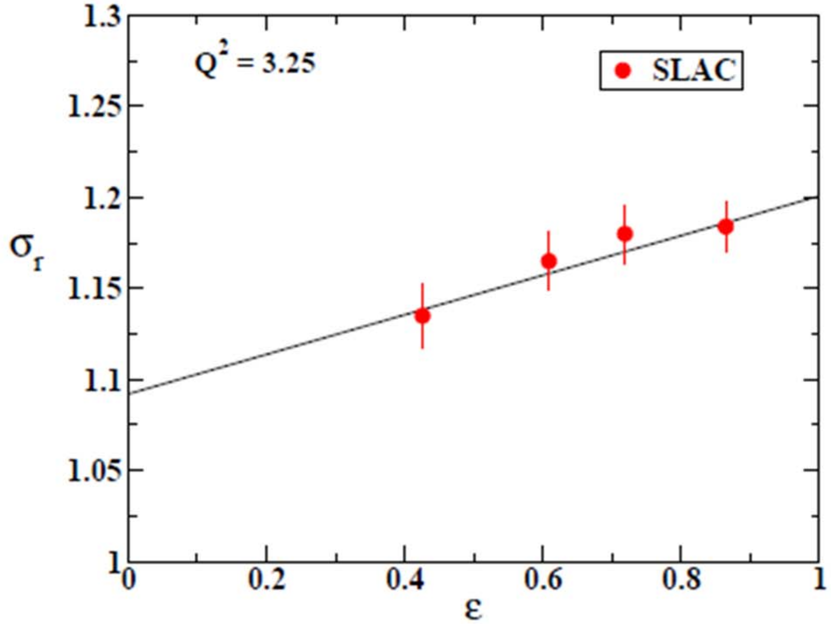
$$\sigma_r \propto \frac{\epsilon}{\tau}\left(\frac{G_E}{G_D}\right)^2 + \left(\frac{G_M}{G_D}\right)^2$$

$G_D = (1 + Q^2/.71)^{-2} \qquad \tau = Q^2/4M^2$

At fixed Q^2: Vary ϵ

G_E is slope

G_M is intercept



For $Q^2 > 1$ measuring G_E becomes more difficult

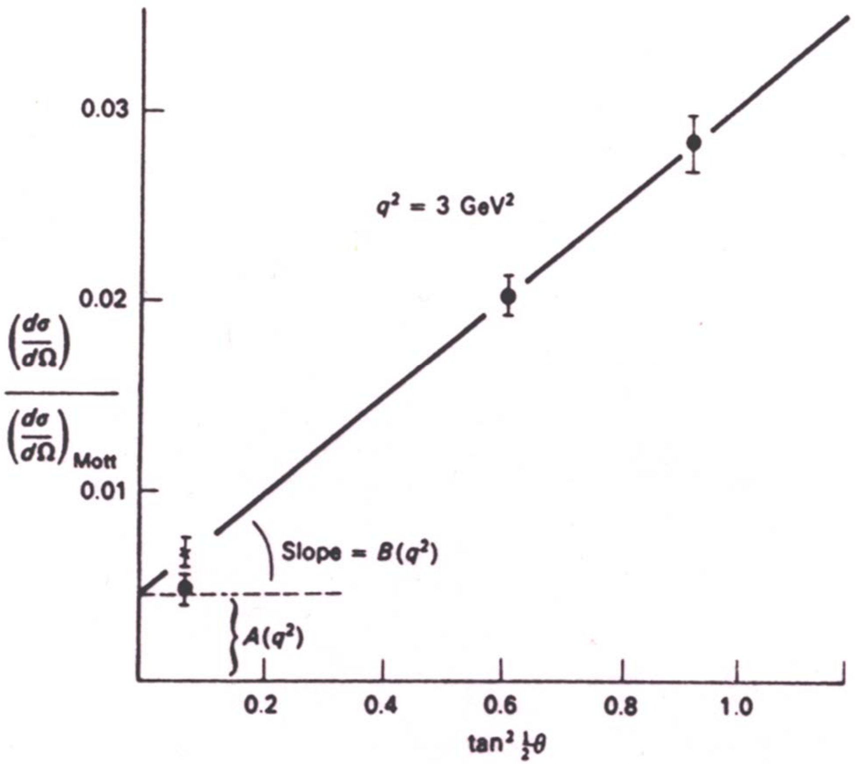
- G_E becomes a smaller fraction of σ
- At $Q^2 = 5$, G_E maximum 8% contribution to σ

(assuming $\mu G_E/G_M = 1$)

$$G_E = G_E(q^2) \qquad G_M = G_M(q^2) \qquad G_E^p(0) = 1 \qquad G_E^n(0) = 0$$

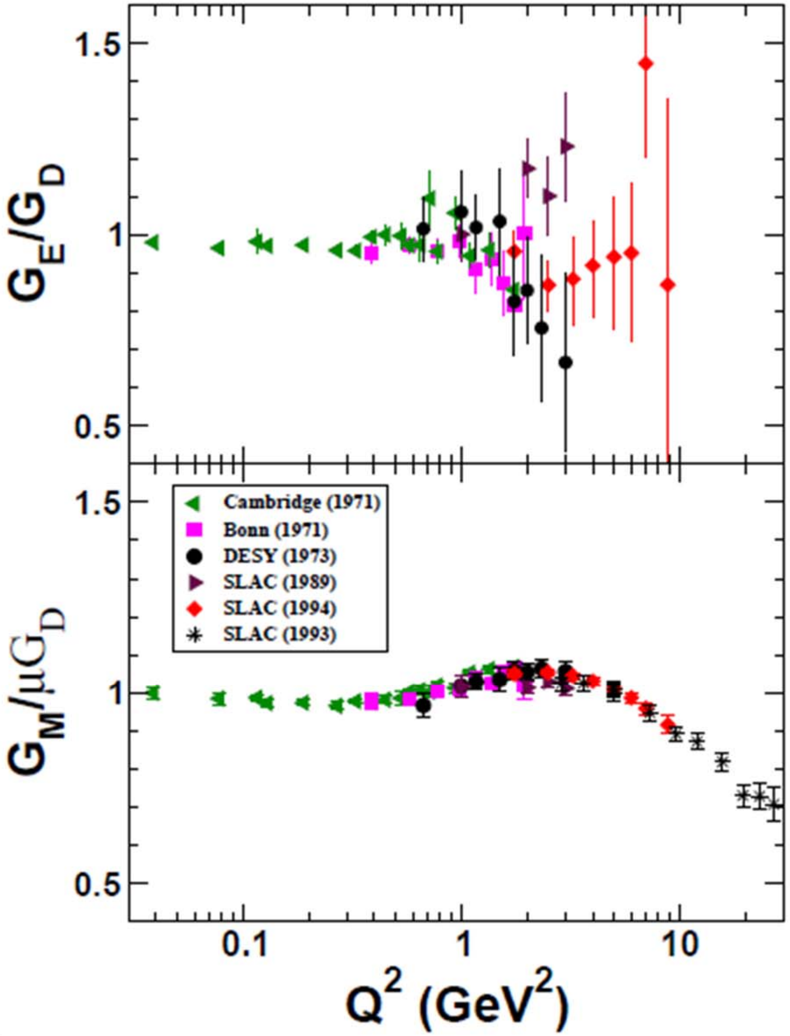
$$G_M^p(0) = +2.79 \qquad G_M^n(0) = -1.91$$

