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HADRON STRUCTURE and RADIATIVE CORRECTIONS

*Egle Tomasi-Gustafsson
Saclay, France*



**Nucleon Structure at FAIR,
Ferrara, 15 –X – 2007**

PLAN

Experimental View and Models

- *space-like (ep-scattering)*
- *time-like (e^+e^- or $ppbar$ annihilation)*

Model Independent Statements

- *Symmetry properties of fundamental interactions*
- *Kinematical constraints*

Exact Calculations ?

- *QED 'exact' calculations*
- *Radiative corrections*

Nucleon Structure and/or Reaction Mechanism?

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Hadron Electromagnetic Form factors

- Characterize the internal structure of a particle (\neq point-like)
- In a P- and T-invariant theory, the EM structure of a particle of spin S is defined by $2S+1$ form factors.
- Neutron and proton form factors are different.
- Elastic form factors contain information on the hadron ground state.
- Playground for theory and experiment.
- New interest due to polarization data

Space-like and time-like regions

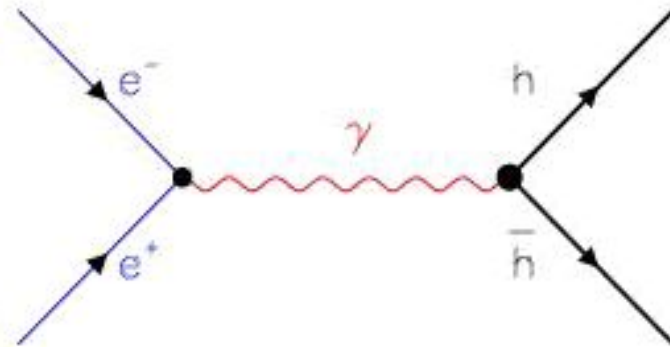
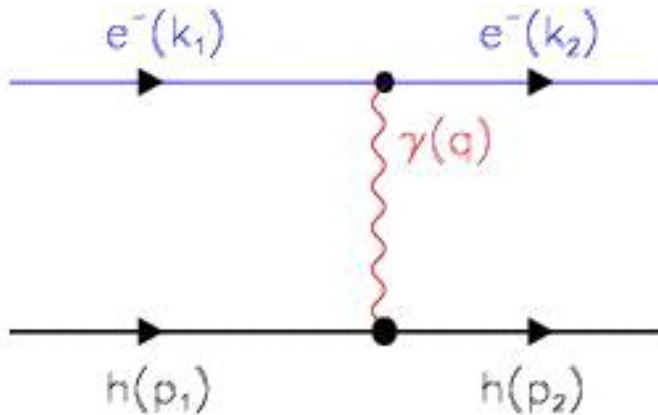
- FFs are analytical functions.

- In framework of one photon exchange, FFs are functions of the momentum transfer squared of the virtual photon, $t = q^2 = -Q^2$.

$t < 0$

$t > 0$

Scattering



$$e^- + h \Rightarrow e^- + h$$

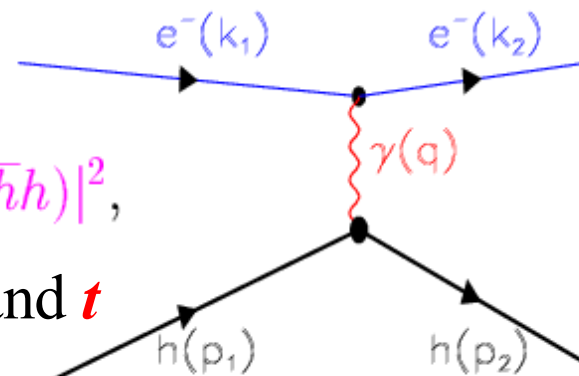
$$e^+ + e^- \Rightarrow \bar{h} + h$$

Form factors are *real in the space-like region*

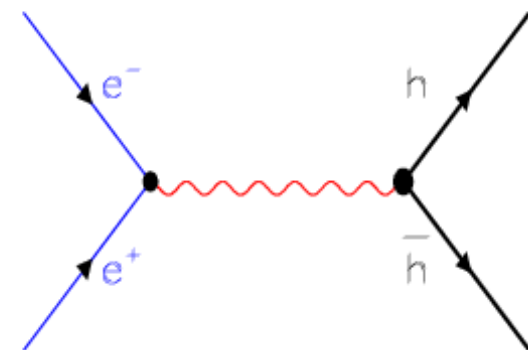
and *complex in the time-like region.*

Crossing Symmetry

$$e^- + h \rightarrow e^- + h$$



$$e^- + e^+ \rightarrow \bar{h} + h$$



Scattering and annihilation channels:

- Described by the same amplitude :

$$|\overline{\mathcal{M}}(e^\pm h \rightarrow e^\pm h)|^2 = f(s, t) = |\overline{\mathcal{M}}(e^+ e^- \rightarrow \bar{h} h)|^2,$$

- function of two kinematical variables, s and t

$$s = (k_1 + p_1)^2$$

$$t = (k_1 - k_2)^2$$

- which scan different kinematical regions

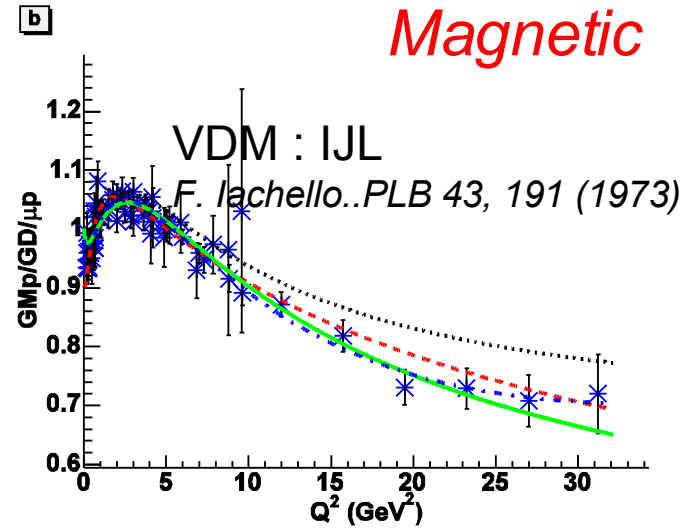
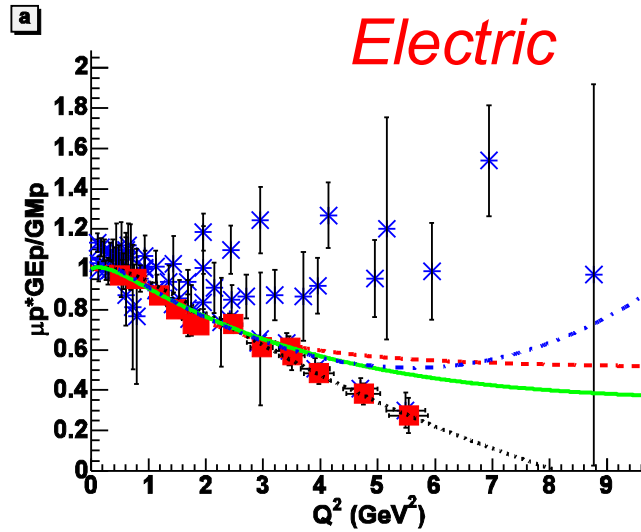
$$\cos^2 \tilde{\theta} = 1 + \frac{st + (s - M^2)^2}{t\left(\frac{t}{4} - M^2\right)} \rightarrow 1 + \frac{ctg^2 \frac{\theta}{2}}{1 + \tau}$$

The nucleon form factors

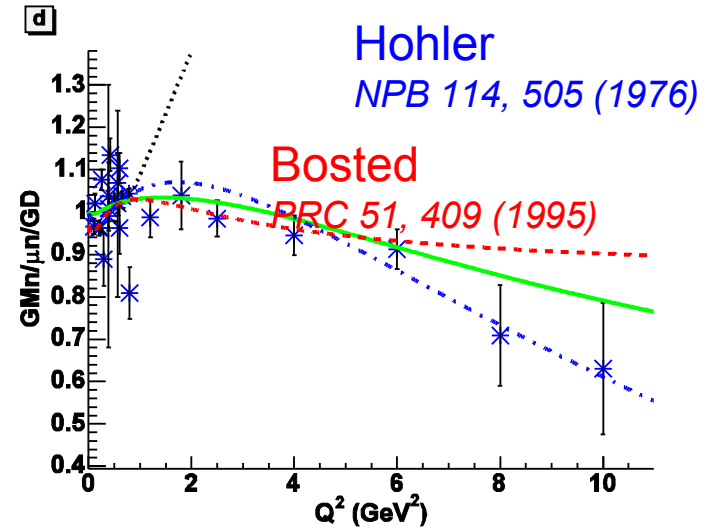
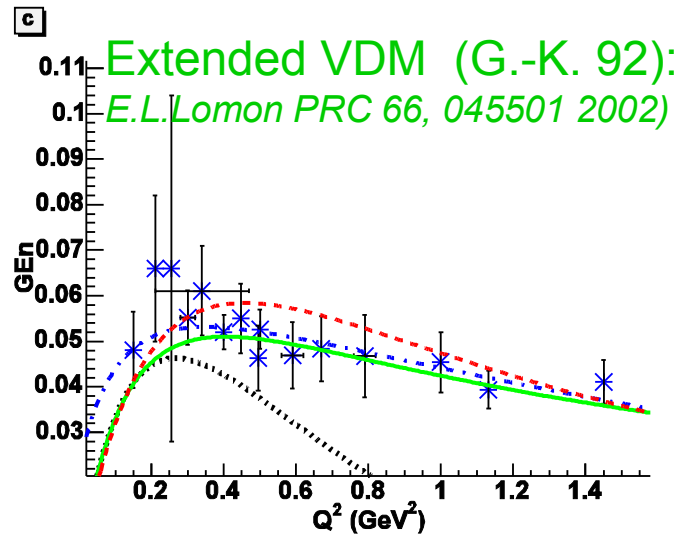
E. T.-G., F. Lacroix, Ch. Duterte, G.I. Gakh, EPJA 24, 419 (2005)

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proton



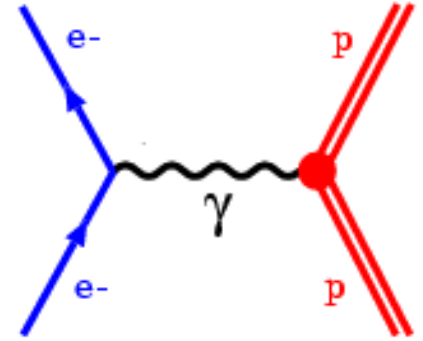
neutron



The Rosenbluth separation (1950)

- Elastic **ep** cross section (1- γ exchange)

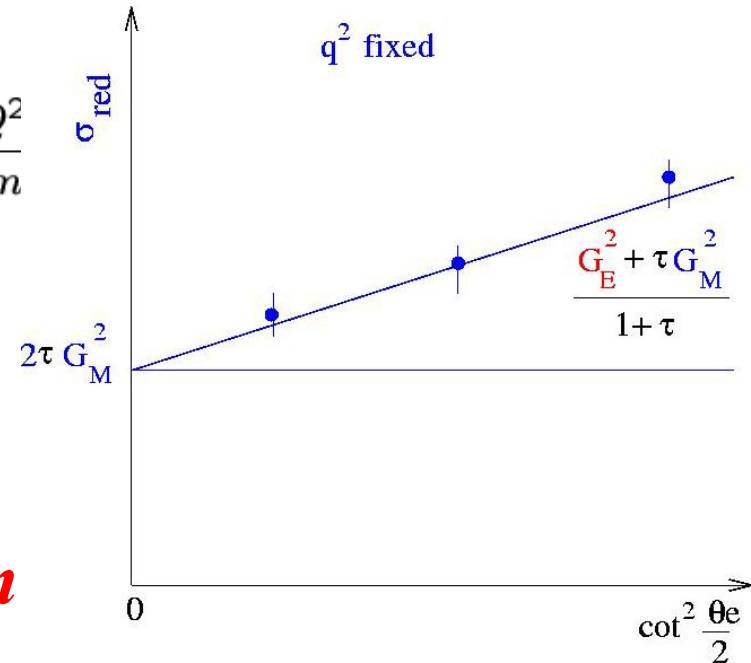
$$\frac{d\sigma}{d\Omega_e} = \sigma_M \left[2\tau G_M^2 \tan^2 \frac{\theta_e}{2} + \frac{G_E^2 + \tau G_M^2}{1 + \tau} \right]$$



- point-like particle: σ Mott

$$\sigma_M = \frac{4\alpha^2}{(-q^2)^2} \frac{\epsilon_2^3}{\epsilon_1} \cos^2 \frac{\theta_e}{2} = \frac{4\alpha^2}{(-q^2)^2} \frac{\epsilon_2^2 \cos^2 \frac{\theta_e}{2}}{1 + 2\frac{\epsilon_1}{m} \sin^2 \frac{\theta_e}{2}}, \quad \tau = \frac{Q^2}{4m^2}$$

$$\sigma_{red} = \frac{\frac{d\sigma}{d\Omega_e}}{\frac{\alpha^2}{-q^2} \left(\frac{\epsilon_2}{\epsilon_1} \right)^2}$$



Linearity of the reduced cross section

The polarization method (1967)

SOVIET PHYSICS - DOKLADY

VOL. 13, NO. 6

DECEMBER, 1968

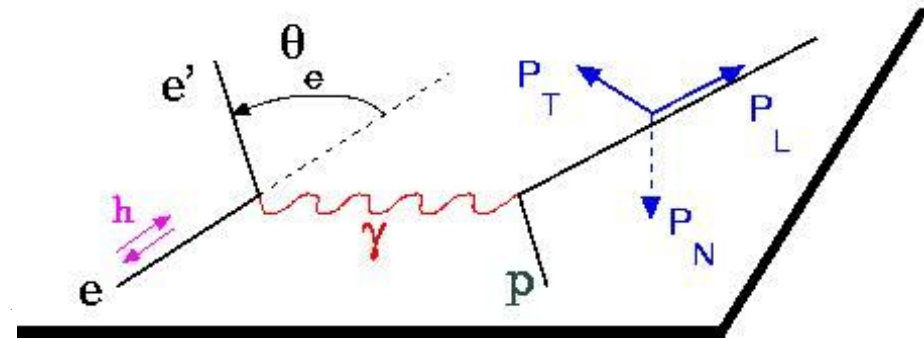
PHYSICS

POLARIZATION PHENOMENA IN ELECTRON SCATTERING BY PROTONS IN THE HIGH-ENERGY REGION

Academician A. I. Akhiezer* and M. P. Rekalov

Physicotechnical Institute, Academy of Sciences of the Ukrainian SSR
Translated from Doklady Akademii Nauk SSSR, Vol. 180, No. 5,
pp. 1081-1083, June, 1968
Original article submitted February 26, 1967

$$s_2 \frac{d\sigma}{d\Omega_R} = 4p_2 \frac{(\mathbf{s} \cdot \mathbf{q})}{1 + \tau} \Gamma(\theta, \varepsilon_1) \left[\tau G_M (G_M + G_E) - \frac{1}{4\varepsilon_1} G_M (G_E - \tau G_M) \right],$$



The polarization induces a term in the cross section proportional to $G_E G_M$
Polarized beam and target or
polarized beam and recoil proton polarization

THE RESULTS

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Linear deviation
from dipole

$$\mu G_{Ep} \neq G_{Mp}$$

Jlab E93-027, E99-007

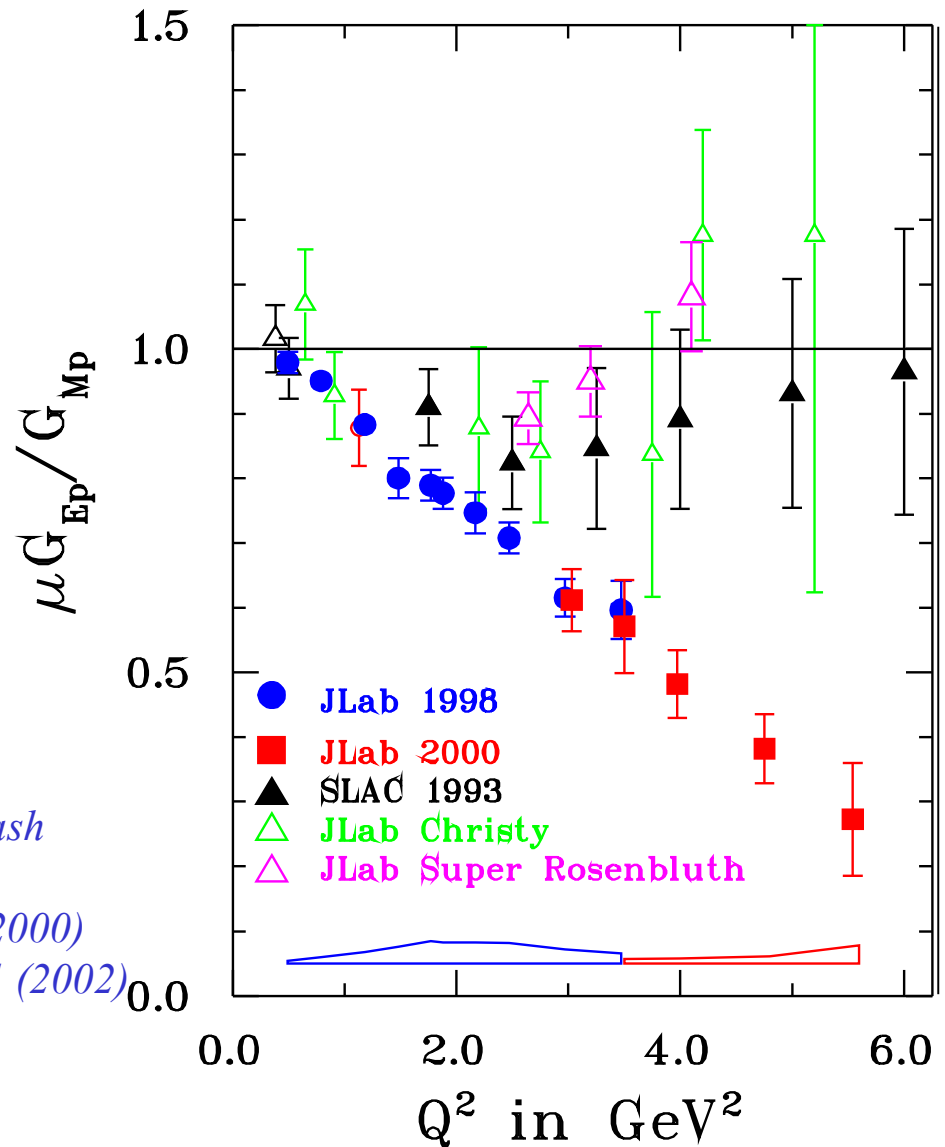
Spokepersons:

