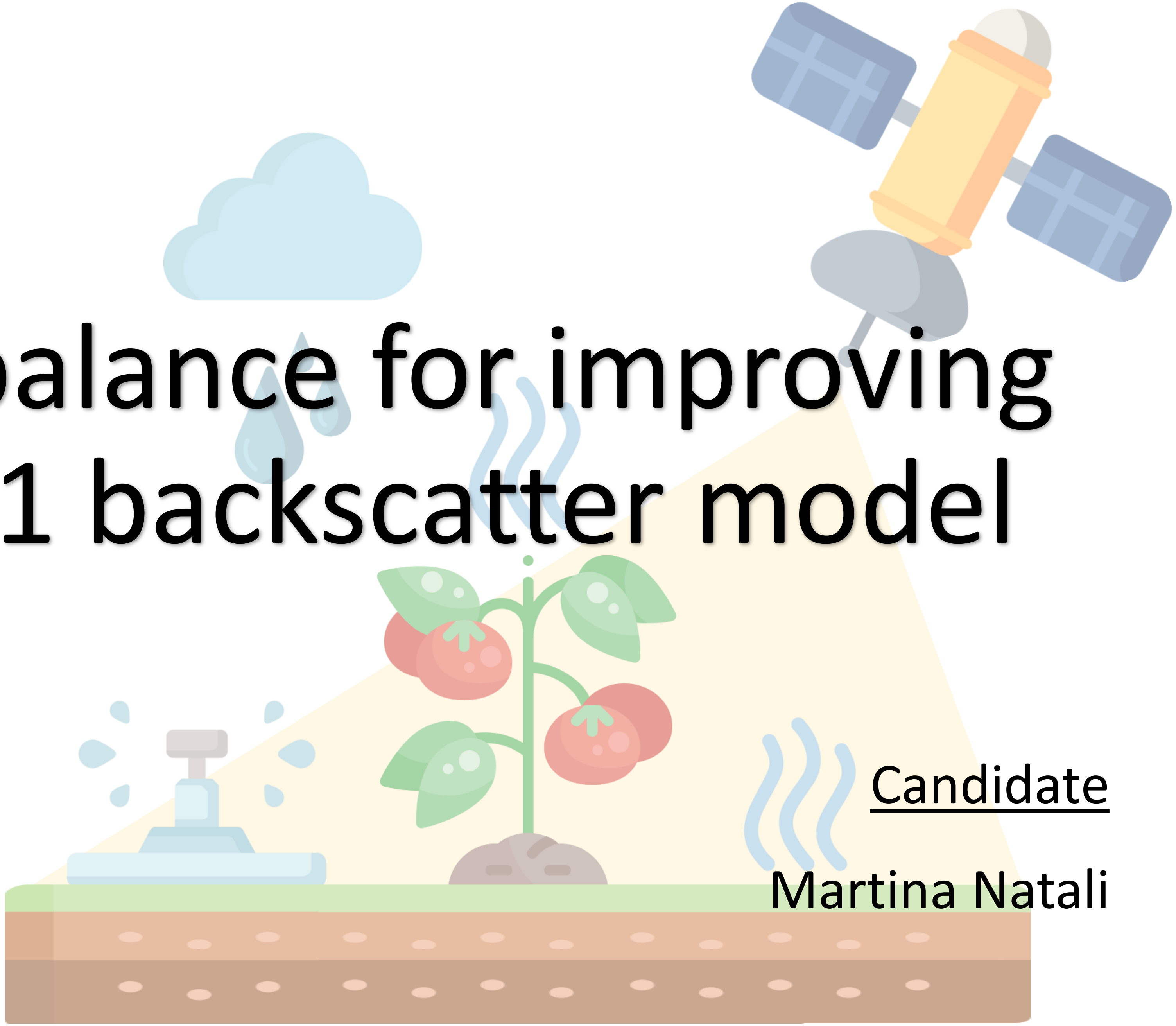
Università degli Studi di Ferrara



Soil water balance for improving a Sentinel-1 backscatter model

Supervisor

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Candidate

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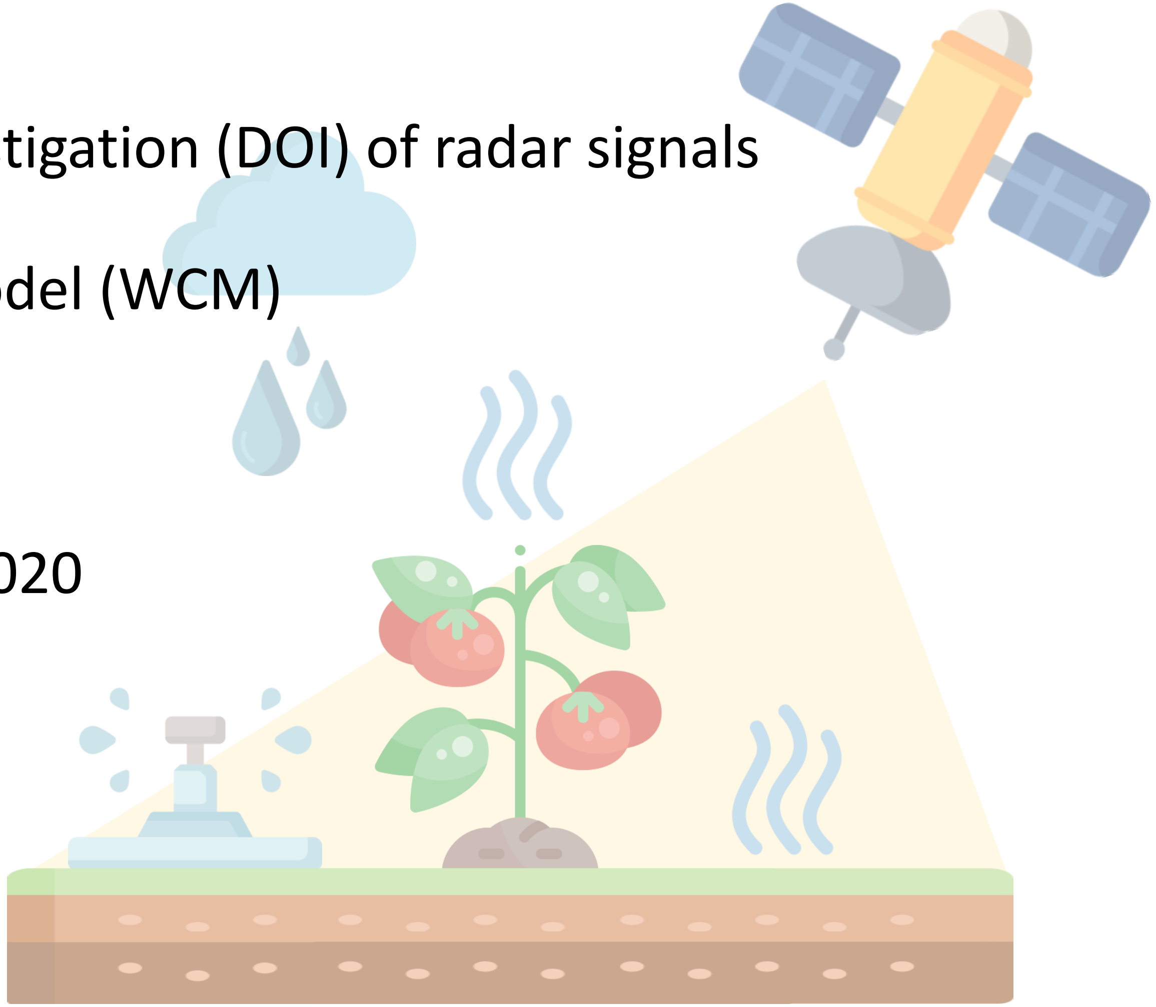
Summary

- Basic idea and implementation
- Study site and experimental setup
- Field of view of in-situ γ station and depth of investigation (DOI) of radar signals
- Soil water balance (SWB) and the Water Cloud Model (WCM)
- Analysis of simulated backscatter (σ^0)
- Results on 2017 campaign and comparison with 2020
- Conclusions and future perspectives



The basic idea…

- The **Water Cloud Model (WCM)** is a radiative transfer model which is used to calculate **σ^0 values over vegetated fields** by using inputs of **soil moisture (SM [m^3m^{-3}])** and vegetation descriptors (e.g. **Normalized Differential Vegetation Index, NDVI [-]**)

- **σ^0 [dB]** is the backscattering coefficient, which is proportional to the ratio of the power received and transmitted by a radar antenna (e.g. Sentinel-1's antenna at 5.4 GHz), normalized over the area of incidence.

