A. Bianconi -- SMALL FACTORIZATION BREAKING -- Ferrara 15.10.07

After recent works (2005-2007) by several authors on factorization violating processes, I consider a class of rescattering processes for which:

(i) rescattering potentially undermines factorization
(ii) rescattering effects are small (the meaning of "small" is discussed later).
(iii) certain regularity properties are respected.

The presented arguments presently apply to initial state interactions only, i.e. to Interactions between systems that exist at the same time. E.g., Drell-Yan.

Relevant definitions.

$$q(x,k_T) = P^+ \int dz^- d^2b \, \exp(-ixP^+z^-) \; \exp(ik_T b) \, g(z^-,b)$$

$$= \int d\xi \, d^2b \, \exp(-ix\xi) \; \exp(ik_T b) \, G(\xi,b)$$

"rescaled" coordinate and distribution**:** $\xi = P^+ z^-$ and $G(\xi,b) = g(z^-,b)$.

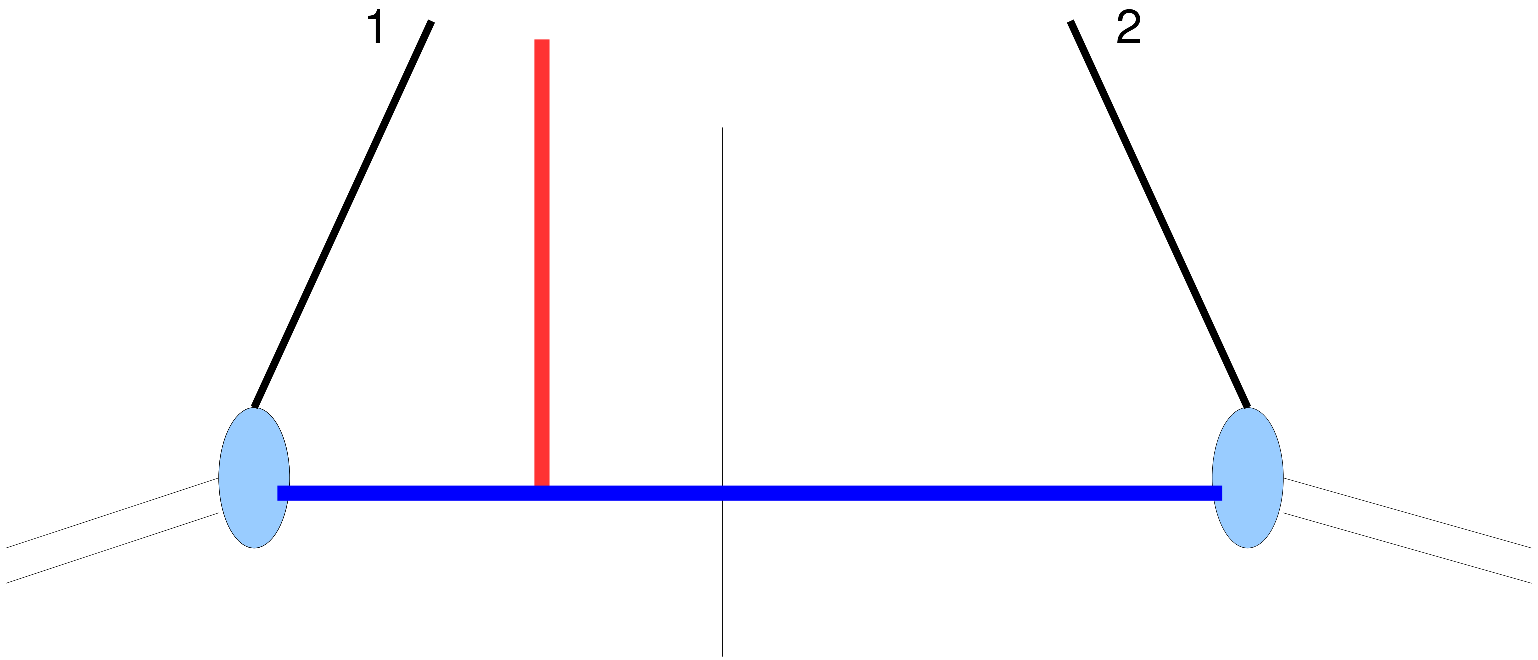
Rescaled quantities are not singular when $P^+ \rightarrow \infty$, $x = O(1)$ (scaling limit).