Ch. Perdrisat, V. Punjabi, M. Jones, E. Brash

M. Jones et al., Phys. Rev. Lett. 84,1398 (2000)

O. Gayou et al., Phys. Rev. Lett. 88,092301 (2002)

V. Punjabi et al., Phys. Rev. C (2006)



Electric NEUTRON Form Factor

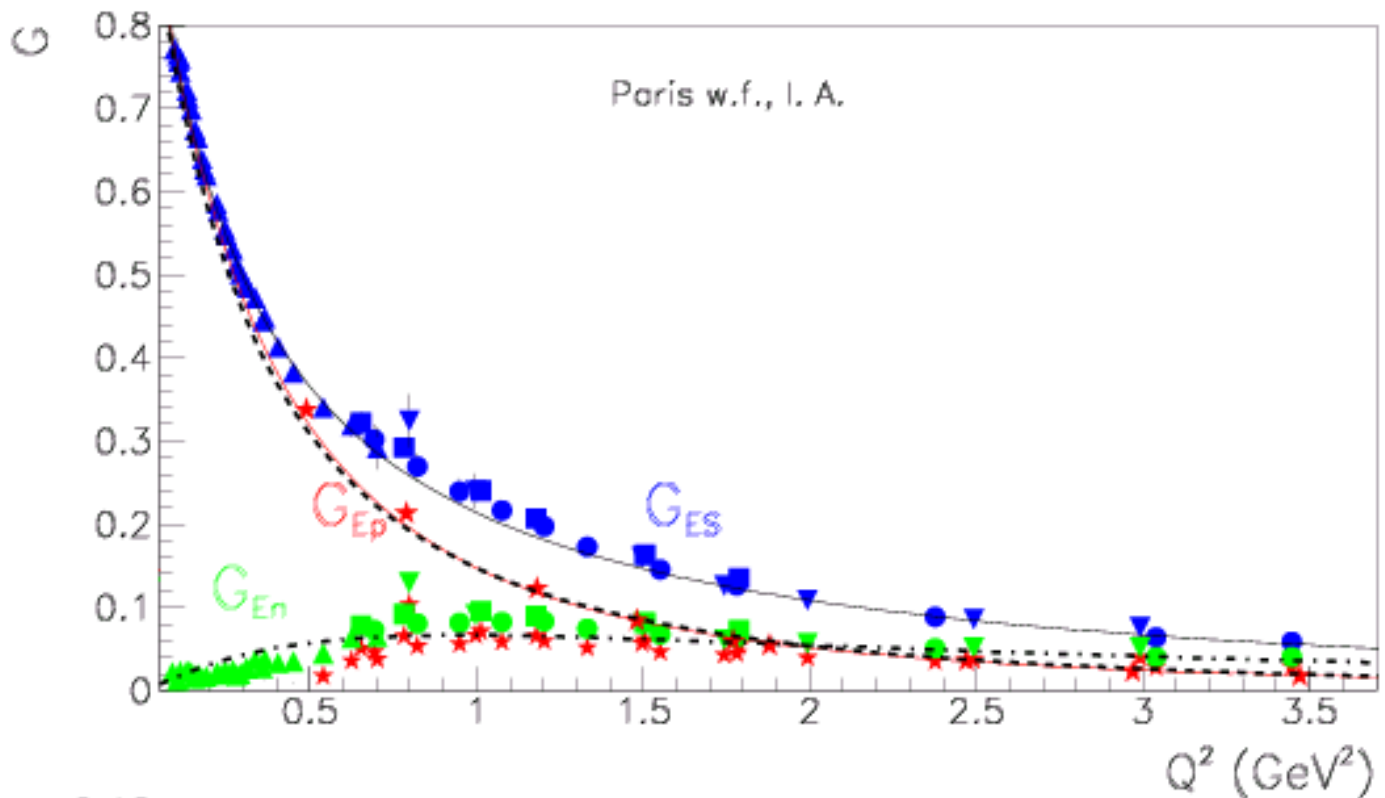
Smaller than for proton, but not so small

New results, also based on polarization method

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E. T-G. and M. P. Rekalo, Europhys. Lett. 55, 188 (2001)

STATUS on EM Form factors

Space-like region

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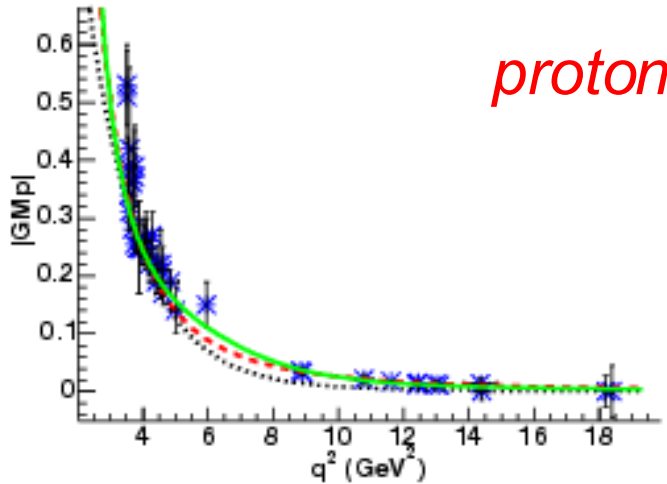
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- 3) "standard" **dipole function** for the nucleon magnetic FFs **G_M^p** and **G_M^n**
- 2) **linear deviation** from the dipole function for the electric proton FF **G_E^p**
- 3) *contradiction between polarized and unpolarized measurements*
- 4) **non vanishing** electric neutron FF, **G_E^n** .

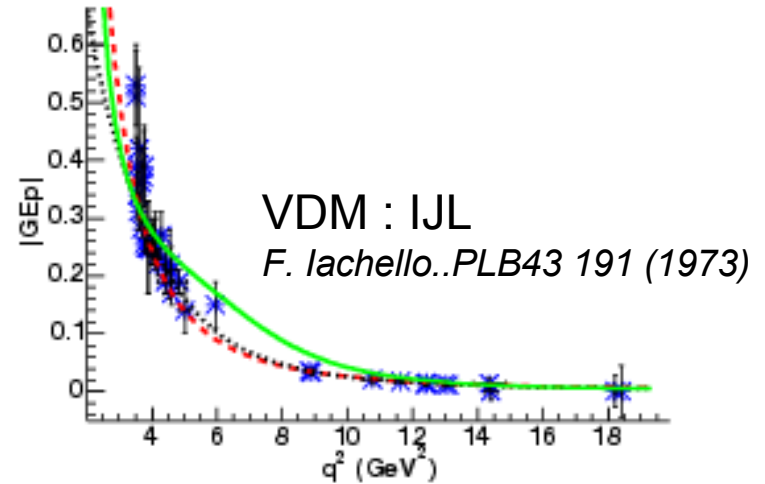
Time-Like Region

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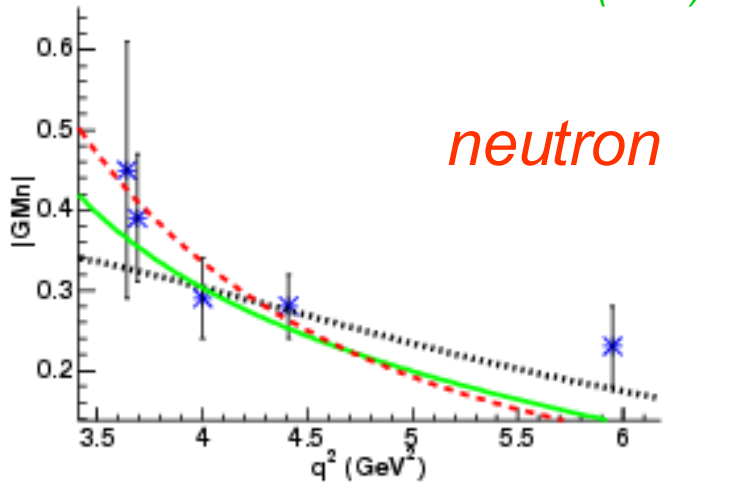
a



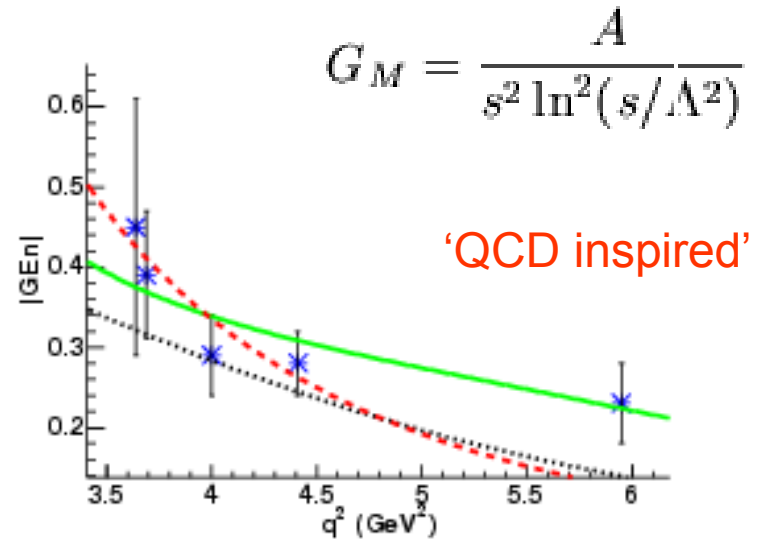
b



c



d



$$G_M = \frac{A}{s^2 \ln^2(s/\Lambda^2)}$$

E. T-G., F. Lacroix, C. Duterte, G.I. Gakh, EPJA 24, 419 (2005)

STATUS on EM Form factors

Time-like region

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4) No individual determination of GE and GM

5) Assume $GE=GM$ (valid only at threshold) VMD or pQCD inspired parametrizations (for p and n):

$$G_M = \frac{A}{s^2 [\pi^2 + \ln^2(s/\Lambda^2)]}$$

$$\bullet A(p) = 56.3 \text{ GeV}^4$$

$$\bullet A(n) = 77.15 \text{ GeV}^4$$

$\Lambda=0.3 \text{ GeV}$ is the QCD scale parameter

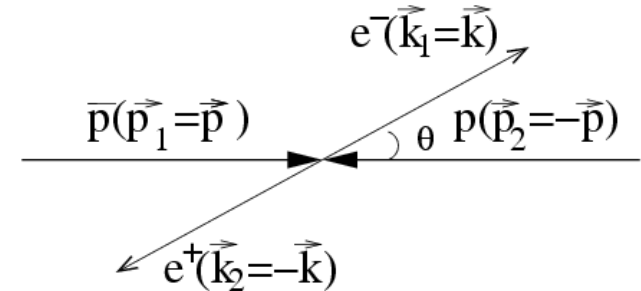
3) TL nucleon FFs are twice larger than SL FFs

4) Recent data from Babar (radiative return) :

- interesting structures in the Q^2 dependence of $GM(=GE)$
- $GM \neq GE$.

Spin Observables

Analyzing power, A



$$\frac{d\sigma}{d\Omega}(P_y) = \left(\frac{d\sigma}{d\Omega}\right)_0 [1 + \mathcal{A}P_y],$$

$$\mathcal{A} = \frac{\sin 2\theta \operatorname{Im} G_E^* G_M}{D\sqrt{\tau}}, \quad D = |G_M|^2(1 + \cos^2 \theta) + \frac{1}{\tau}|G_E|^2 \sin^2 \theta$$

Double spin observables

$$\left(\frac{d\sigma}{d\Omega}\right)_0 A_{xx} = \sin^2 \theta \left(|G_M|^2 + \frac{1}{\tau} |G_E|^2 \right) \mathcal{N},$$

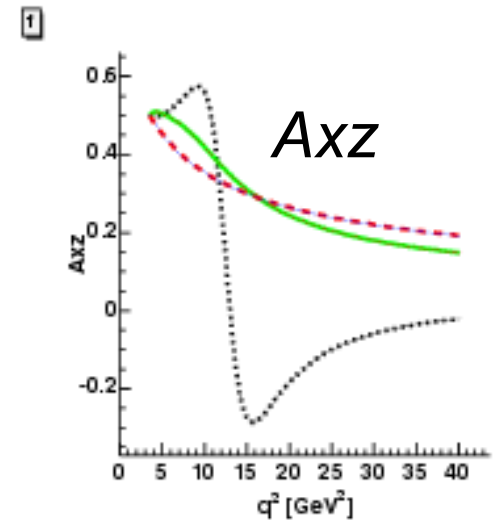
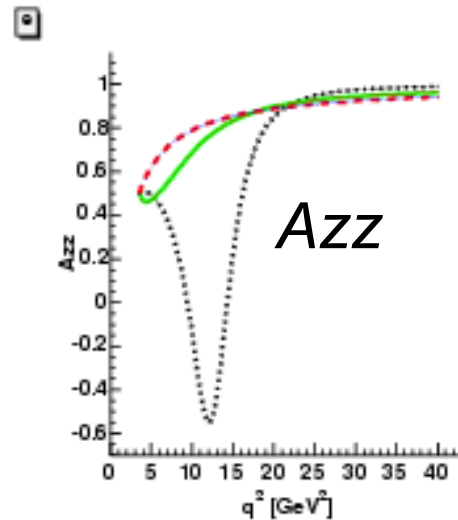
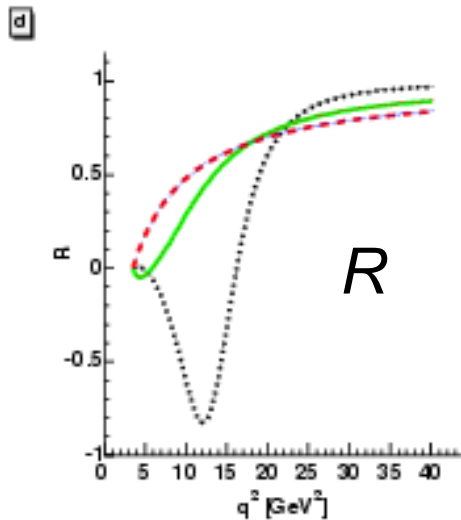
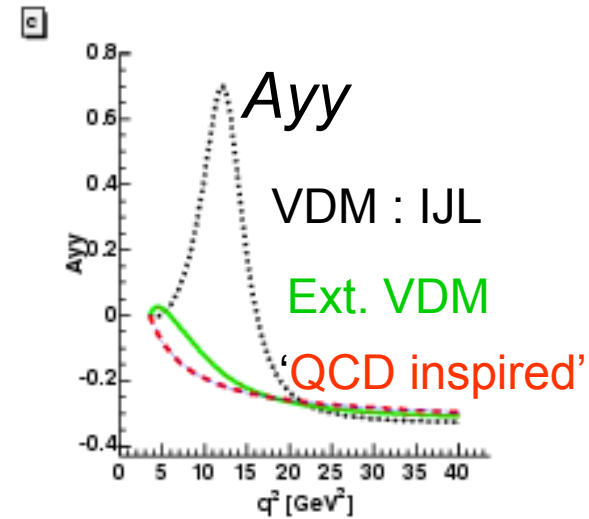
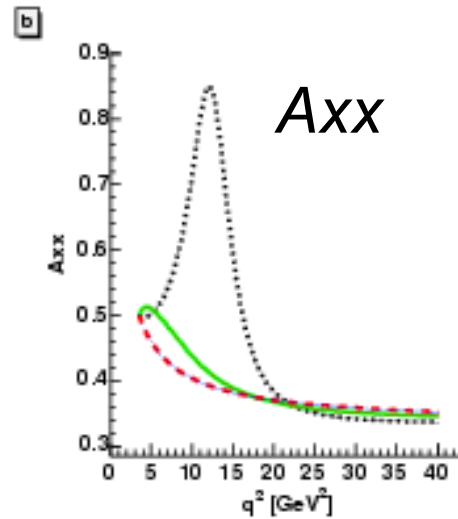
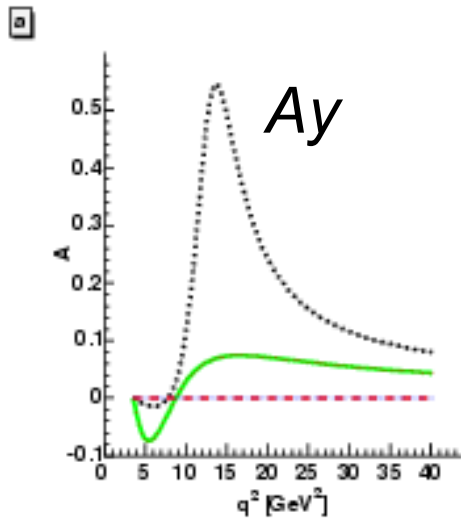
$$\left(\frac{d\sigma}{d\Omega}\right)_0 A_{yy} = -\sin^2 \theta \left(|G_M|^2 - \frac{1}{\tau} |G_E|^2 \right) \mathcal{N},$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0 A_{zz} = \left[(1 + \cos^2 \theta) |G_M|^2 - \frac{1}{\tau} \sin^2 \theta |G_E|^2 \right] \mathcal{N},$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0 A_{xz} = \left(\frac{d\sigma}{d\Omega}\right)_0 A_{zx} = \frac{1}{\sqrt{\tau}} \sin 2\theta \operatorname{Re} G_E G_M^* \mathcal{N}.$$