$$\left(\frac{d\sigma}{d\Omega}\right) \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{Mott} = A(q^2) + B(q^2)\tan^2\frac{\theta}{2}$$



Rosenbluth Plot

Proton Form Factors : G_M and G_E



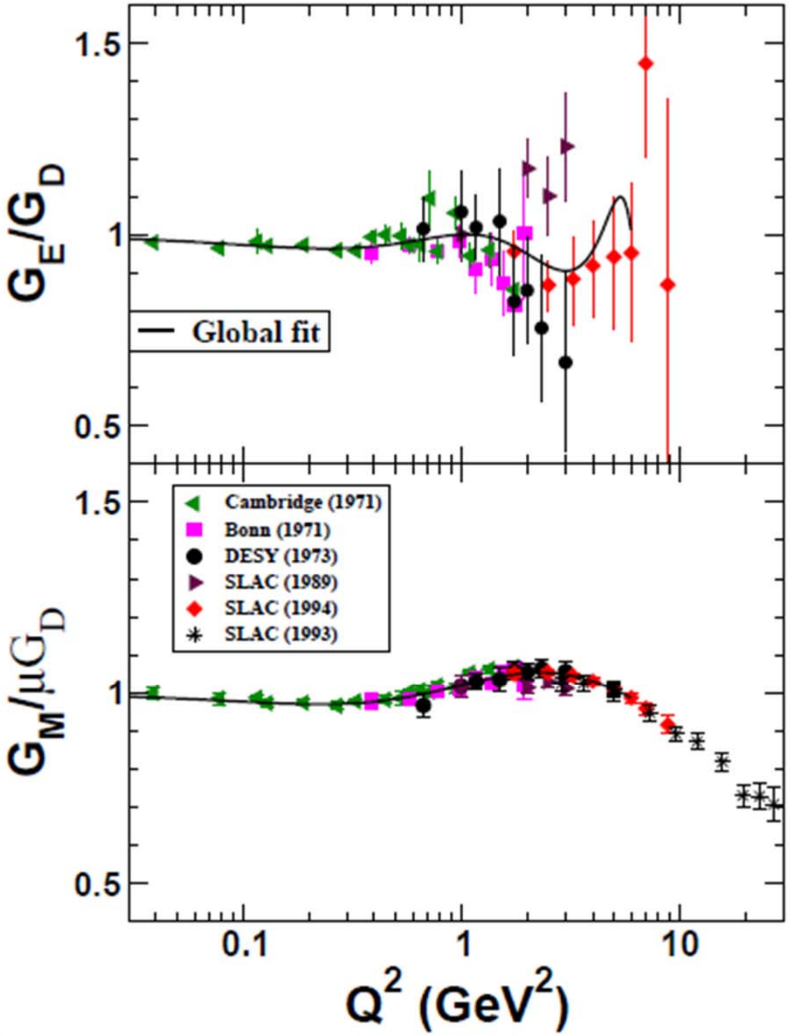
At all Q^2

→ G_M well measured

At $Q^2 > 1$

→ Error on G_E large

Proton Form Factors : G_M and G_E



At all Q^2

→ G_M well measured

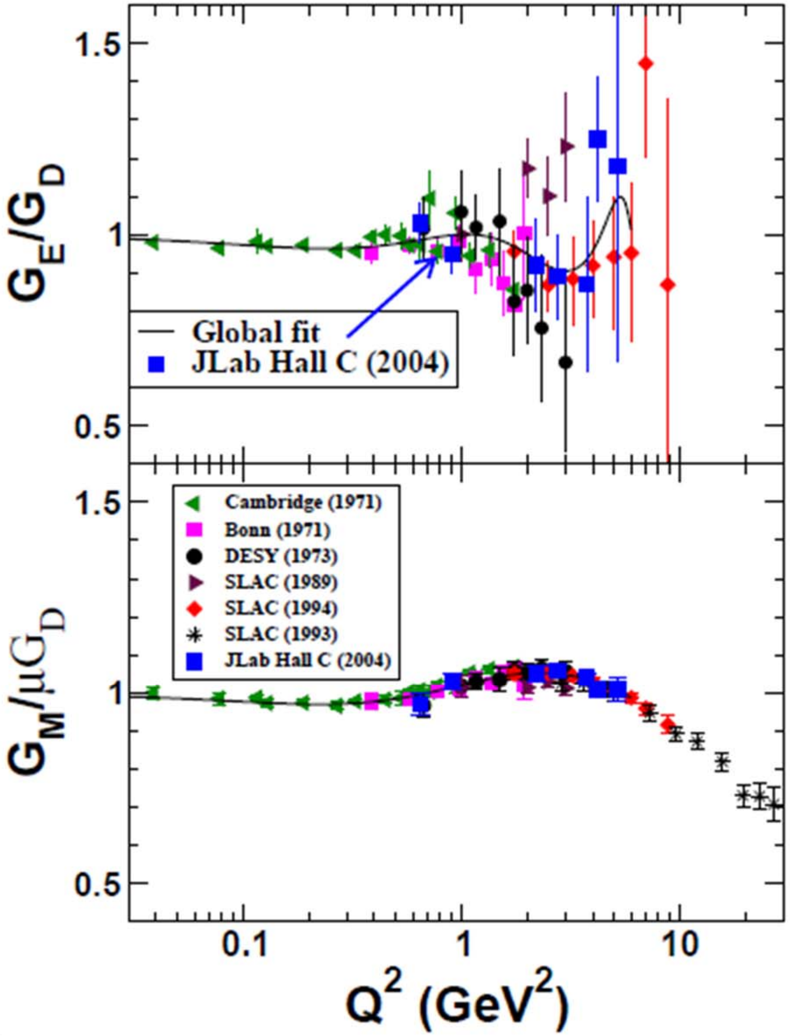
At $Q^2 > 1$

→ Error on G_E large

→ Recent global fit to world cross section data

