Baryon form factors from initial state radiation processes and some phenomenological considerations

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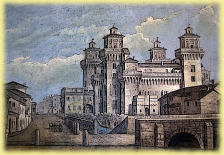


Nucleon Structure at FAIR

IUSS, Ferrara, 15-16 October, 2007

Outline

- **Initial State Radiation main features**

- *BaBar* $\sigma(e^+e^- \to p\bar{p}\gamma)$ and **Coulomb correction**
- *BaBar* $\sigma(e^+e^- \to \Lambda\bar{\Lambda}\gamma)$

- Space and time G_E^p/G_M^p via dispersion relations

- “Baryonium” and dips in $e^+e^- \to$ hadronic channels?

- G_M^p asymptotic behavior from a dispersive sum rule

BaBar:

Official results approved by the *BaBar* Collaboration

Pheno:

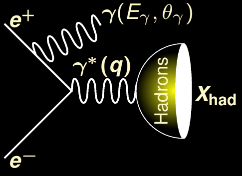
Phenomenological analysis by R. Baldini, C. Bini, P. Gauzzi, M. Mirazita, M. Negrini and S.P.

BABAR: Initial State Radiation

ISR main features

ISR studies at the $\Upsilon(4S)$ mass can yield the same observables as the low energy e^+e^- experiments

- Precise measurements on e^+e^- cross sections at low CM energy
- Hadron spectroscopy for $1 < \sqrt{s} < 5\ GeV$
- Form factors
- Discovery of new states [e.g. $Y(4260)$]



$e^+e^- \longrightarrow \gamma X_{had}$

Born cross section

$$\frac{d^2\sigma(e^+e^- \to \gamma X_{had})}{dx d\theta_\gamma} = W(x,\theta_\gamma)\,\sigma_{e^+e^- \to X_{had}}(s)$$

$$x = \frac{2E_\gamma}{E_{CM}} \qquad s = q^2 = E_{CM}^2(1-x)$$

Advantages

- All q at the same time $\Longrightarrow$ Better control on systematics (e.g. greatly reduced point to point)
- CM boost $\Longrightarrow$ at threshold $\epsilon \neq 0$ + $\sigma_W \sim 1\,MeV$
- Detected ISR γ $\Longrightarrow$ full X_{had} angular coverage

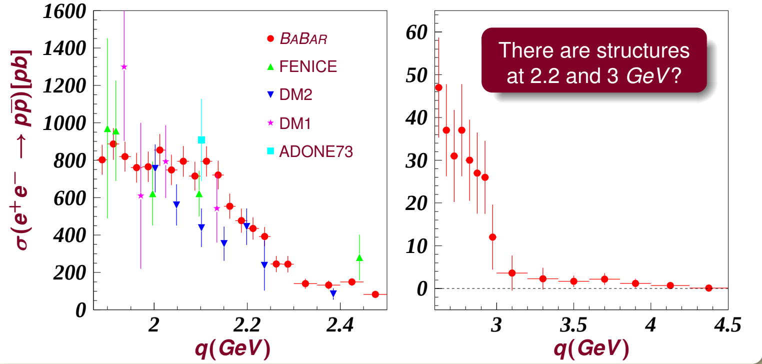
Drawbacks

- $\mathcal{L} \propto \Delta s$ bin width
- More backgrounds

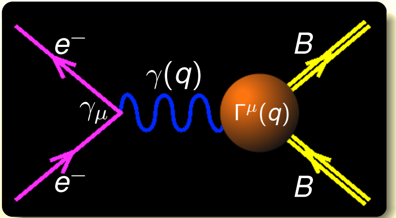
BaBar: $\sigma(e^+e^- \to p\bar{p}\gamma)$

- Analyzed 232 fb^{-1}
- $p\bar{p}\gamma$ kinematic fit
- 4025 selected events
- $\epsilon \sim 18 \pm 1\%$
- $\sim 6\%$ background mainly due to non ISR $e^+e^- \to p\bar{p}\pi^0$

BaBar cross section from threshold to 4.5 *GeV* [PRD73 (2006) 012005]



Baryon Form Factors and cross sections

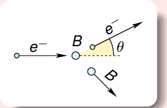


Baryon current operator (Dirac & Pauli)

$$\Gamma^\mu(q) = \gamma^\mu F_1(q^2) + \frac{i}{2M_B}\sigma^{\mu\nu} q_\nu F_2(q^2)$$

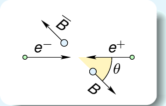
Electric and Magnetic Form Factors

$$G_E(q^2) = F_1(q^2) + \tau F_2(q^2) \qquad \tau = \frac{q^2}{4M_B^2}$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$



Elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}} \left[G_E^2 - \tau\left(1 + 2(1-\tau)\tan^2\frac{\theta}{2}\right) G_M^2 \right] \frac{1}{1-\tau}$$



Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2\right]$$

Coulomb correction for charged B: $C \approx \dfrac{y}{1-e^{-y}}$ $\qquad y = \dfrac{\pi\alpha}{\beta}$

Pheno: Coulomb correction in $p\bar{p}$ at threshold

Coulomb correction at threshold

$$C = \frac{\frac{\pi\alpha}{\beta}}{1 - \exp\left(-\frac{\pi\alpha}{\beta}\right)} \xrightarrow[\beta\to 0]{} \frac{\pi\alpha}{\beta}$$

This factor compensates for phase space and gives a constant value at threshold

Coss section at threshold ($\beta \to 0$, $\tau \to 1$, $s \to 4M_p^2$)

$$\lim_{\text{threshold}} \sigma(s) = \frac{4\pi^2\alpha^3}{3 \cdot 4M_p^2} \frac{3}{2} |G^p(4M_p^2)|^2 \approx 0.8\, nb\, |G^p(4M_p^2)|^2$$

Coulomb correction

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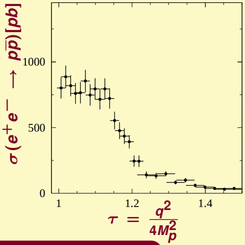
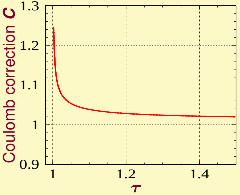
Non-zero value for $\tau = 1$

