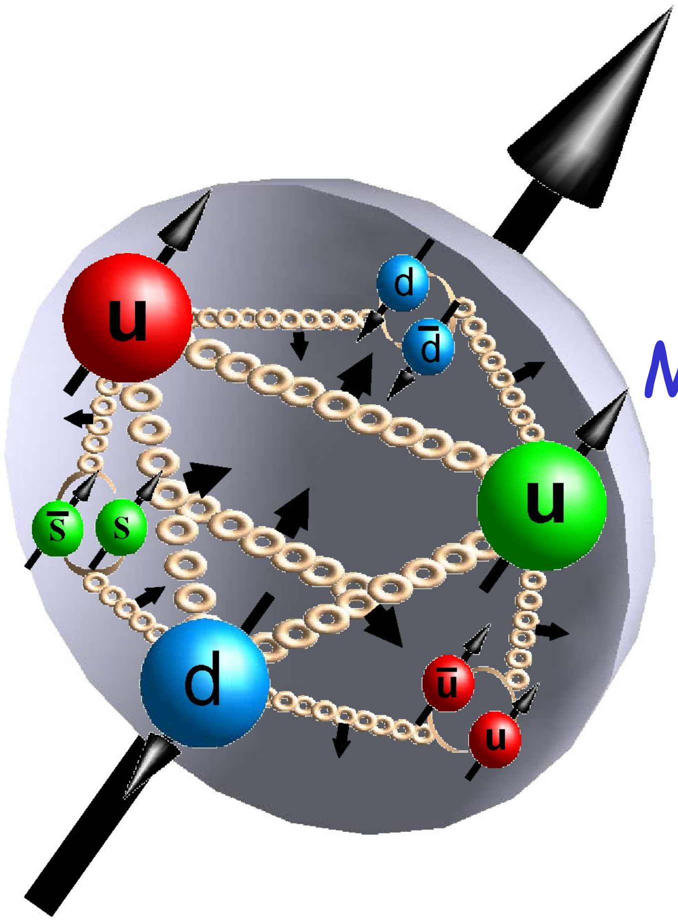


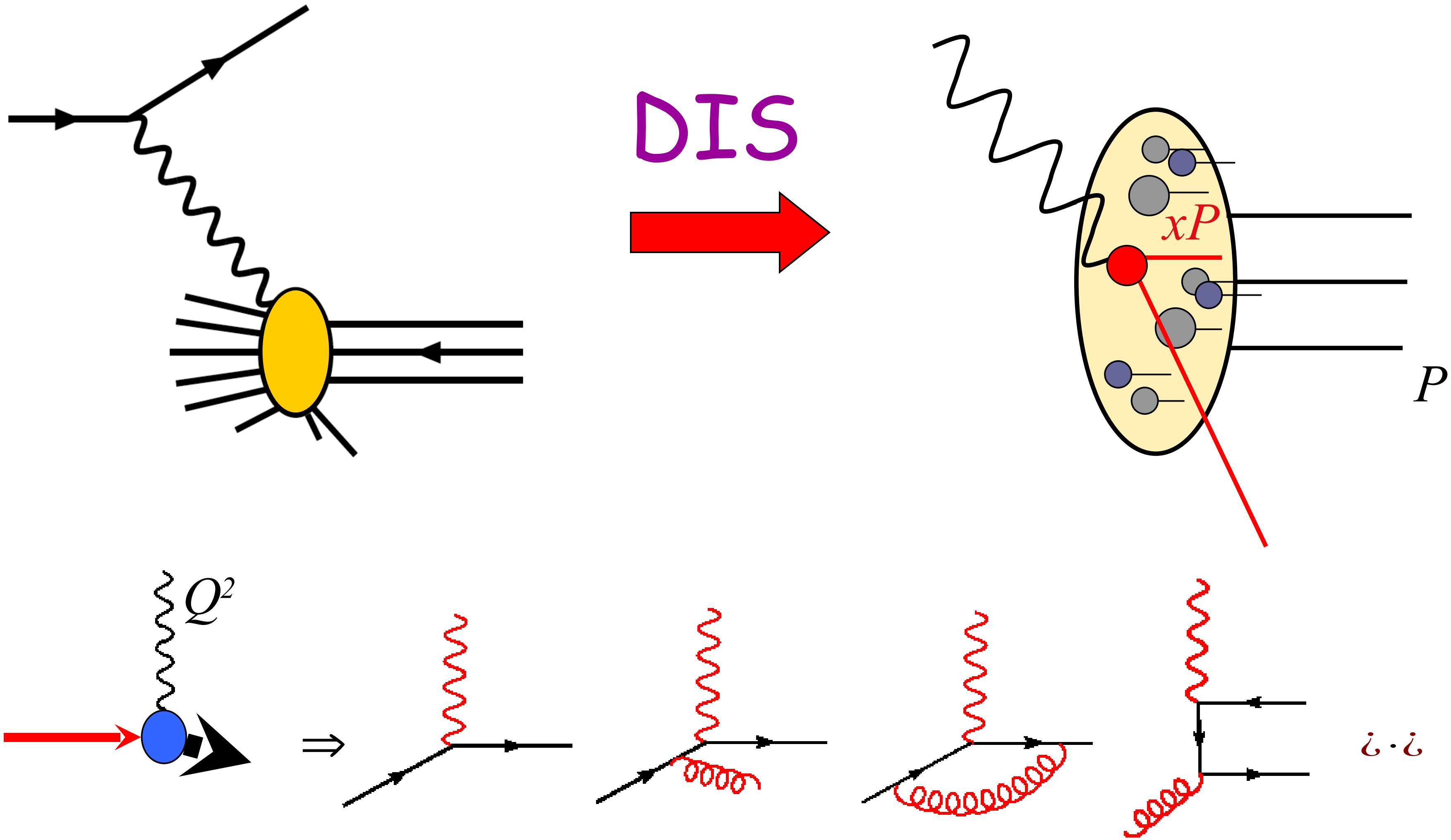
The transverse nucleon structure

Mauro Anselmino,
Torino University and INFN

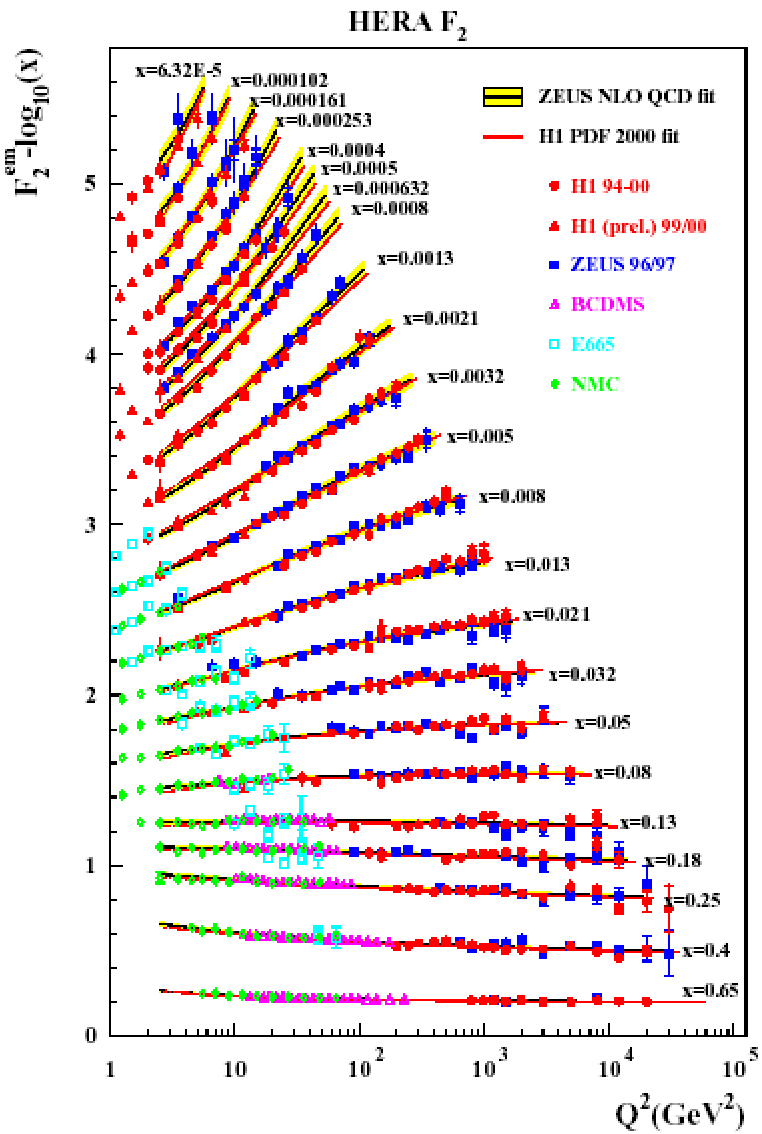
Nucleon Structure at FAIR

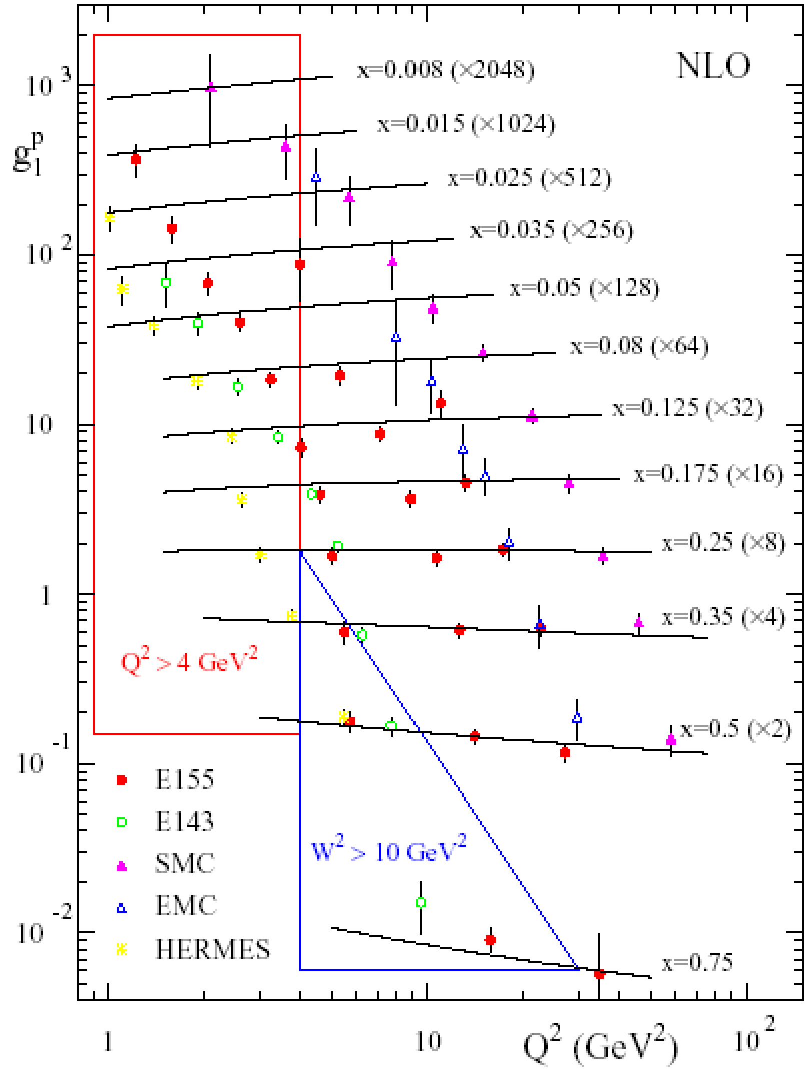
15-16 September, 2007 - Ferrara

The longitudinal structure of nucleons is "simple"
It has been studied for almost 40 years



essentially x and Q^2 degrees of freedom ….





Very good or good knowledge of unpolarized, $q(x,Q^2)$, $g(x,Q^2)$, and longitudinally polarized, $\Delta q(x,Q^2)$, partonic distributions; poor knowledge of $\Delta g(x,Q^2)$

$$\Delta q = q_+^+ - q_-^+ \qquad \Delta g = g_+^+ - g_-^+$$

helicity distributions

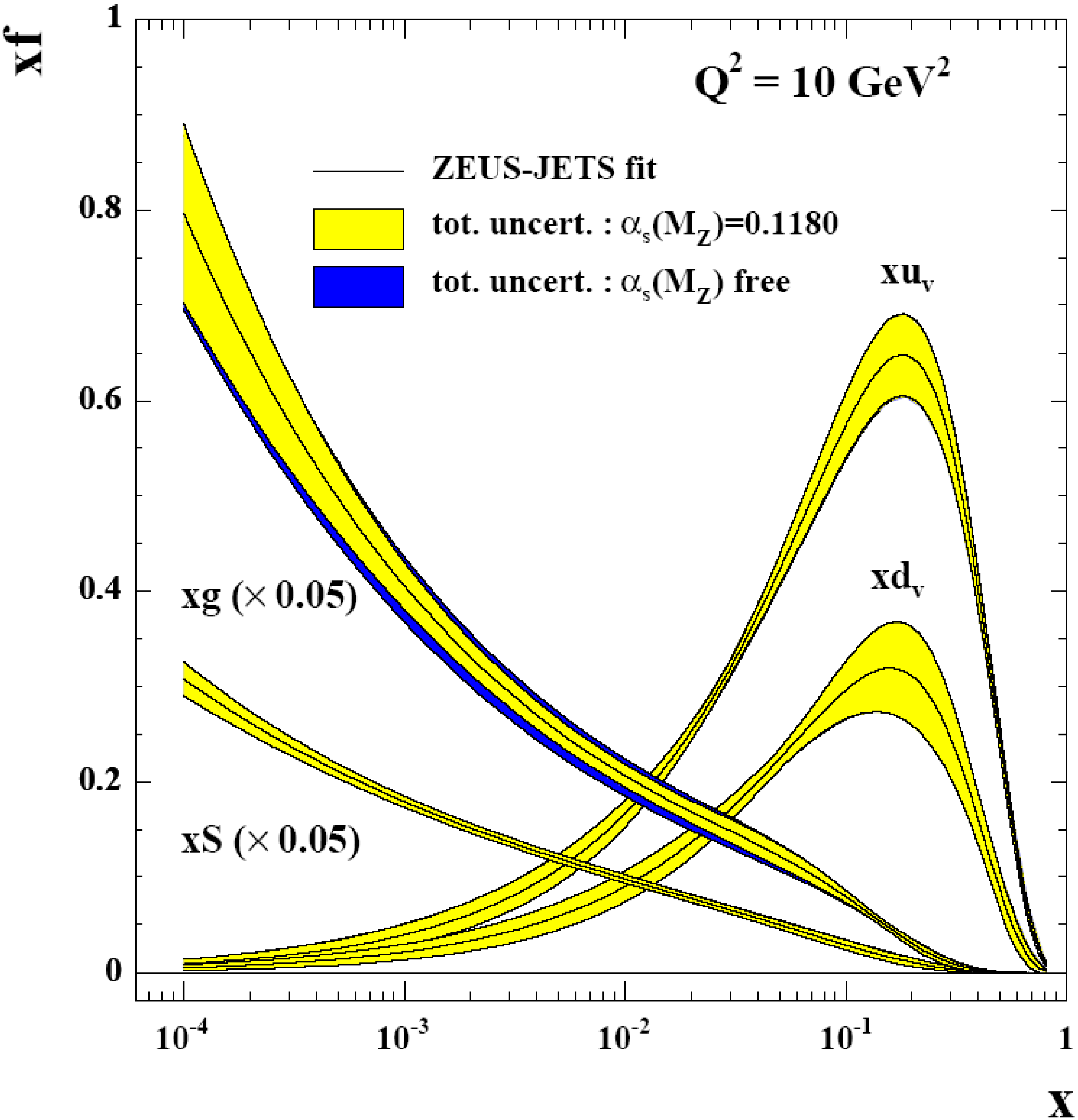
$$q = q_+^+ + q_-^+ \qquad g = g_+^+ + g_-^+$$

partonic distributions

What is the total amount of longitudinal spin carried by partons (inside a longitudinally polarized proton)?