J. Arrington PRC 69, 02201R (2004)

Proton Form Factors : G_M and G_E



At all Q^2

→ G_M well measured

At $Q^2 > 1$

→ Error on G_E large

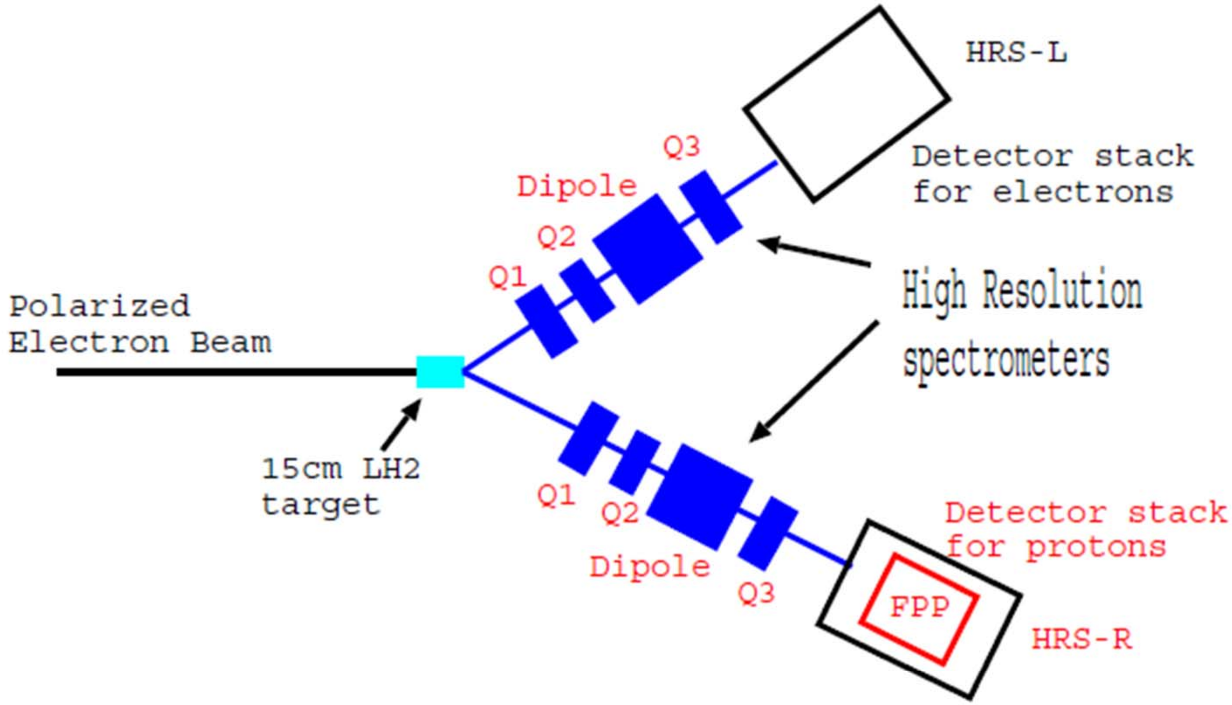
→ Recent global fit to world cross section data

J. Arrington PRC 69, 02201R (2004)

→ Recent Hall C data

M. E. Christy, PRC 70, 015206 (2004)

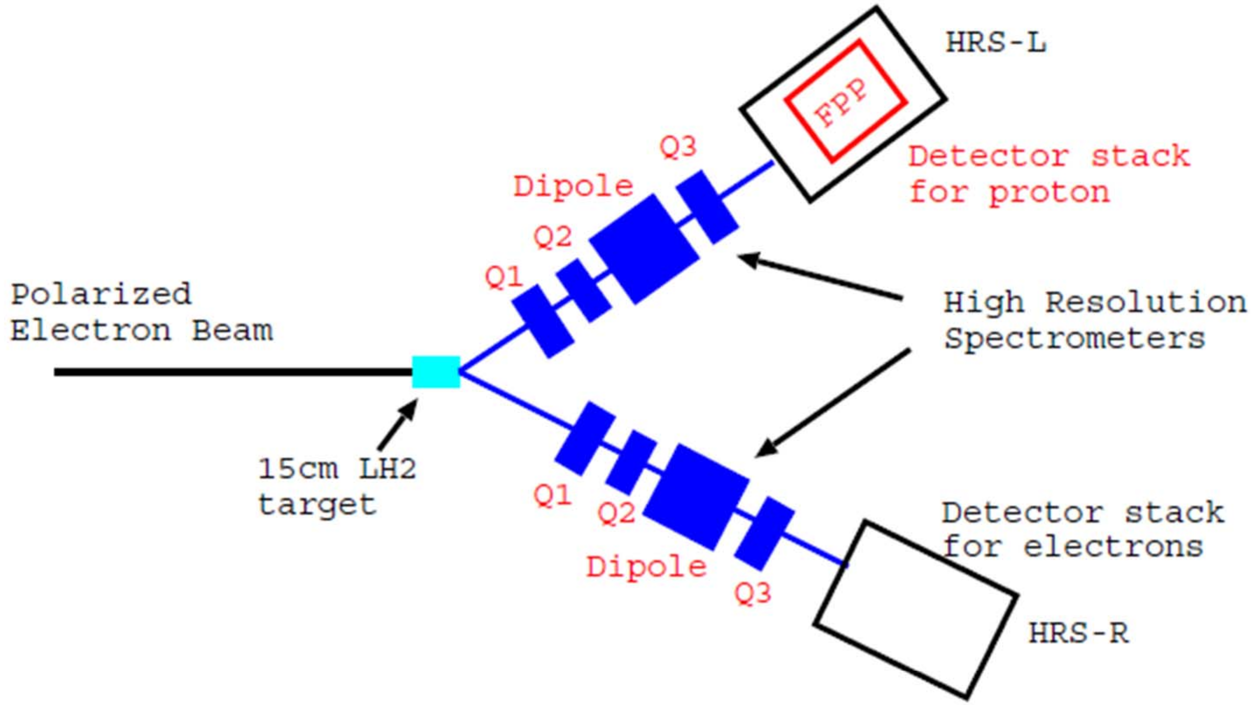
JLab Recoil Polarization Experiments



1st JLab experiment Electrons detected HRS-L
Protons detected HRS-R

Covered Q^2 from 0.5 to 3.5 GeV^2.