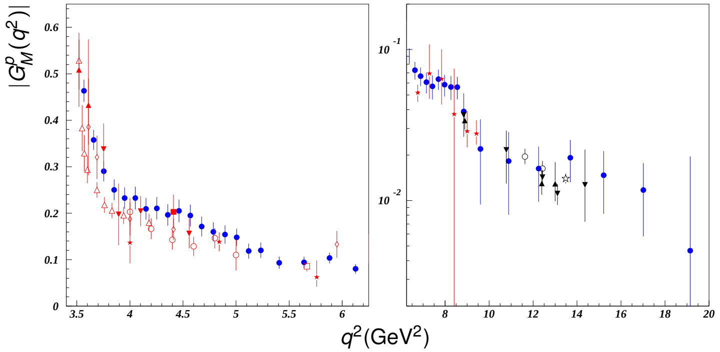
Data show unexplained plateau



$$|G_E^p(4M_p^2)| \equiv |G_M^p(4M_p^2)| \approx 1$$

The Coulomb correction does not explain the plateau for $\tau > 1$





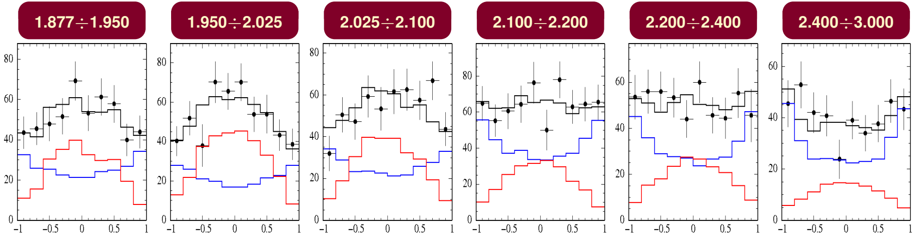
- ● '05 *BaBar* $e^+e^- \to p\bar{p}$ with ISR
- ■ ◇ '73, '94 ADONE, FENICE
- ▲ '77 ELPAR ($p\bar{p}$ at rest)
- ▼ ○ □ '79, '83, '90 DM1, DM2
- △ '94 PS170 ($p\bar{p}$ at rest, p stopped in liquid H)
- ▲ ▼ ○ '93, '99, '03 E760, E835 ($p\bar{p}$ at rest)
- ☆ ★ '05 CLEO, BES

All these data have been obtained assuming $|G_M^p| = |G_E^p| \equiv |G^p|$

$$|G^p|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{16\pi\alpha^2 C}{3}\frac{\sqrt{1-1/\tau}}{4q^2}(1+1/2\tau)}$$

BABAR: $\sigma(e^+e^- \to p\bar{p}\gamma)$ angular distribution

$\cos\theta_p$ distributions form threshold up to 3 GeV [intervals in $E_{CM} \equiv q$ (GeV)]

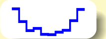


Events/0.2 vs. $\cos\theta_p$

$$\frac{d\sigma}{d\cos\theta_p} = A\left[H_E(\cos\theta_p, q^2)\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right|^2 + H_M(\cos\theta_p, q^2)\right]$$

H_E and H_M from MC

Histograms show contributions from

- G_E 
- G_M 

At low q

$\sin^2\theta_p > 1 + \cos^2\theta_p$

$\Rightarrow$

First observation!

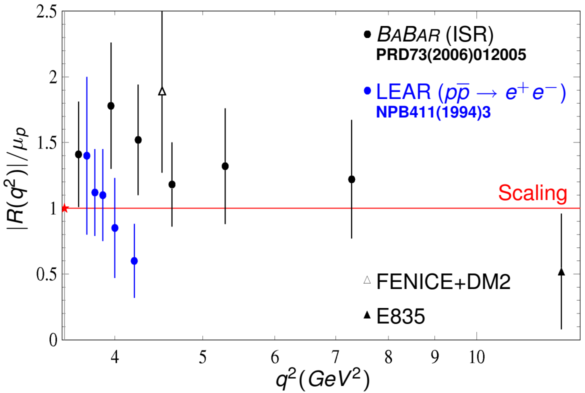
$|G_E^p| > |G_M^p|$

At higher q, $|G_E^p| \to |G_M^p|$

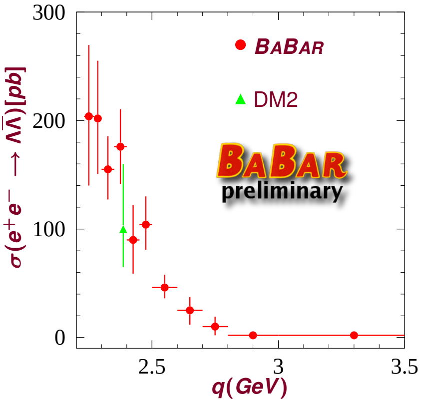
BaBar: Time-like $|G_E^p / G_M^p|$ measurements

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2}|G_M^p|^2\left[(1+\cos^2\theta) + \frac{4M_p^2}{q^2\mu_p^2}\sin^2\theta|R|^2\right]$$

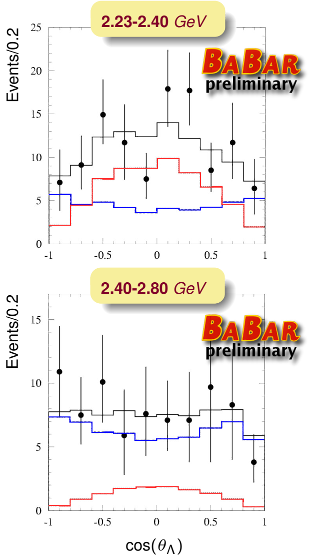
$$R(q^2) = \mu_p \frac{G_E^p(q^2)}{G_M^p(q^2)}$$



Cross section



Angular distributions



$\Lambda\bar{\Lambda}\gamma$ channel

- **Analyzed:** $230\ fb^{-1}$
- **Signal:** 204 ± 19
- **Background:** 15 ± 3

Pheno: Coulomb correction in $\Lambda\bar{\Lambda}$ at threshold?

Extended charge density

$$\rho(\vec{r}) = \frac{1}{(2\pi)^3}\int G_E^{\Lambda}(q^2) e^{i\vec{q}\cdot\vec{r}} d^3\vec{q}$$

The screened α' is ($R \sim R_\Lambda$)

$$\frac{\alpha'}{R} = \alpha \int \frac{\rho(\vec{r}')\rho_R(\vec{r}'')}{|\vec{r}' - \vec{r}''|} d^3\vec{r}' d^3\vec{r}''$$

Parton interaction after confinement should be negligible...