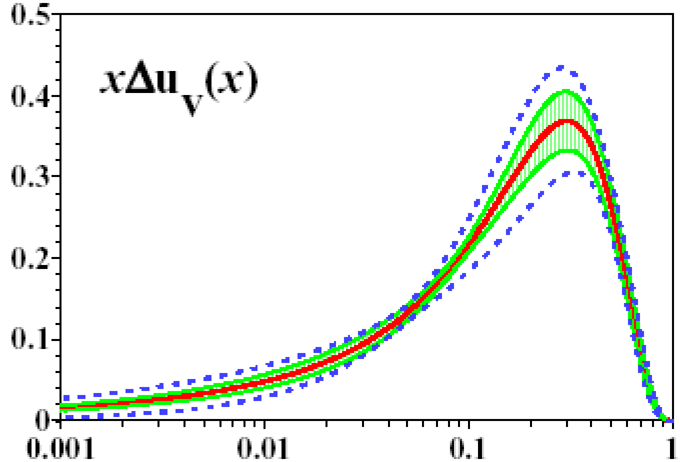
$$\frac{1}{2} = \langle S_q \rangle + \langle S_g \rangle + \langle L_q \rangle + \langle L_g \rangle \quad ?$$

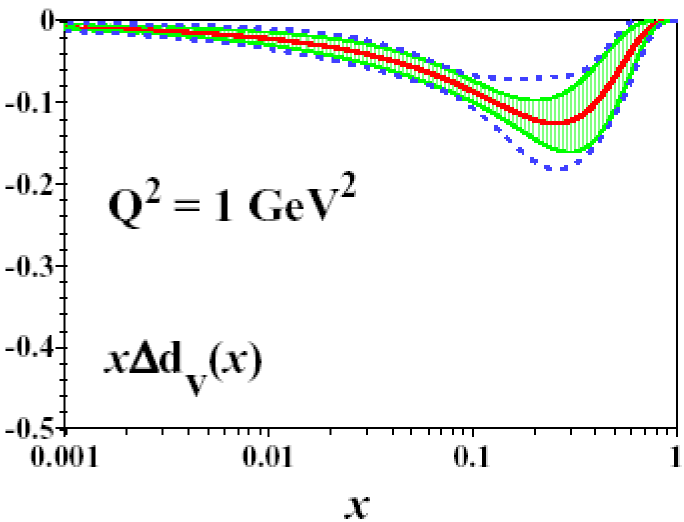
$$\left(\langle S_q \rangle = \frac{1}{2} \int_0^1 \Delta\Sigma(x,Q^2)\, \mathrm{d}x \qquad \langle S_g \rangle (Q^2) = \int_0^1 \Delta g(x,Q^2)\, \mathrm{d}x \qquad \Delta\Sigma = \sum_q \Delta q \right)$$

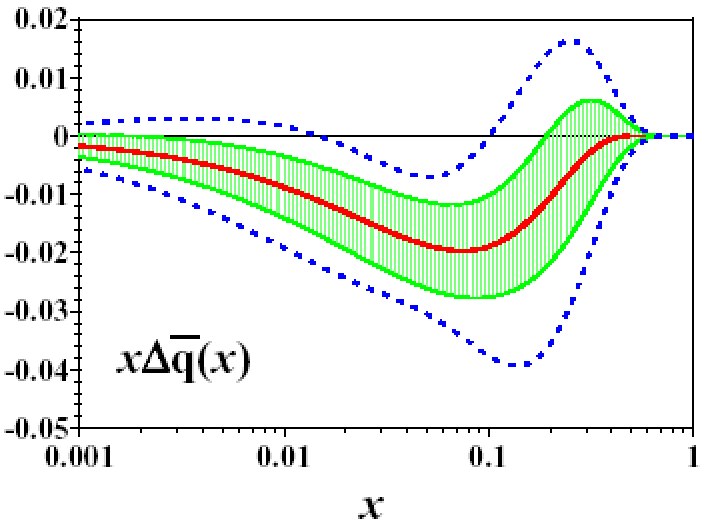


xf
Q^2 = 10 GeV^2
ZEUS-JETS fit
tot. uncert. : αs(MZ)=0.1180
tot. uncert. : αs(MZ) free
xuv
xdv
xg (× 0.05)
xS (× 0.05)
0
0.2
0.4
0.6
0.8
1
10-4
10-3
10-2
10-1
1
x

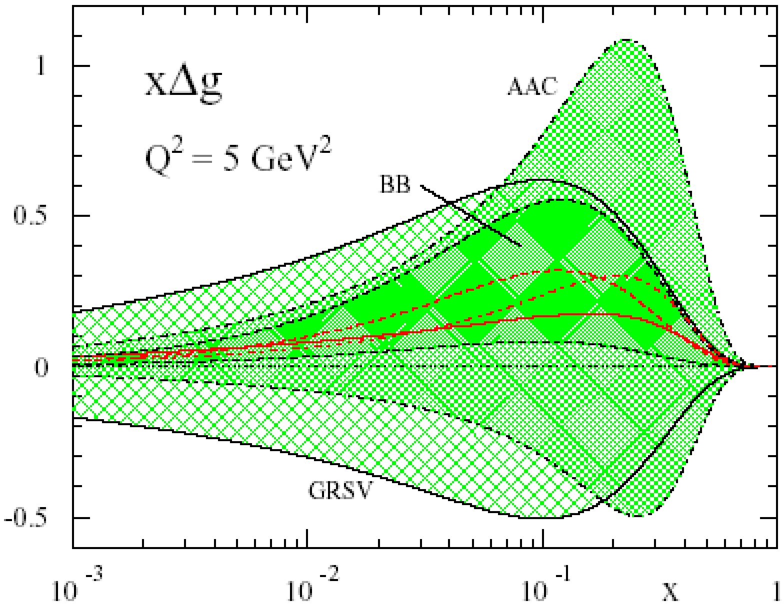
helicity distributions,
AAC collaboration

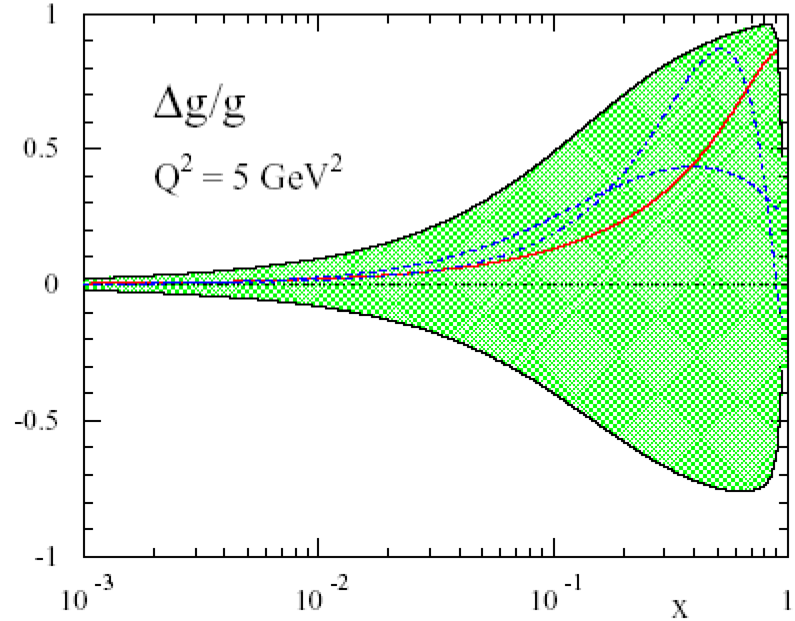






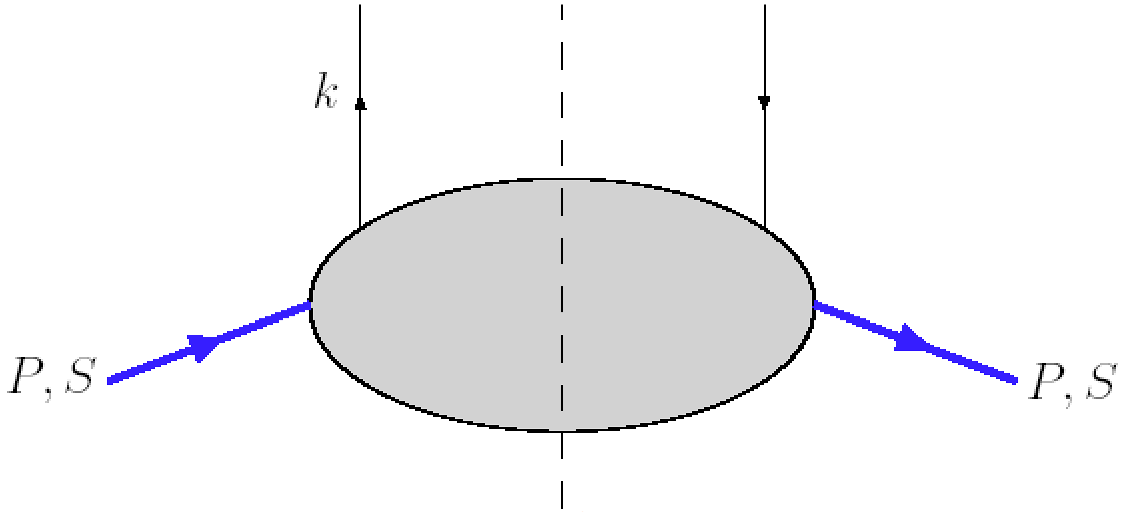
still large uncertainty in gluon helicity distribution,
although recent data seem to indicate small values





(Research Plan for Spin Physics at RHIC)

Even at leading twist, in collinear configuration, $q(x,Q^2)$ and $\Delta q(x,Q^2)$ are not the whole story



Correlator:

$$\Phi_{ij}(k;P,S)=\sum_X \frac{d^3P_X}{(2\pi)^3 2E_X}(2\pi)^4\delta^4(P-k-P_X)\langle PS|\Psi_j(0)|X\rangle\langle X|\Psi_i(0)|PS\rangle$$

$$=\int d^4\xi \quad e^{ik\cdot\xi}\langle PS|\bar{\Psi}_j(0)\Psi_i(\xi)|PS\rangle$$

$$\Phi(x,k)=\frac{1}{2}\left[f_1\not{n}_+ + g_{1L}\gamma^5\not{n}_+ P_L + h_{1T} i\sigma_{\mu\nu}\gamma^5\not{n}_+^\mu P_T^\nu\right]$$

f_1: q g_{1L}: Δq h_{1T}: $\Delta_T q$

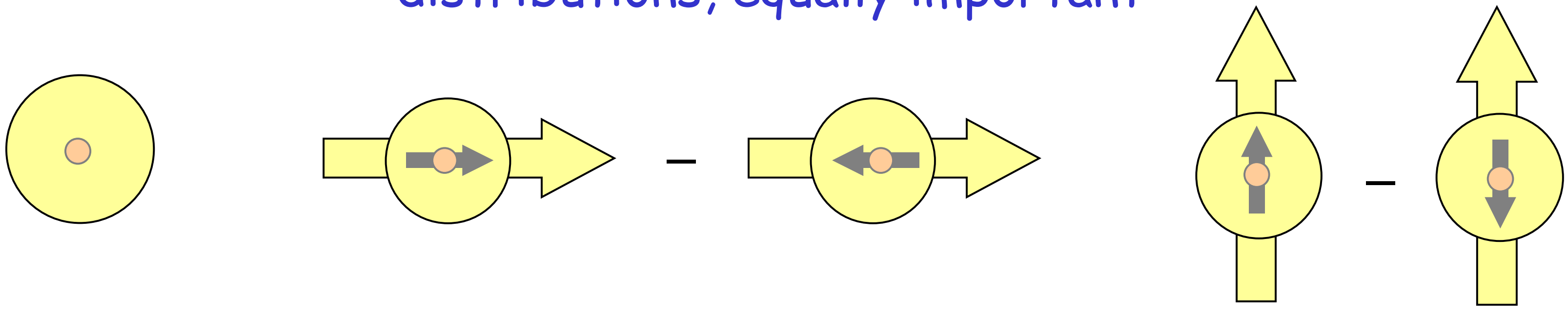
Transversity distribution

$$\Delta_T q(x) = q_{\uparrow}^{\uparrow}(x) - q_{\downarrow}^{\uparrow}(x)$$

$\Delta_T q$ also denoted as h_{1q} or δq

$q(x, Q^2)$, $\Delta q(x, Q^2)$ and $\Delta_T q(x, Q^2)$

are all fundamental, and different, leading-twist quark distributions, equally important

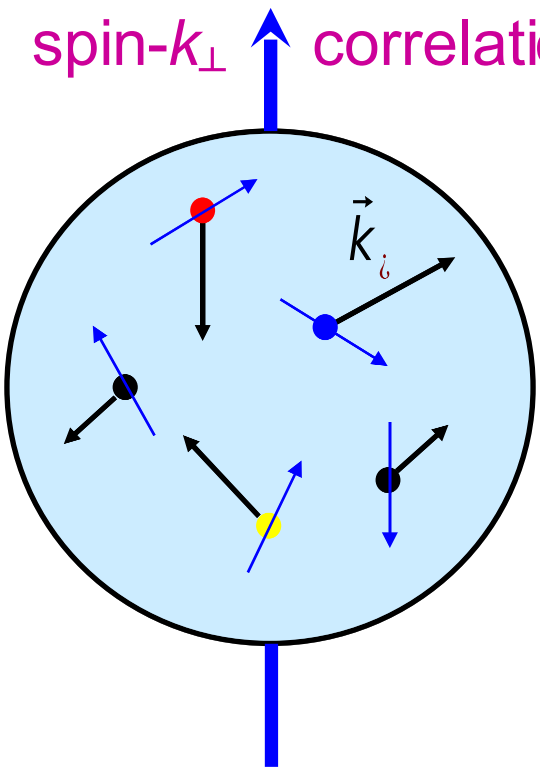


$\Delta_T q = \Delta q$ only for a proton at rest

Transversity decouples from Deep Inelastic Scattering

The transverse structure is much more interesting and less studied (not only because of transversity)

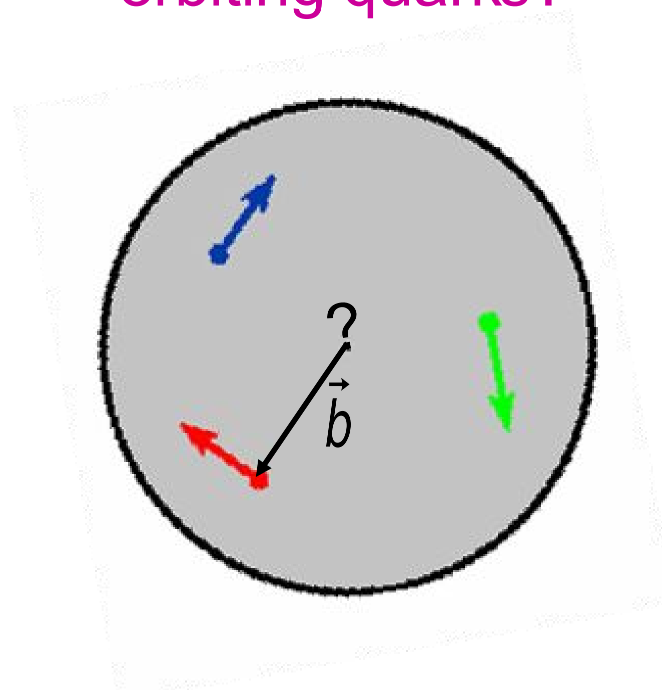
spin-$k_\perp$ correlations?



Transverse Momentum Dependent distribution functions

$$q(x, k_{¿}; Q^2)$$

orbiting quarks?

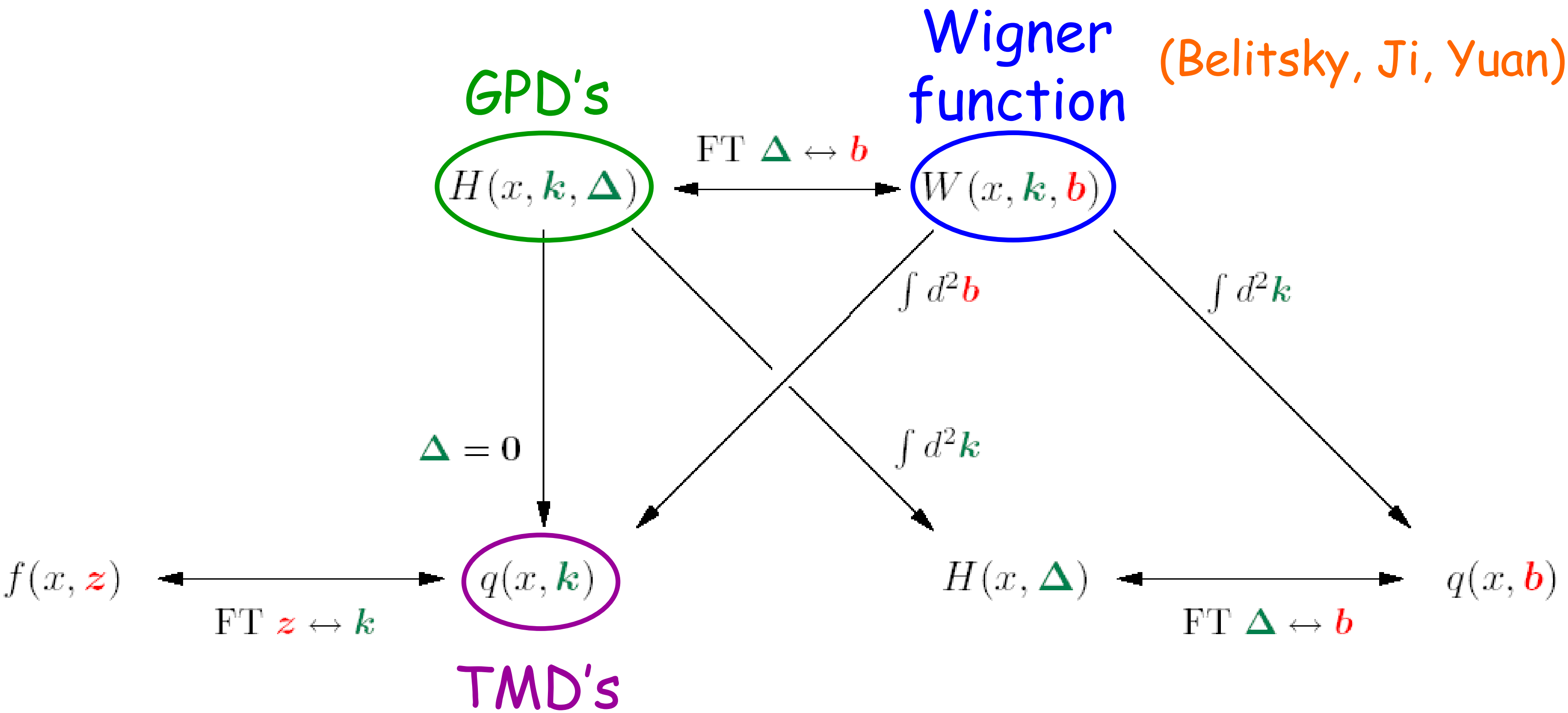


Space dependent distribution functions

$$q(x, b; Q^2)$$

The mother of all functions

M. Diehl, Trento workshop, June 07

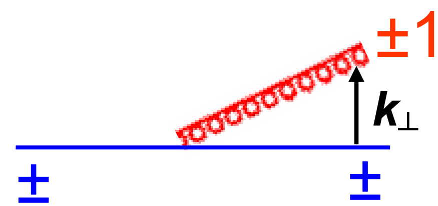


Partonic intrinsic motion

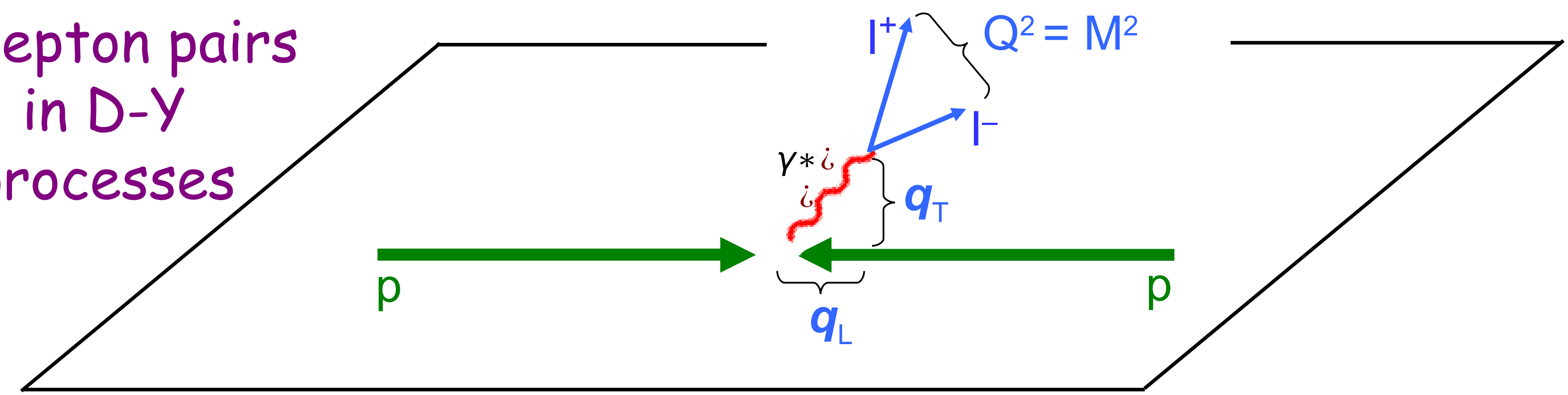
Plenty of theoretical and experimental evidence for transverse motion of partons within nucleons and of hadrons within fragmentation jets