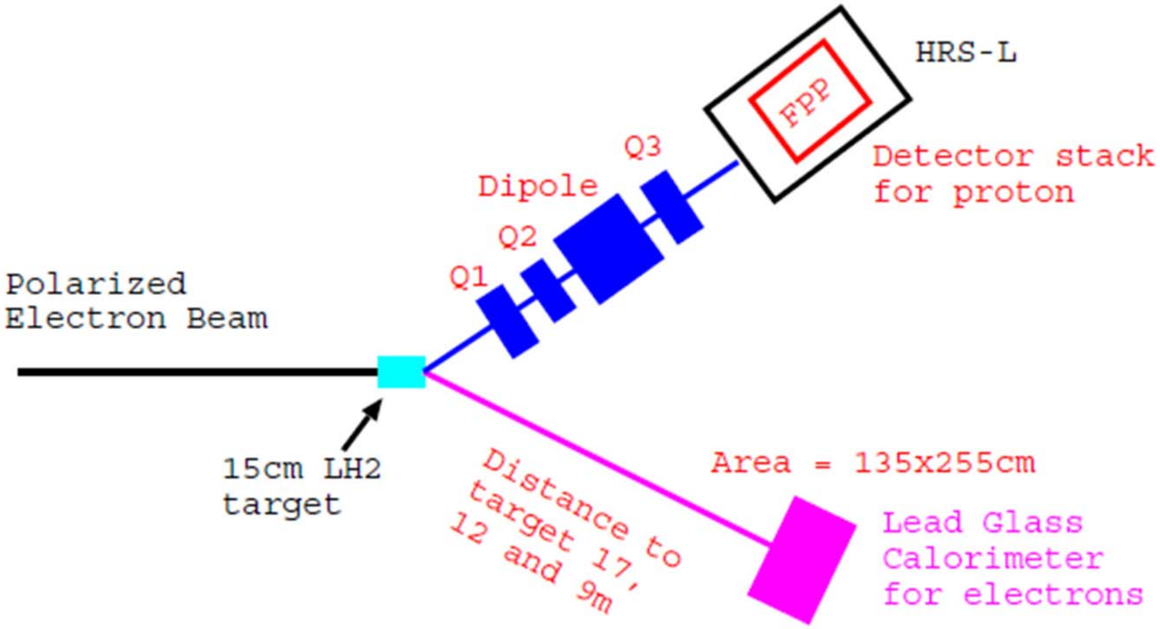
JLab Recoil Polarization Experiments



2nd JLab experiment   Protons detected HRS-L

Covered Q^2 from 3.5 to 5.6 GeV^2.

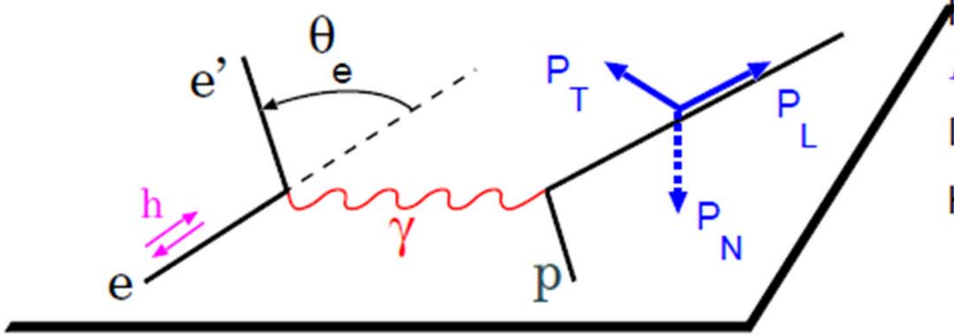
JLab Recoil Polarization Experiments



2nd JLab experiment   Protons detected HRS-L
Electrons detected calorimeter

Covered Q^2 from 3.5 to 5.6 GeV^2.