Limitations of the WCM

SM inputs: in-situ punctual measurements, can't cover large areas

SM inputs: investigate deep soil layer (~ 10 cm)

Vegetation description based on grasslands

Site-dependent, period-dependent

The basic idea…

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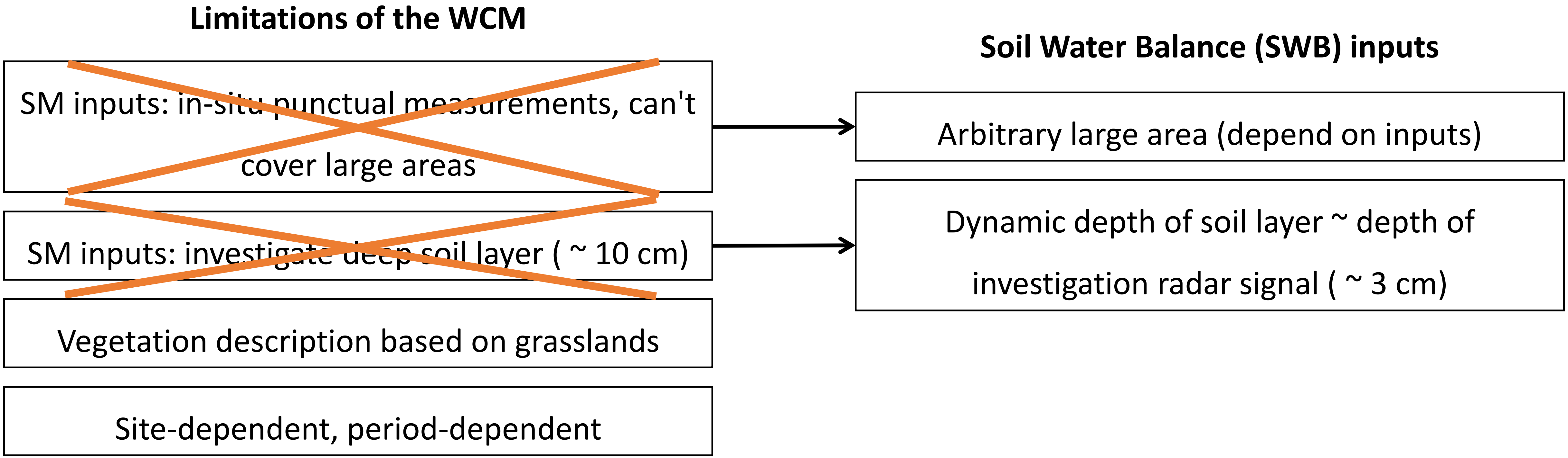


Limitations of the WCM

~~SM inputs: in-situ punctual measurements, can't cover large areas~~

~~SM inputs: investigate deep soil layer (~ 10 cm)~~

Vegetation description based on grasslands

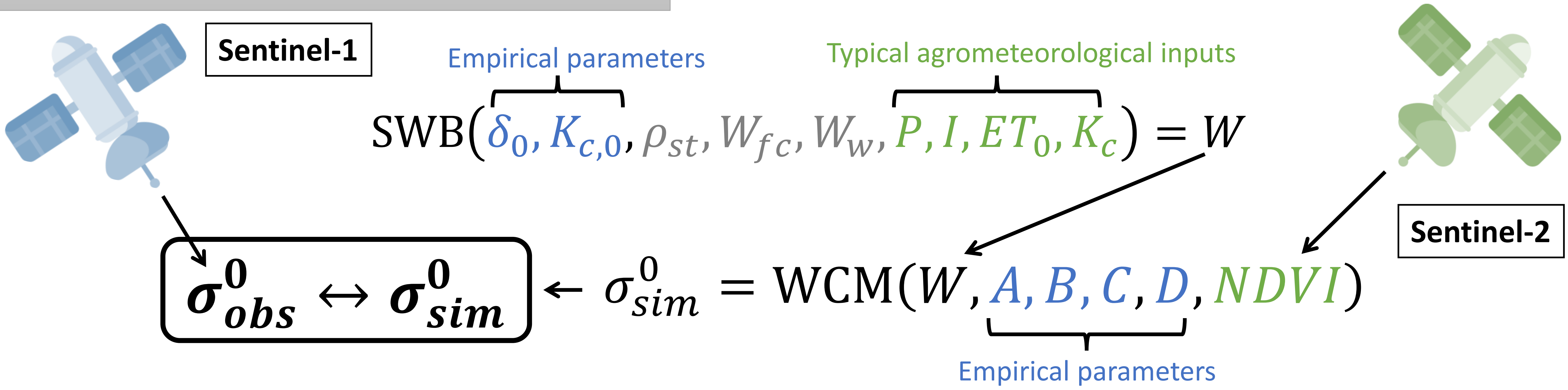
Site-dependent, period-dependent

Soil Water Balance (SWB) inputs

Arbitrary large area (depend on inputs)