Other threshold effects

- QCD Coulomb-like correction $\alpha \longrightarrow C_F\alpha_S$
- $\Lambda\bar{\Lambda}$ production through $p\bar{p}$ rescattering

Threshold correction for $\Lambda\bar{\Lambda}$ channel

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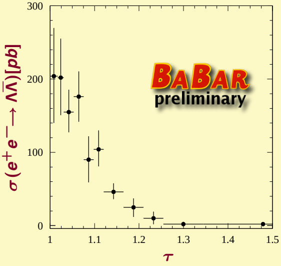
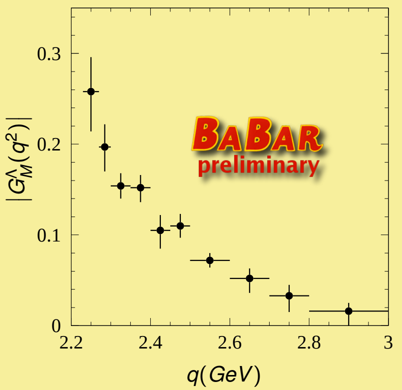
Non-zero value for $\tau = 1$

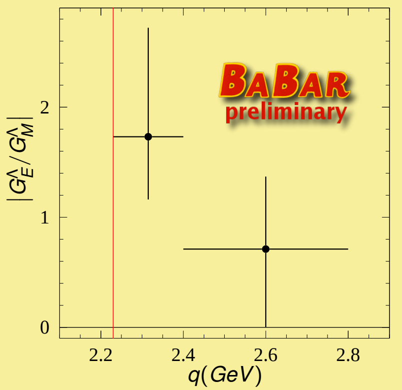
Data show no plateau (see $p\bar{p}$)



Form Factor ($|G_E^\Lambda| = |G_M^\Lambda|$)

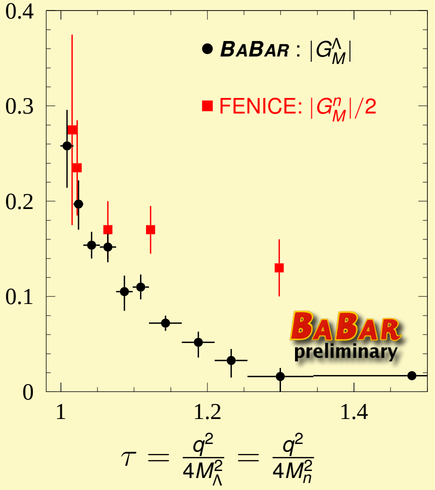


Ratio $|G_E^\Lambda/G_M^\Lambda|$



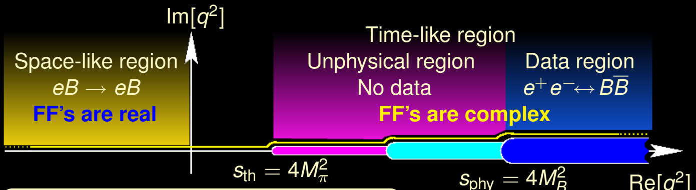
$|G_M^\Lambda|$ and $|G_M^n|$ comparison through U-spin

U-spin $\Longrightarrow G_M^\Lambda = G_M^n/2$



Additional corrections are needed to account for the $SU(3)$ flavor symmetry breaking

q^2-complex plane



Crossing: tot. helicity $= \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$

$$G_E(4M_B^2) = G_M(4M_B^2)$$

Perturbative QCD constrains the asymptotic behaviour

Brodsky, Farrar, Lepage

pQCD: $q^2 \to -\infty$

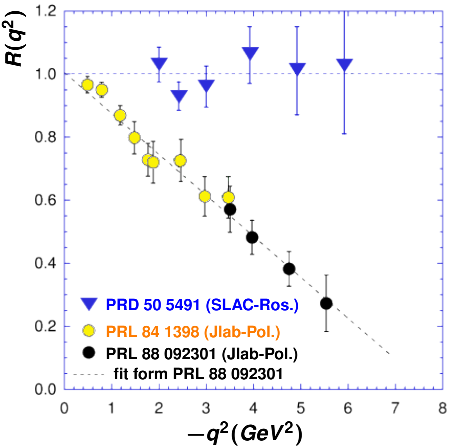
$$F_i(q^2) \propto (-q^2)^{-(i+1)} \Rightarrow G_{E,M} \propto (-q^2)^{-2}$$

Analyticity: $q^2 \to \pm\infty$

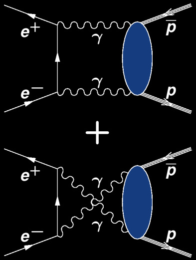
$$G_{E,M}(-\infty) = G_{E,M}(+\infty)$$

Space-like *R* measurements

Space-like data



$\gamma\gamma$ exchange



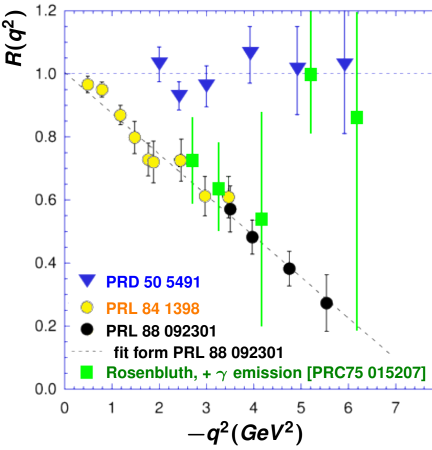
$\gamma\gamma$ exchange interferes with the Born term

Asymmetry in angular distributions

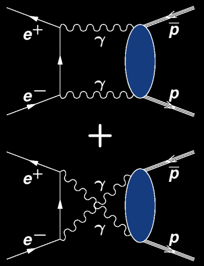
Egle, previuos talk [arXiv:0710.0454]

Space-like R measurements

Space-like data



$\gamma\gamma$ exchange



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Asymmetry in angular distributions

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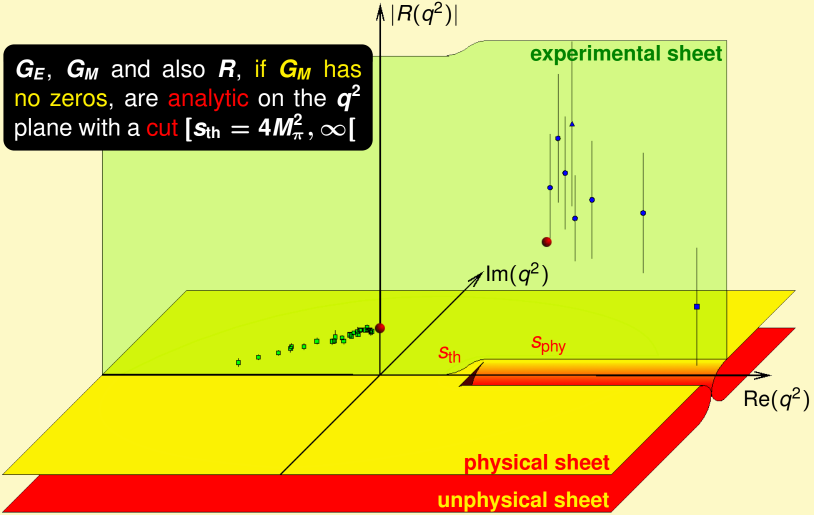
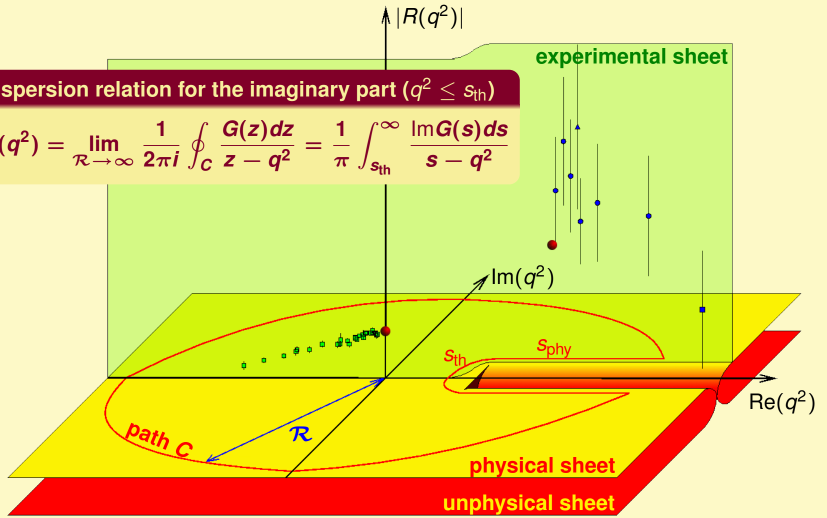
$R(q^2)$ in the complex plane