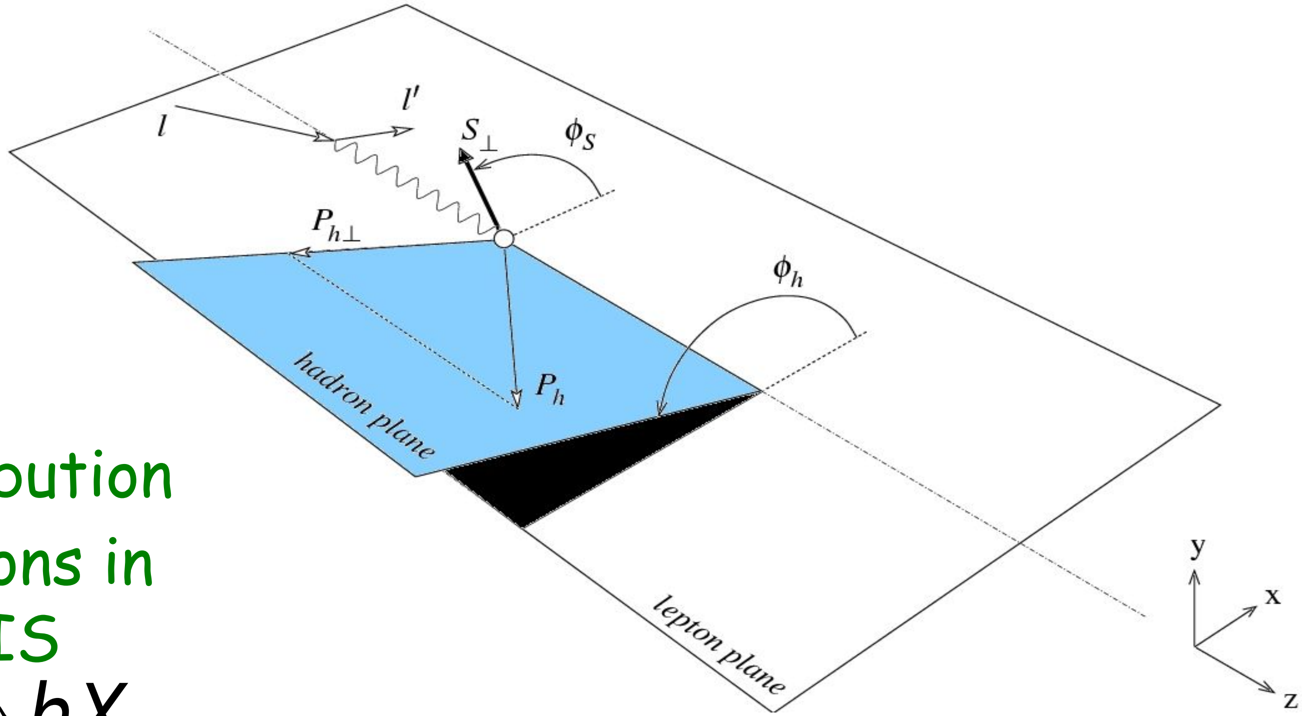
uncertainty principle $\Delta x \approx 1$ fm $\Rightarrow$ $\Delta p \approx 0.2$ GeV/c

gluon radiation



q_T distribution of lepton pairs in D-Y processes

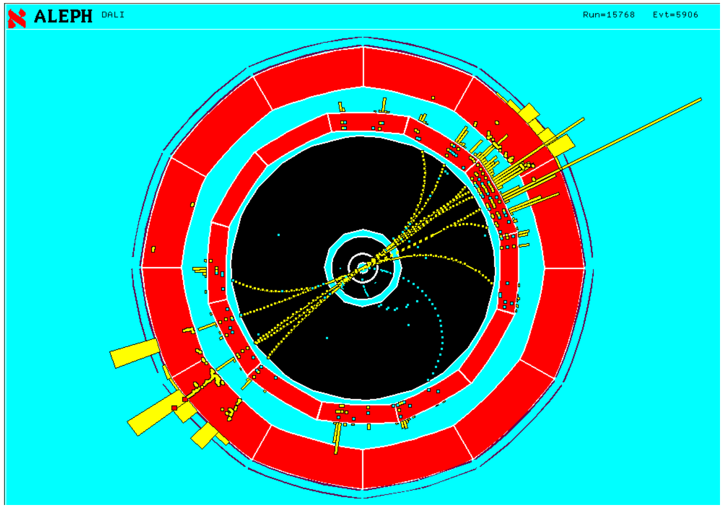




p_T distribution of hadrons in SIDIS

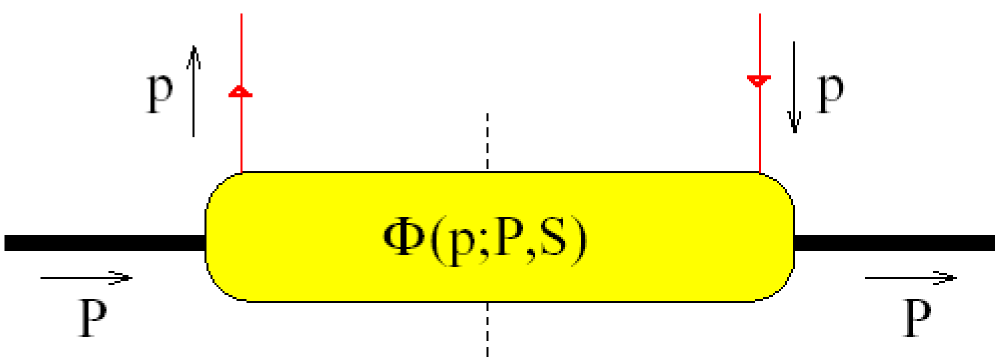
$$\gamma * p \rightarrow hX$$

Hadron distribution in jets in e^+e^- processes

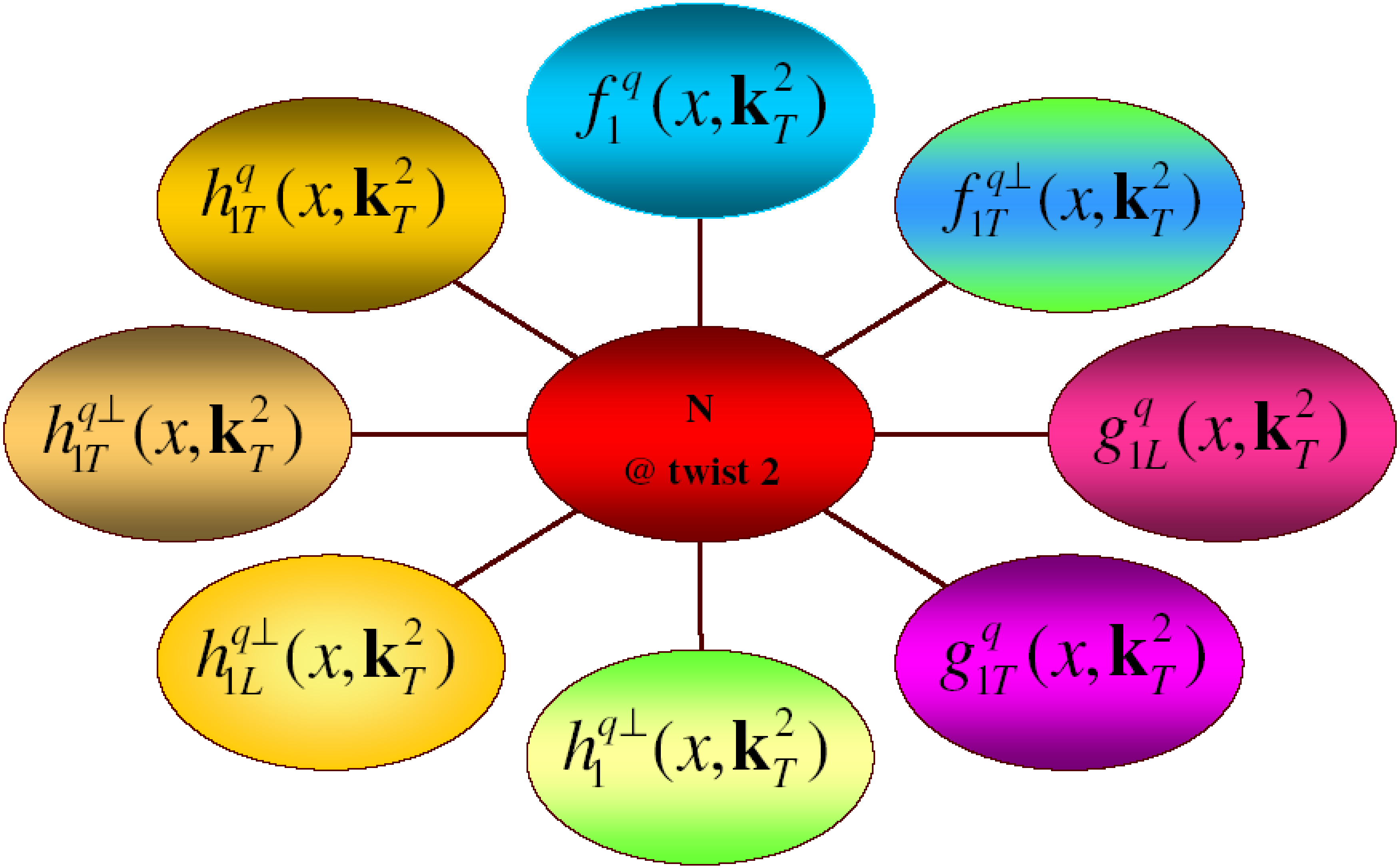


The leading-twist correlator, with intrinsic $k_\perp$, contains several other functions .....

$$\Phi(x_a, \boldsymbol{k}_{\perp a}) = \frac{1}{2}\Big[f_1 \not{n}_+ + f_{1T}^\perp \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu k_{\perp a}^\rho (P_T^A)^\sigma}{M} + \Big(P_L^A g_{1L} + \frac{\boldsymbol{k}_{\perp a}\cdot \boldsymbol{P}_T^A}{M} g_{1T}^\perp \Big)\gamma^5 \not{n}_+$$
$$+ h_{1T}\, i\sigma_{\mu\nu}\gamma^5 n_+^\mu (P_T^A)^\nu + \Big(P_L^A h_{1L}^\perp + \frac{\boldsymbol{k}_{\perp a}\cdot \boldsymbol{P}_T^A}{M} h_{1T}^\perp \Big) \frac{i\sigma_{\mu\nu}\gamma^5 n_+^\mu k_{\perp a}^\nu}{M}$$
$$+ h_1^\perp \frac{\sigma_{\mu\nu} k_{\perp a}^\mu n_+^\nu}{M} \Big] .$$



8 leading-twist spin-$k_\perp$ dependent distribution functions



Courtesy of Aram Kotzinian

How and what do we know about TMDs and what do we learn from them?

TMDs in SIDIS

Single Spin Asymmetries (SSA) in SIDIS:

Sivers functions

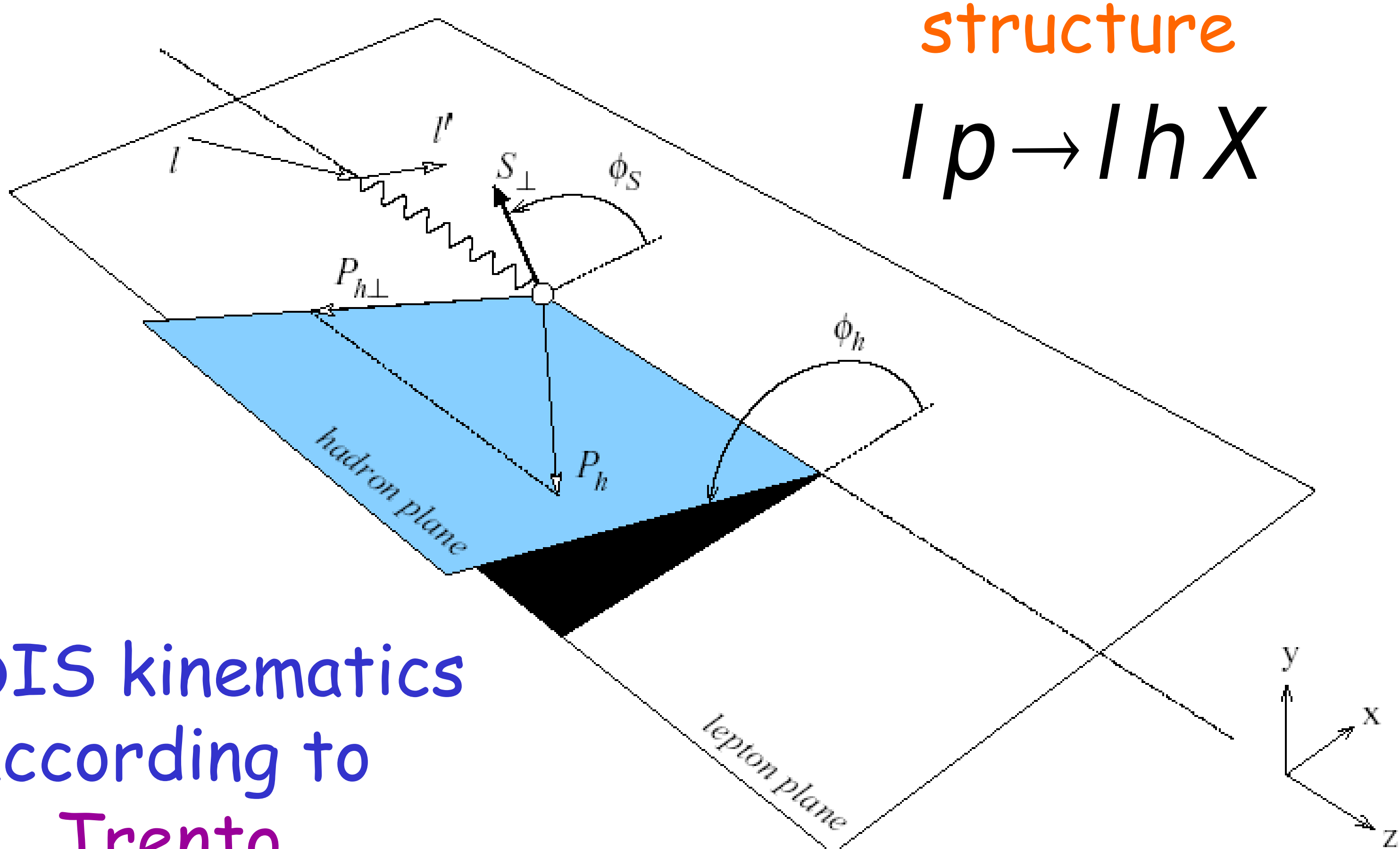
[Collins function from e^+e^- unpolarized processes (Belle) and first extraction of transversity]

SSA in hadronic processes

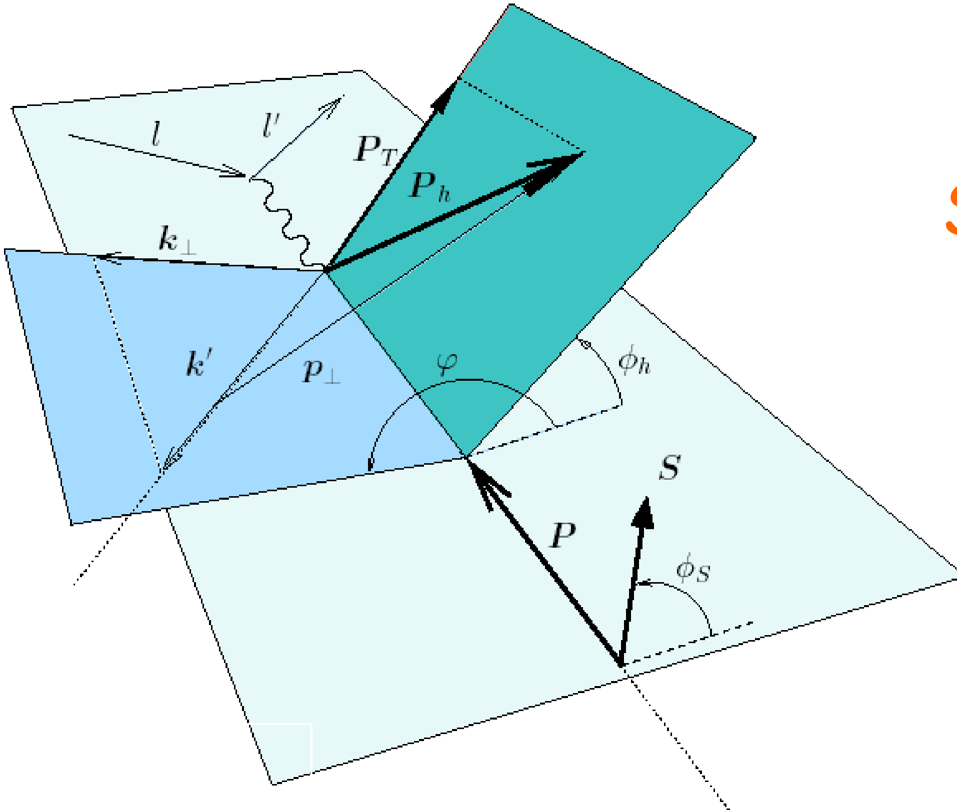
Future measurements and transversity

Main source of information on transverse nucleon structure

$$l\,p \to l\,h\,X$$



SIDIS kinematics according to Trento conventions (2004)