Spin Transfer Reaction $^1\mathbf{H}(\vec{e}, e'\,\vec{p})$



Helicity dependent : P_N, P_T, P_L

$P_N = 0$ for elastic

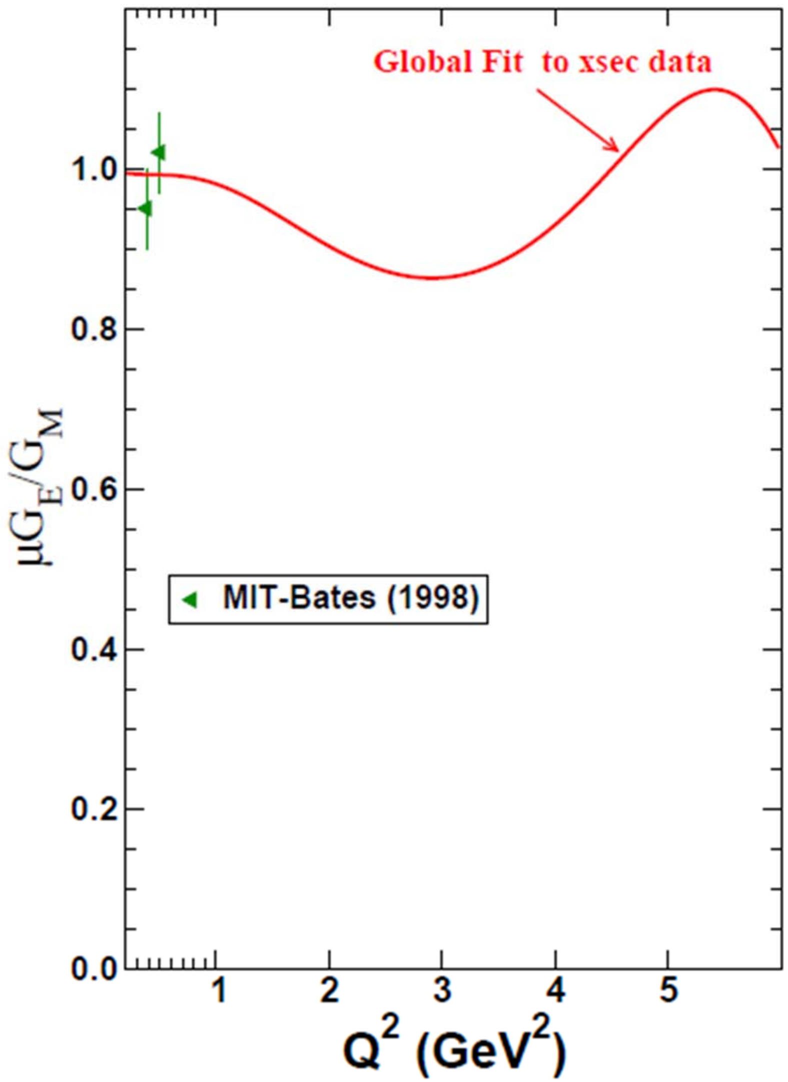
In 1γ approx.

Helicity independent $P_N = 0$

$$\boxed{\frac{G_E}{G_M} = -\,\frac{P_T}{P_L}\,\frac{E_e+E_{e'}}{2M}\,\tan\left(\frac{\theta_e}{2}\right)}$$

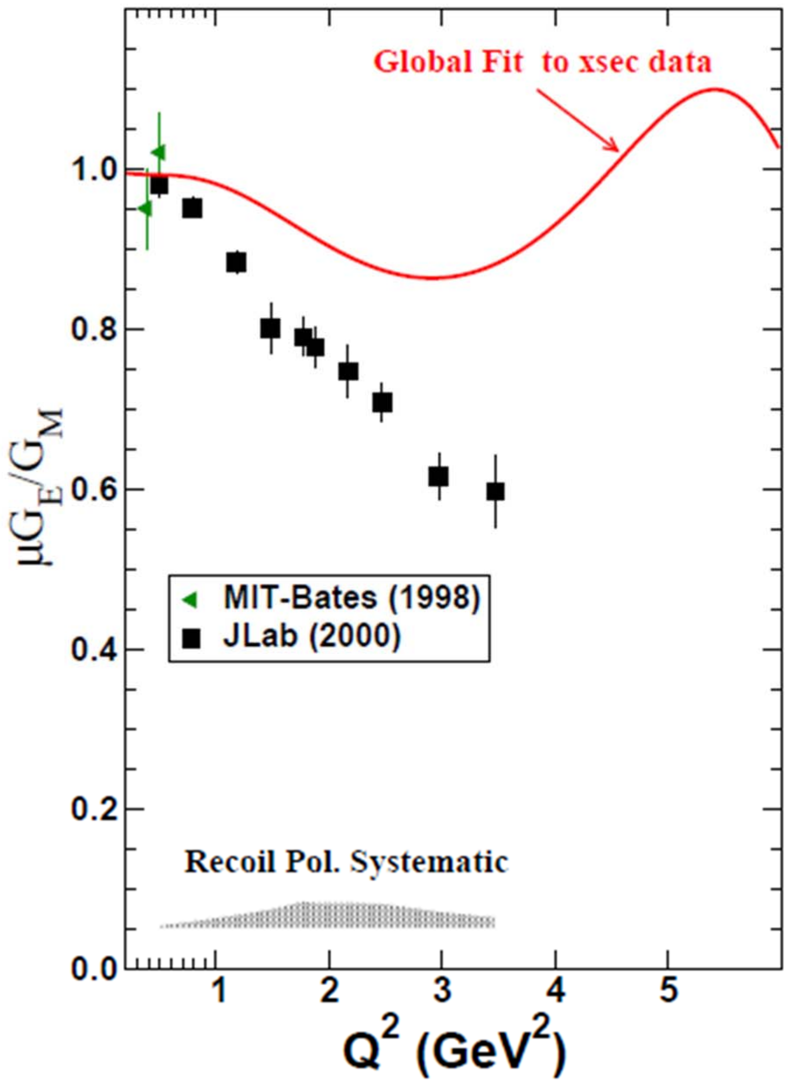
- Experiments detect the electron and proton in coincidence
- Proton spin measured by second scattering in polarimeter
- Experiments done at MIT-Bates, Mainz and JLab