Models in T.L. Region (polarization)

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E. T-G., F. Lacroix, C. Duterte, G.I. Gakh, EPJA 24, 419(2005)

Issues

- Some models (IJL 73, Di-quark, soliton..) predicted such behavior before the data appeared

BUT

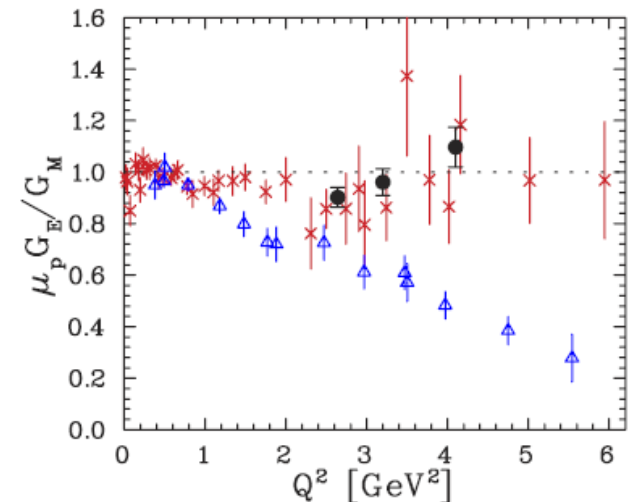
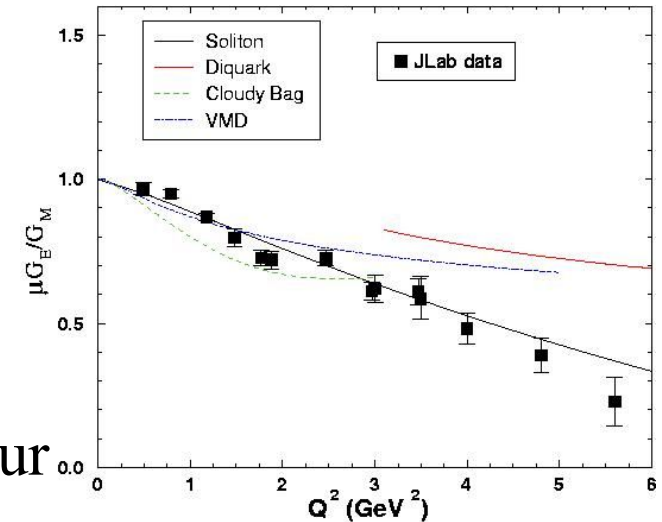
- Simultaneous description of the four nucleon form factors...

- ...in the space-like and in the time-like regions

- Consequences for the light ions description

- When pQCD starts to apply?

- Source of the discrepancy



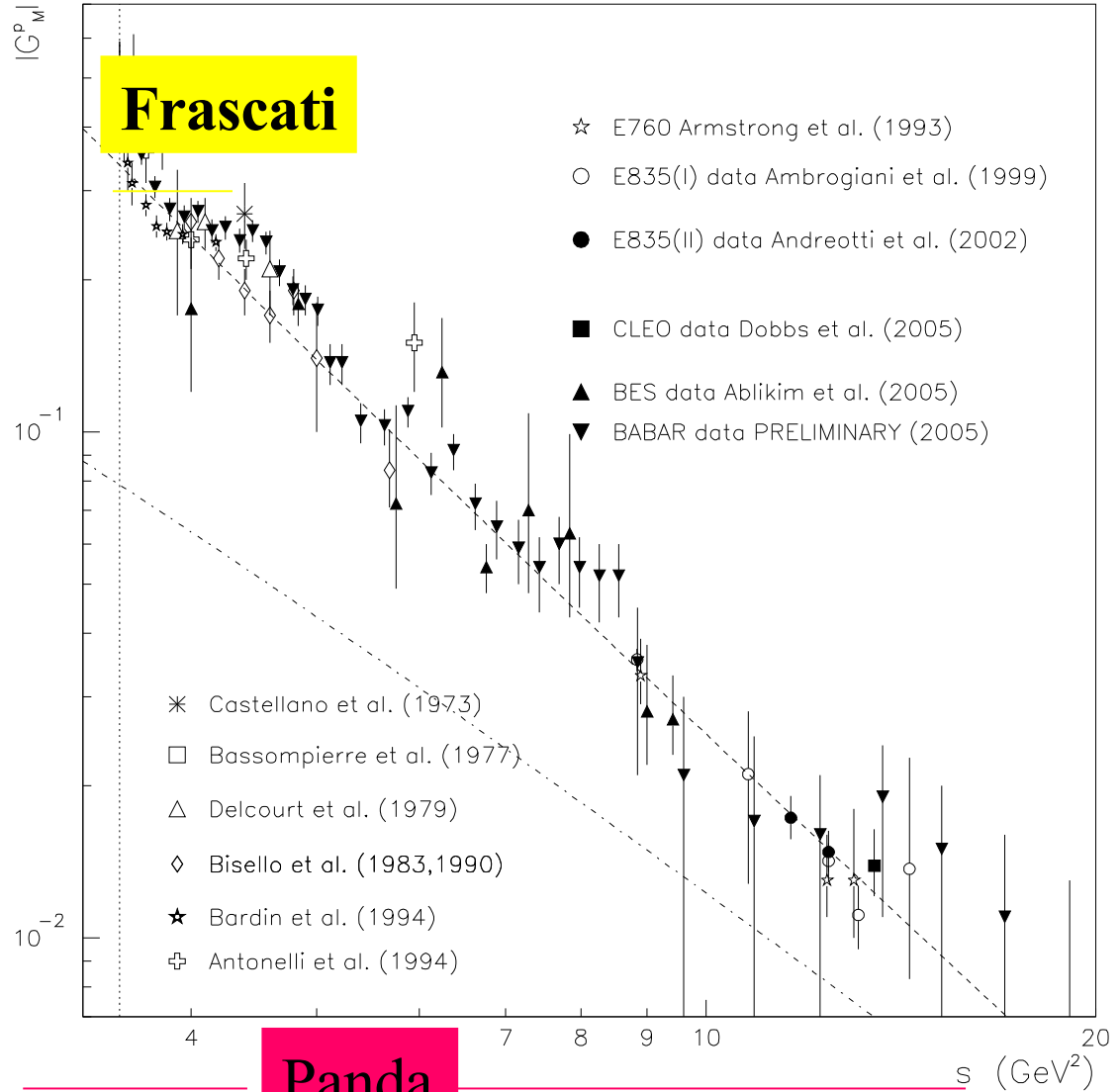
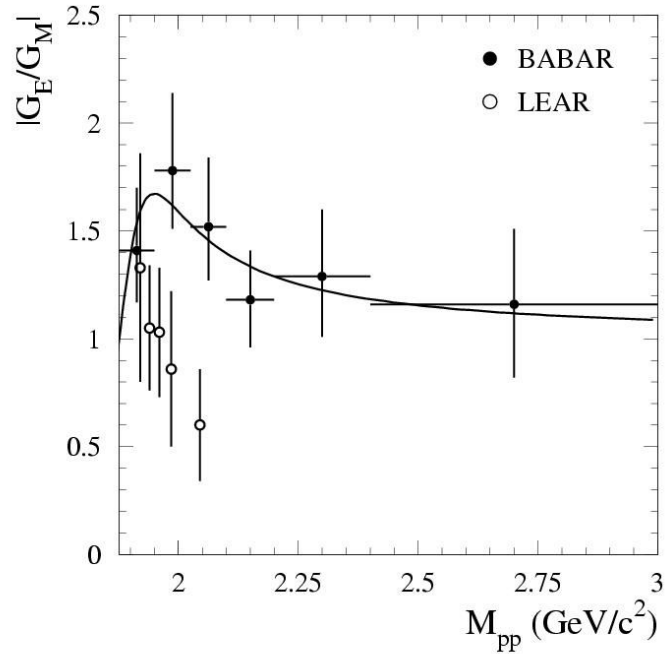
Perspectives in Time-Like region

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$$G_E = G_M ?$$



Time-like observables: $|G_E|^2$ and $|G_M|^2$.

-The cross section for $\bar{p} + p \rightarrow e^+ + e^-$ (1 γ -exchange):

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{8m^2\sqrt{\tau-1}} [\tau|G_M|^2(1 + \cos^2\theta) + |G_E|^2\sin^2\theta]$$

θ : angle between e^- and \bar{p} in cms.

A. Zichichi, S. M. Berman, N. Cabibbo, R. Gatto, Il Nuovo Cimento XXIV, 170 (1962)

B. Bilenkii, C. Giunti, V. Wataghin, Z. Phys. C 59, 475 (1993).

G. Gakh, E.T-G., Nucl. Phys. A761,120 (2005).

As in SL region:

- Dependence on q^2 contained in FFs
- Even dependence on $\cos^2\theta$ (1 γ exchange)
- No dependence on sign of FFs
- Enhancement of magnetic term

but TL form factors are complex!

Time-like observables: $|G_E|^2$ and $|G_M|^2$.

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-The Total Cross Section



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$$\sigma(q^2) = \mathcal{N} \frac{8}{3} \pi \left[2|G_M|^2 + \frac{1}{\tau} |G_E|^2 \right]. \quad \mathcal{N} = \frac{\alpha^2}{4\sqrt{q^2(q^2 - 4m^2)}}$$

-The angular asymmetry, R

$$\frac{d\sigma}{d(\cos\theta)} = \sigma_0 \left[1 + \mathcal{R} \cos^2\theta \right], \quad \mathcal{R} = \frac{\tau|G_M|^2 - |G_E|^2}{\tau|G_M|^2 + |G_E|^2}$$

Cross section at 90°

Due to limited statistics, no experimental determination of individual FFs in TL region, yet: $G_E = G_M$ or $G_E = 0$

Time-Like Region

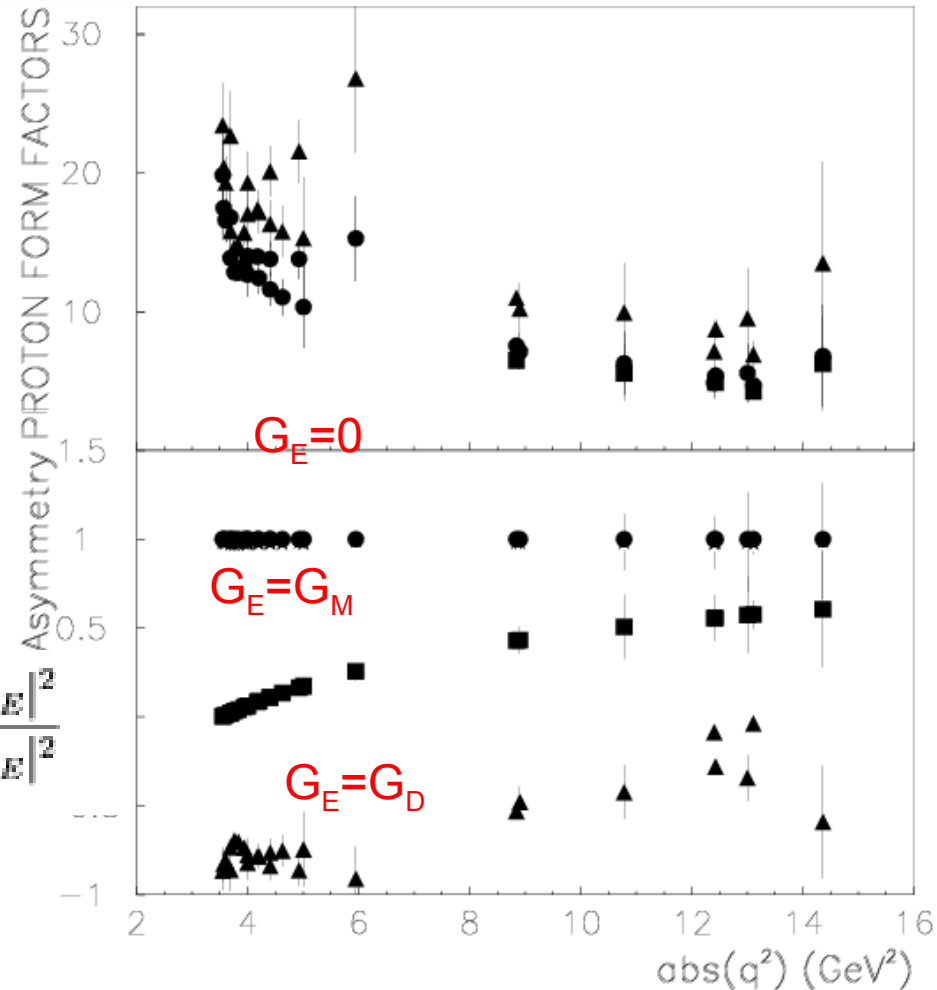
$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{8m^2\sqrt{\tau-1}} [\tau|G_M|^2(1 + \cos^2\theta) + |G_E|^2\sin^2\theta]$$

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$$|G_M|^2$$

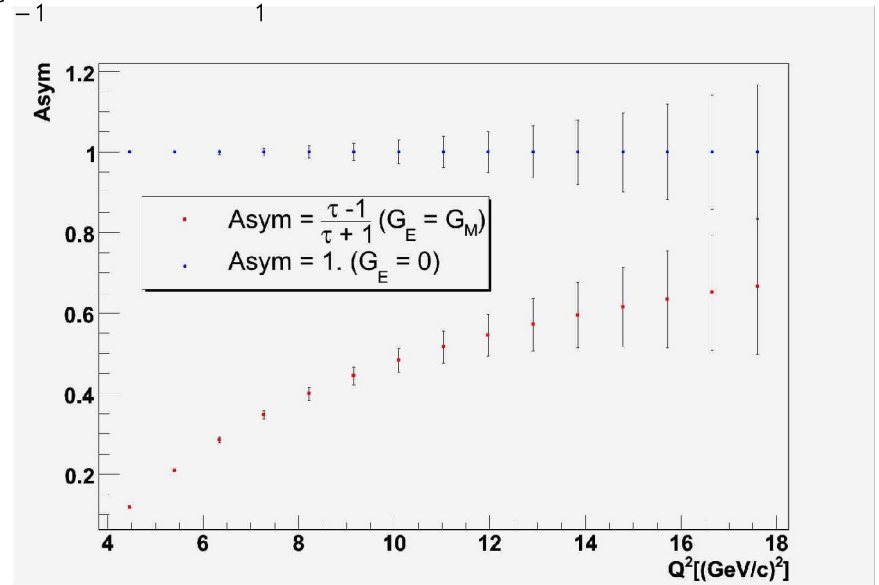
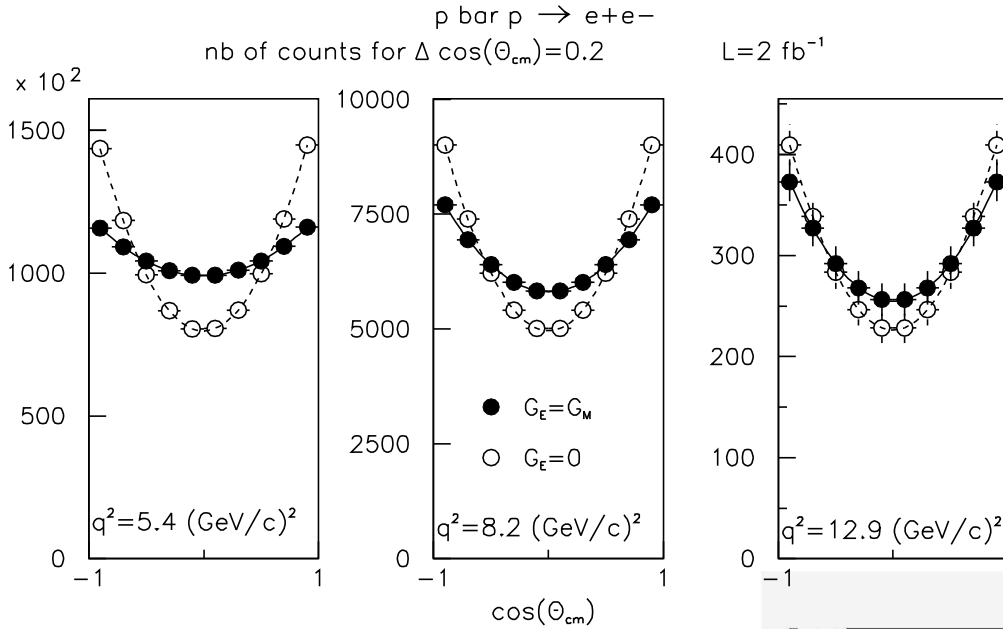
Asym

$$\frac{d\sigma}{d(\cos\theta)} = \sigma_0 [1 + \mathcal{R} \cos^2\theta], \quad \mathcal{R} = \frac{\tau|G_M|^2 - |G_E|^2}{\tau|G_M|^2 + |G_E|^2}$$



E. T-G. and M. P. Rekalo, Phys. Lett. B 504, 291 (2001)

Predictions for PANDA



B. Ramstein, D. Marchand ...

Two-photon exchange?