G_E, G_M and also R, if G_M has no zeros, are analytic on the q^2 plane with a cut $[s_{th} = 4M_\pi^2, \infty[$



$R(q^2)$ in the complex plane



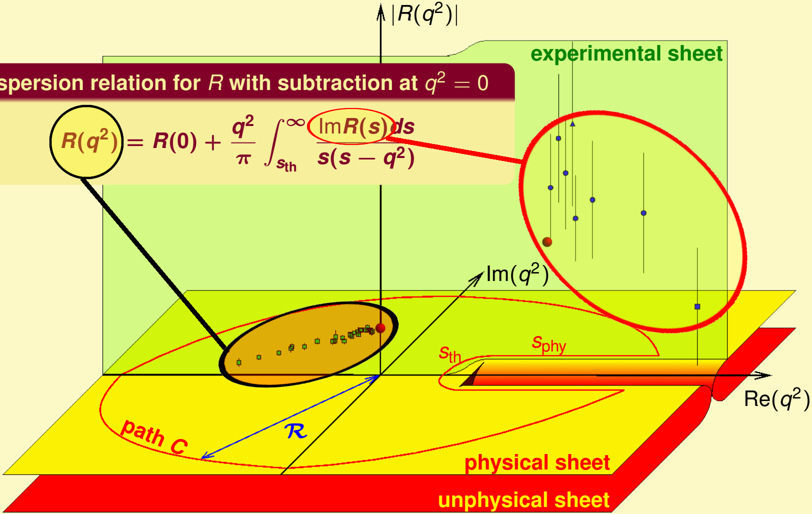
Dispersion relation for the imaginary part ($q^2 \leq s_{\rm th}$)

$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i} \oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi} \int_{s_{\rm th}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s - q^2}$$

$R(q^2)$ in the complex plane

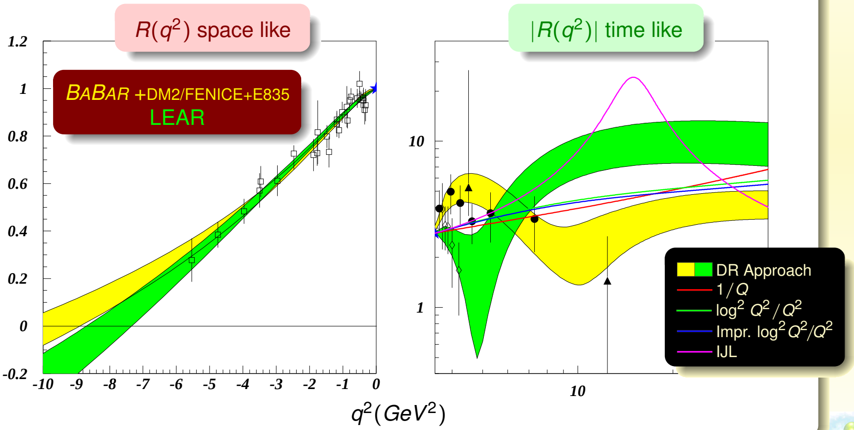
Dispersion relation for R with subtraction at $q^2 = 0$

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}R(s)\,ds}{s(s-q^2)}$$



Using a Dispersion Relation (DR) formalism, we fit data in the time- and space-like regions and extrapolate into the whole q^2-complex plane

Reconstructed R in space and time regions



NPB(Proc.Supp.)162(2006)46

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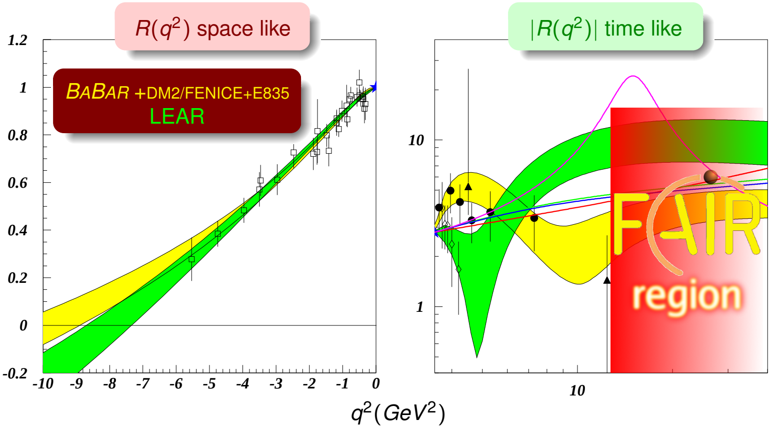
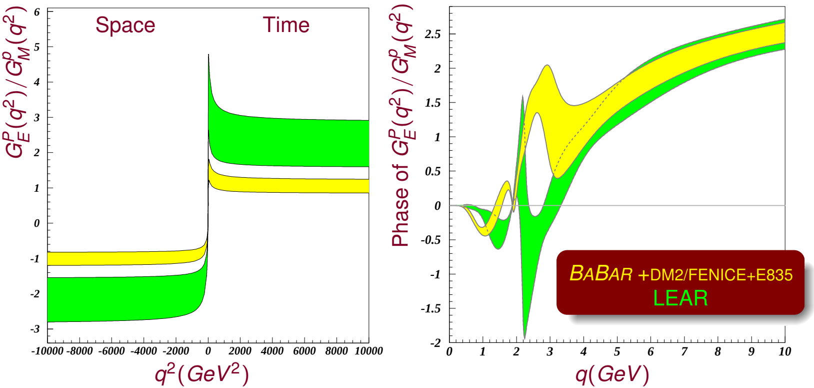
Reconstructed R in space and time regions