G_E/G_M from recoil polarization



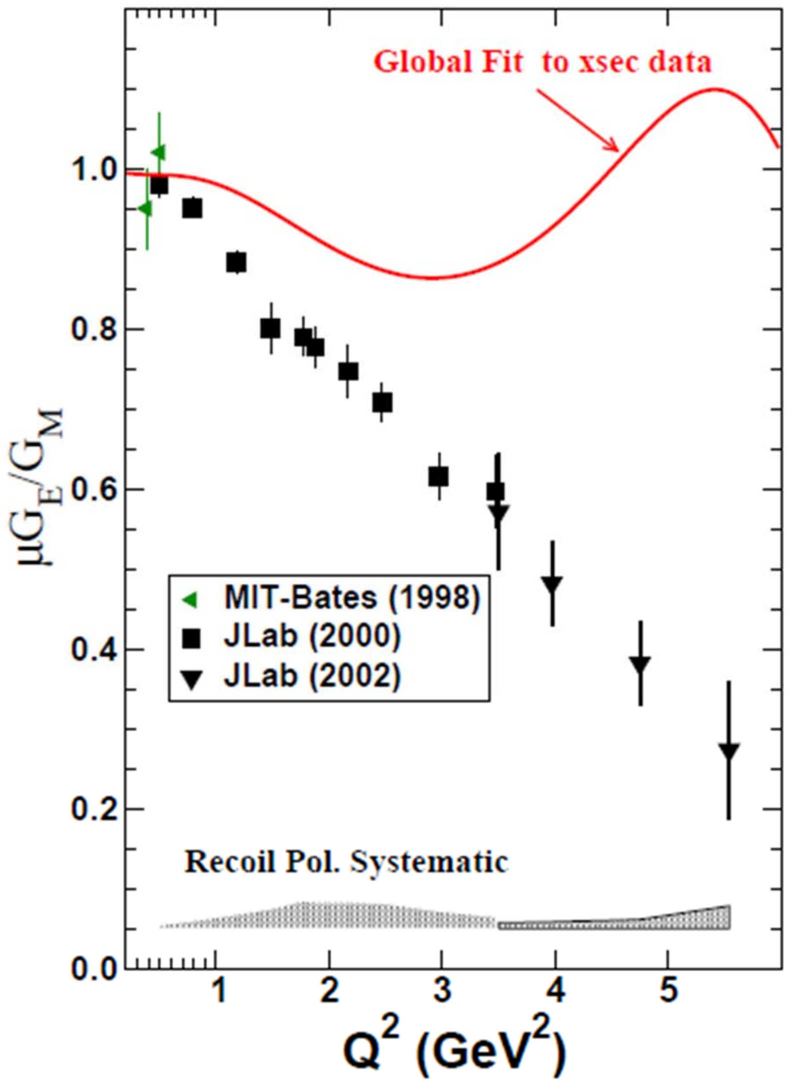
- MIT-Bates experiment

G_E/G_M from recoil polarization



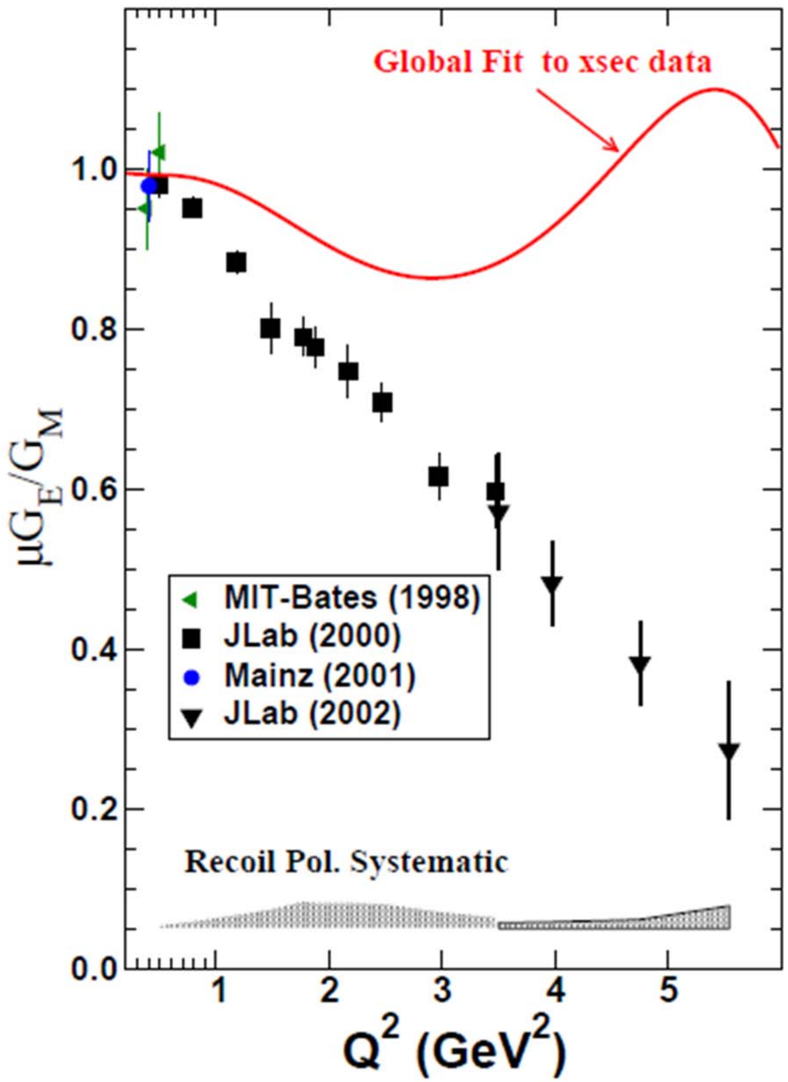
- MIT-Bates experiment
- 1st JLab experiment
  Used two HRS

G_E/G_M from recoil polarization



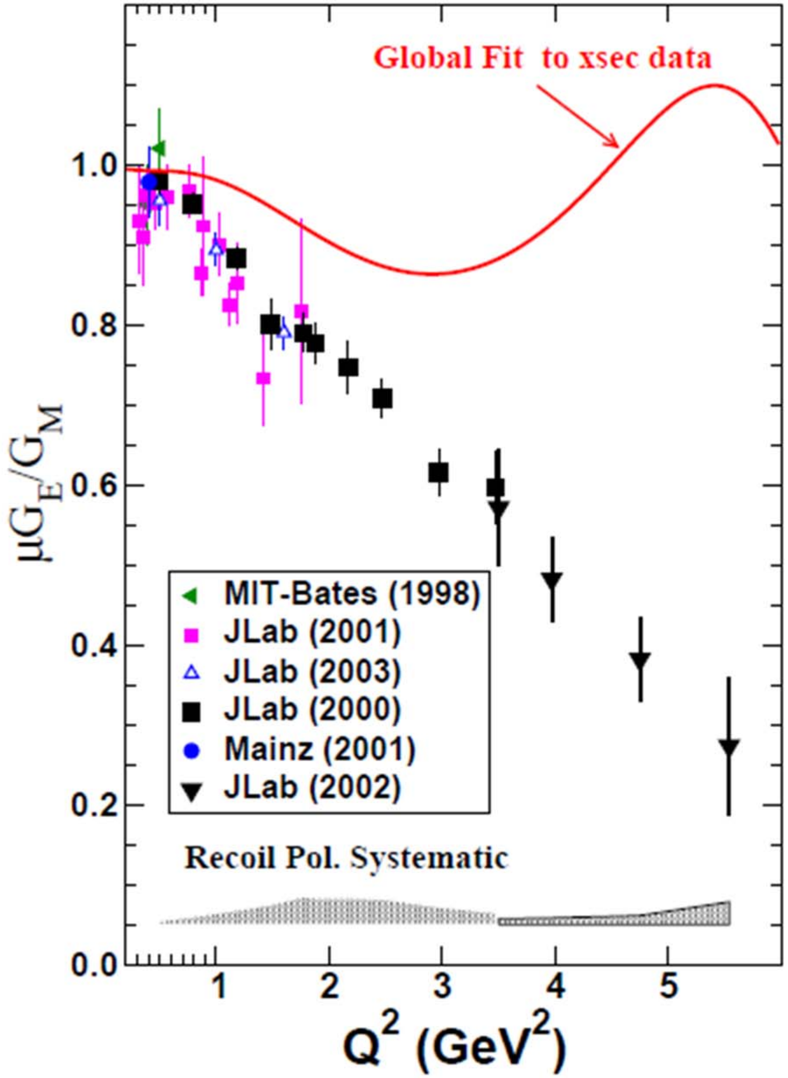
- MIT-Bates experiment
- 1st JLab experiment
  Used two HRS
- 2nd JLab experiment
  Used HRS+Calo