Electric proton FF

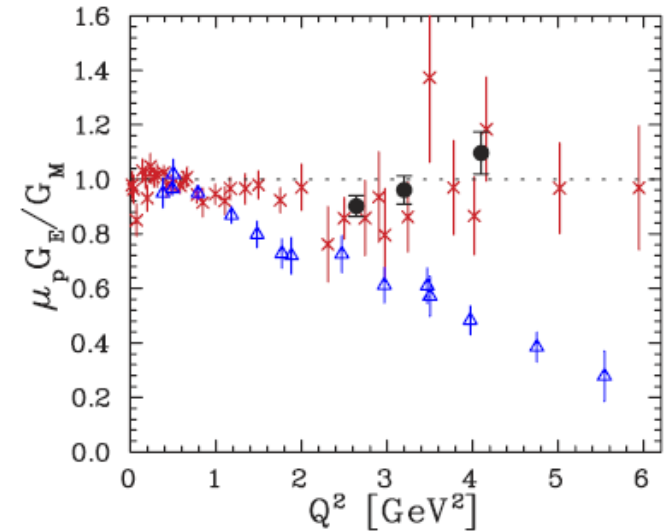
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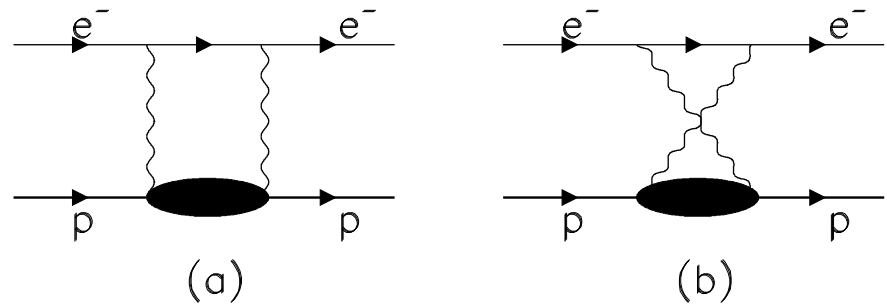
Different results with different experimental methods !!

- Both methods based on the same formalism
- Experiments repeated



New mechanism?

- $1\gamma\text{-}2\gamma \sim \alpha = e^2/4\pi = 1/137$
- 1970's: Gunion, Lev...



Two-Photon exchange

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- 1γ - 2γ interference is of the order of $\alpha=e^2/4\pi=1/137$ (in usual calculations of radiative corrections, one photon is 'hard' and one is 'soft')

- In the 70's it was shown [*J. Gunion and L. Stodolsky, V. Franco, F.M. Lev, V.N. Boitsov, L. Kondratyuk and V.B. Kopeliovich, R. Blankenbecker and J. Gunion*] that, at large momentum transfer, due to the sharp decrease of the FFs, if the momentum is shared between the two photons, the 2γ contribution can become very large.

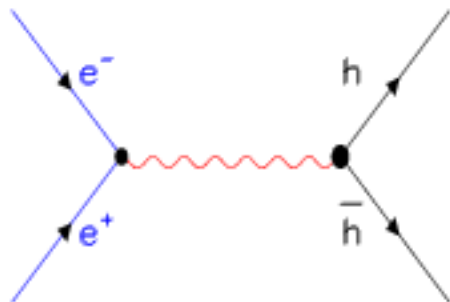
1 γ -2 γ interference

M. P. Rekalov, E. T.-G. and D. Prout, Phys. Rev. C (1999)

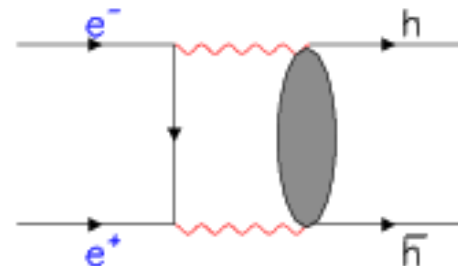
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$$C(\gamma) = -1$$



$$C(2\gamma) = +1$$

$S = 1, \ell = 0$ and $S = 1, \ell = 2$ with $\mathcal{J}^P = 1^-$,

$$|\overline{\mathcal{M}}_1(e^+e^- \rightarrow \bar{h}h)|^2 = a(t) + \cos^2 \tilde{\theta} b(t)$$

$$\text{Re} \mathcal{M}_1 \mathcal{M}_2^* = \cos \tilde{\theta} (a_0 + a_1 \cos^2 \tilde{\theta} + \dots)$$

$$\frac{d\sigma}{d\Omega_e}(e^-h \rightarrow e^-h) = \sigma_0 \left(\underbrace{A \cot^2 \frac{\theta_e}{2}}_{1\gamma} + B + \underbrace{C \cot \frac{\theta_e}{2}}_{2\gamma} + D \cot^3 \frac{\theta_e}{2} + \dots \right)$$

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***The 1γ - 2γ interference
destroys the linearity
of the Rosenbluth plot!***

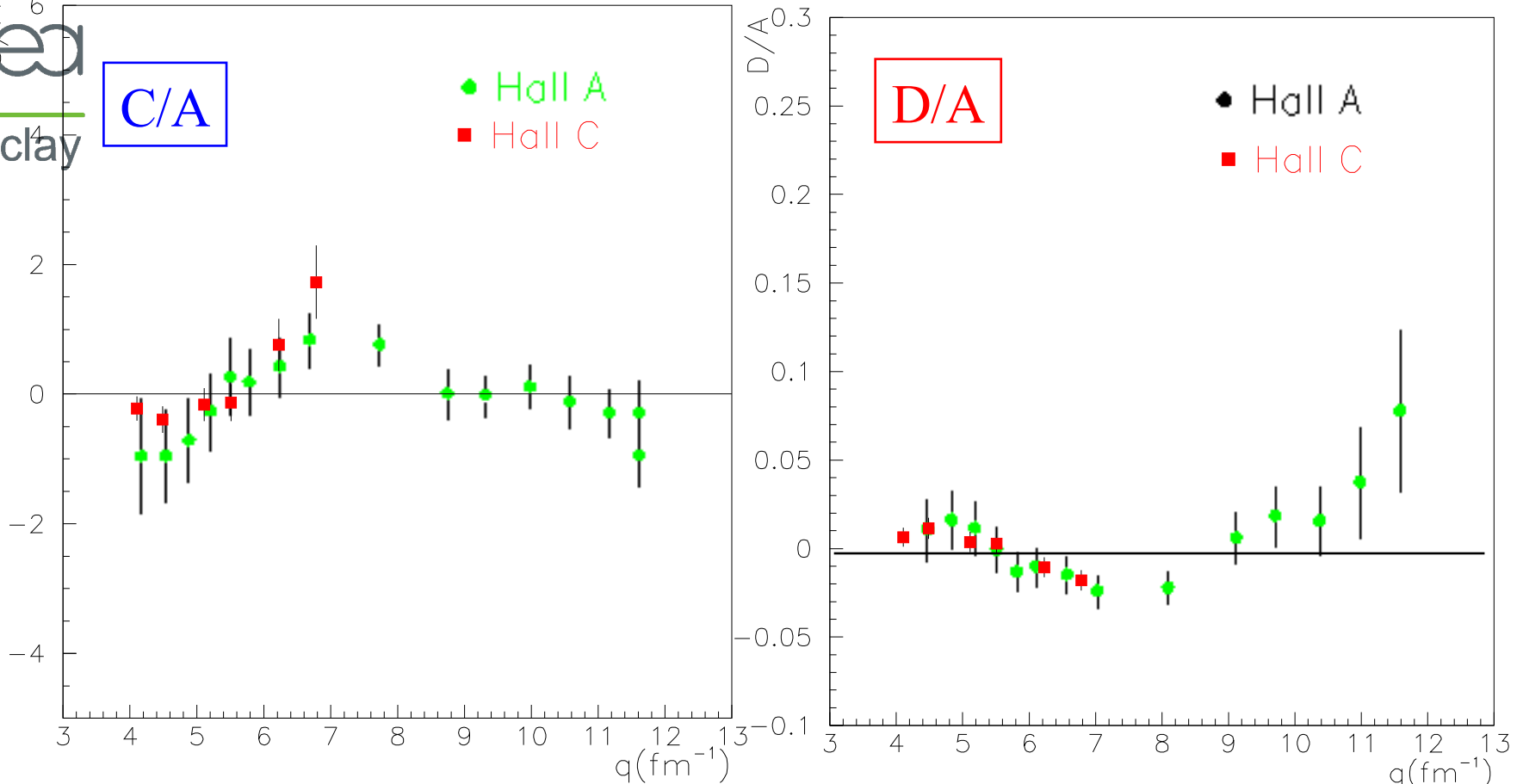
1 γ -2 γ interference (e-d)

$$\frac{d\sigma}{d\Omega_e}(e^-h \rightarrow e^-h) = \sigma_0 \left(A \cot^2 \frac{\theta_e}{2} + B + C \cot \frac{\theta_e}{2} + D \cot^3 \frac{\theta_e}{2} + \dots \right)$$

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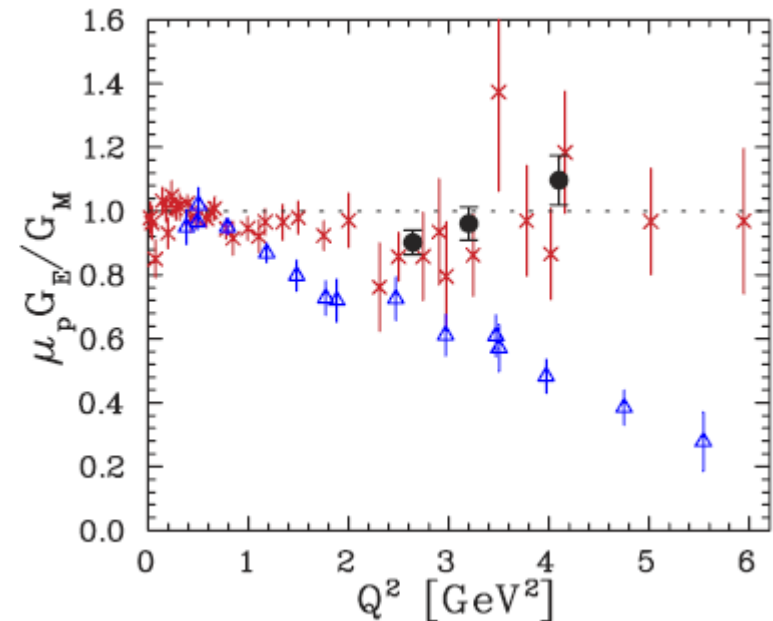
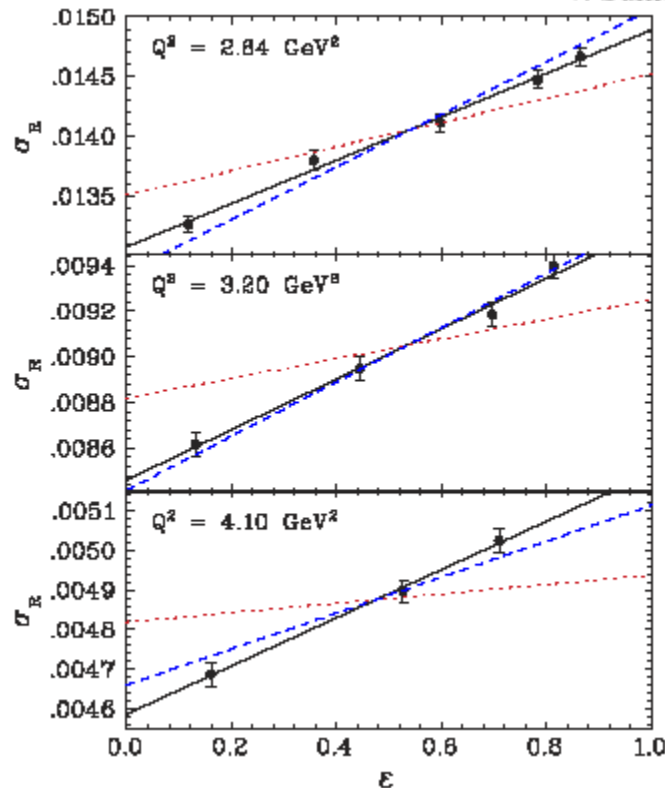
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M. P. Rekalov, E. T-G and D. Prout, Phys. Rev. C60, 042202 (1999)

Precision Rosenbluth Measurement of the Proton Elastic Form Factors

I. A. Qattan,^{1,2} J. Arrington,² R. E. Segel,¹ X. Zheng,² K. Aniol,³ O. K. Baker,⁴ R. Beams,² E. J. Brash,⁵ J. Calarco,⁶ A. Camsonne,⁷ J.-P. Chen,⁸ M. E. Christy,⁴ D. Dutta,⁹ R. Ent,⁸ S. Frullani,¹⁰ D. Gaskell,¹¹ O. Gayou,¹² R. Gilman,^{13,8} C. Glashauser,¹³ K. Hafidi,² J.-O. Hansen,⁸ D. W. Higinbotham,⁸ W. Hinton,¹⁴ R. J. Holt,² G. M. Huber,⁵ H. Ibrahim,¹⁴ L. Jisonna,¹ M. K. Jones,⁸ C. E. Keppel,⁴ E. Kinney,¹¹ G. J. Kumbartzki,¹³ A. Lung,⁸ D. J. Margaziotis,³ K. McCormick,¹³ D. Meekins,⁸ R. Michaels,⁸ P. Monaghan,⁹ P. Moussiegt,¹⁵ L. Pentchev,¹² C. Perdrisat,¹² V. Punjabi,¹⁶ R. Ransome,¹³ J. Reinhold,¹⁷ B. Reitz,⁸ A. Saha,⁸ A. Sarty,¹⁸ E. C. Schulte,² K. Slifer,¹⁹ P. Solvignon,¹⁹ V. Sulkosky,¹² K. Wijesooriya,² and B. Zeidman²



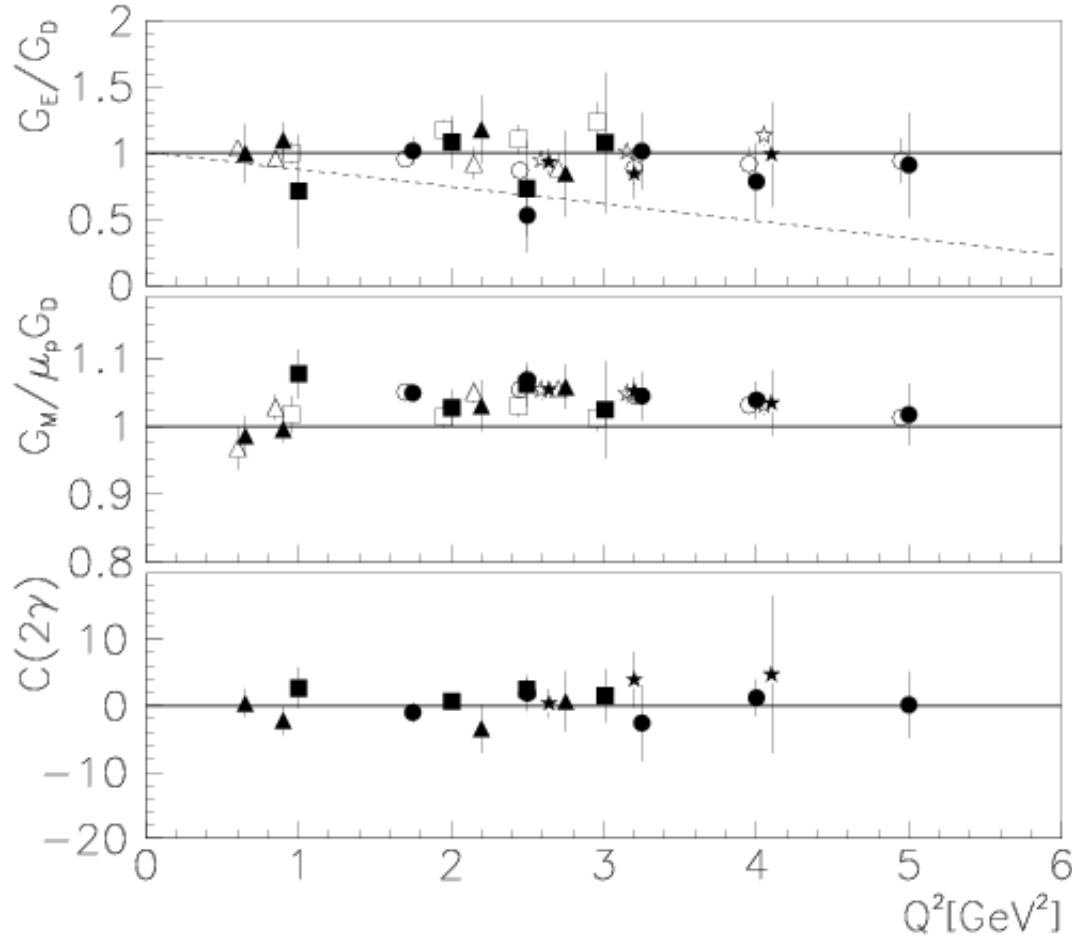
Parametrization of 2γ -contribution for $e+p$

$$\sigma^{red}(Q^2, \epsilon) = \epsilon G_E^2(Q^2) + \tau G_M^2(Q^2) + \alpha F(Q^2, \epsilon),$$

$$F(Q^2, \epsilon) \rightarrow \sqrt{\frac{1+\epsilon}{1-\epsilon}} f^{(a)}(Q^2)$$

$$f^{(a)}(Q^2) = \frac{C_{2\gamma} G_D}{[1 + Q^2 [\text{GeV}]^2 / m_a^2]^2}$$

**From the data:
deviation from linearity
<< 1%!**



E. T.-G., G. Gakh, Phys. Rev. C (2005)

Two-Photon exchange

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- The 2γ amplitude is expected to be mostly imaginary.
- In this case, the 1γ - 2γ interference is more important in time-like region, as the Born amplitude is complex.