Pheno: Asymptotic $G_E^P(q^2)/G_M^p(q^2)$ and phase

NPB(Proc.Supp.)162(2006)46

Asymptotic behaviour of $G_E^P(q^2)/G_M^p(q^2)$



pQCD prediction

$$\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right| \xrightarrow[|q^2|\to\infty]{} 1$$

Pheno: Asymptotic $G_E^P(q^2)/G_M^p(q^2)$ and phase

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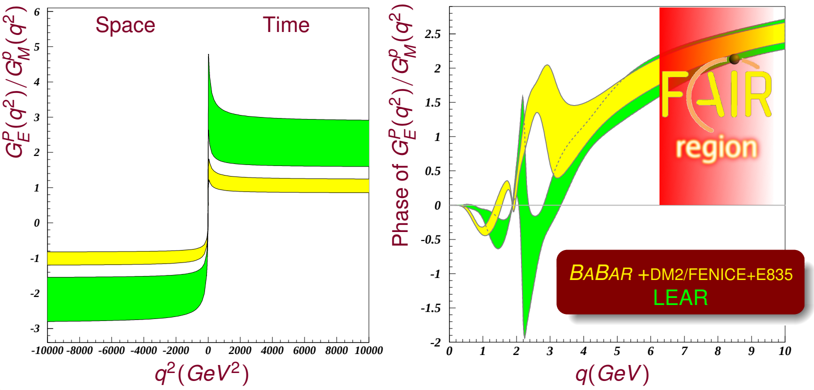
Asymptotic behaviour of $G_E^P(q^2)/G_M^p(q^2)$

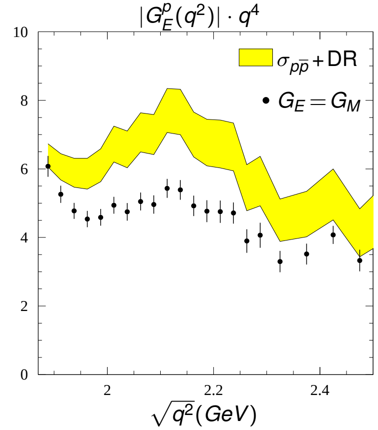


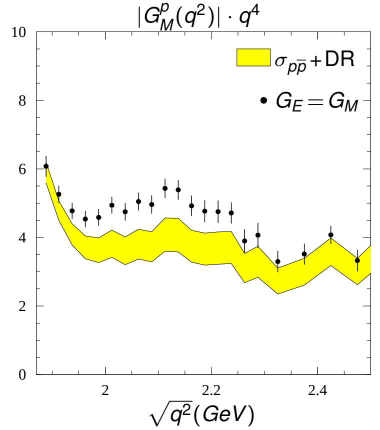
pQCD prediction

$$\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right| \xrightarrow[|q^2|\to\infty]{} 1$$

Pheno: $|G_E^p(q^2)|$ and $|G_M^p(q^2)|$ from $\sigma_{p\bar{p}}$ and DR

BABAR $\sigma(e^+e^- \to p\bar{p}\gamma)$ + DR



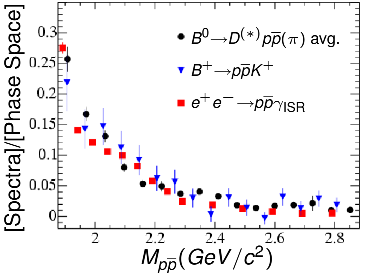


G_M^p very steep at threshold $\Longrightarrow$ vector "Baryonium" ?

Steep rising behaviours in other $p\bar{p}$ spectra

BABAR: $p\bar{p}$ spectra in some B decays compared with $|G_M^p|$

PRD74 051101

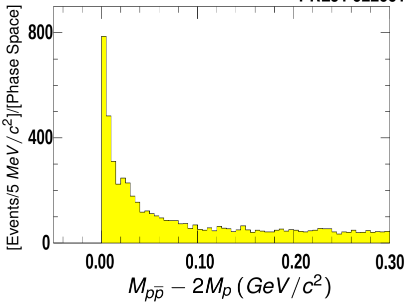


Similar results found by Belle PRL88 181803, PRL89 151802

BES in $J/\Psi \longrightarrow p\bar{p}\gamma$

Opposite C parity in the $p\bar{p}$ channel

PRL91 022001

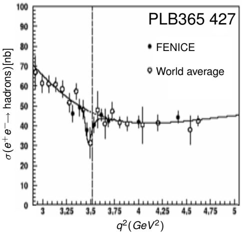


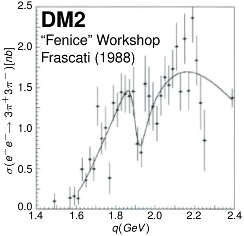
Sub-threshold resonance

Preferred $J^P = 0^{\pm}$, $C = +$
$M \approx 1860\ MeV/c^2$
$\Gamma < 30\ MeV$

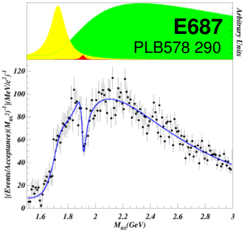
Dips in multihadronic reactions

e^+e^- annihilation processes

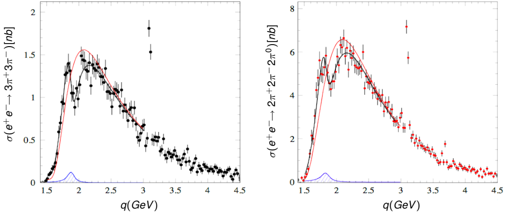




Diffractive photoproduction



e^+e^- annihilation processes with ISR at BABAR [PRD73 052003]



V_0	M(MeV)	Γ(MeV)
hadrons	∼1870	10÷20
DM2	1930(30)	35(20)
E687	1910(10)	37(13)
BABAR	1880(50)	130(30)
BABAR (π^0)	1860(20)	160(20)

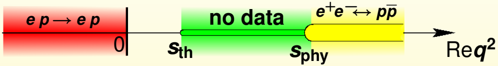
The parameters depend on the model used for the background

Dispersion relations and sum rules

Geshkenbeĭn, Ioffe, Shifman ’74

- **DR’s connect space and time values of a form factor $G(q^2)$**

$$G(q^2) = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} \frac{{\rm Im}G(s)ds}{s - q^2}$$



Drawbacks

- **The imaginary part is not experimentally accessible**
- **There are no data in the unhysical region $[s_{\rm th}, s_{\rm phy}]$**
- **We need to know the asymptotic behavior**