I will use the **mixed fourier transform h(x,b)**:

$$h(x,b) = \int d\xi \; \exp(-ix\xi) \; G(\xi,b) \qquad\qquad q(x,k_T) = \int d^2b \; \exp(ibk_T) \; h(x,b)$$

h(x,b) and G(ξ,b) will be indicated as **h(x)** and **G(ξ)** where b is not explicitly needed.

G(ξ) = (Imaginary part of the) amplitude for creating a **quark hole in a hadron** in the spacetime point 1 and propagating it up to 2. $\xi \equiv \xi(2) - \xi(1)$.



The longitudinal fraction x is the same in 1 and 2 by definition.
In absence of rescattering (red line) x is also **locally** conserved.

Factorization breaking rescattering removes local x-conservation.
I use the **magnitude of x-changes** to estimate the size of rescattering **effects**.

Standard definition of parton distribution and of rescattering operator :

$$\mathbf{h(x)} \equiv \int d\xi \; e^{-i x \xi} < P \,|\, \Psi(\xi) \; F_\epsilon(\xi) \; \Psi^+(0) \,|\, P >$$

$\mathbf{F_\epsilon(\xi)}$ is taken as **scalar**. Else, a **sum** accompanies all the following passages.

ϵ gives the overall strength of the interactions, with the condition $\mathbf{F_\epsilon = 1}$ **for** $\mathbf{\epsilon = 0}$.

To quantify the rescattering effects, I introduce the frequency spectrum of F_ϵ:

$$F_\epsilon(\xi) = \int dy \; e^{-i y \xi} \; f_\epsilon(y)$$

In particular, $f_\epsilon(y)$ must coincide with a delta function for $\epsilon = 0$.

If $f_\epsilon(y)$ is nonzero for a given finite y, it means that **changes x → x+y are present with weight** $\mathbf{f_\epsilon(y)}$ **in rescattering processes.**

Getting back to the starting relations, I insert the definition

$$F_\epsilon(\xi) = \int dy\ e^{-i y \xi}\ f_\epsilon(y)$$

into $\quad h(x) = \int d\xi\ e^{-i x \xi} < P \,|\, \Psi(\xi)\ F_\epsilon(\xi)\ \Psi^+(0) \,|\, P >$

$$\mathbf{h(x)} = \int d\xi\ e^{-i x \xi} < P \,|\, \Psi(\xi) \int dy\ e^{-i y \xi}\ f_\epsilon(y)\ \Psi^+(0) \,|\, P > =$$

$$= \int dy\ f_\epsilon(y) \int d\xi\ e^{-i (x+y) \xi} < P \,|\, \Psi(\xi)\ \Psi^+(0) \,|\, P >$$

$$= \int dy\ f_\epsilon(y)\ h_0(x+y) \quad = \boldsymbol{\int dy\ f_\epsilon(x\text{-}y)\ h_0(y)}$$

where $\mathbf{h_0}$ is the distribution function **in absence of rescattering**.

Now a set of requirements on the effects of rescattering may be formulated in terms of the frequency spectrum $f_\epsilon(x\text{-}x')$.

1) Although $f_\epsilon(y)$ is not a delta function, I require it to consist of a **peak at y = 0**.

2) The peak is **regular on the real y-axis**.

3) The **width ϵ** of the peak is small: **$\epsilon \ll 1$**.

4) $f_\epsilon(y)$ is **negligible for $|y| \gg \epsilon$**.

5) For $\epsilon = 0$, $f_\epsilon(y) = \delta(y)$.