Unpolarized cross section

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$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4q^2} \sqrt{\frac{\tau}{\tau-1}} D$$

$$D = (1 + \cos^2 \theta)(|G_M|^2 + 2\text{Re}G_M\Delta G_M^*) + \frac{1}{\tau} \sin^2 \theta(|G_E|^2 + 2\text{Re}G_E\Delta G_E^*) + 2\sqrt{\tau(\tau-1)} \cos \theta \sin^2 \theta \text{Re}\left(\frac{1}{\tau}G_E - G_M\right)F_3^*$$

2 γ -contribution:

- **Induces four new terms**
- **Odd function of θ :**
- **Does not contribute at $\theta=90^\circ$**

Symmetry relations

- Properties of the TPE amplitudes with respect to the transformation: $\cos \theta = -\cos \theta$ i.e., $\theta \rightarrow \pi - \theta$
(equivalent to non-linearity in Rosenbluth fit)

$$\Delta G_{E,M}(q^2, -\cos\theta) = -\Delta G_{E,M}(q^2, \cos\theta),$$
$$F_3(q^2, -\cos\theta) = F_3(q^2, \cos\theta)$$

- Based on these properties one can **remove** or **single out** TPE contribution

Symmetry relations

- Differential cross section at **complementary** angles:

The SUM cancels the 2γ contribution:

$$\frac{d\sigma_+}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi - \theta) = 2 \frac{d\sigma^{Born}}{d\Omega}(\theta)$$

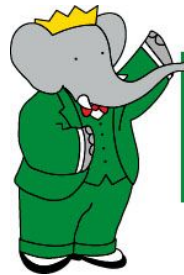
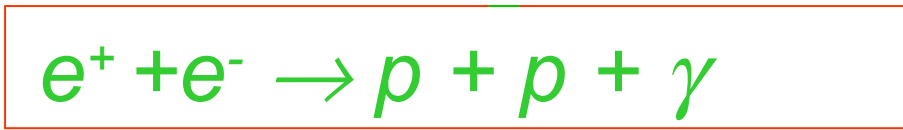
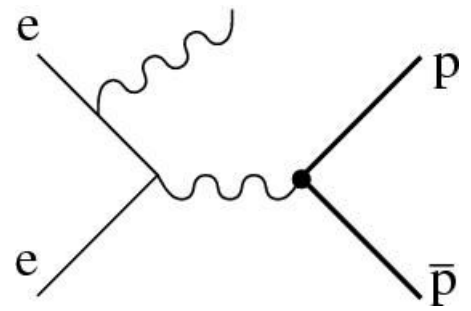
The DIFFERENCE enhances the 2γ contribution:

$$\frac{d\sigma_-}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta) = 4N \left[(1 + x^2) \text{Re}G_M \Delta G_M^* + \right. \\ \left. + \frac{1 - x^2}{\tau} \text{Re}G_E \Delta G_E^* + \sqrt{\tau(\tau - 1)} x (1 - x^2) \text{Re}\left(\frac{1}{\tau} G_E - G_M\right) F_3^* \right]$$

$$\tau = \frac{q^2}{4m^2}, \quad x = \cos\theta$$

Radiative Return (ISR)

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$$\frac{d\sigma(e^+ e^- \rightarrow p \bar{p} \gamma)}{dm d\cos\theta} = \frac{2m}{s} W(s, x, \theta) \sigma(e^+ e^- \rightarrow p \bar{p})(m), \quad x = \frac{2E_\gamma}{\sqrt{s}} = 1 - \frac{m^2}{s},$$

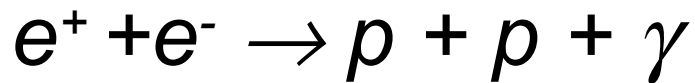
$$W(s, x, \theta) = \frac{\alpha}{\pi x} \left(\frac{2 - 2x + x^2}{\sin^2 \theta} - \frac{x^2}{2} \right), \quad \theta \gg \frac{m_e}{\sqrt{s}}.$$

Structure Function method

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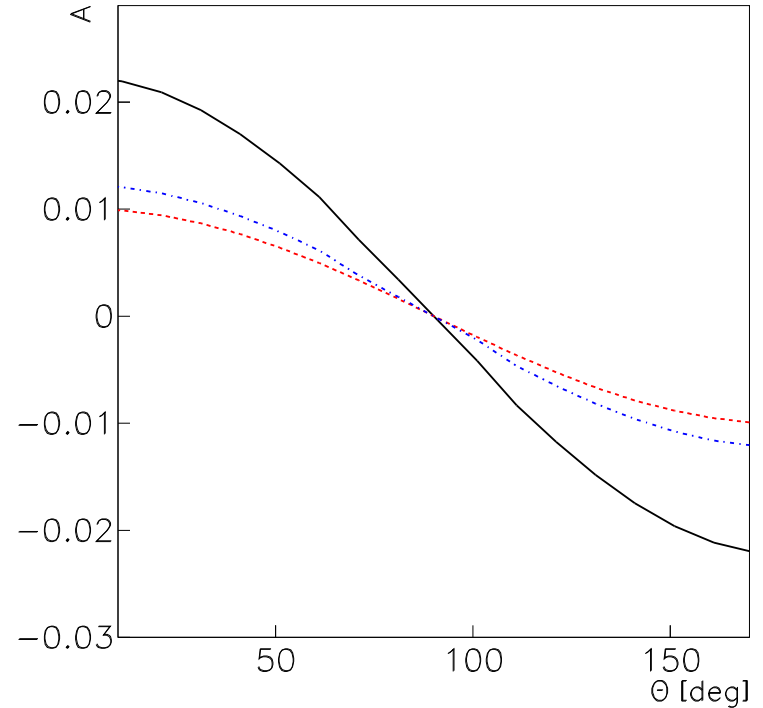


$$A^{soft}(E) \simeq \frac{2\alpha}{\pi} \left(\ln \frac{1 + \beta c}{1 - \beta c} \ln \frac{\Delta E}{E} \right)$$

$$A^{tot} = A^{soft} + A^{hard} = \frac{2\alpha}{\pi} \psi(c, \beta), \quad |A^{tot}| \leq 2\%$$

$$\frac{d\sigma}{d\Omega}(c) \pm \frac{d\sigma}{d\Omega}(-c) \sim \int dx_1 \mathcal{D}(x_1, L) \mathcal{D}(x_2, L) dx_2 \left(1 + \frac{\alpha}{\pi} K \right)$$

$$\frac{d\sigma}{d\Omega}(c) + \frac{d\sigma}{d\Omega}(-c) = 2 \frac{d\sigma_0}{d\Omega} \left[1 + \frac{\alpha}{\pi} \left(\frac{3}{2} L - 2(L-1) \ln \frac{\Delta E}{E} + \frac{\pi^2}{3} - 2 \right) \right], \quad L = \ln \frac{t}{m^2},$$

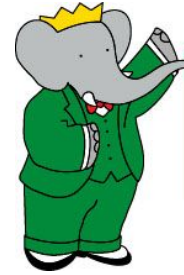
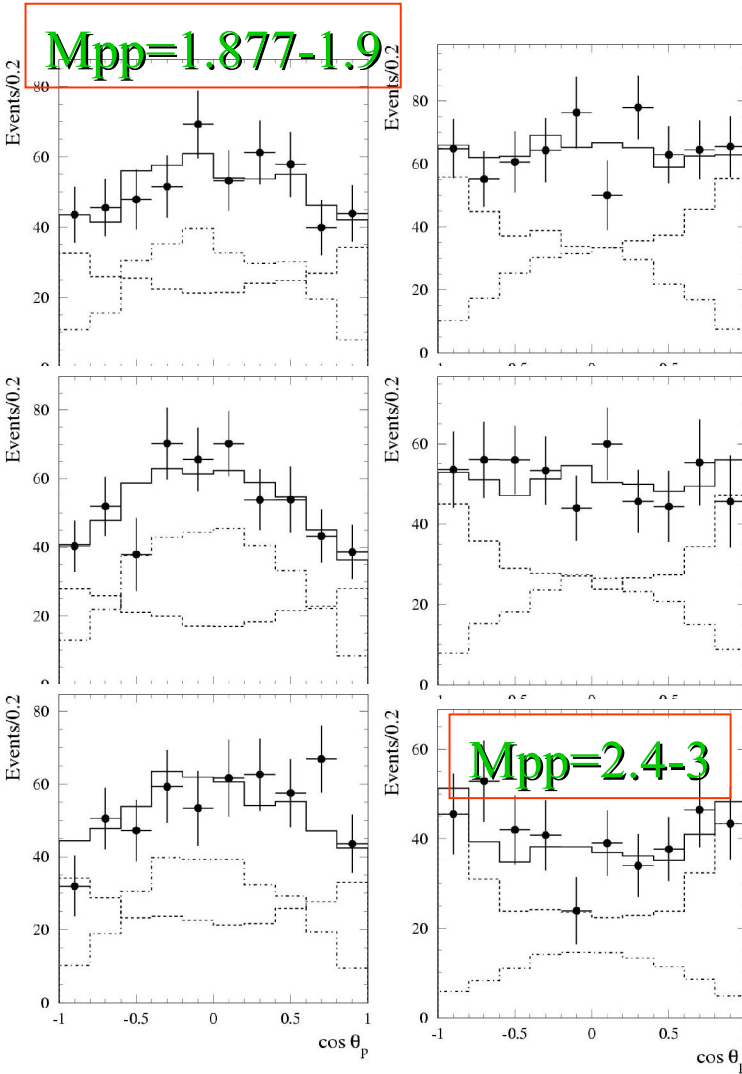


Angular distribution

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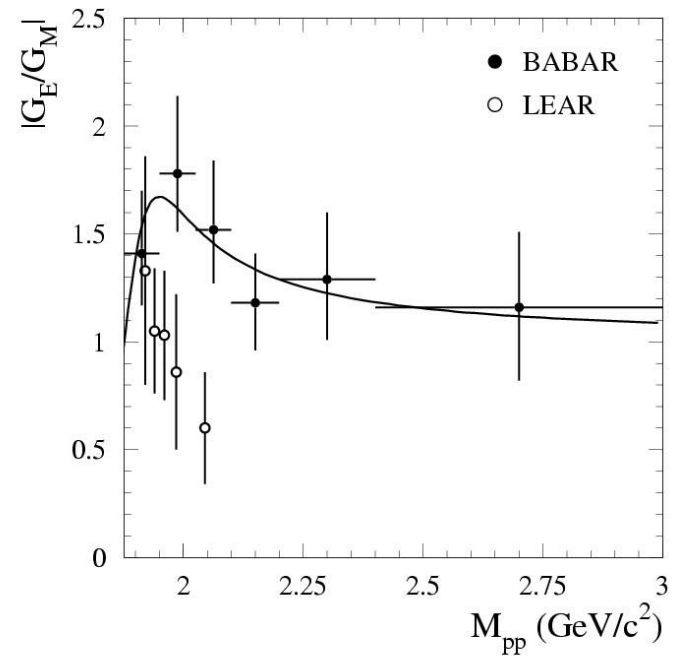
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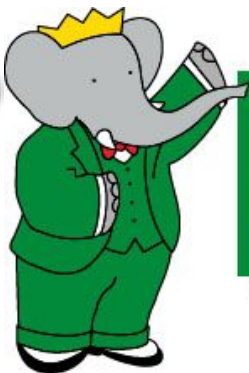
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$$\frac{dN}{d \cos \theta_p} = A \left[H_M(\cos \theta, M_{pp}) + \left| \frac{G_E}{G_M} \right|^2 H_E(\cos \theta, M_{pp}) \right]$$





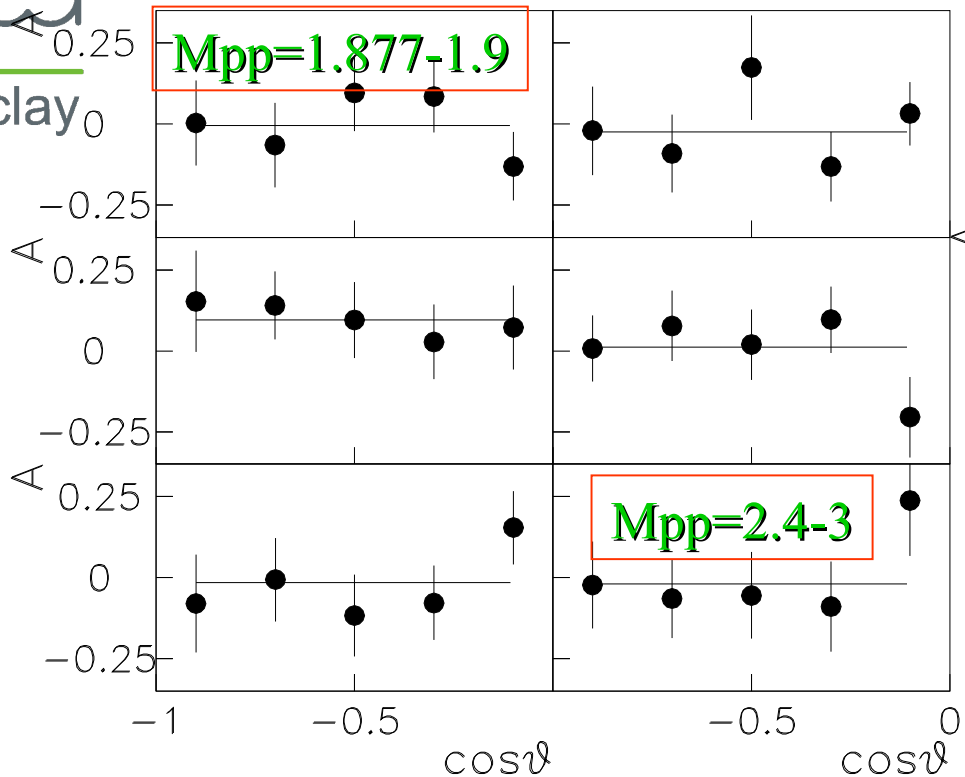
BABAR

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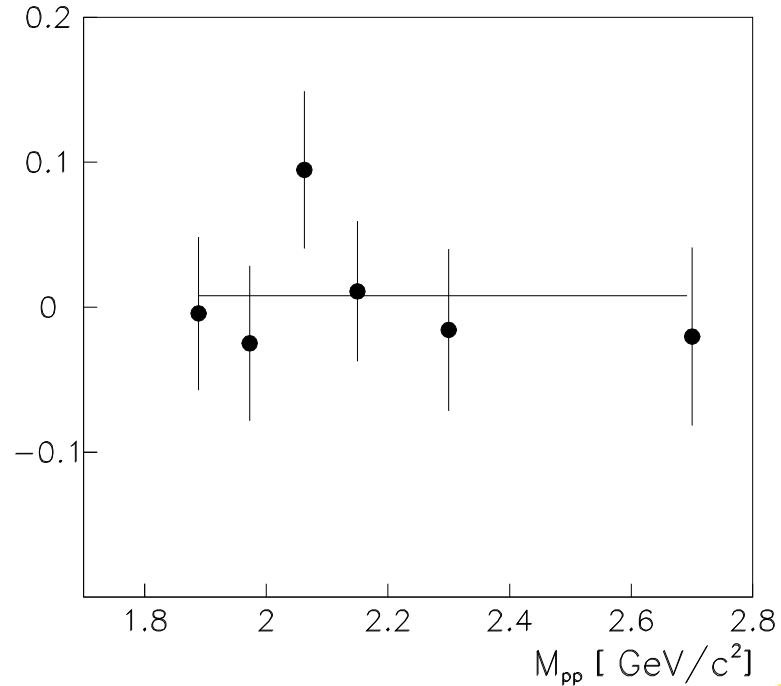
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$A=0.01 \pm 0.02$