- **They applied the DR for the imaginary part to the function**

$$\phi(z) = f(z)\frac{\ln G(z)}{z\sqrt{s_{\rm th} - z}} \quad \text{with} \quad \int_0^{s_{\rm phy}} f^2(z)dz \ll 1$$

Advantages

- **The DR integral contains the modulus $|G(s)|$**
- **The unhysical region contribution is suppressed**

Drawback

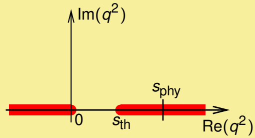
- **Zeros of $G(z)$ are poles for $\phi(z)$**

Attenuation of the unphysical region

Strategy

- Use the function $\phi(z) = f(z)\dfrac{\ln G(z)}{z\sqrt{s_{\rm th} - z}}$
- $f(z)$, is analytic with the cut $(-\infty, 0)$
- $f(z) = f_L(w) = \sum_{l=0}^{L} \dfrac{2l+1}{(L+1)^2} P_l(1-2w)$, $w = \dfrac{\sqrt{s_{\rm phy}} - \sqrt{z}}{\sqrt{s_{\rm phy}} + \sqrt{z}}$

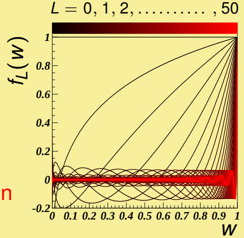
q^2-complex plane



- This function, with $f_L(0)=1$, minimizes:

$$\int_0^1 f_L^2(w)\,dw$$

and suppresses the contribution in the unphysical region



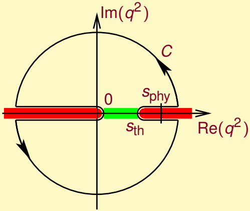
Attenuated DR and sum rule

New DR with variable suppressed region $[0, s_{phy}]$ $[G(q^2)$ has no zeros$]$

$$\oint_C \phi(z)dz = 0$$

$$\Downarrow$$

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)] \ln G(t)}{t\sqrt{s_{th} - t}} dt}_{\text{Space-like}} = \underbrace{\int_{s_{th}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{th}}} ds}_{\text{Time-like}}$$



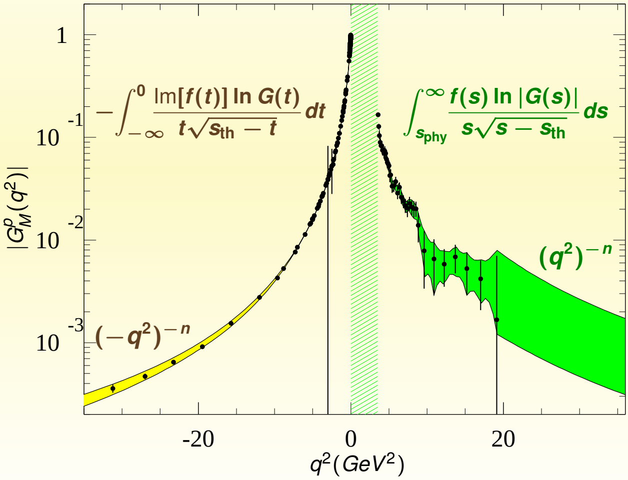
Convergence relation to test asymptotic power behaviour of G_M^p

$$\underbrace{-\int_{-\infty}^{0} \frac{\mathrm{Im}[f(t)] \ln G(t)}{t\sqrt{s_{th} - t}} dt}_{\text{Space-like data} + (-t)^{-n}} = \int_{s_{th}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{th}}} ds \approx \underbrace{\int_{s_{phy}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{th}}} ds}_{\text{Time-like data} + s^{-n}}$$

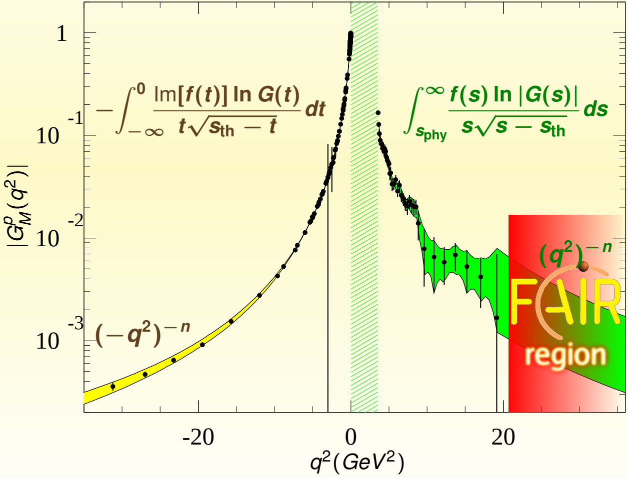
***n* is the free parameter**

Pheno: Sum rule: result

$$G_M^p(q^2) \underset{|q^2|\to\infty}{\propto} (q^2)^{-(2.27\pm0.36)}$$



$$G_M^p(q^2) \underset{|q^2|\to\infty}{\propto} (q^2)^{-(2.27\pm 0.36)}$$



Conclusions

BaBar:

- Structured $\sigma(e^+e^- \to p\bar{p}\gamma)$ (Best measurement ever done!)
- $\sigma(e^+e^- \to \Lambda\bar{\Lambda}\gamma) \approx 0.2\,nb$ at threshold
- $|G_E^p| > |G_M^p|$ and $|G_E^\Lambda| \gtrsim |G_M^\Lambda|$ above threshold

Pheno:

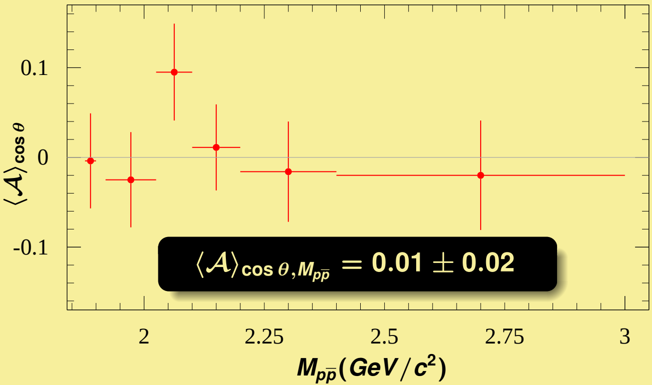
- Coulomb correction $\Rightarrow \sigma_{B\bar{B}} > 0$ at threshold
- Space and time $|G_E^p/G_M^p|$ via DR
  - asymptotic behaviour and space-like zero
  - results on G_E^p and G_M^p
  - "Baryonium" and dips in $e^+e^- \to$ hadronic channels ?