SIDIS in parton model with intrinsic $k_\perp$

factorization holds at large Q^2, and $P_T \approx k_\perp \approx \Lambda_{QCD}$ Ji, Ma, Yuan

$$d\sigma^{lp\to lhX} = \sum_q f_q(x, k_\perp; Q^2) \otimes d\hat{\sigma}^{lq\to lq}(y, k_\perp; Q^2) \otimes D_q^h(z, p_\perp; Q^2)$$

Polarized SIDIS cross section, up to subleading order in 1/Q

$$
\begin{aligned}
d\sigma = {} & d\sigma_{UU}^{0} + \cos 2\Phi_h \, d\sigma_{UU}^{1} + \frac{1}{Q}\cos\Phi_h \, d\sigma_{UU}^{2} + \lambda_e \frac{1}{Q}\sin\Phi_h \, d\sigma_{LU}^{3} \\
& + S_L \left\{ \sin 2\Phi_h \, d\sigma_{UL}^{4} + \frac{1}{Q}\sin\Phi_h \, d\sigma_{UL}^{5} + \lambda_e \left[d\sigma_{LL}^{6} + \frac{1}{Q}\cos\Phi_h \, d\sigma_{LL}^{7} \right] \right\} \\
& + S_T \left\{ \sin(\Phi_h - \Phi_S) \, d\sigma_{UT}^{8} + \sin(\Phi_h + \Phi_S) \, d\sigma_{UT}^{9} + \sin(3\Phi_h - \Phi_S) \, d\sigma_{UT}^{10} \right. \\
& + \frac{1}{Q} \left[\sin(2\Phi_h - \Phi_S) \, d\sigma_{UT}^{11} + \sin\Phi_S \, d\sigma_{UT}^{12} \right] \\
& \left. + \lambda_e \left[\cos(\Phi_h - \Phi_S) \, d\sigma_{LT}^{13} + \frac{1}{Q} \left(\cos\Phi_S \, d\sigma_{LT}^{14} + \cos(2\Phi_h - \Phi_S) \, d\sigma_{LT}^{15} \right) \right] \right\}
\end{aligned}
$$

SIDISLAND

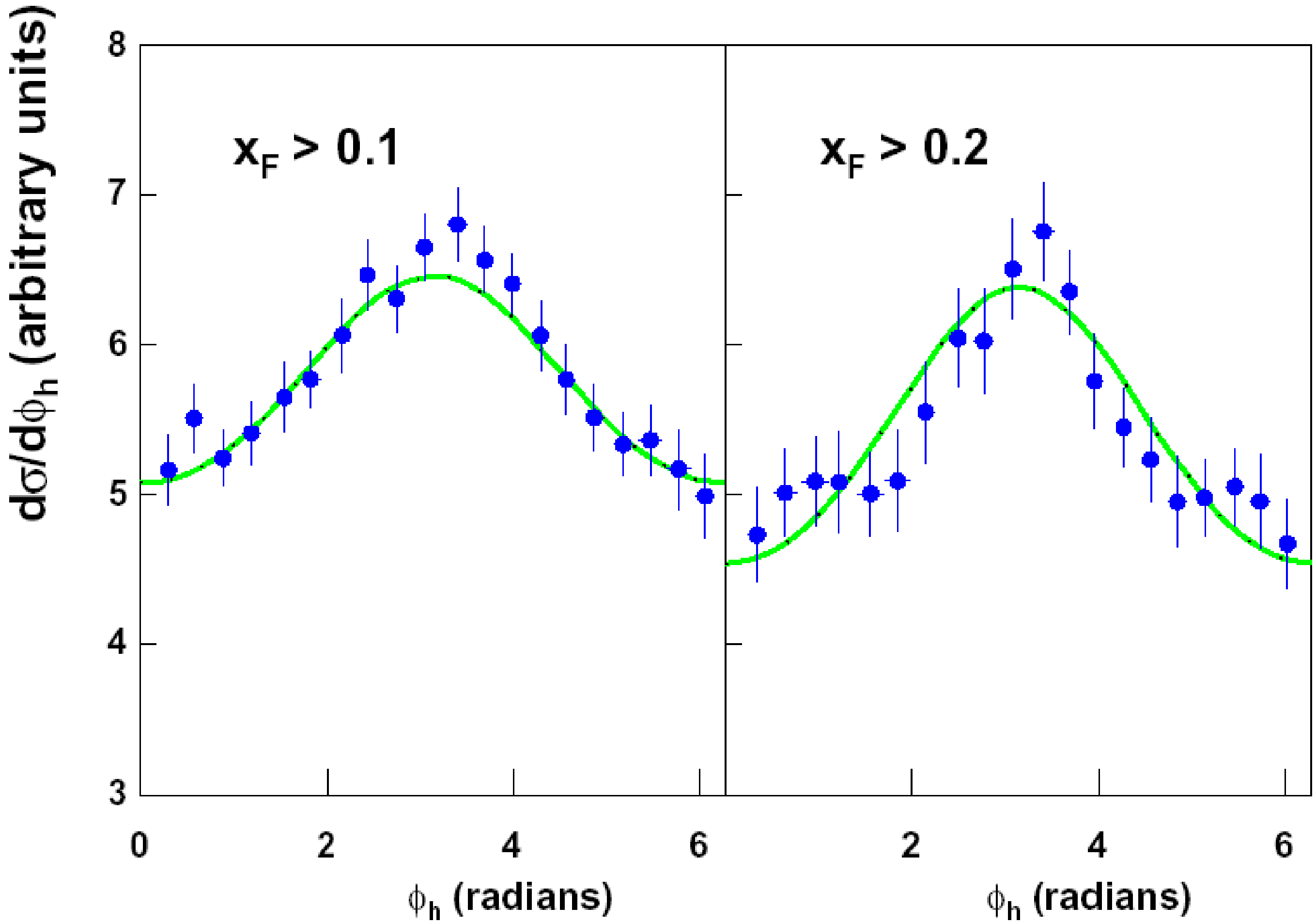
Kotzinian, **NP B441** (1995) 234

Mulders and Tangermann, **NP B461** (1996) 197

Boer and Mulders, **PR D57** (1998) 5780

Bacchetta et al., **PL B595** (2004) 309

Bacchetta et al., **JHEP 0702** (2007) 093



Azimuthal dependence induced by quark intrinsic motion

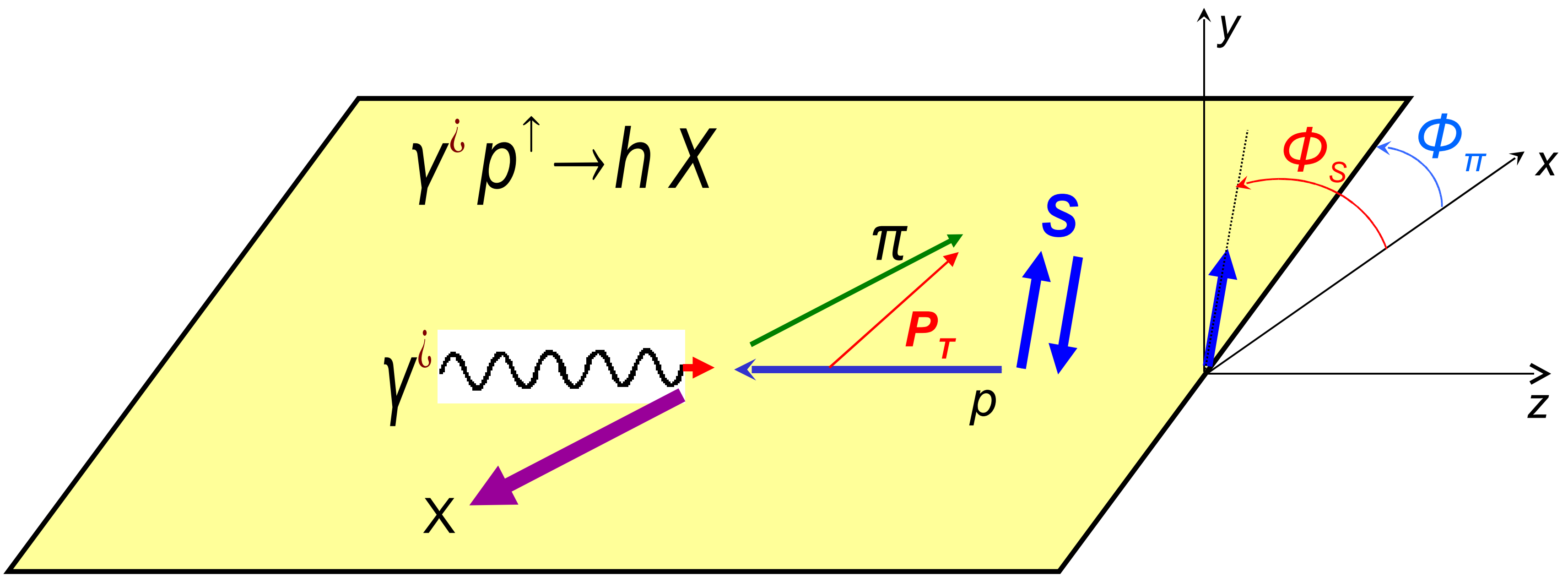
EMC data, μp and μd, *E* between 100 and 280 GeV

$$\langle k_\perp^2 \rangle = 0.28 \ (\mathrm{GeV})^2 \qquad \langle p_\perp^2 \rangle = 0.25 \ (\mathrm{GeV})^2$$

M.A., M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia and A. Prokudin

Transverse single spin asymmetries in SIDIS, experimentally observed

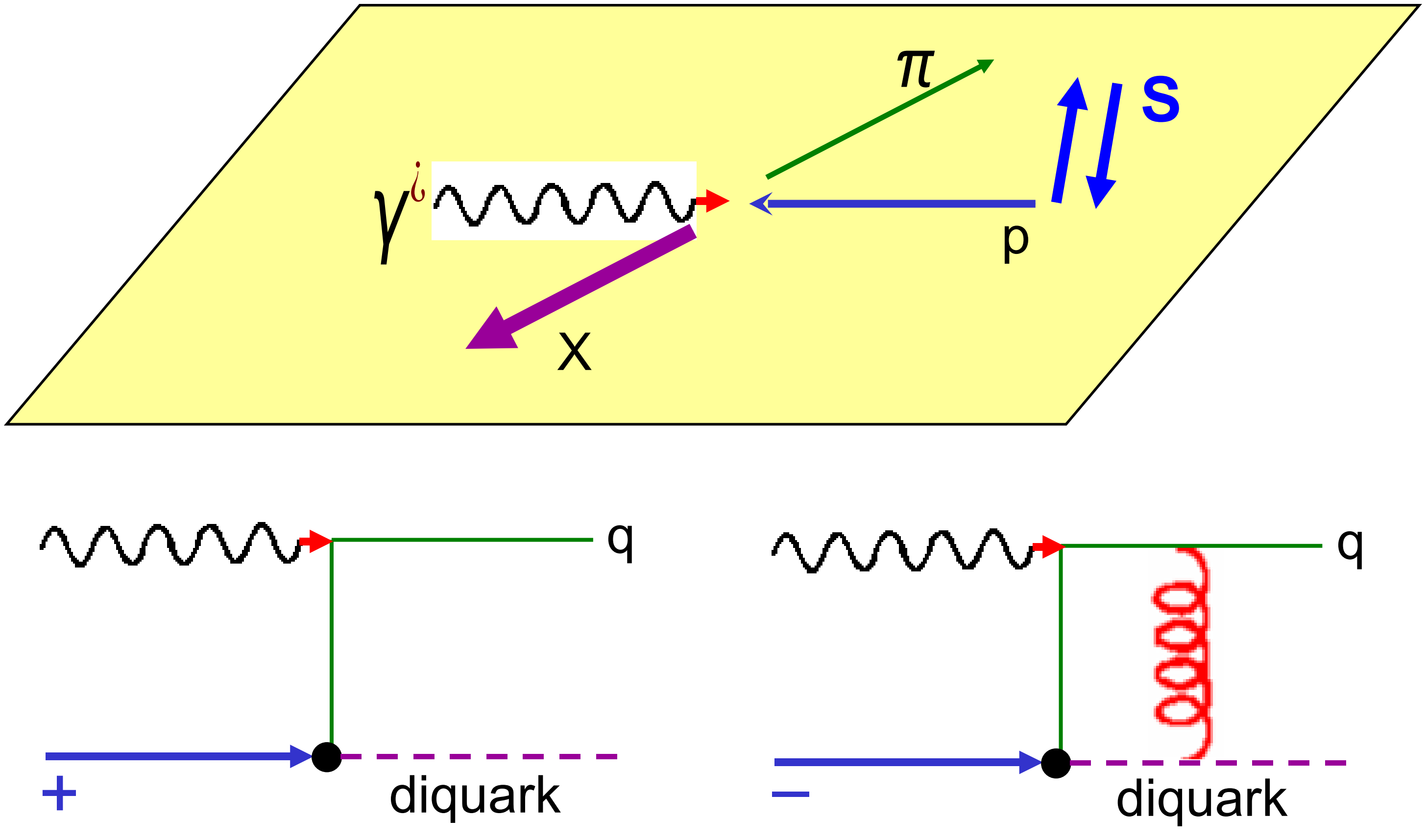
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$



$$A_N \propto S \cdot \left(p \times P_T \right) \propto P_T \sin(\Phi_\pi - \Phi_S) \qquad \gamma * - p \;\; \mathrm{c.}m\mathrm{.} \;\; \mathrm{frame}$$

in collinear configurations there cannot be (at LO) any P_T

Brodsky, Hwang, Schmidt model for Sivers function



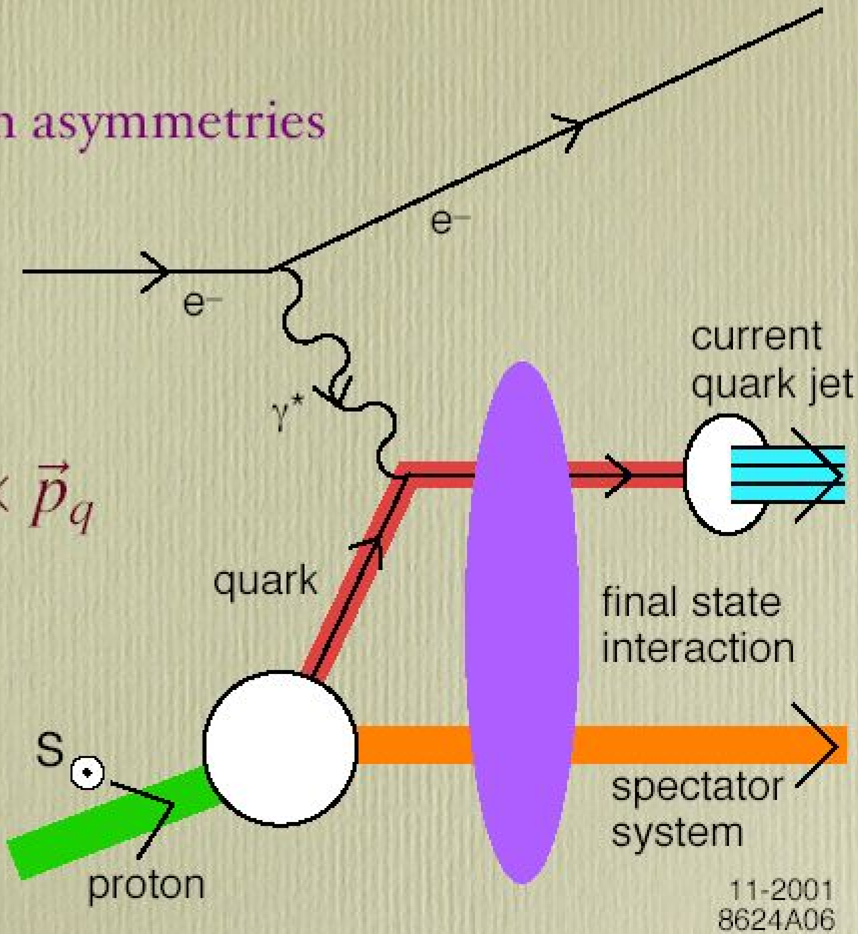
$$S\cdot\left(p\times P_T\right)\propto P_T\sin\left(\Phi_\pi-\Phi_S\right)$$

needs $k_\perp$ dependent quark distribution in $p^\uparrow$: Sivers function

Single-spin asymmetries

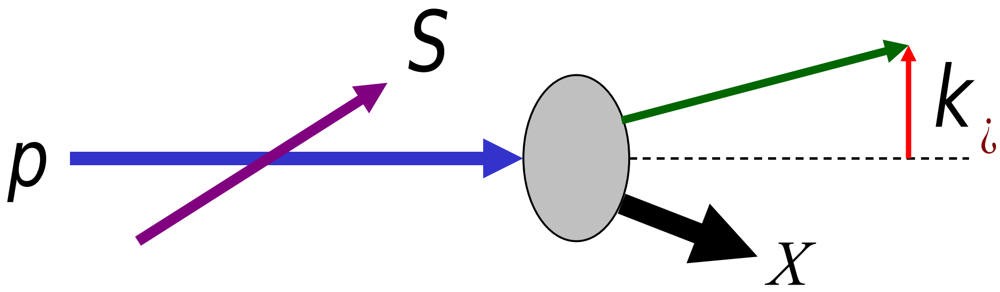
Sivers Effect

$\vec{S}_p \cdot \vec{q} \times \vec{p}_q$



How does intrinsic motion help with SSA?