Radiative Corrections to the data

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- RC can reach 40% on σ

- Declared error $\sim 1\%$

- Same correction for G_E and G_M

- Have a large ϵ -dependence

- Affect the slope

$$\sigma_{el} = \sigma_{meas} \cdot RC$$



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$$\begin{aligned} \sigma_{meas}^{red}(Q^2, \epsilon) &= \sigma^{red}(Q^2, \epsilon)[1 - \delta_R(Q^2, \epsilon)] = G_M^2(Q^2) \left(\tau(Q^2) + \epsilon \frac{G_E^2(Q^2)}{G_M^2(Q^2)} \right) [1 - \epsilon \delta'(Q^2)] \\ &= G_M^2(Q^2) \left[\tau + \epsilon \left(\frac{G_E^2(Q^2)}{G_M^2(Q^2)} - \tau \delta'(Q^2) \right) \right] \end{aligned}$$

↑
slope

Slope negative if : $\delta' \geq \frac{G_E^2}{\tau G_M^2}$

The slope is negative starting from 2-3 GeV²

Rosenbluth separation

Contribution of the *electric term*

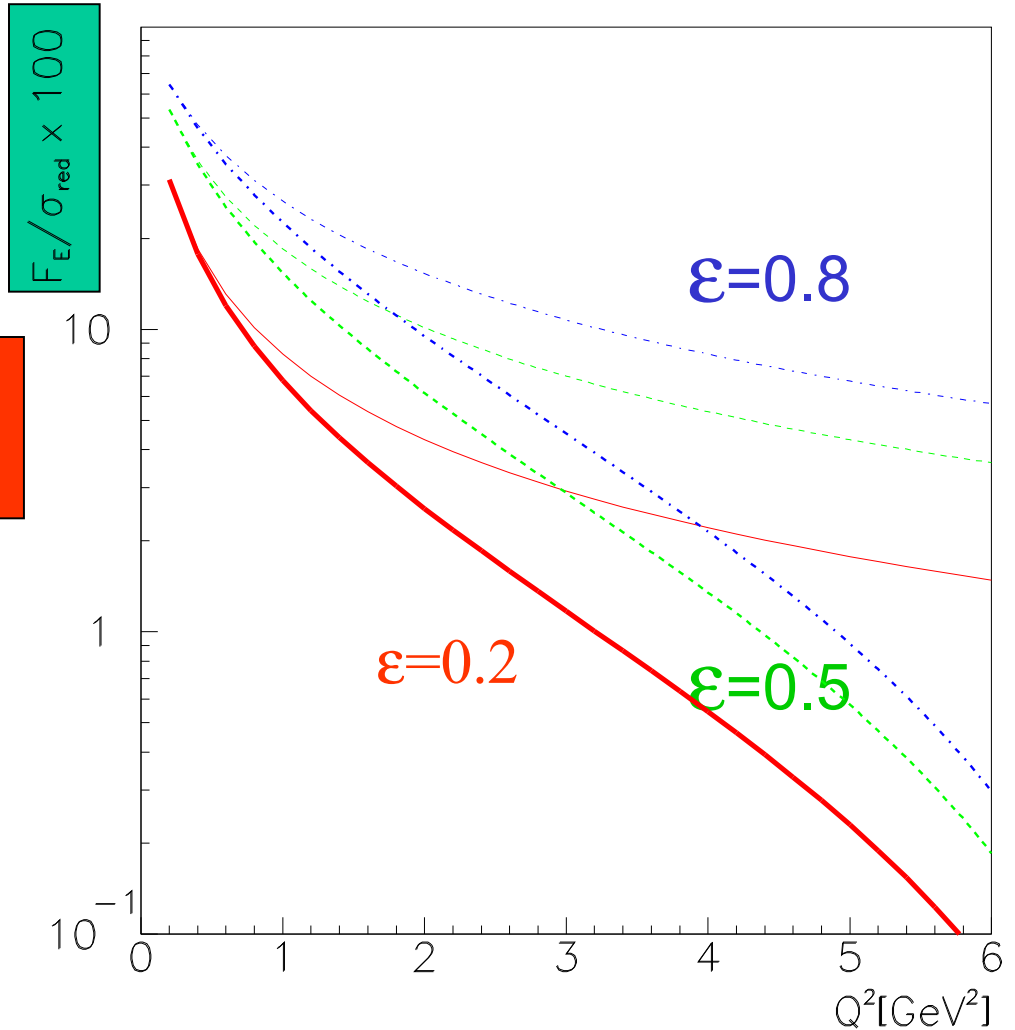
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$$\sigma_{red} = \tau G_{MP}^2 + \epsilon G_{EP}^2$$

...to be compared to the absolute value of σ and to the *size* and ϵ dependence of RC



The proton magnetic form factor

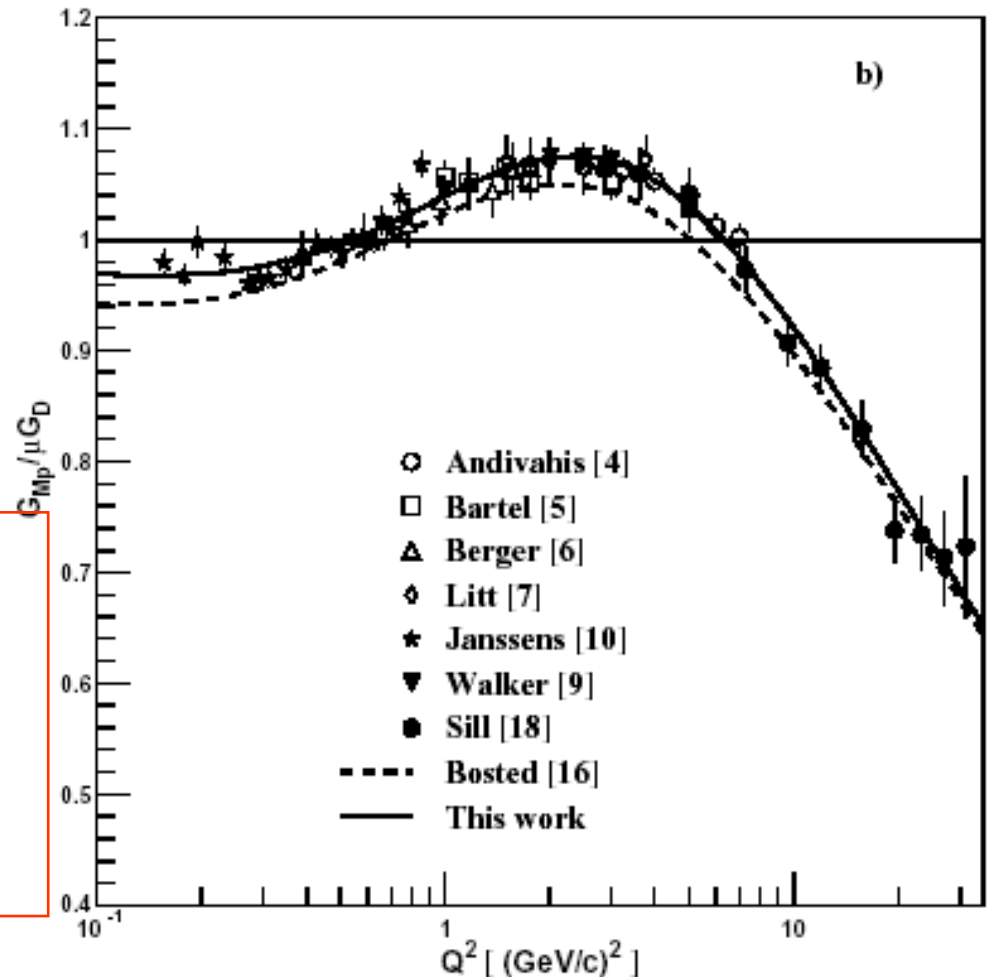
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The polarization
results induce
1.5-3% global effect

*The difference is not at
the level of the
measured observables,
but on the slope
(derivative)!*



E. Brash et al. Phys. Rev. C65, 051001 (2002)

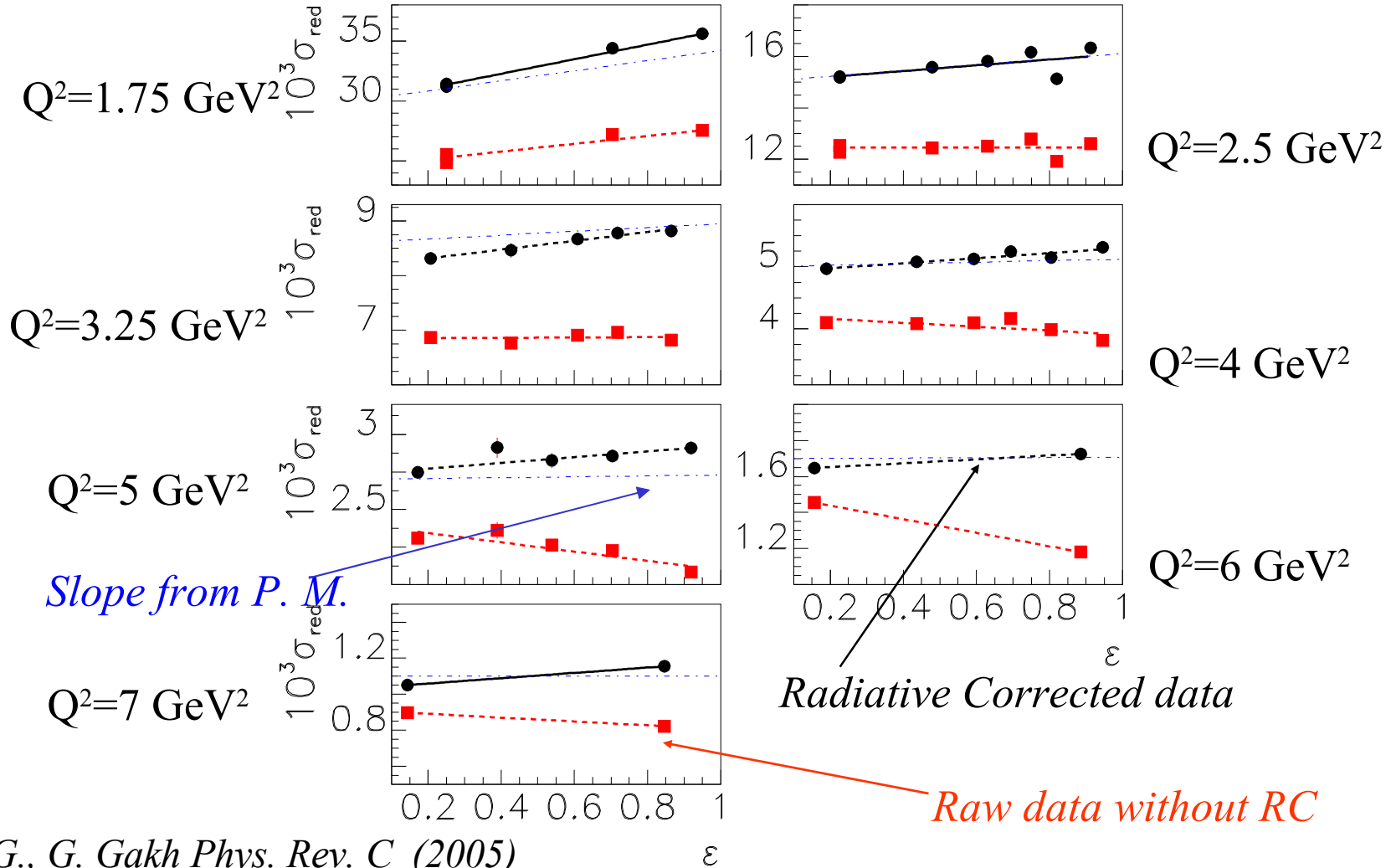
Reduced cross section and RC

Data from L. Andivahis et al., Phys. Rev. D50, 5491 (1994)

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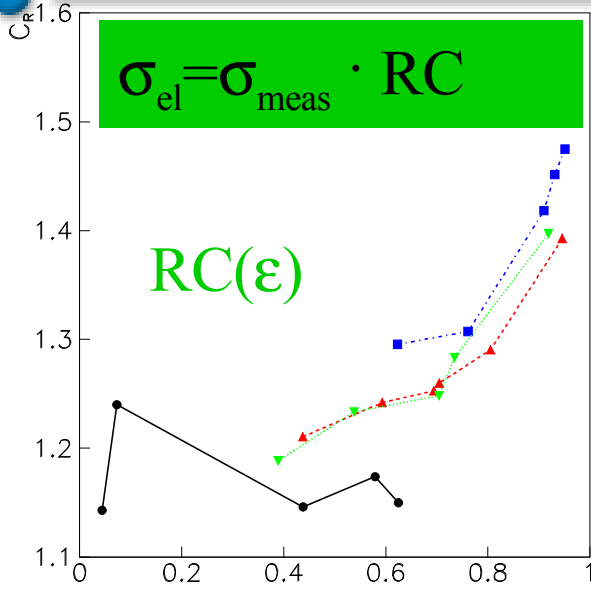
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E. T.-G., G. Gakh Phys. Rev. C (2005)

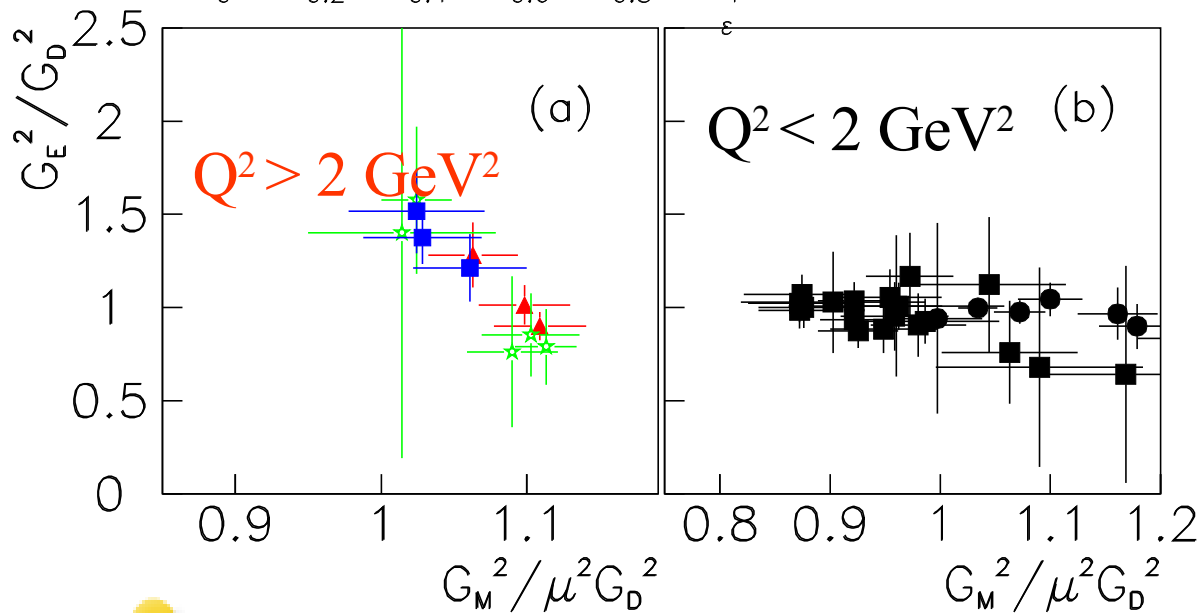
Experimental correlation

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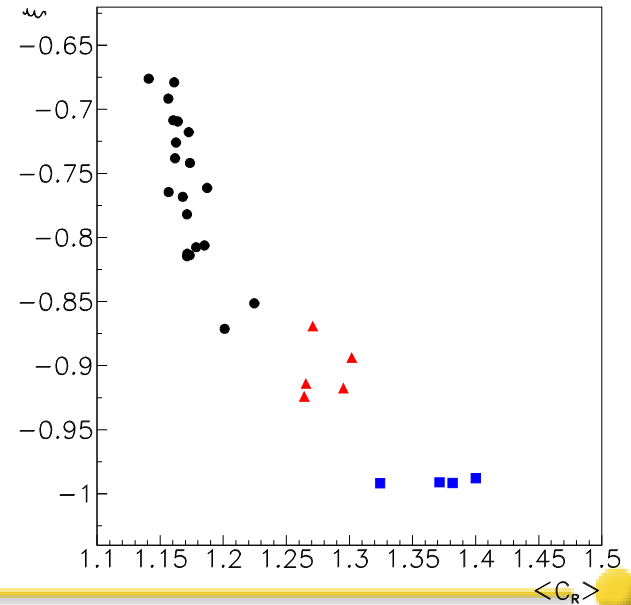


$$\sigma_{red} = \tau G_{MP}^2 + \epsilon G_{EP}^2$$

only published values!!