Dynamic depth of soil layer ~ depth of investigation radar signal (~ 3 cm)

…and its implementation



Sentinel-1

Sentinel-2

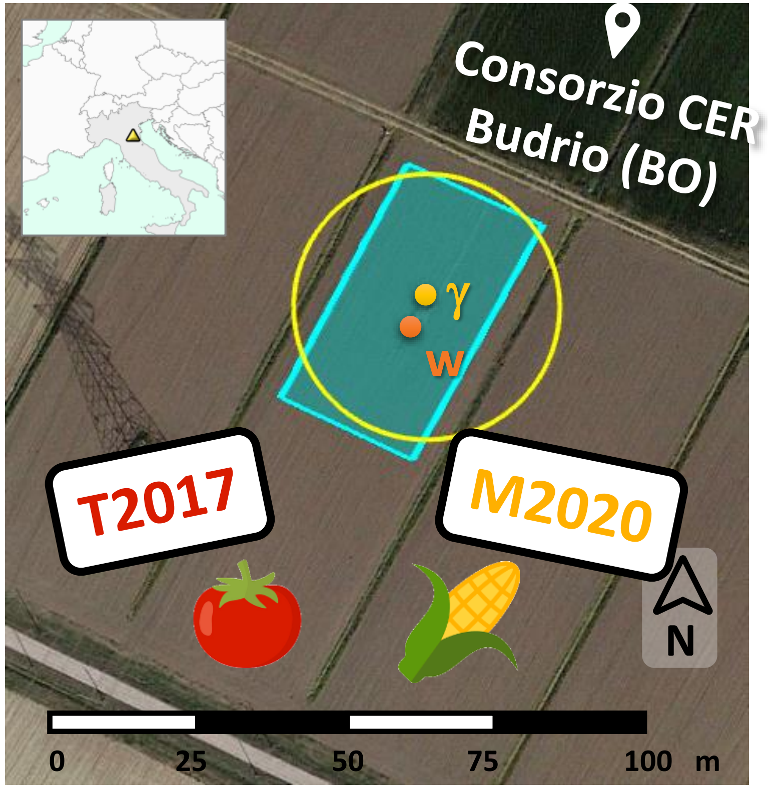
$$\mathrm{SWB}(\overbrace{\delta_0, K_{c,0}}^{\text{Empirical parameters}}, \rho_{st}, W_{fc}, W_w, \overbrace{P, I, ET_0, K_c}^{\text{Typical agrometeorological inputs}}) = W$$

$$\boxed{\boldsymbol{\sigma^0_{obs}} \leftrightarrow \boldsymbol{\sigma^0_{sim}}} \leftarrow \sigma^0_{sim} = \mathrm{WCM}(W, \underbrace{A, B, C, D}_{\text{Empirical parameters}}, NDVI)$$

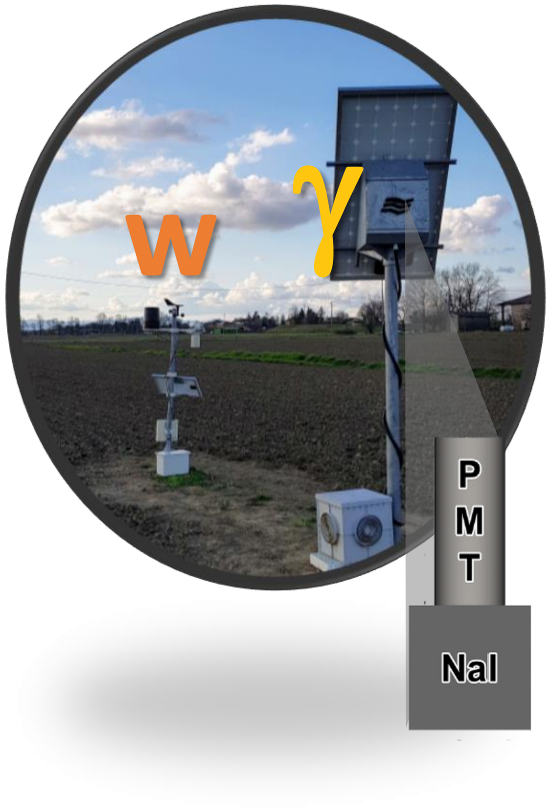
Physical observables

- ρ_{st} = standard potential depletion fraction [-], i.e. the fraction of soil moisture depleted in 1 hour, depends on weather conditions
- W_{fc} = field capacity [m^{3}m^{-3}], i.e. soil moisture for which plants do not suffer water stress, depends on soil texture
- W_w = wilting point [m^{3}m^{-3}], i.e. soil moisture for which plants suffer the most stress, depends on soil texture