One can introduce spin-$\boldsymbol{k}_\perp$ correlation in the Parton Distribution Functions (PDFs) and in the parton Fragmentation Functions (FFs)



Only possible (scalar) correlation is

$$S\cdot(p\times k_\perp)$$

Sivers function

$$f_{q/p,S}(x,k_{¿})=f_{q/p}(x,k_{¿})+\frac{1}{2}\Delta^{N}f_{q/p^{\uparrow}}(x,k_{¿})S\cdot(\hat{p}\times\hat{k}_{¿})$$

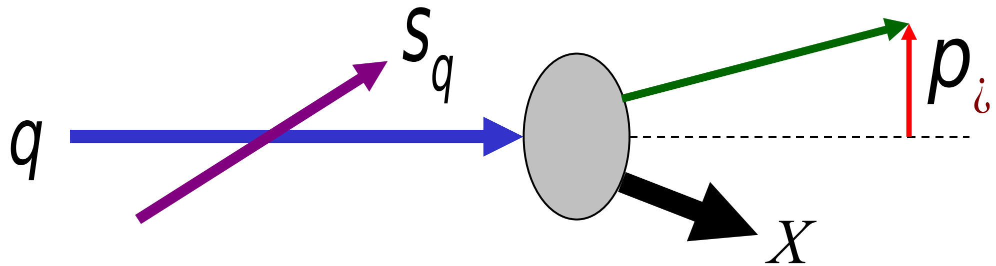
$$=f_{q/p}(x,k_{¿})-\frac{k_{¿}}{M}f_{1T}^{¿q}(x,k_{¿})S\cdot(\hat{p}\times\hat{k}_{¿})$$

Boer-Mulders function

$$f_{q,s_q/p}(x,k_{¿})=\frac{1}{2}f_{q/p}(x,k_{¿})+\frac{1}{2}\Delta^{N}f_{q^{\uparrow}/p}(x,k_{¿})s_q ¿(\hat{p}\times\hat{k}_{¿})$$

$$=\frac{1}{2}f_{q/p}(x,k_{¿})-\frac{1}{2}\frac{k_{¿}}{M}h_{1}^{¿q}(x,k_{¿})s_q\cdot(\hat{p}\times\hat{k}_{¿})$$

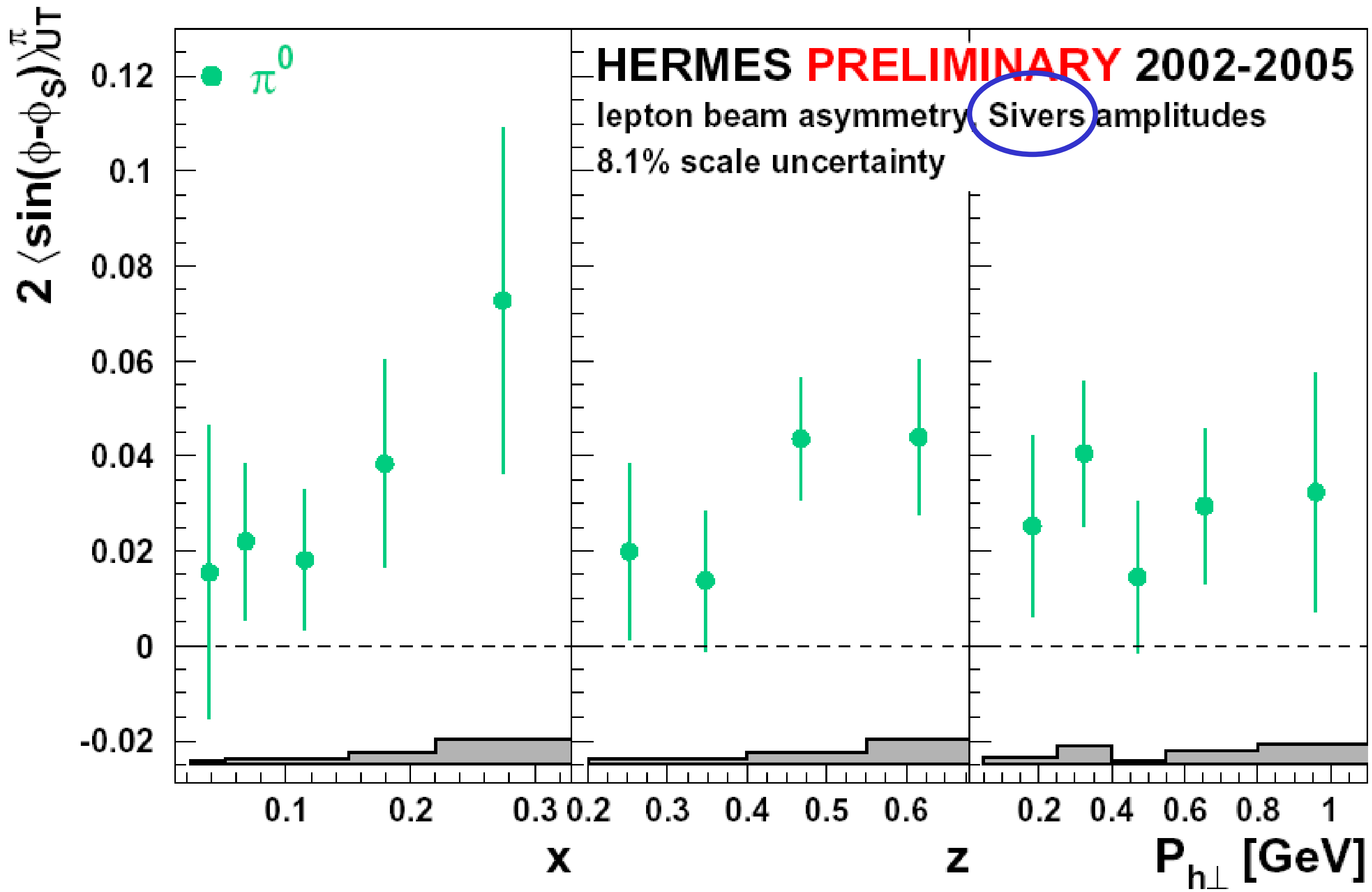
Spin-k$_\perp$ correlations in fragmentation process (case of final spinless hadron)

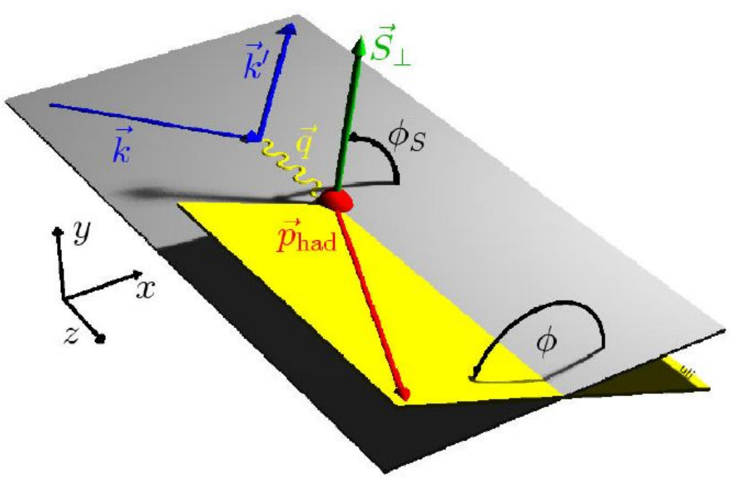


Collins function

$$D_{h/q,s_q}(z,p_¿)=D_{h/q}(z,p_¿)+\frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_¿)s_q¿(\hat{p}_q¿\hat{p}_¿)$$

$$=D_{h/q}(z,p_¿)+\frac{p_¿}{zM_h}H_1^{¿q}(z,p_¿)s_q\cdot(\hat{p}_q\times\hat{p}_¿)$$

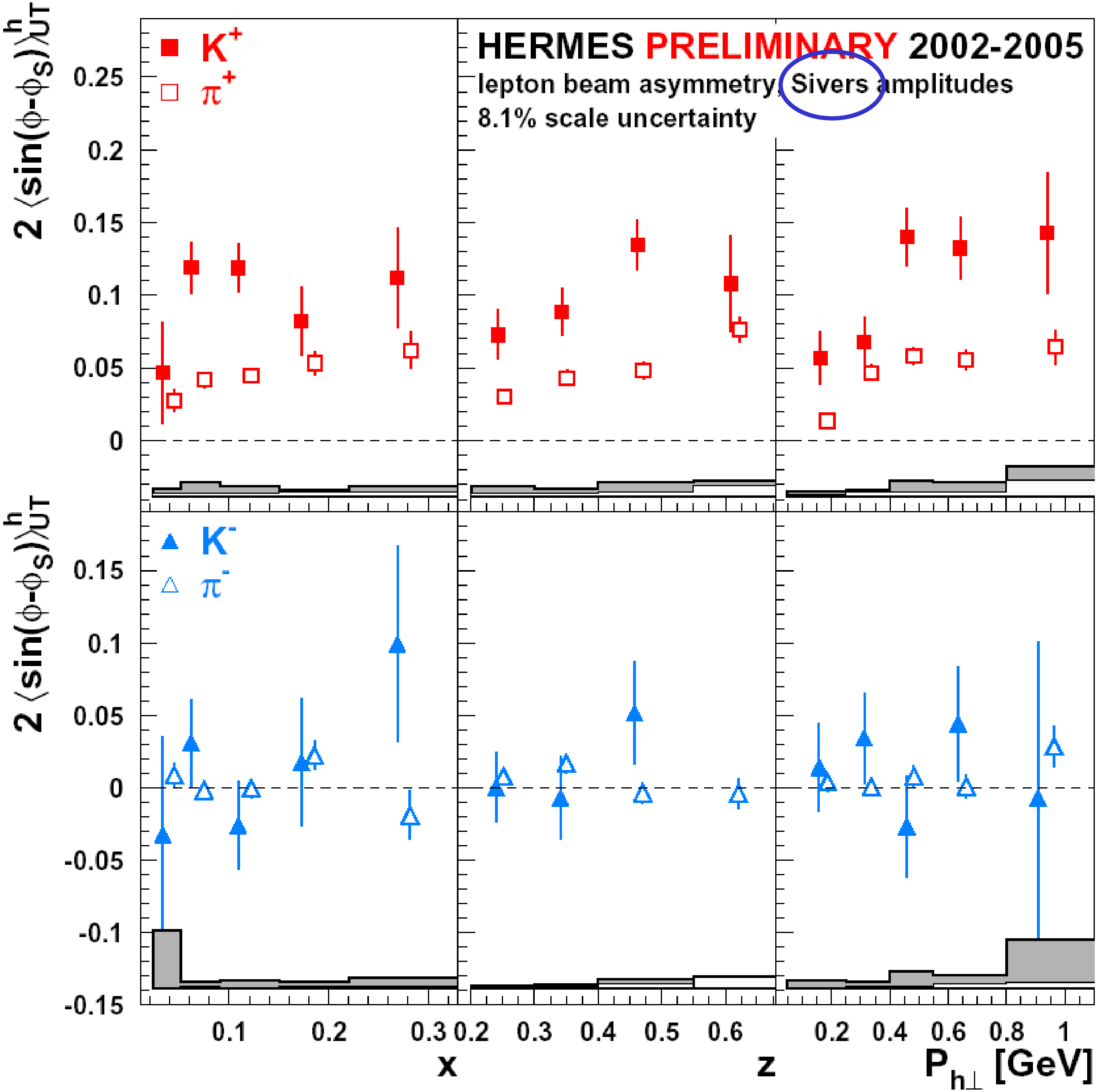




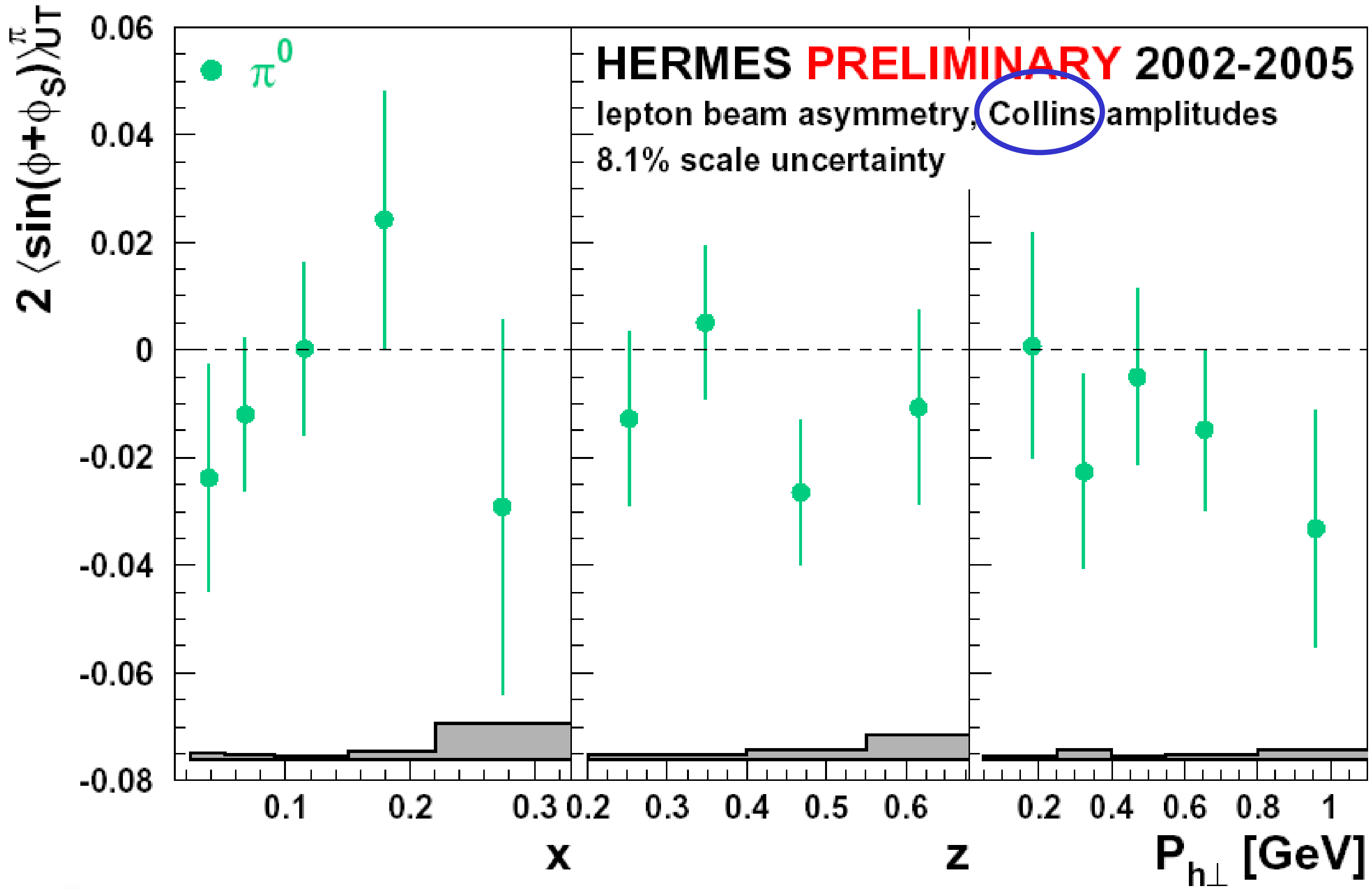
$$2\langle \sin(\Phi-\Phi_S)\rangle = A_{UT}^{\sin(\Phi-\Phi_S)}$$

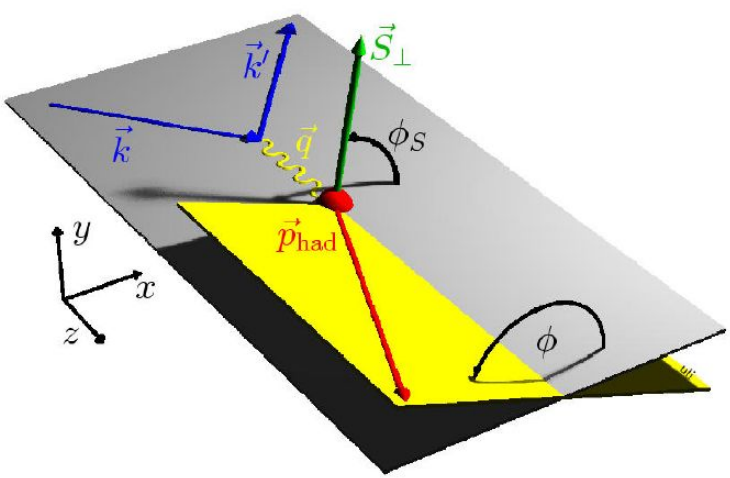
$$\text{¿}\, 2\frac{\int d\Phi\ d\Phi_S (d\sigma^{\uparrow}-d\sigma^{\downarrow})\sin(\Phi-\Phi_S)}{\int d\Phi\ d\Phi_S (d\sigma^{\uparrow}+d\sigma^{\downarrow})}$$

$$\text{¿}\, \sum_q e_q^2 \Delta^N f_{q/p^{\uparrow}}(x, k_{\text{¿}}) \otimes D_{\pi/q}(z, p_{\text{¿}})$$



Large K^+ asymmetry!