Correlation ($\langle RC \cdot \epsilon \rangle$)



Scattered electron energy

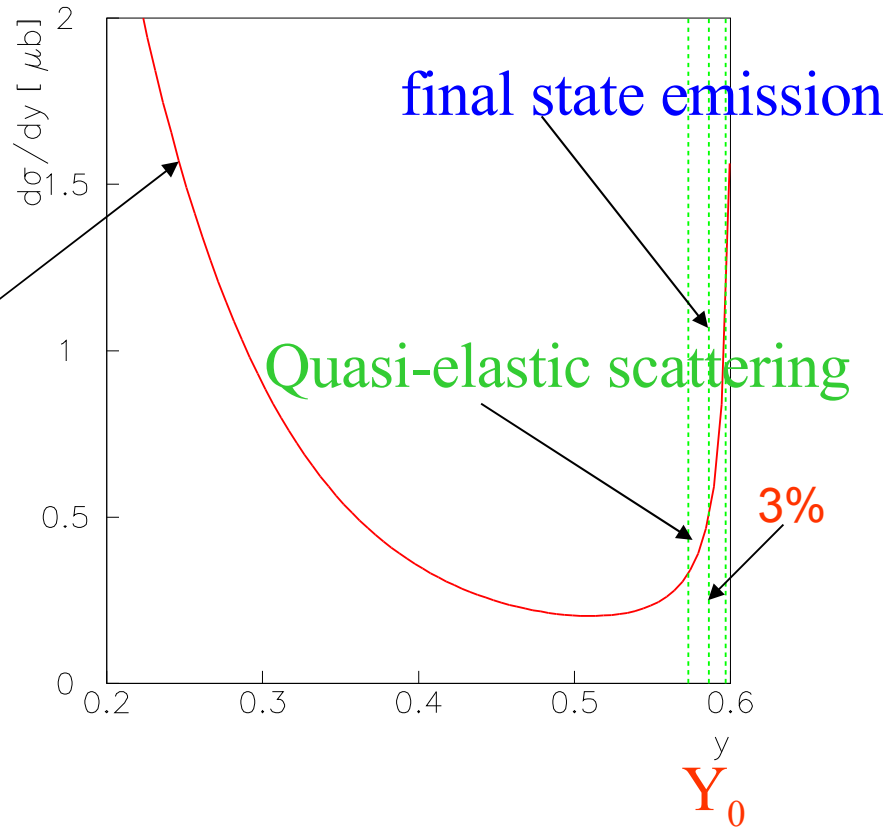
$$E'/E = y \quad ; \quad y_0 = \frac{1}{\beta}$$

$$\beta = 1 + \frac{2E}{m} \frac{\hbar^2 \theta^2}{2}$$

Initial state emission

$$\Delta \frac{d\sigma}{d\Omega} \sim \frac{d\sigma_0}{d\Omega} \cdot \frac{2}{\pi} \hbar \frac{E}{\Delta E} \hbar \frac{2EM}{m_e^2}$$

Not so small!



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Shift to LOWER Q^2

All orders of PT needed \rightarrow

beyond Mo & Tsai approximation

Structure Function method

E. A. Kuraev and V.S. Fadin, Sov. J. of Nucl. Phys. 41, 466 (1985)

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- SF method applied to QED processes: calculation of radiative corrections with precision of 0.1%.
- Takes into account the dynamics of the process
- Formulated in terms of parton densities (leptons, antileptons, photons) *Lipatov equations (1975)*
- Many applications to different processes

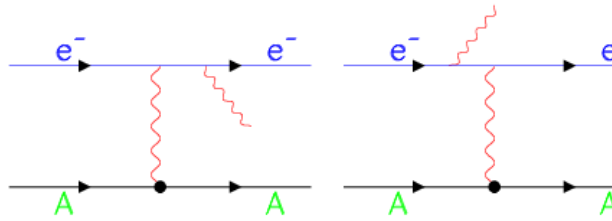
$$d\sigma^V(y) = \int_{x_0}^1 \frac{dx}{x} \rho_x \frac{d\sigma_0(EK)}{|1 - \pi(Q^2/x)|^2} \mathcal{D}(x, L) \mathcal{D}\left(\frac{y/x}{x}, L\right) \left(1 + \frac{2}{\pi} \mathcal{K}\right)$$

$$L = \ln \frac{Q^2}{m_e^2}$$

Electron SF: probability to ‘find’ electron in the initial electron, with energy fraction x and virtuality up to Q^2

Results

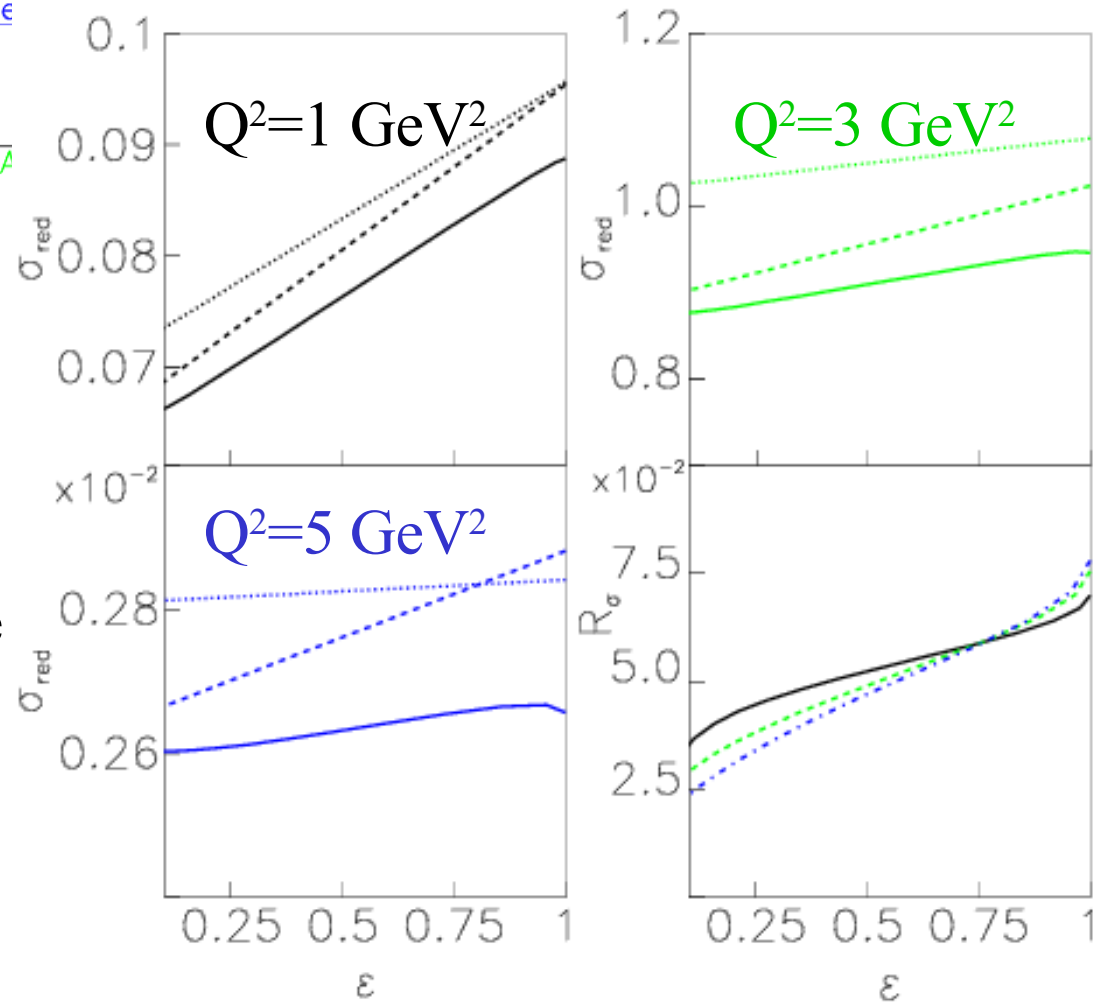
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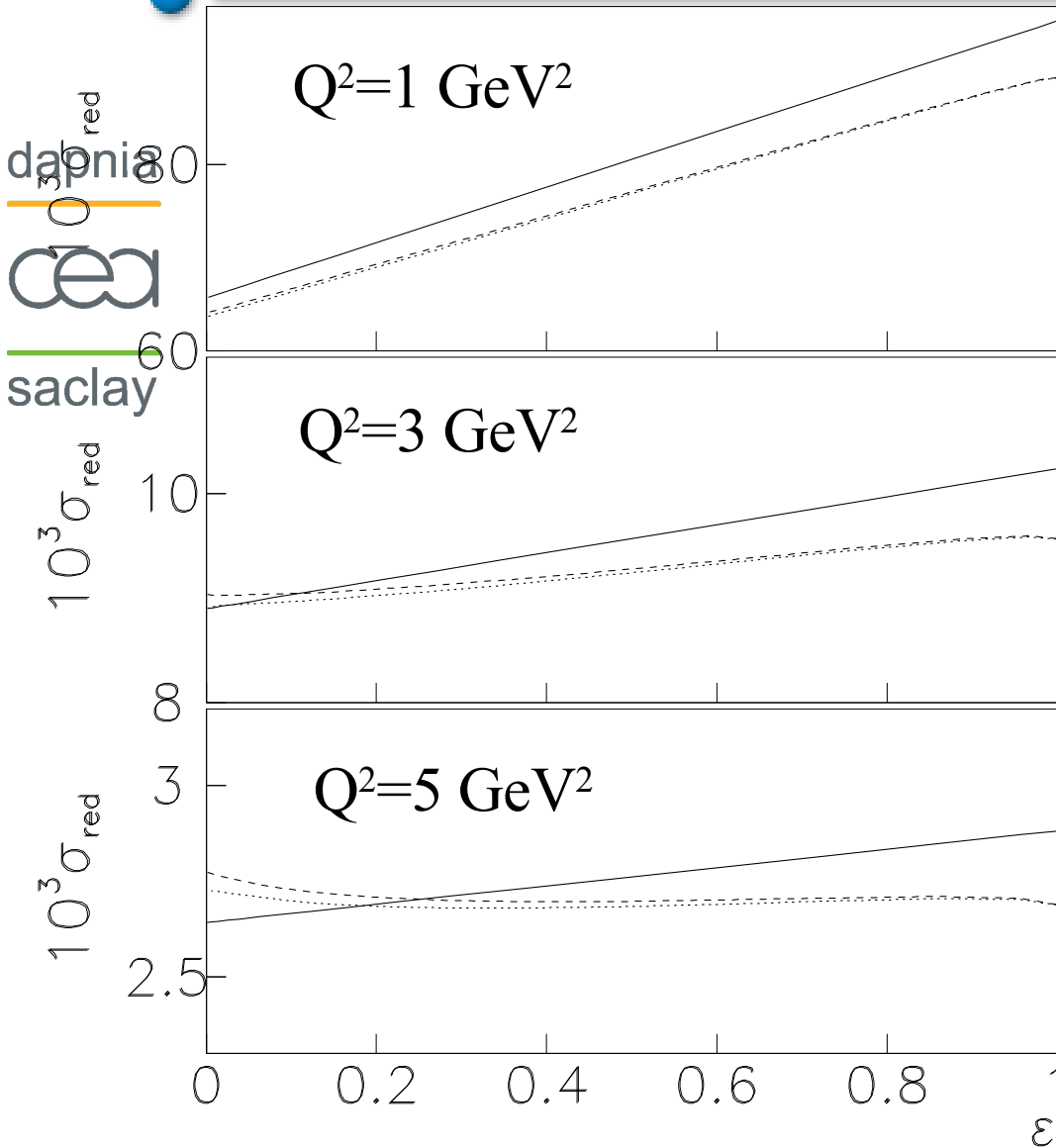
- SF Born
- RC Born
- Polarization

Both calculations assume dipole FFs

*The slope changes
(due to different RC)*



Unpolarized Cross section



$$\sigma_{red} = \tau G_{MP}^2 + \epsilon G_{EP}^2$$

Born +dipole FFs (=unpolarized experiment+Mo&Tsai)

SF (with dipole FFs)

SF+2γ exchange

SF: change the slope!

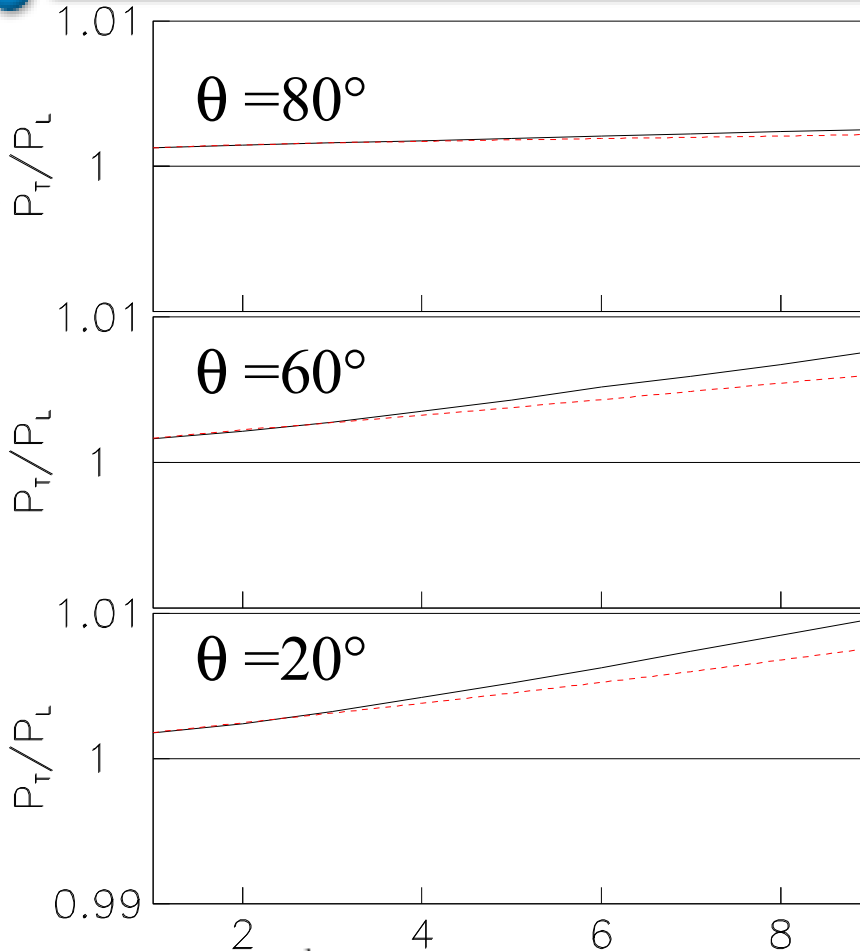
2γ exchange very small!