6) $f_\epsilon(y)$ is **causality-modified**, so that its anti-transform $F_\epsilon(\xi)$ contains $\theta(\xi)$.
(this factor is present in the ξ-fourier transform defining q(x), so it can be added or removed from $F_\epsilon(\xi)$ without consequences, since $\theta(\xi)\theta(\xi) = \theta(\xi)$)

Properties (1) to (5) allow one to approximate: $\boldsymbol{\pi f_\epsilon(y) \cong \epsilon/(\epsilon^2+y^2)}$

For |y| up to a few ϵ-units, any regular peak may be approximated this way, and $\epsilon/(\epsilon^2+y^2) \to \pi\,\delta(y)$ for $\epsilon \to 0$.

The approximation has two poles $\mathbf{y = \pm\, i\, \epsilon}$.

Property (6) (causality) means to remove one of the two poles. So the used approximation is $\mathbf{f_\epsilon(y) \cong i \,/\, [2\pi\,(y + i\,\epsilon)]}$.

The fourier antitransform of this is $\mathbf{F_\epsilon(\xi) = \theta(\xi)\,\exp(-\,\epsilon\xi)}$.

The step function contains causality.
The exponential term suggests that the selected approximation for rescattering contains long-range self-neutralization properties (filtering).
This is ordinary for chaotic interactions. Here the relevant requirements are finiteness and smallness of $\boldsymbol{\epsilon}$.

$\mathbf{h(x) = \int dy\, f_\epsilon(x\text{-}y)\, h_0(y)}$ is dominated by the pole of $f_\epsilon(x\text{-}y)$ at $y = x + i\,\epsilon$.

The integration range [0,1] may be mapped onto $[-\infty , \infty]$ via $x = e^z/(1+e^z)$, and for the chosen form of $f_\epsilon(x\text{-}y)$ the pole contribution to the integral is the same as if the y-range were infinite.

More simply, one may assume that all that happens far from the range [0-1] has no relevance, and make as if the range were infinite.

$$\mathbf{h(x) = \int dy\, f_\epsilon(x\text{-}y)\ h_0(y) \ \simeq\ h_0(\, x + i\,\epsilon\,) \ \simeq\ h_0(x) + i\,\epsilon\,(d\, h_0(x) / dx) + O(\epsilon^2)}$$

The first correction to the real part is **second order in ϵ**.

This means that unless ϵ is really $\sim$ 1, the leading term gets **small corrections** from rescattering.

The imaginary correction contributes to two terms:

1) to the real part of the amplitude for quark hole propagation (h_0 comes from the imaginary part).

2) to a T-odd contribution to the imaginary part of the quark propagation amplitude.

To see this we remind that we are working on the mixed fourier transform **h(x,b)**:

$$\mathbf{q(x,k_T) = \int d^2b\ \exp(-ibk_T)\ h(x,b)}$$

The Trento definition of Sivers function for a proton polarized along y:

$$\mathbf{q(x,k_x,k_y) = q(x,k_T) + k_x\ s(x,k_T) / M}$$

may be b-fourier-tranformed to $\mathbf{h_1(x,b^2) + i\ b_x\ h_2(x,b^2)}$

Comparing it with **$h_0(x) + i\ \epsilon\ (d\ h_0(x)$ / $dx)$** we see that **IF** ϵ contains an **odd dependence on b_x**, we have an $O(\epsilon)$ Sivers asymmetry.

This odd-dependence produces a phase shift 90^0 in the b_x-fourier transform. Then the imaginary term $i\ \epsilon\ (d\ h_0(x)\ /\ dx)$ acquires the same phase as $q(x,k_T)$. The b_x-even part of ϵ contributes to the real part of the quark hole propagation amplitude.

Summarizing:

Rescattering whose frequency spectrum is a narrow and regular peak with width ϵ near the x-conservation condition, have average effect $O(\epsilon^2)$ on the T-even distributions, and $O(\epsilon)$ on the T-odd ones.