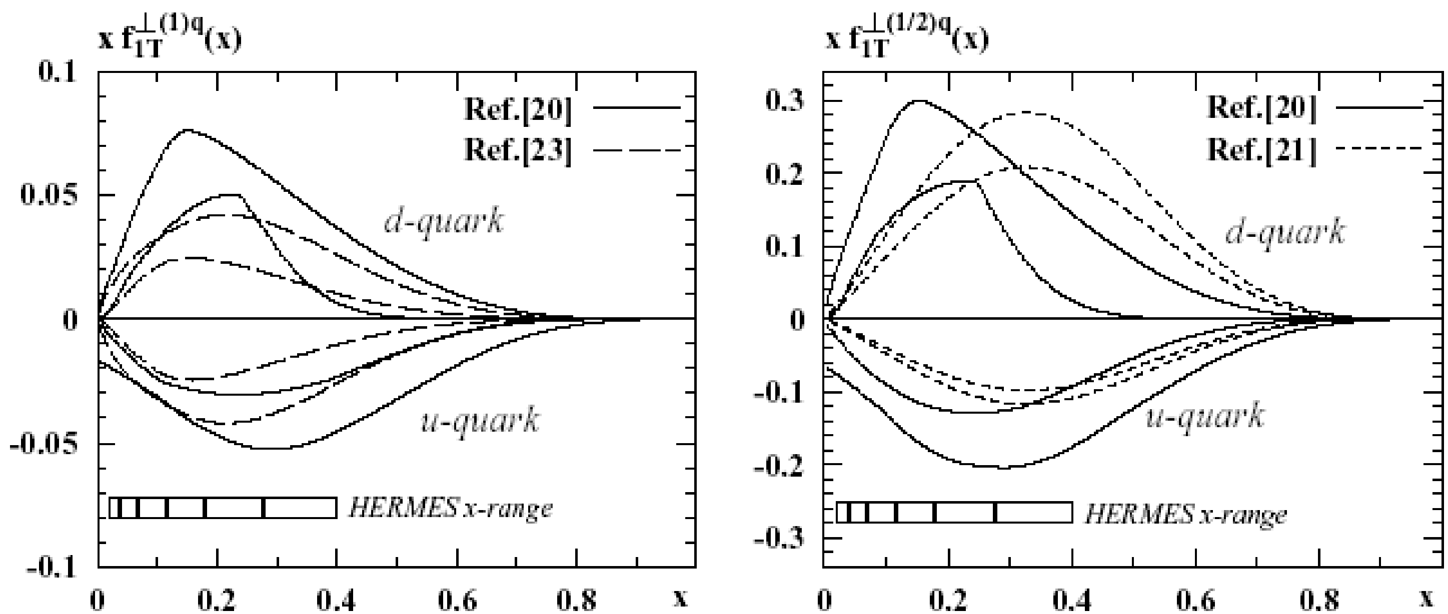
$$2\langle \sin(\Phi+\Phi_S)\rangle = A_{UT}^{\sin(\Phi+\Phi_S)}$$

$$\text{¿}\, 2\,\frac{\int d\Phi\ \mathrm{d}\Phi_S (d\sigma^{\uparrow}-d\sigma^{\downarrow})\sin(\Phi+\Phi_S)}{\int d\Phi\ \mathrm{d}\Phi_S (d\sigma^{\uparrow}+d\sigma^{\downarrow})}$$

$$\text{¿}\, \sum_q e_q^2 h_{1q}(x, k_{\text{¿}}) \otimes \Delta^N D_{h/q^{\uparrow}}(z, p_{\text{¿}})$$

Present knowledge of Sivers function (u,d)

M. Anselmino, M. Boglione, J.C. Collins, U. D'Alesio, A.V. Efremov, K. Goeke, A. Kotzinian, S. Menze, A. Metz, F. Murgia, A. Prokudin, P. Schweitzer, W. Vogelsang, F. Yuan

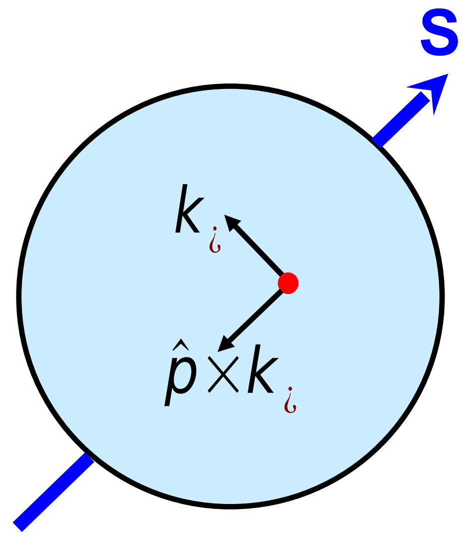


The first and 1/2-transverse moments of the Sivers quark distribution functions. The fits were constrained mainly (or solely) by the preliminary HERMES data in the indicated x-range. The curves indicate the 1-σ regions of the various parameterizations.

$$f_{1T}^{¿(1)q} = \int d^2k_{¿} \frac{k_{¿}^2}{2M^2} f_{1T}^{¿q}(x, k_{¿}) \qquad f_{1T}^{¿(1/2)q}(x) = \int d^2k_{¿} \frac{k_{¿}}{M} f_{1T}^{¿q}(x, k_{¿})$$

What do we learn from the Sivers distribution?

number density of partons with longitudinal momentum fraction x and transverse momentum $\boldsymbol{k}_\perp$, inside a proton with spin $\boldsymbol{S}$



$$\sum_a \int dx\, d^2 k_\perp\, k_\perp\, f_{a/p^\uparrow}(x, k_\perp) = 0$$

M. Burkardt, PR **D69**, 091501 (2004)

Total amount of intrinsic momentum carried by partons of flavour a

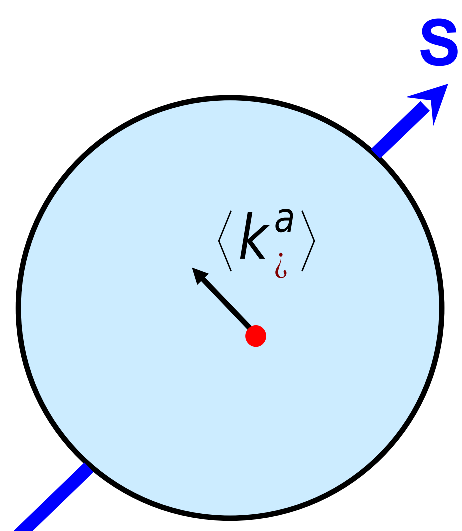
$$\langle \mathbf{k}_\perp^a \rangle = \int dx\, d^2k_\perp\, \mathbf{k}_\perp \left[\hat{f}_{a/p}(x, k_\perp) + \frac{1}{2} \Delta^N \hat{f}_{a/p^\uparrow}(x, k_\perp)\, \mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) \right]$$

$$= \left(\sin\Phi_S\, \hat{\mathbf{i}} - \cos\Phi_S\, \hat{\mathbf{j}} \right) \frac{\pi}{2} \int dx\, dk_\perp\, k_\perp^2\, \Delta^N \hat{f}_{a/p^\uparrow}(x, k_\perp)$$

for a proton moving along the +z-axis and polarization vector

$$\mathbf{S} = \left(\cos\Phi_S\, \hat{\mathbf{i}} + \sin\Phi_S\, \hat{\mathbf{j}} \right)$$

$$\mathbf{S} \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp) = \sin(\Phi_S - \phi)$$



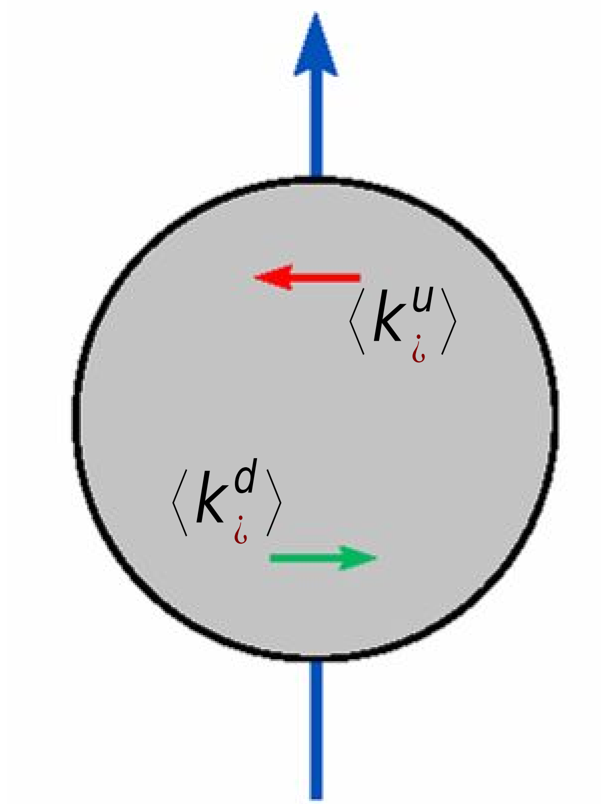
Numerical estimates from SIDIS data

U. D'Alesio

$$\langle k^{u}_{¿} \rangle \simeq +0.14^{+0.05}_{-0.06} \left(\sin\Phi_S \hat{i} - \cos\Phi_S \hat{j} \right) \quad \text{GeV/c}$$

$$\langle k^{d}_{¿} \rangle \simeq -0.13^{+0.03}_{-0.02} \left(\sin\Phi_S \hat{i} - \cos\Phi_S \hat{j} \right) \quad \text{GeV/c}$$

$$\langle k^{u}_{¿} \rangle + \langle k^{d}_{¿} \rangle = 0\,?$$



Burkardt sum rule saturated by u and d quarks?

Valence quark dominance?

Sivers function and orbital angular momentum

D. Sivers

Sivers mechanism originates from $S \cdot L_q$
then it is related to the quark orbital angular momentum

For a proton moving along *z* and polarized along *y*

$$\int_0^1 dx\, \Delta^N f_{q/p^{\uparrow}}(x, k_{¿}) = \frac{\langle L_y^q \rangle}{2} \qquad ?$$

Sivers function and proton anomalous magnetic moment

M. Burkardt, S. Brodsky, Z. Lu, I. Schmidt

Both the Sivers function and the proton anomalous magnetic moment are related to correlations of proton wave functions with *opposite helicities*

$$\int_0^1 dx\, d^2 k_{\text{¿}} \Delta^N f_{q/p^\uparrow}(x, k_{\text{¿}}) = C\kappa_q \qquad ?$$

in qualitative agreement with large z data:

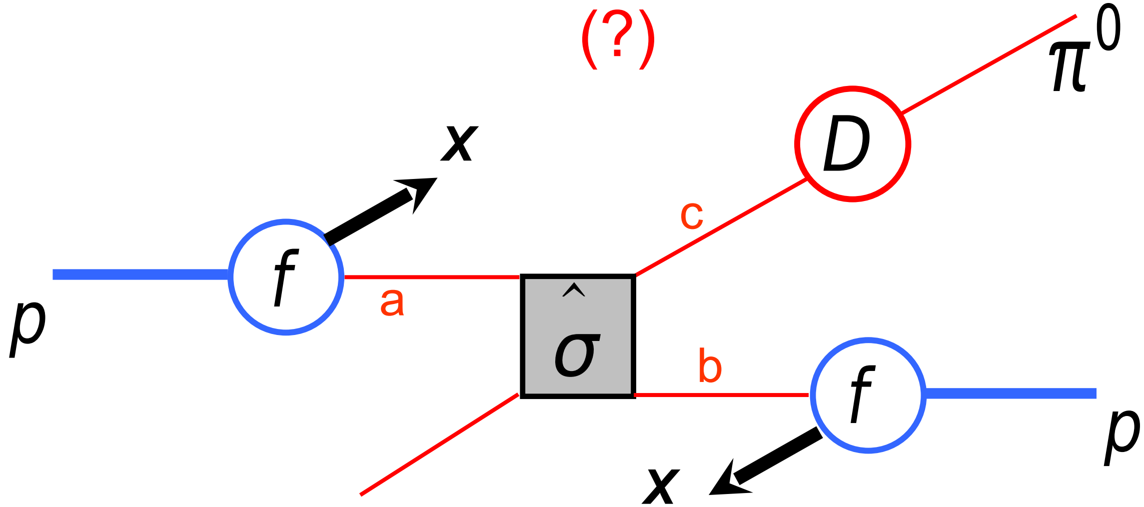
$$\frac{A_{UT}^{\sin(\Phi_{\pi^+} - \Phi_S)}}{A_{UT}^{\sin(\Phi_{\pi^-} - \Phi_S)}} \longrightarrow \frac{\kappa_u}{\kappa_d}$$

TMDs and SSAs in hadronic collisions
(abandoning safe grounds ....)

$pp \to \pi^0 X$ (collinear configurations)

factorization theorem

(?)

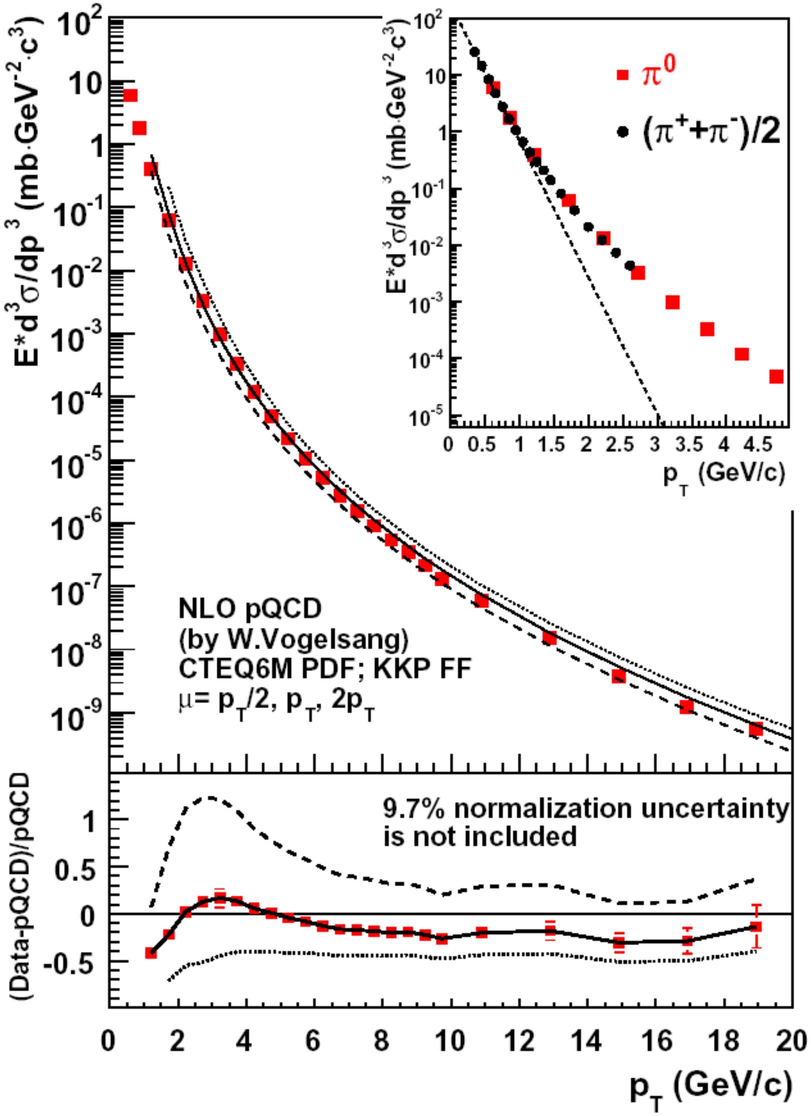


$$d\sigma = \sum_{a,b,c,d=q,\bar{q},g} \underbrace{f_{a/p}(x_a) \otimes f_{b/p}(x_b)}_{\text{PDF}} \otimes d\hat{\sigma}^{ab\to cd} \otimes \underbrace{D_{\pi/c}(z)}_{\text{FF}}$$

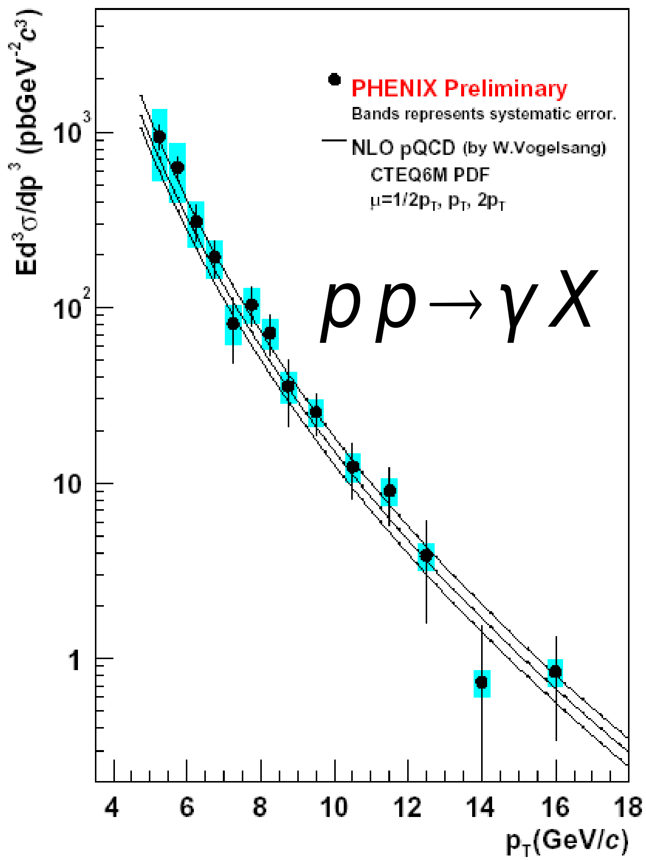
PDF

pQCD elementary interactions

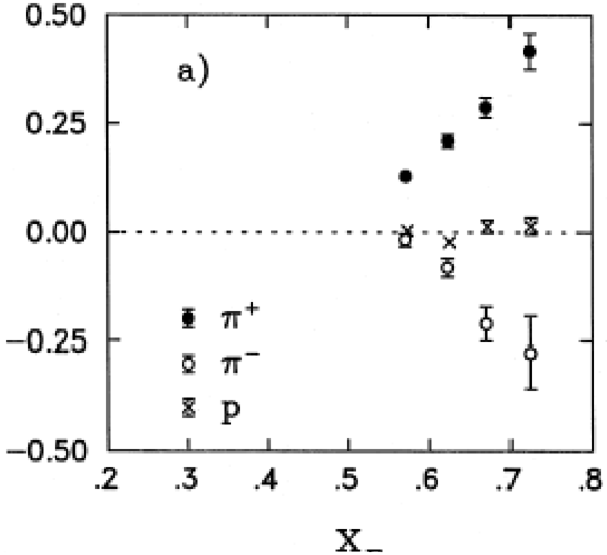
FF



RHIC data $\sqrt{s} = 200$ GeV



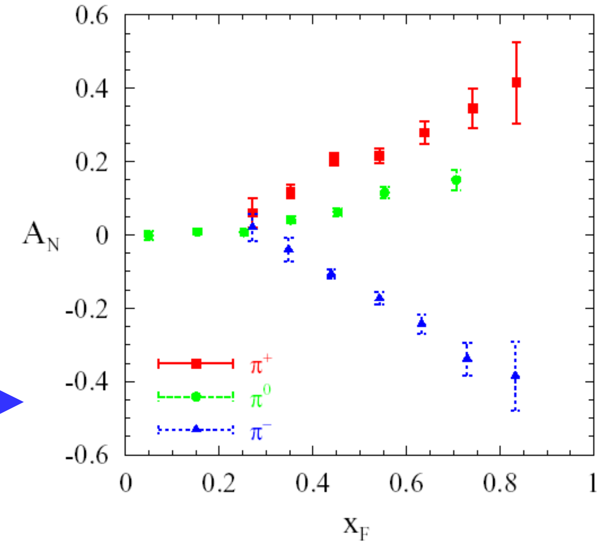
excellent agreement with data for
unpolarized cross section, but no SSA