- Space and time data on G_M^p connected via analyticity confirm the pQCD asymptotic behavior $G_M^p \propto (q^2)^{-2}$

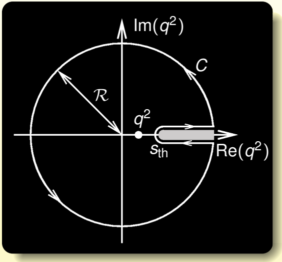
BACK-UP SLIDES

$\gamma\gamma$ exchange from $e^+e^- \to p\bar{p}\gamma$ BABAR data

$$\mathcal{A}(\cos\theta, M_{p\bar{p}}) = \frac{\frac{d\sigma}{d\Omega}(\cos\theta, M_{p\bar{p}}) - \frac{d\sigma}{d\Omega}(-\cos\theta, M_{p\bar{p}})}{\frac{d\sigma}{d\Omega}(\cos\theta, M_{p\bar{p}}) + \frac{d\sigma}{d\Omega}(-\cos\theta, M_{p\bar{p}})}$$



$G_E(q^2)$, $G_M(q^2)$ and also $R(q^2)$, if G_M has no zeros, are analytic on the q^2 plane with a cut $[s_{th} = 4M_\pi^2, \infty[$



Dispersion relation for the imaginary part ($q^2 \leq s_{th}$)

$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i} \oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi} \int_{s_{th}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s - q^2}$$

Subtraction at $q^2 = 0$ because of a non-vanishing asymptotic limit of the ratio

For $q^2 \leq s_{th}$ R is real

$$R(q^2) = R(0) + \frac{q^2}{\pi} \int_{s_{th}}^{\infty} \frac{\mathrm{Im}R(s)ds}{s(s - q^2)}$$

For $q^2 > s_{th}$ R is complex

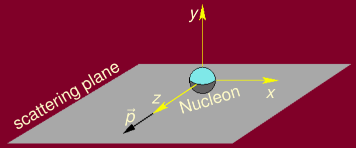
$$\mathrm{Re}R(q^2) = R(0) + \frac{q^2}{\pi} \,\mathrm{Pr} \int_{s_{th}}^{\infty} \frac{\mathrm{Im}R(s)ds}{s(s - q^2)}$$

Polarization formulae in the time-like region

The ratio $R(q^2)$ is complex for $q^2 \geq s_{th}$

$$R(q^2) = \mu_p \frac{G_E^p(q^2)}{G_M^p(q^2)} = |R(q^2)|e^{i\rho(q^2)}$$

The polarization depends on the phase ρ



Polarization components and single spin asymmetry

$$\mathcal{P}_y = -\frac{\sin(2\theta)|R|\sin(\rho)}{\mu_p D\sqrt{\tau}} = \left\{ \begin{array}{c} \text{Does not depend on } P_e \\ \text{in } p^\uparrow \overline{p} \longrightarrow e^+e^- \end{array} \right\} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \equiv \mathcal{A}_y$$

$$\mathcal{P}_x = -P_e \frac{2\sin(2\theta)|R|\cos(\rho)}{\mu_p D\sqrt{\tau}}$$

$$\mathcal{P}_z = P_e \frac{2\cos(\theta)}{D} = \left\{ \text{Does not depend on the phase } \rho \right.$$

$$D = 1 + \cos^2\theta + \frac{1}{\tau\mu_p^2}|R|^2\sin^2\theta \qquad \tau = \frac{q^2}{4M_N^2} \qquad P_e = \text{electron polarization}$$

Pheno: Parameterization and constraints

The imaginary part of R is parameterized by two series of orthogonal polynomials $T_i(x)$

$$\mathrm{Im}R(q^2) \equiv I(q^2) = \begin{cases} \sum_i C_i T_i(x) & x = \dfrac{2q^2 - s_{\rm phy} - s_{\rm th}}{s_{\rm phy} - s - 0} & s_{\rm th} \le q^2 \le s_{\rm phy} \\ \sum_j D_j T_j(x') & x' = \dfrac{2s_{\rm phy}}{q^2} - 1 & q^2 > s_{\rm phy} \end{cases}$$

$$s_{\rm th} = 4M_\pi^2 \qquad s_{\rm phy} = 4M_N^2$$

Theoretical conditions on $ImR(q^2)$

- $R(4M_\pi^2)$ is real $\Longrightarrow I(4M_\pi^2) = 0$
- $R(4M_N^2)$ is real $\Longrightarrow I(4M_N^2) = 0$
- $R(\infty)$ is real $\Longrightarrow I(\infty) = 0$

Theoretical conditions on $R(q^2)$

- Continuity at $q^2 = 4M_\pi^2$
- $R(4M_N^2)$ is real and $\mathrm{Re}R(4M_N^2) = \mu_p$

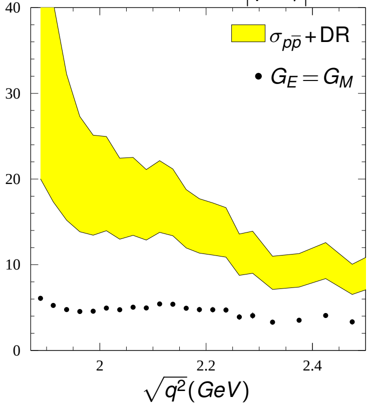
Experimental conditions on $R(q^2)$ and $|R(q^2)|$

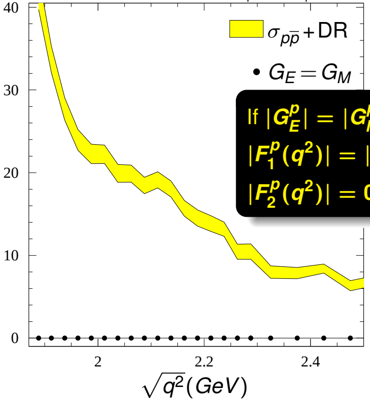
- Space-like region ($q^2 < 0$) data for R from TJNAF and MIT-Bates
- Time-like region ($q^2 \ge 4M_N^2$) data for $|R|$ from FENICE+DM2, *BABAR* , E835 and LEAR

