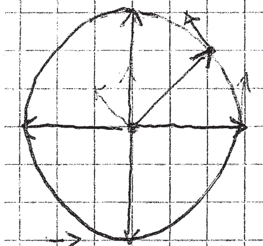
Moto circolare uniforme in coordinate cartesiane



$$ds = r\,d\varphi$$

$$v = r\frac{d\varphi}{dt} = r\omega_0$$

$$\frac{d\varphi}{dt} = \dot\varphi$$

$$\vec r = x(t)\,\hat i + y(t)\,\hat j$$

$$x(t) = r_0 \cos\omega_0 t$$
$$y(t) = r \sin\omega_0 t$$

$$\vec v = \frac{d\vec r}{dt} = -\omega_0 r \sin\omega_0 t\,\hat i + \omega_0 r \cos\omega_0 t\,\hat j$$

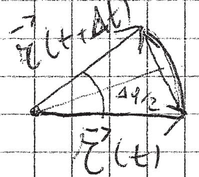
$$|\vec v| = v_s = \omega_0 r$$

$$\vec a = \frac{d\vec v}{dt} = -\omega_0^2 \vec r$$

$$\vec a_c = -\omega_0^2 \vec r$$

$$\frac{d^2\vec r}{dt^2} = -\omega_0^2 \vec r \qquad \begin{cases} \frac{d^2x}{dt^2} = -\omega_0^2 x \\ \frac{d^2y}{dt^2} = -\omega_0^2 y \end{cases}$$

Moto circolare uniforme solo calcolo vettoriale



$$\Delta\varphi$$

$$\vec v = \lim_{\Delta t\to 0}\frac{\Delta\vec r}{\Delta t} = \frac{d\vec r}{dt}$$

r costante $\quad \frac{d\vec r}{dt} \perp r$

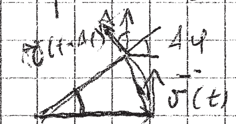
$$\Delta r = 2r\sin\frac{\Delta\varphi}{2}$$

$$v = \lim_{\Delta t\to 0}\frac{\Delta r}{\Delta t} \qquad \frac{\Delta r}{\Delta t} = \frac{\Delta r}{\Delta\varphi}\frac{\Delta\varphi}{\Delta t}$$

$$\frac{\Delta r}{\Delta t} = r\sin\left(\frac{\Delta\varphi}{2}\right)\Big/\frac{\Delta\varphi}{2}\cdot\frac{\Delta\varphi}{\Delta t}$$

$$\lim_{\Delta t\to 0}\frac{\Delta r}{\Delta t} = r\dot\varphi$$

$\dot\varphi = \omega_0$ = costante $\quad r\omega_0$

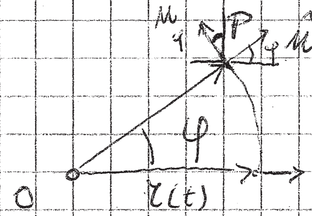


$$\frac{\Delta r}{r} = \Delta\varphi = \frac{\Delta v}{v}$$

$$\lim_{\Delta t\to 0}\frac{\Delta \vec v}{\Delta t} = \perp \vec v$$

$$\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta r}{\Delta t} = \frac{v^2}{r}$$

Moto circolare uniforme in coordinate polari



$$\vec r = r\,\hat u_r(P)$$

$$\hat u_r = \cos\varphi\,\hat i + \operatorname{sen}\varphi\,\hat j$$

$$u_\varphi = -\operatorname{sen}\varphi\,\hat i + \cos\varphi\,\hat j$$

$$\vec v = \frac{d\vec r}{dt} = \frac{dr}{dt}\hat u_r + r\frac{d\hat u_r}{dt}$$

$$\frac{d\hat u_r}{dt} = -\operatorname{sen}\varphi\,\hat i\,\dot\varphi + \cos\varphi\cdot\hat j\,\dot\varphi = \dot\varphi\,\hat u_\varphi$$

$$\vec v = \dot r\,\hat u_r + r\dot\varphi\,\hat u_\varphi$$

$r = \text{cost} \quad \dot r = 0 \qquad \dot\varphi = \text{cost} = \omega_0$

$$\vec v = r\dot\varphi\,\hat u_\varphi = r\omega_0\,\hat u_\varphi$$

Continuo con moto circ uniforme

$$\vec v = r\dot\varphi\,\hat u_\varphi$$

$$\vec a_c = \frac{dr}{dt}\,\omega_0\,\hat u_\varphi + r\frac{d\dot\varphi}{dt}\hat u_\varphi + r\dot\varphi\frac{d\hat u_\varphi}{dt}$$

$$\frac{du_\varphi}{dt} = \frac{d\hat u_\varphi}{d\varphi}\frac{d\varphi}{dt} = \dot\varphi\left(-\cos\varphi\,\hat i - \operatorname{sen}\varphi\,\hat j\right) = -\dot\varphi\,u_r$$

$$\vec a_c = -r\dot\varphi^2\,\hat u_r$$

$$\vec a = \underset{(\ddot r - r\dot\varphi^2)}{a_r}\,\hat u_r + \underset{(2\dot r\dot\varphi + r\ddot\varphi)}{a_\varphi}\,\hat u_\varphi = \frac{1}{r}\frac{d}{dt}\left(r^2\dot\varphi\right)$$