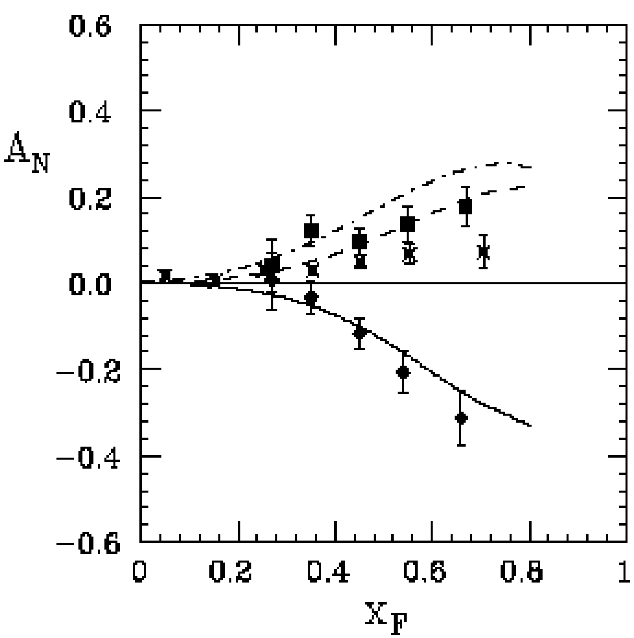


BNL-AGS $\sqrt{s}$ = 6.6 GeV
$0.6 < p_T < 1.2$

$$p^{\uparrow} p \rightarrow \pi X$$

E704 $\sqrt{s}$ = 20 GeV
$0.7 < p_T < 2.0$





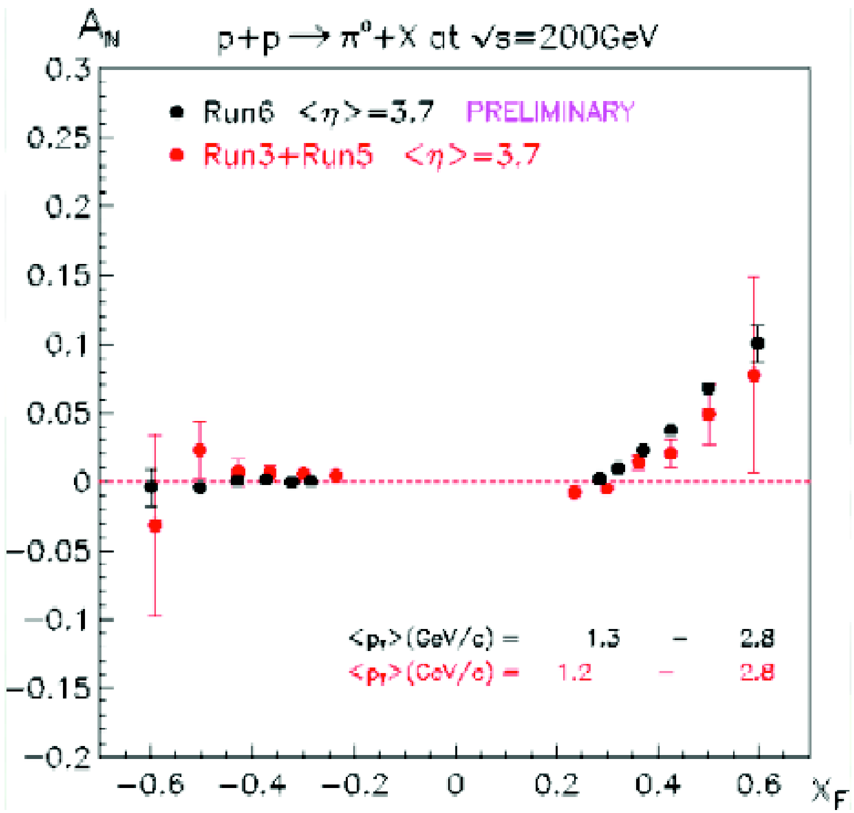
E704 $\sqrt{s}$ = 20 GeV
$0.7 < p_T < 2.0$

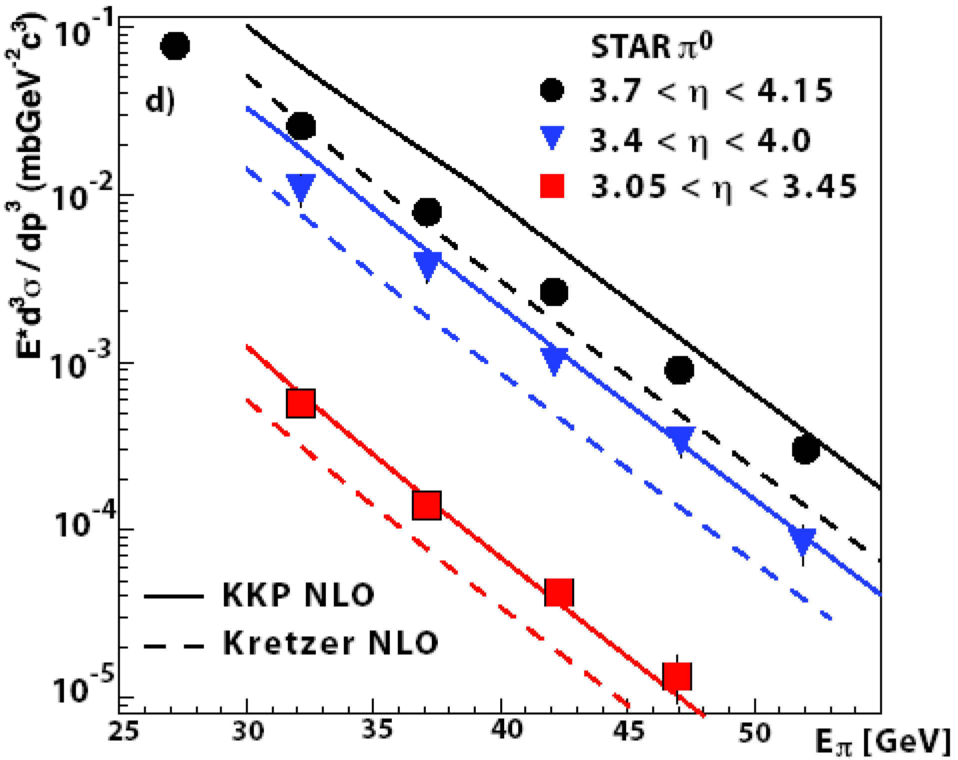
$$\bar{p}^{\uparrow} p \rightarrow \pi X$$

observed transverse
Single Spin Asymmetries

$$A_N \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

experimental
data on SSA





STAR-RHIC $\sqrt{s}$ = 200 GeV 1.2 < p_T < 2.8

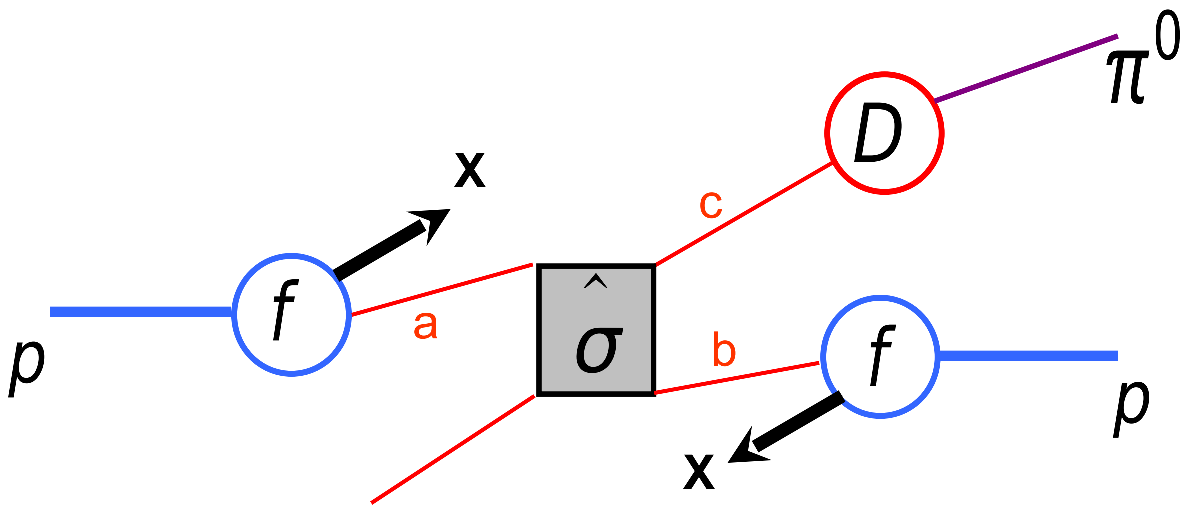
and A_N stays at high energies ….

SSA in hadronic processes: intrinsic $k_\perp$, factorization?

Two main different (?) approaches

Generalization of collinear scheme

(M. A., M. Boglione, U. D'Alesio, E. Leader, S. Melis, F. Murgia)



$$d\sigma = \sum_{a,b,c,d=q,\bar{q},g} f_{a/p}(x_a, k_{\perp a}) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes d\hat{\sigma}^{ab\to cd}(k_{\perp a}, k_{\perp b}) \otimes D_{\pi/c}(z, p_{\perp \pi})$$

It generalizes to polarized case

$$d\sigma^{A,S_A+B,S_B\to C+X}=\sum \rho^{a/A,S_A}_{\lambda_a\lambda'_a} f_{a/A,S_A}(x_a,k_{¿a})\otimes \rho^{b/B,S_B}_{\lambda_b\lambda'_b} f_{b/B,S_B}(x_b,k_{¿b})$$

$$\otimes \hat{M}^{ab\to cd}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} M^{ab\to c\hat{d}}_{\lambda'_c,\lambda_d;\lambda'_a,\lambda'_b}(k_{¿a},k_{¿b})\otimes D^{\lambda_C,\lambda_C}_{\lambda_c,\lambda'_c}(z,p_{¿\pi})$$

plenty of phases

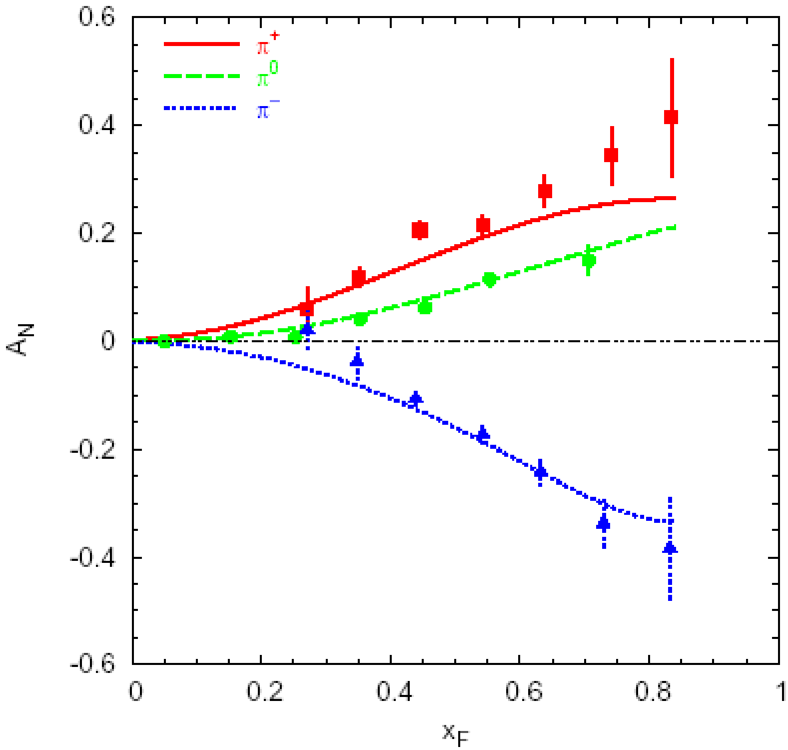
main remaining contribution to SSA from Sivers effect

$$d\Delta\sigma^{p,S+p\to\pi+X}=\sum_q \Delta^N f_{q/p^\uparrow}(x_a,k_{¿a})\otimes f_{b/p}(x_b,k_{¿b})$$

$$\otimes d\hat{\sigma}^{ab\to cd}(k_{¿a},k_{¿b})\otimes D_{\pi/c}(z,p_{¿\pi})$$

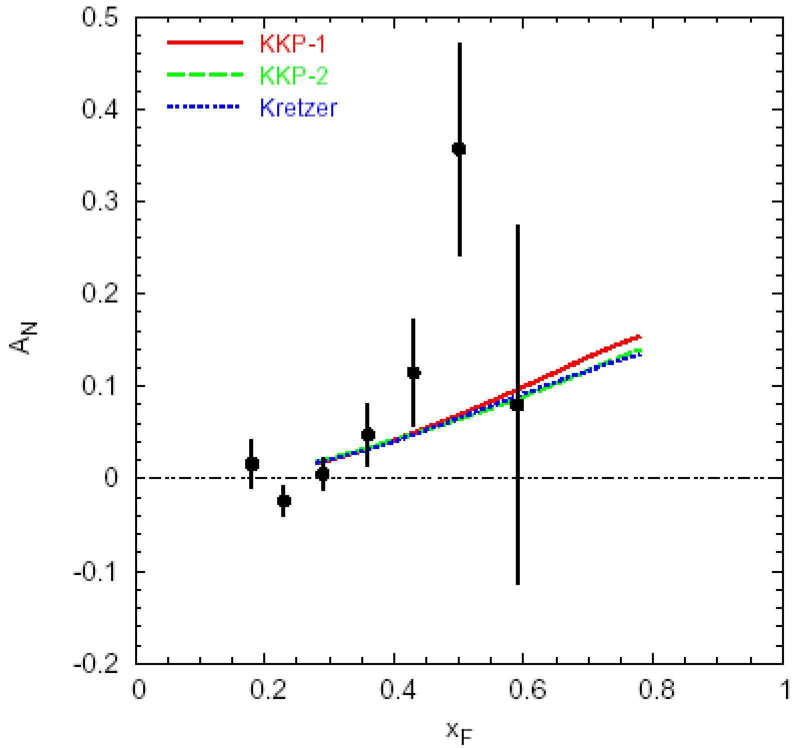
U. D'Alesio, F. Murgia

E704 data



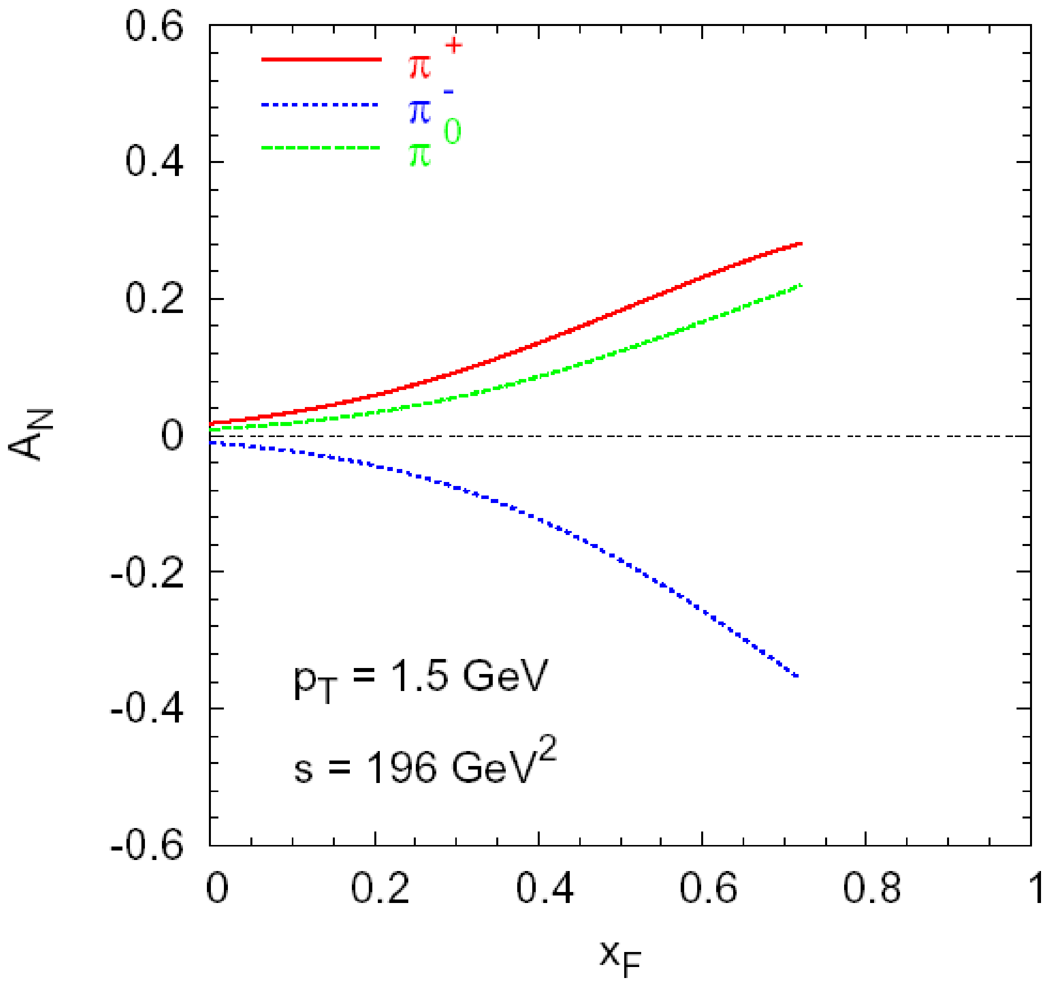
fit

STAR data



prediction

Predictions for PAX



predictions based on Sivers functions from E704 data

$$p^{\uparrow}\bar{p} \to \pi^{+,0,-} X$$

Higher-twist partonic correlations

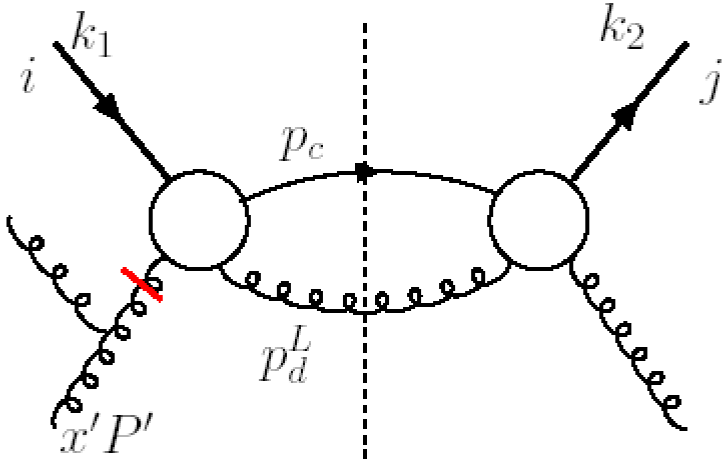
(Efremov, Teryaev; Qiu, Sterman; Kouvaris, Vogelsang, Yuan)