Polarization ratio

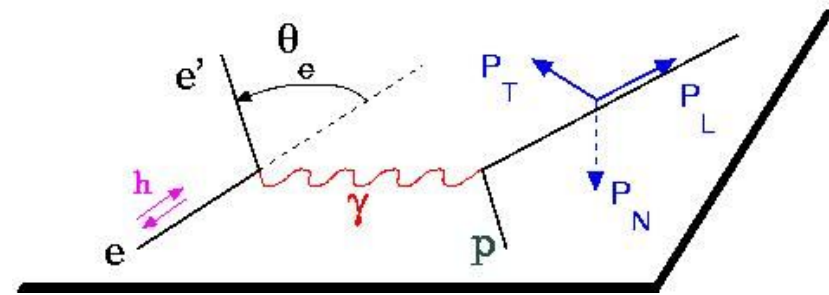
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Born —
SF



2γ exchange very small!

$$\left(\mathcal{P}_t \frac{d\sigma}{d\Omega} \right)_{corr} = -\lambda \int_{z_1}^1 dz \frac{D(z, \beta)}{[1 - \Pi(Q_z^2)]^2} \frac{\alpha^2}{Q_z^2} \left(\frac{1}{\rho_z} \right)^2 \frac{10^2 \text{GeV}^2}{\sqrt{\tan^2(\theta/2)(1 + \tau_z)}} G_E(Q_z^2) G_M(Q_z^2) \left(1 + \frac{\alpha}{\pi} K_t \right);$$

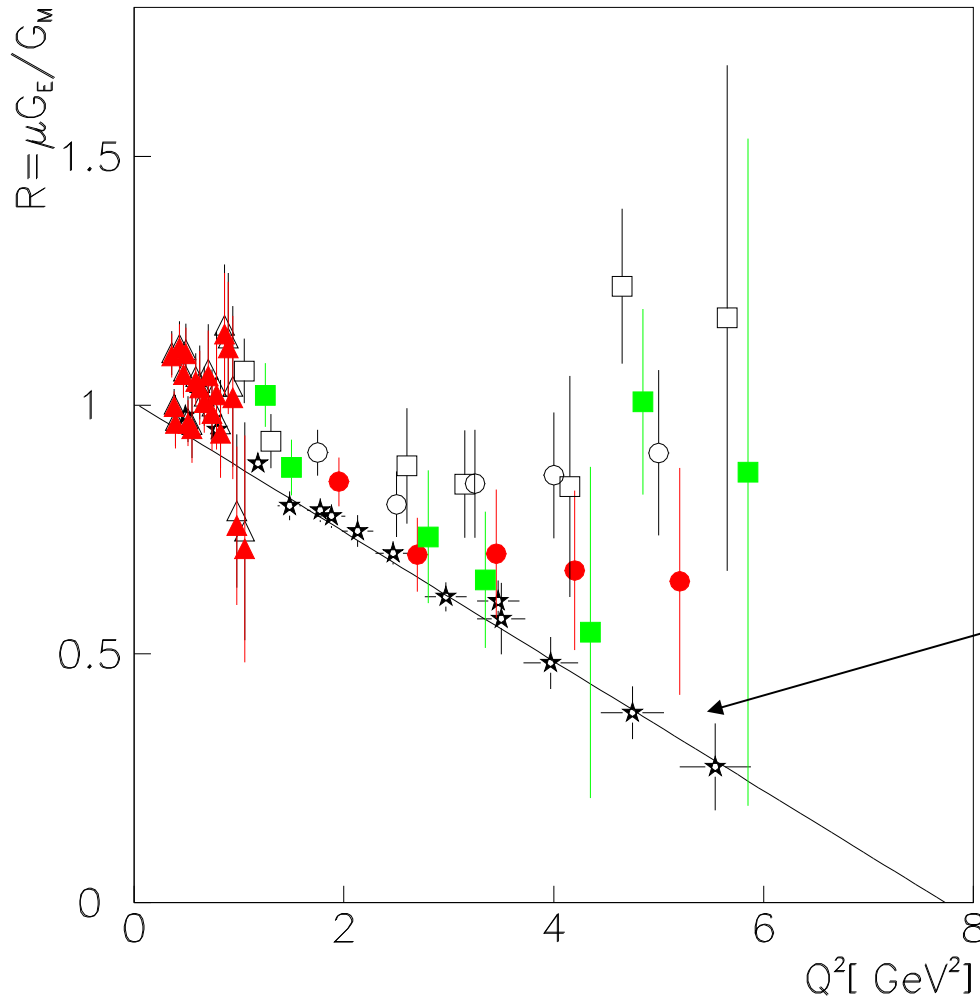
$$\left(\mathcal{P}_\ell \frac{d\sigma}{d\Omega} \right)_{corr} = -\lambda \int_{z_1}^1 dz \frac{D(z, \beta)}{[1 - \Pi(Q_z^2)]^2} \frac{\alpha^2}{2M^2} \left(\frac{1}{\rho_z} \right)^2 \sqrt{1 + \frac{1}{\tan^2(\theta/2)(1 + \tau_z)}} G_M^2(Q_z^2) \left(1 + \frac{\alpha}{\pi} K_\ell \right).$$

Correction (SF method)

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SLAC data

JLab data

Polarization data

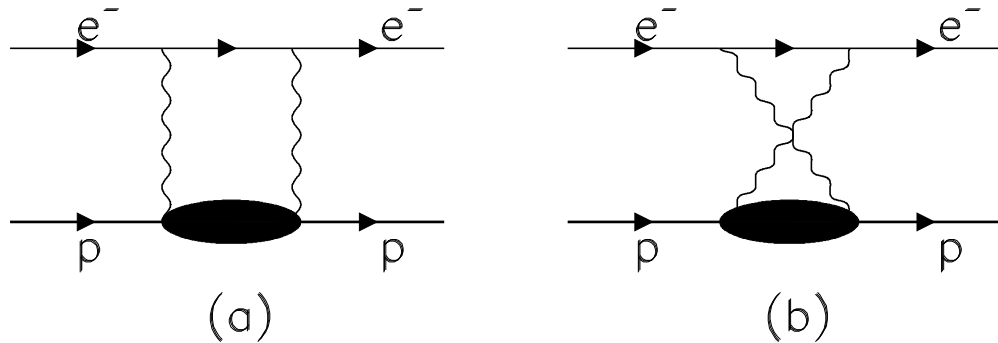
Yu. Bystricky, E.A.Kuraev, E. T.-G, Phys. Rev. C 75, 015207 (2007)

Interference of $1\gamma \otimes 2\gamma$ exchange

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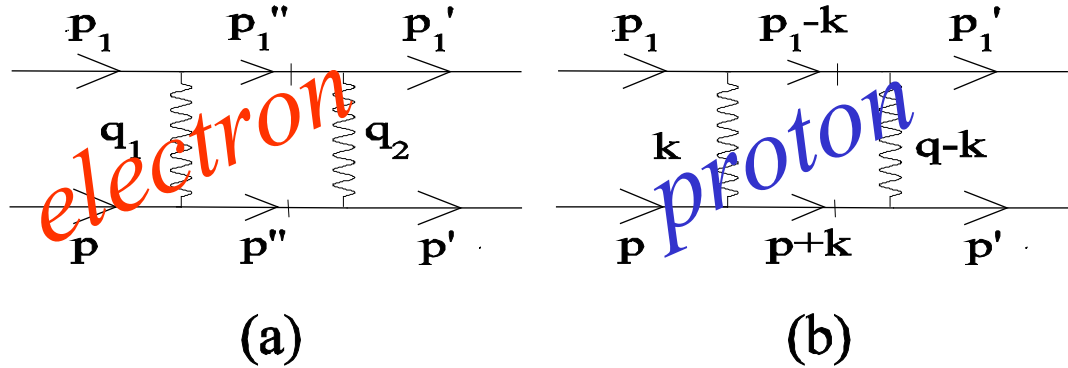
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- Explicit calculation for structureless proton
 - The contribution is small, for unpolarized and polarized ep scattering
 - Does not contain the enhancement factor L
 - The relevant contribution to K is ~ 1

E.A.Kuraev, V. Bytev, Yu. Bystricky, E.T-G Phys. Rev. D74 013003 (1076)



Imaginary part of the 2γ amplitude

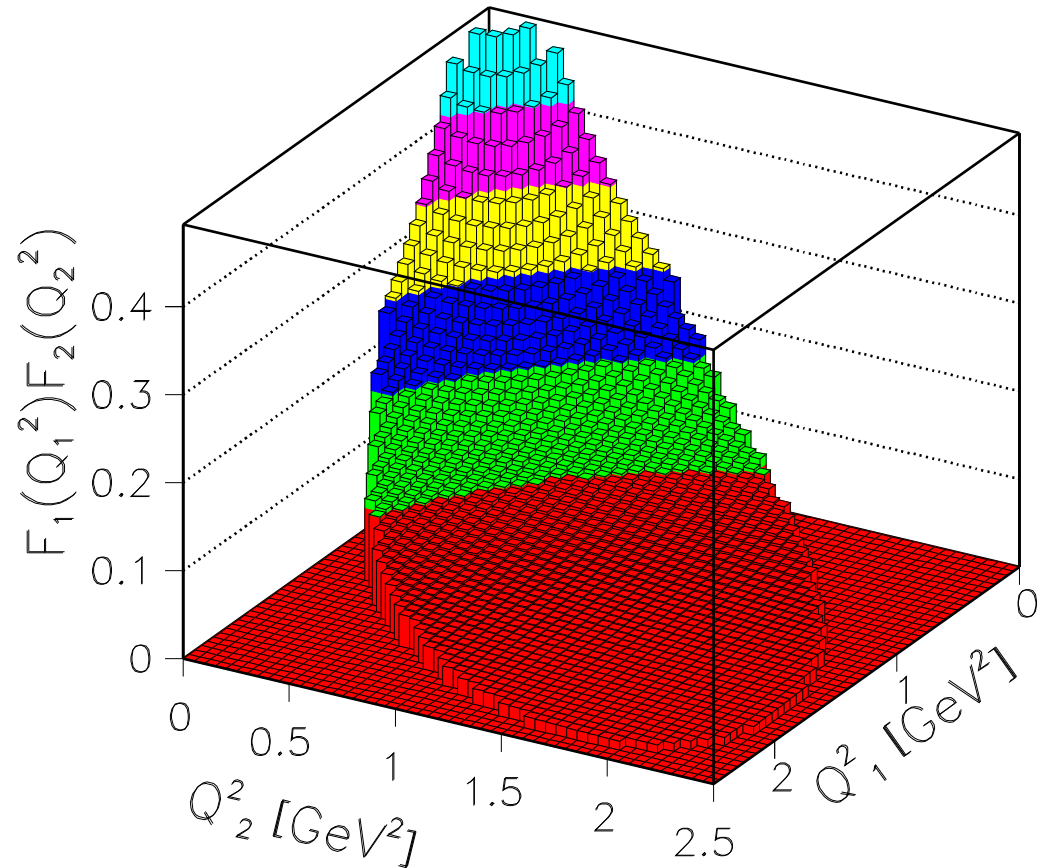
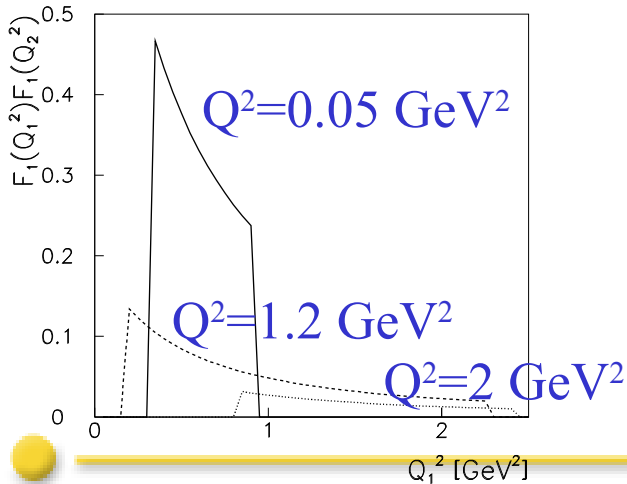
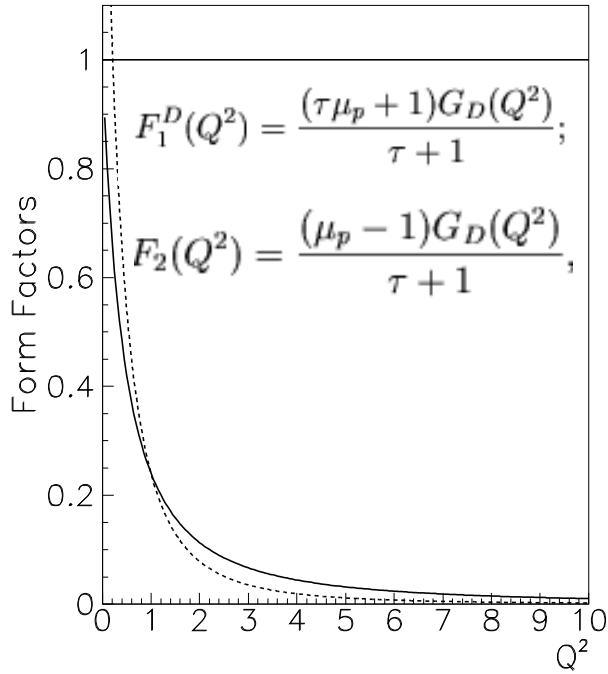
$$\mathcal{M}_{1a} = \frac{1}{\sqrt{8s}} \int \frac{dQ_1^2 dQ_2^2}{\sqrt{\mathcal{D}_1} (Q_1^2 + \lambda^2) (Q_2^2 + \lambda^2)}$$

$$\mathcal{M}_{1b} = \frac{1}{\sqrt{8s}} \int \frac{dQ_1^2 dQ_2^2 F(Q_1^2) F(Q_2^2)}{\sqrt{\mathcal{D}_1} (Q_1^2 + \lambda^2) (Q_2^2 + \lambda^2)}$$

$$dO'' = \frac{2dQ_1^2 dQ_2^2}{\sqrt{\mathcal{D}_1} Q_0^2}, \quad \mathcal{D}_1 = 2(Q_1^2 + Q_2^2)Q^2 Q_0^2 - 2Q^2 Q_1^2 Q_2^2 - (Q_1^2 - Q_2^2)Q_0^2 - (Q^2)^2 Q_0^2$$

QED versus QCD

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$$F(Q_1^2)F(Q_2^2) < 1$$

Perspectives and Conclusions

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• Fundamental measurement: the electric and the magnetic distributions of the proton are different in SL region.

cea

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What about TL ? *Separation of G_E and G_M via angular dependence of differential cross section*

- Clarify reaction mechanism: *2γ exchange by model independent symmetry requirements*
- Unified description in TL and SL region : *zero of GEp ?*
- Asymptotic properties : *QCD and analyticity*

*Model independent properties
Lessons from QED*

Nucleon form factor ratio

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• The ratio of the FFs moduli is given by the following expression:

$$\frac{d\sigma_+(\theta_1)}{d\Omega} : \frac{d\sigma_+(\theta_2)}{d\Omega} = \frac{\tau(1+x_1^2) + (1-x_1^2)R^2}{\tau(1+x_2^2) + (1-x_2^2)R^2}$$

$$x_i = \cos\theta_i, \quad R = \frac{|G_E|}{|G_M|}$$

Model independent considerations for $\bar{p} + p \rightarrow e^+ + e^-$

M. L. Goldberger, Y. Nambu and R. Oehme, Ann. Phys 2, 226 (1957)

P. Guichon and M. Vanderhaeghen, P. R.L. 91, 142303 (2003)

M.P. Rekalo and E. Tomasi-Gustafsson, EPJA 22, 331 (2004)

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The hadronic current:

$$J_\mu = \bar{u}(-p_2) [\tilde{G}_M(q^2, t) \gamma_\mu + \frac{P_\mu}{m} \tilde{F}_2(q^2, t) + \frac{P_\mu}{m^2} \hat{K} F_3(q^2, t)] u(p_1)$$

Decomposition of the amplitudes:

$$K = \frac{k_1 + k_2}{2}, \quad \mathcal{P} = \frac{p_1 + p_2}{2},$$

$$\tilde{G}_M(q^2, t) = G_M(q^2) + \Delta G_M(q^2, t);$$

$$\tilde{G}_E(q^2, t) = G_E(q^2) + \Delta G_E(q^2, t).$$

For 1γ -exchange:

$$\tilde{G}_M^{\text{Born}}(q^2, t) = G_M(q^2), \quad \tilde{F}_2^{\text{Born}}(q^2, t) = F_2(q^2), \quad F_3^{\text{Born}}(q^2, t) = 0$$

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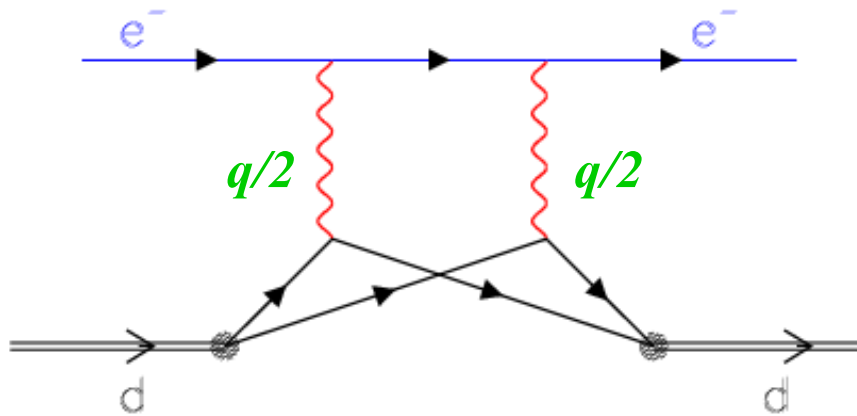
The work presented here was initiated in a collaboration with Prof. M. P. REKALO

Qualitative estimation of Two-Photon exchange (for ed)

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$$\mathcal{M}_1 = \alpha F_d(t).$$

$$\mathcal{M}_2 = \alpha^2 F_N^2 \left(\frac{t}{4} \right).$$

Form factors \rightarrow quark counting rules: $F_d \sim t^5$ and $F_N \sim t^2$

$$\frac{\mathcal{M}_2}{\mathcal{M}_1} = \alpha F_N^2 / F_d(t) = 256 \alpha t / m_x^2$$

For $t = 4 \text{ GeV}^2$, $t / m_x^2 \simeq 6$

$$\frac{\mathcal{M}_2}{\mathcal{M}_1} \simeq 1500 \alpha \rightarrow 10!$$

*For d, ^3He , ^4He , 2γ effect should appear at $\sim 1 \text{ GeV}^2$,
for protons $\sim 10 \text{ GeV}^2$*