G_E/G_M from recoil polarization



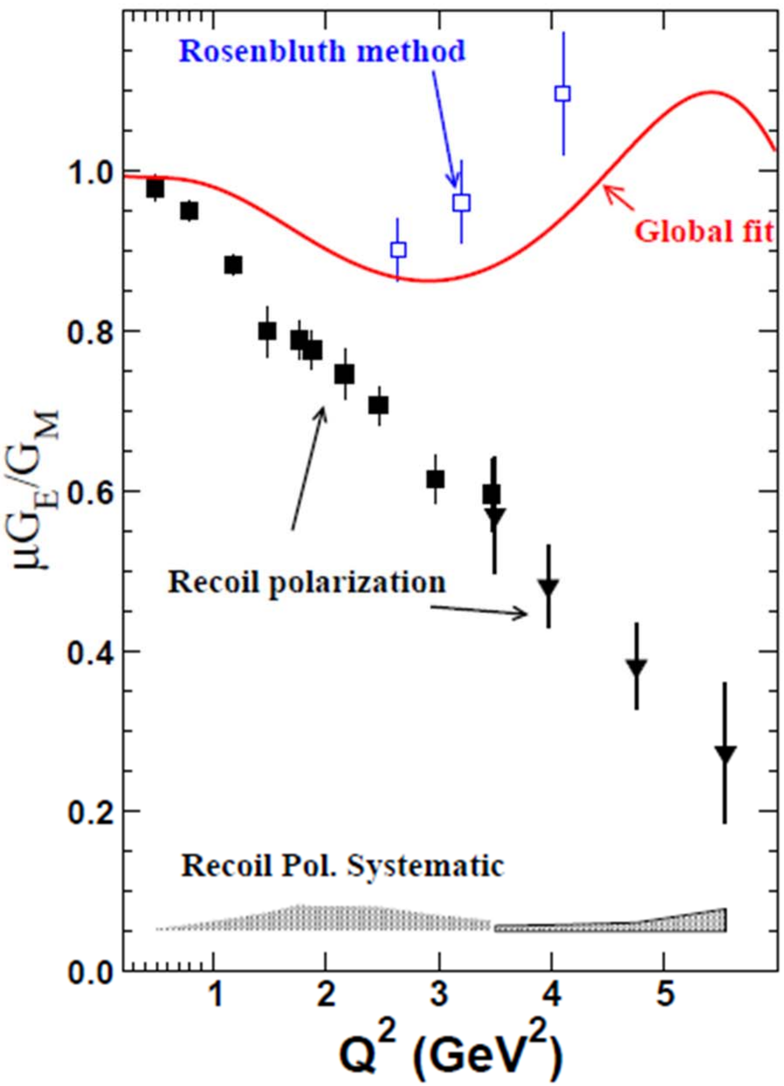
- MIT-Bates experiment
- 1st JLab experiment
  Used two HRS
- 2nd JLab experiment
  Used HRS+Calo
- Mainz experiment

G_E/G_M from recoil polarization



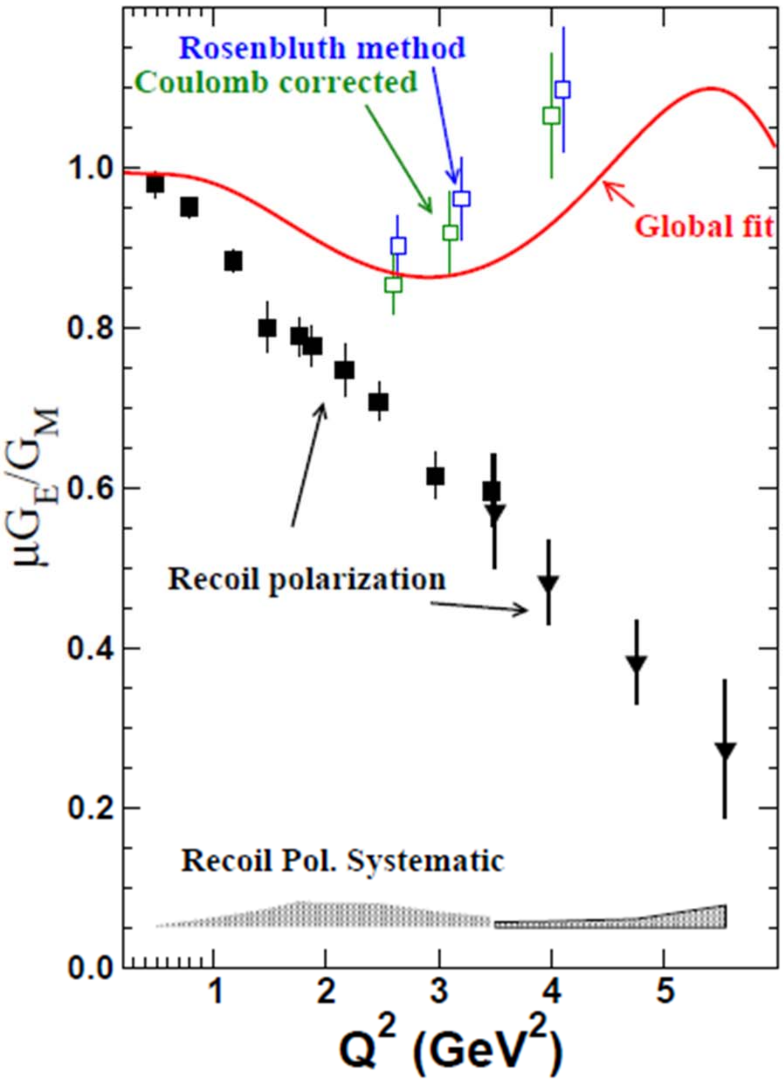
- MIT-Bates experiment
- 1st JLab experiment
  Used two HRS
- 2nd JLab experiment
  Used HRS+Calo
- Mainz experiment
- Secondary part of other JLab experiments.