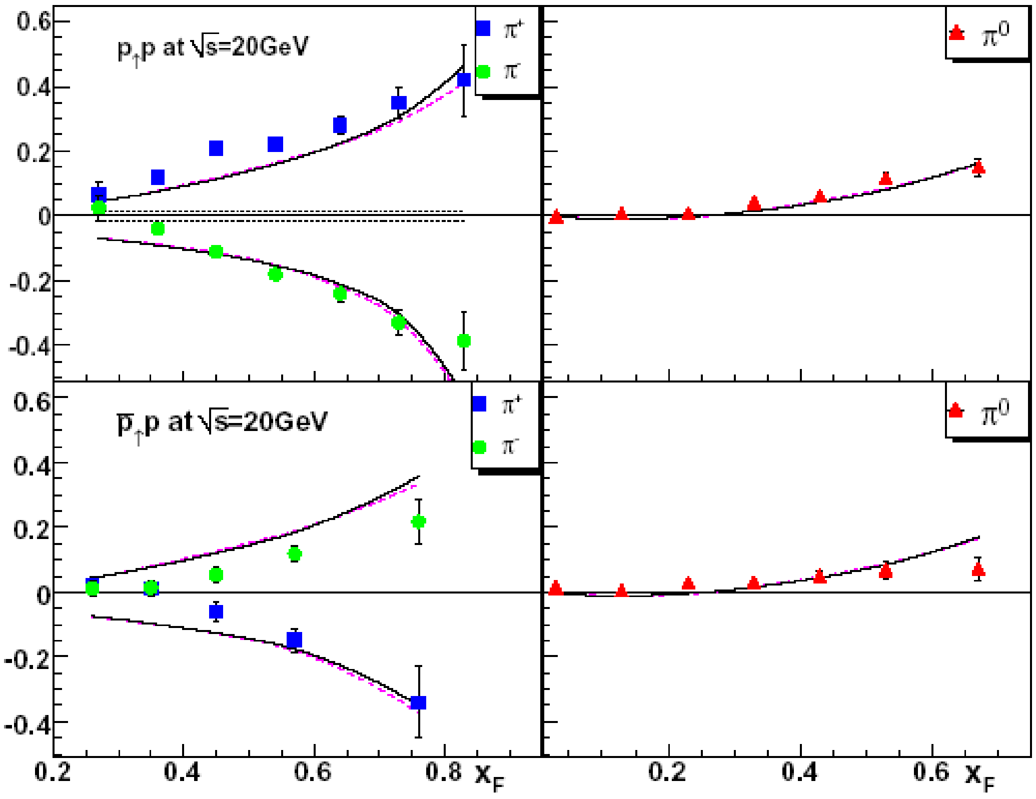
contribution to SSA $(A^{\uparrow} B \rightarrow h X)$

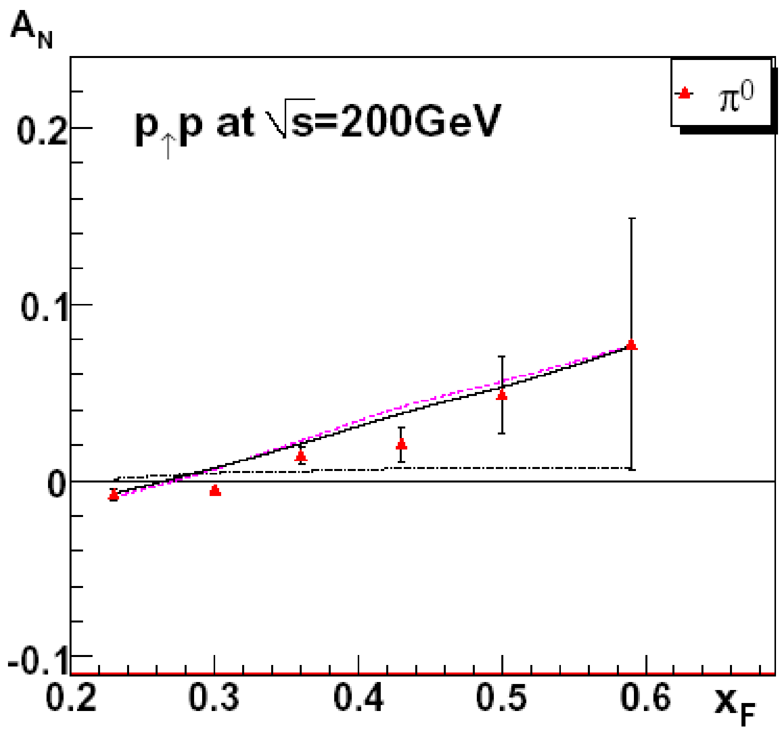
$$d\Delta\sigma \propto \sum_{a,b,c} \underbrace{T_a(k_1, k_2, S_{¿})}_{\text{twist-3 functions}} \otimes f_{b/B}(x_b) \otimes \underbrace{H^{ab\rightarrow c}(k_1, k_2)}_{\text{hard interactions}} \otimes D_{h/c}(z)$$

"collinear expansion" at order $k_{i\perp}$

$$T_a = N_a x^{\alpha_a} (1-x)^{\beta_a} f_{a/A}(x)$$
$$\propto f_{1T}^{¿(1)}$$





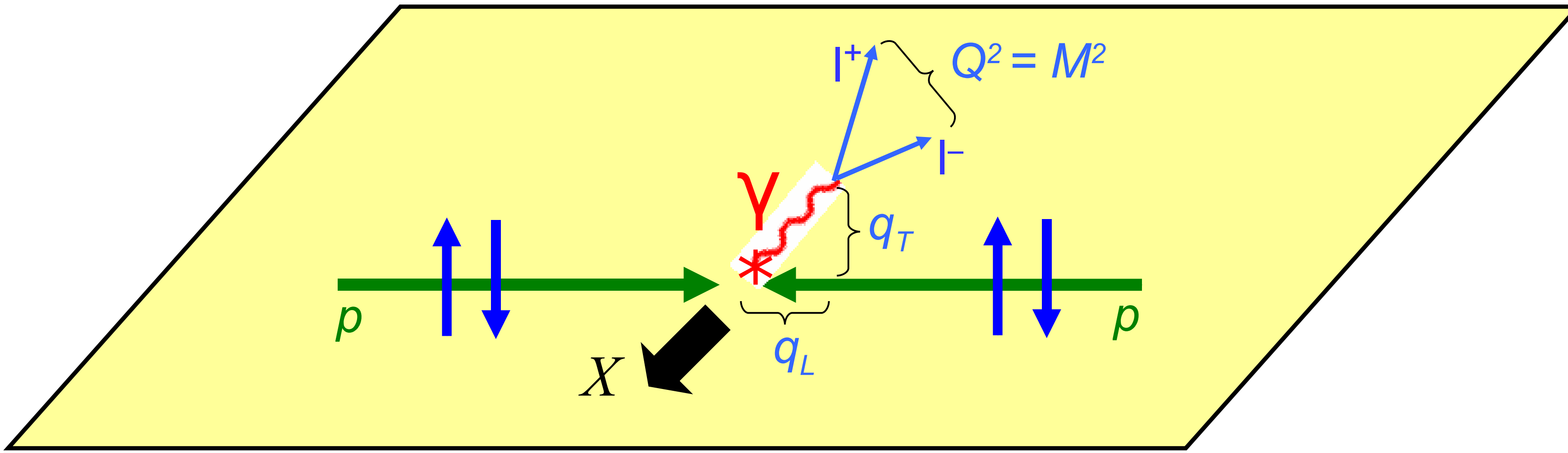


fits of E704 and STAR data

Kouvaris, Qiu, Vogelsang, Yuan

TMDs and SSAs in Drell-Yan processes

(returning to safer grounds and looking at future ....)



factorization holds, two scales, M^2, and q_T

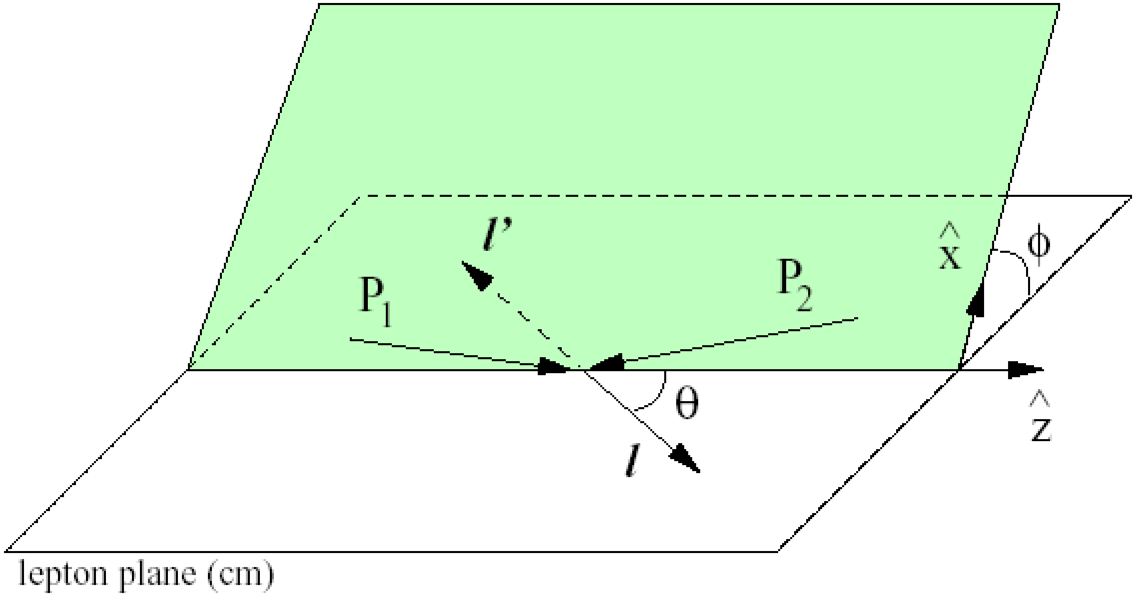
$$d\sigma^{D-Y} = \sum_q f_q(x_1, k_{\text{¿}}; Q^2) \otimes f_{\bar{q}}(x_2, k_{\text{¿}}; Q^2)\ \mathrm{d}\ \{\ \hat{\sigma}^{q\bar{q}\to l^+l^-}\ \text{¿}$$

D-YLAND, similar to SIDISLAND

talk by S. Melis

Unpolarized cross section already very interesting

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega}=\frac{3}{4\pi}\frac{1}{\lambda+3}\left(1+\lambda\cos^2\theta+\mu\sin^2\theta\cos\varphi+\frac{\nu}{2}\sin^2\theta\cos 2\varphi\right)$$



Collins-Soper frame

(Polarized) Drell-Yan cross sections allow to

access many TMDs (Boer-Mulders, ...) D-YLAND

verify whether $f_{1T}^{\text{¿}q}|_{SIDIS} = -f_{1T}^{\text{¿}q}|_{D-Y}$! ! !

and offer the **golden channel** to measure the transversity distribution (talk by A. Drago)

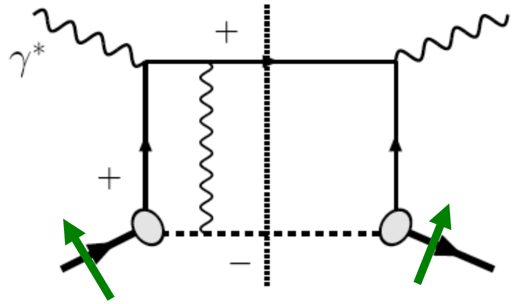
$$A_{TT} \propto \sum_q h_{1q}(x_1) \otimes h_{1\bar{q}}(x_2)$$

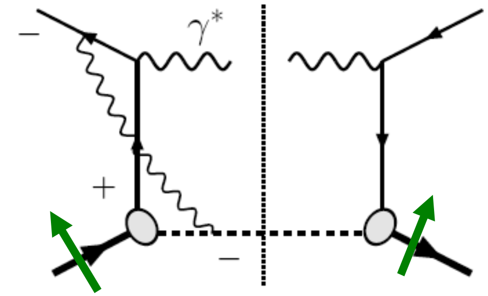
GSI energies: $s = 30 - 210 \ \text{GeV}^2$ $\quad M \geq 2 \ \text{GeV}^2$

Ideal for optimising cross section and probing valence quark region. A_{TT} predicted to be large and safe from pQCD corrections

Non-universality of Sivers Asymmetries: Unique Prediction of Gauge Theory !

Simple QED example:

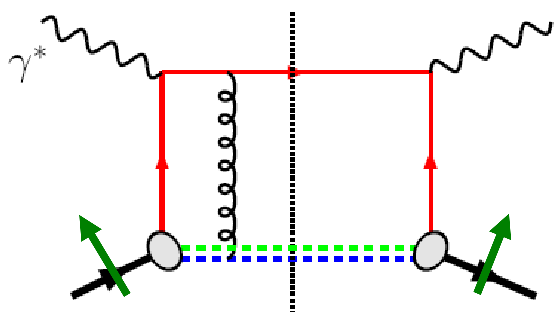


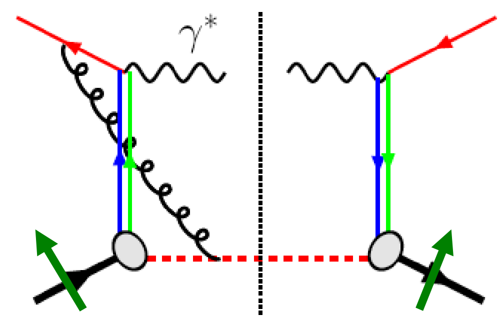


DIS: attractive

Drell-Yan: repulsive

Same in QCD:



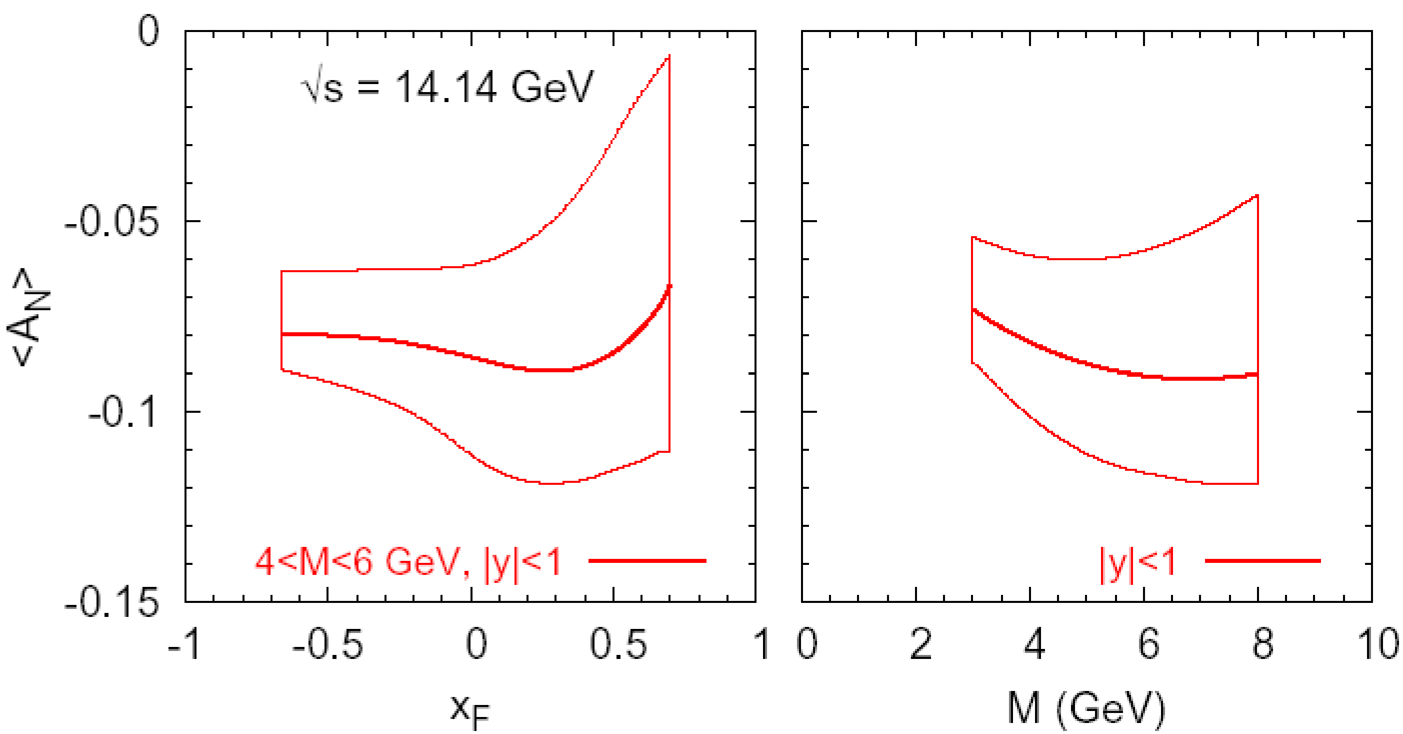


As a result:

$$\text{Sivers}|_{\text{DIS}} = -\text{Sivers}|_{\text{DY}}$$

Predictions for A_N in D-Y processes at PAX

U. D'Alesio



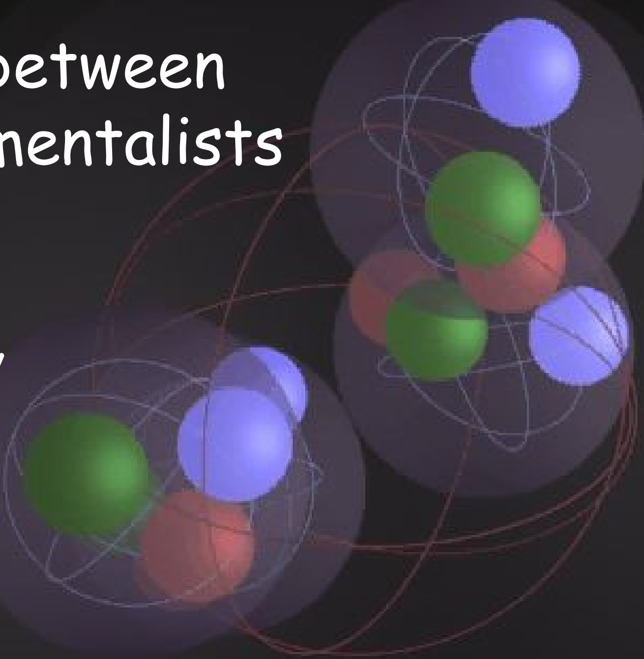
$$p^{\uparrow}\bar{p} \to l^{+}l^{-}X$$

Sivers function from SIDIS data,
with opposite sign

Conclusions and thanks

Disentangling the internal quark structure of nucleons; quark orbital motion, spin orbit correlations, …

Strict collaboration between theorists and experimentalists



Present experiments:
HERMES, COMPASS,
RHIC, JLab, BELLE

Future ones: JLab12,
GSI-FAIR, JPARK

FP6 European I3 project (HadronPhysics)

FP7 European I3 proposal (HadronPhysics2)