BABAR $\sigma(e^+e^- \to p\bar{p})$ + DR

$$|F_1^p(q^2)|q^4 = |G_M^p| \frac{|G_E^p/G_M^p - \tau|}{|1-\tau|} q^4$$

$$|F_2^p(q^2)|q^4 = |G_M^p| \frac{|G_E^p/G_M^p - 1|}{|\tau - 1|} q^4$$





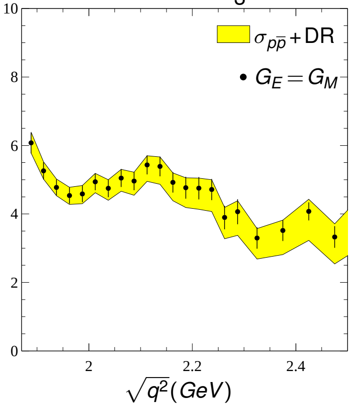
If $|G_E^p| = |G_M^p|$

$|F_1^p(q^2)| = |G_M^p(q^2)|$

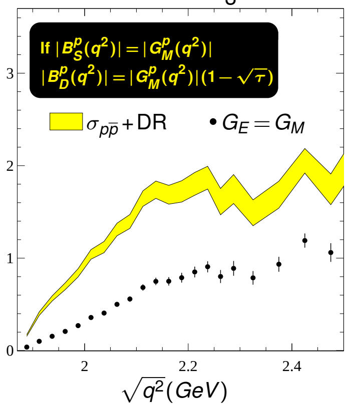
$|F_2^p(q^2)| = 0$

BABAR $\sigma(e^+e^- \to p\bar{p})$ + DR

$$|B_S^p(q^2)|q^4 = \frac{|2\sqrt{\tau}G_M^p + G_E^p|}{3}q^4$$

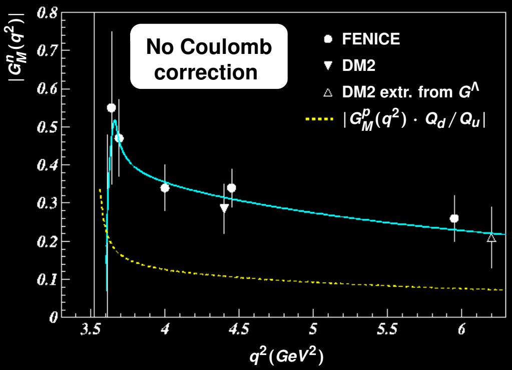


$$|B_D^p(q^2)|q^4 = \frac{|\sqrt{\tau}G_M^p - G_E^p|}{3}q^4$$



Time-like $|G_M^n|$ measurements

Only two measurements by FENICE and DM2



	$\lvert G_M^n/G_M^p \rvert$
Data	~ 1.5
Naively	$\sim \lvert Q_d/Q_u \rvert$
pQCD	< 1
Soliton models	~ 1
VMD	$\gg 1$

Threshold behaviour from angular distribution

$$G_M^n(4M_n^2) = G_E^n(4M_n^2) = 0?$$

BABAR does agree with FENICE

Large $G_M^\Lambda \xrightarrow{\text{U-spin}}$ large G_M^n

We start from the imaginary part of the ratio $\boldsymbol{R(q^2)}$, written in the most general and model-independent way as

$$I(q^2) \equiv \mathrm{Im}[R(q^2)] = \text{series of orthogonal polynomials}$$

Some theoretical constraints can be applied directly on this function $I(q^2)$

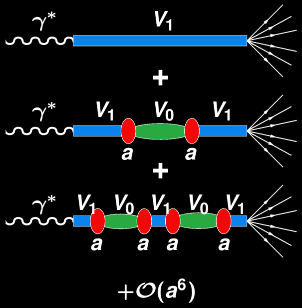
Dispersion Relations

The function $R(q^2)$ is fully reconstructed in both time-like and space-like regions

The other theoretical conditions and the experimental constraints can be imposed on the obtained analytic expression of $\boldsymbol{R(q^2)}$

Pheno: "Baryonium" → dip in multihadronic processes

P.J. Franzini and F.J. Gilman, 1985



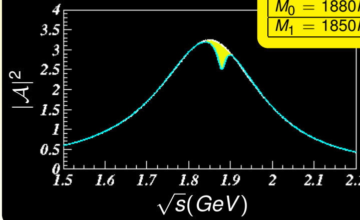
A vector meson V_0 ($J^{PC} = 1^{--}$), with vanishing e^+e^- coupling, which decays through an intermediate broad vector meson V_1

$$\mathcal{A} \propto \frac{1}{s - M_1^2}\left(1 + a\frac{1}{s - M_0^2}a\frac{1}{s - M_1^2} + \cdots\right)$$

$$\mathcal{A} = \frac{s - M_0^2}{(s - M_1^2)(s - M_0^2) - a^2}$$

For instance...

$M_0 = 1880\,MeV$	$\Gamma_0 = 20\,MeV$
$M_1 = 1850\,MeV$	$\Gamma_1 = 300\,MeV$



Pheno: Asymptotic value and space-like zero

Real asymptotic values for R

$$|R_{BABAR}(\infty)| = (1.0 \pm 0.2)\mu_p$$
$$|R_{LEAR}(\infty)| = (2.2 \pm 0.6)\mu_p$$

BABAR is in agreement with the scaling law $|G_E(q^2)| \simeq |G_M(q^2)|$ as $q^2 \to \infty$

Asymptotic behaviour of F_2/F_1

$$\lim_{q^2\to\infty} \frac{q^2}{4M_N^2}\left|\frac{F_2}{F_1}\right| = \left|\frac{R(\infty)}{\mu_p} - 1\right| = \begin{cases} 2.0 \pm 0.2 & \text{BABAR} \\ 3.2 \pm 0.6 & \text{LEAR} \end{cases}$$

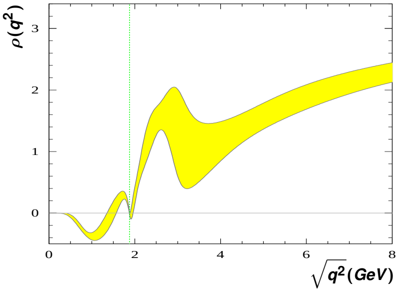
$$\left|\frac{F_2}{F_1}\right| \underset{q^2\to\infty}{\propto} \frac{1}{q^2}$$

Space-like zero

$$t_0^{BABAR} = (-10 \pm 1)\ GeV^2$$
$$t_0^{LEAR} = (-8.0 \pm 0.8)\ GeV^2$$

Phragmèn-Lindelöf theorem

$$\rho(q^2) \underset{q^2\to\infty}{\longrightarrow} \pi$$



BABAR: ISR cross section and luminosity

ISR Cross section

$$\sigma_{X_{\rm had}}(s) = \frac{dN/d\sqrt{s}}{\epsilon(1+\delta_{\rm rad})(dL/d\sqrt{s})}$$

ϵ = reconstruction efficiency

$\delta_{\rm rad}$ = radiative corrections

ISRLuminosity

$$\frac{dL}{d\sqrt{s}} = \frac{\alpha}{\pi x}\left[\left(2-2x+x^2\right)\log\frac{1+C}{1-C} - x^2 C\right]\frac{2\sqrt{s}}{M^2_{\Upsilon(4S)}} L_{ee}$$

- from integration of the radiator function over θ^*_γ
- $20^o < \theta^*_\gamma < 160^o$ acceptance for ISR photon

$C = \cos\theta^*_{\gamma,\rm min}$ and θ^*_γ is the ISR angle in e^+e^- CM