Comparison of G_E/G_M



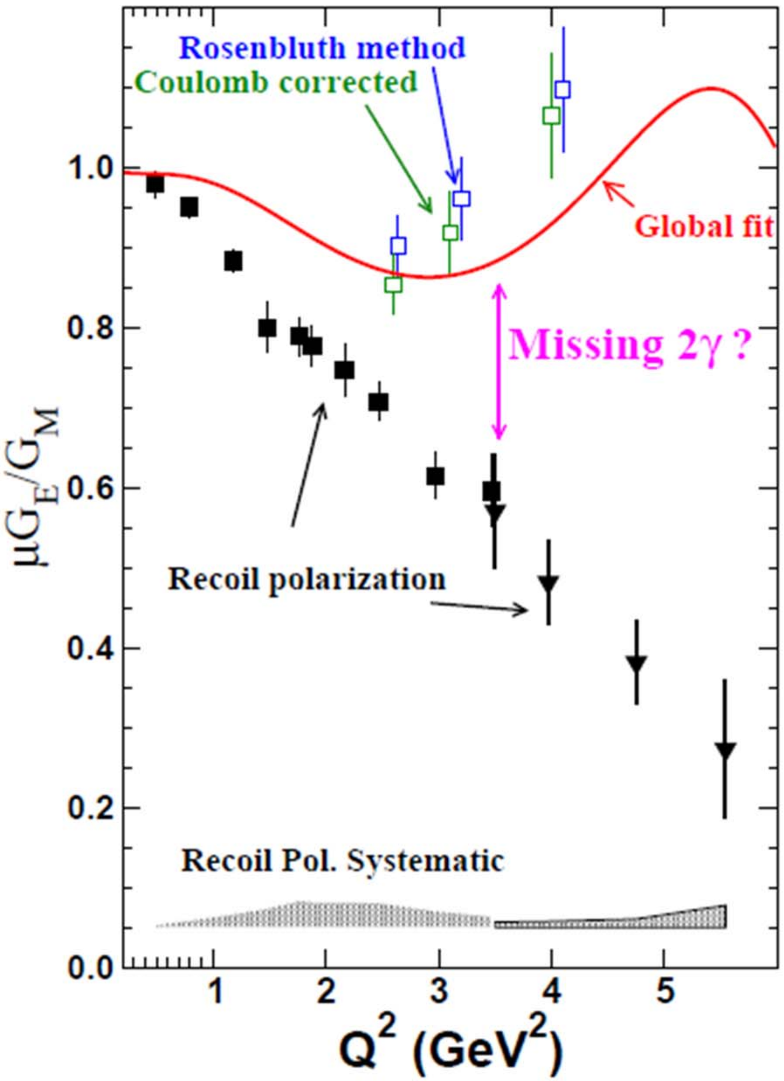
- Hall A Rosenbluth G_E/G_M agrees with previous cross section measurements
- Discrepancy between G_E/G_M from recoil polarization and from cross section measurement persistent

Comparison of G_E/G_M



- Coulomb correction (soft multi-γ) is small correction to xsec data

Comparison of G_E/G_M



- Coulomb correction (soft multi-γ) is small correction to xsec data
- Missing physics from 2γ exchange?

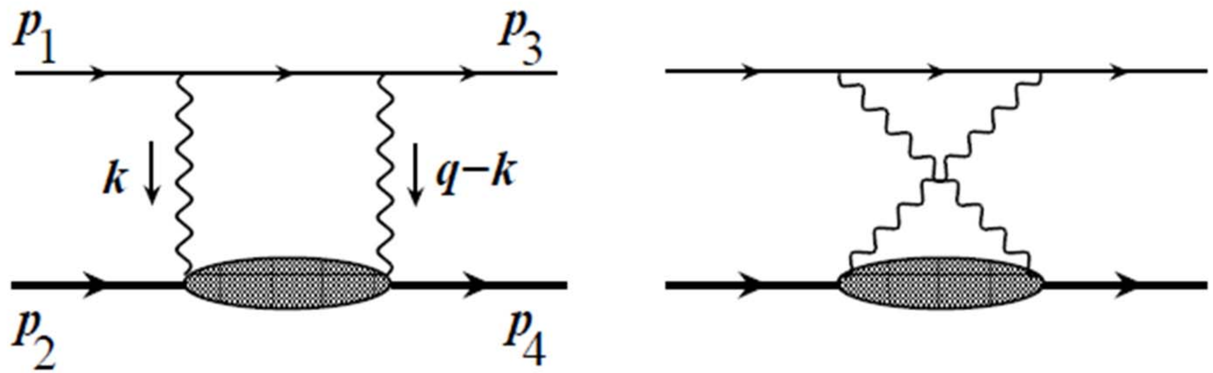
$$\sigma_r \propto \frac{\epsilon}{\tau} G_E^2 + G_M^2 + \epsilon \sigma_{2\gamma}(\epsilon, Q^2)$$

Explain discrepancy with

$$\sigma_{2\gamma}(\epsilon, Q^2) \sim 6\%$$

with small ϵ, Q^2 dependence

2γ exchange contribution



- Nucleon elastic intermediate state *P.G. Blunden, W. Melnitchouk, J.A. Tjon*
- GPD calculation *A. V. Afanasev, S. J. Brodsky, C. E. Carlson, Y. Chen, M. Vanderhaeghen*
- Various proposals to measure 2γ exchange contribution
  - ϵ dependence of $\sigma_{e^+p}/\sigma_{e^-p}$
  - Precision ϵ dependence of σ_{e^-}
  - ϵ dependence of P_T/P_L.
- Talks in the afternoon 2γ session