Study site and field campaigns



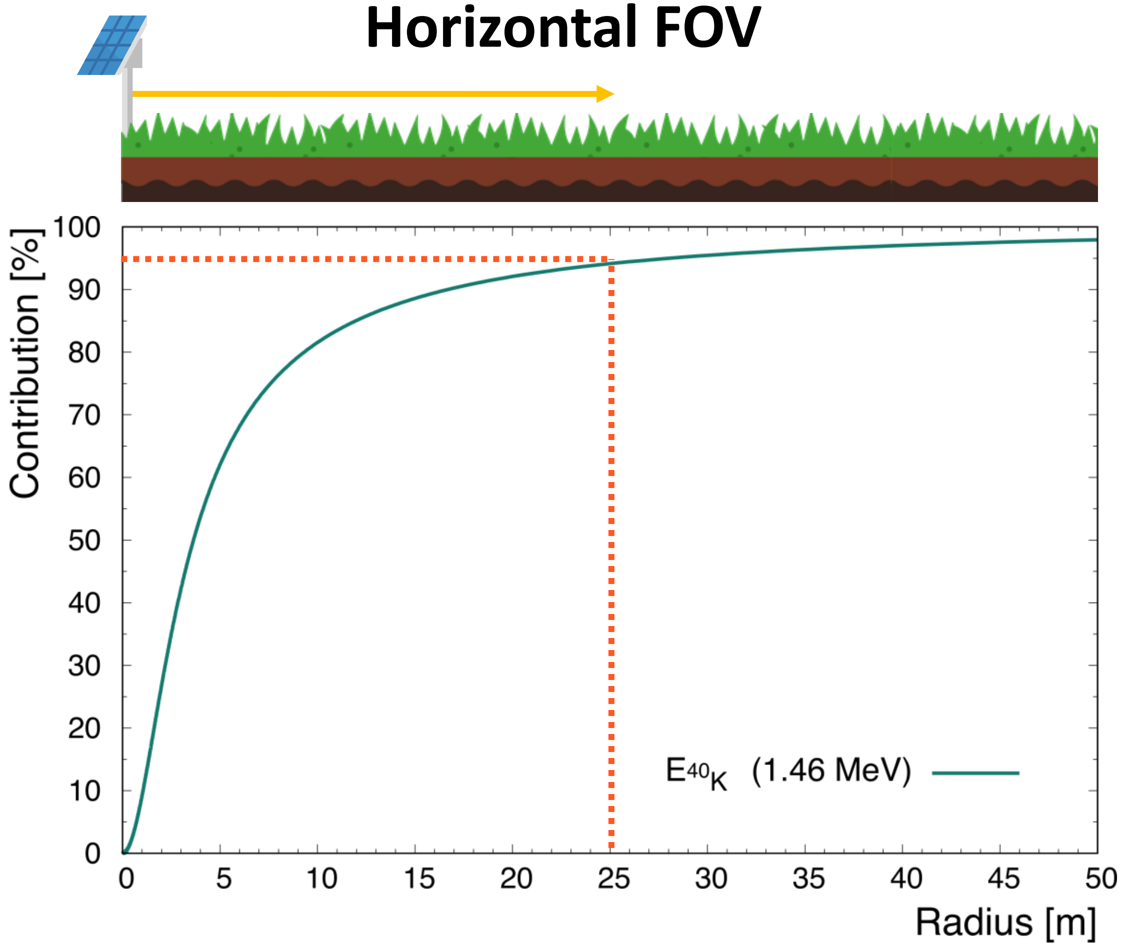
Parameter	Value
Sand [%]	45
Silt [%]	40
Clay [%]	15
Soil textural class	Loamy
Wilting point (W_w) [m^3/ m^3]	0.09 ± 0.01
Field capacity (W_{fc}) [m^3/ m^3]	0.32 ± 0.01
Saturation (W_s) [m^3/ m^3]	0.48 ± 0.01



- 2 field campaigns in a test site in Budrio (BO), Italy, cultivated with **tomato in 2017 (T2017)**, **maize in 2020 (M2020)**, which have different K_c
- **meteorological station (w)** which provides hourly precipitation, wind speed, solar radiation, air temperature, dew point and atmospheric pressure measurements, which are used to compute ET_0
- γ **station (γ)** equipped with a 1L NaI(Tl) scintillator that measures ^{40}K decay spectra; nuclear measurements are used to calculate **hourly SM** which is used as comparison/benchmark for SM estimates from SWB

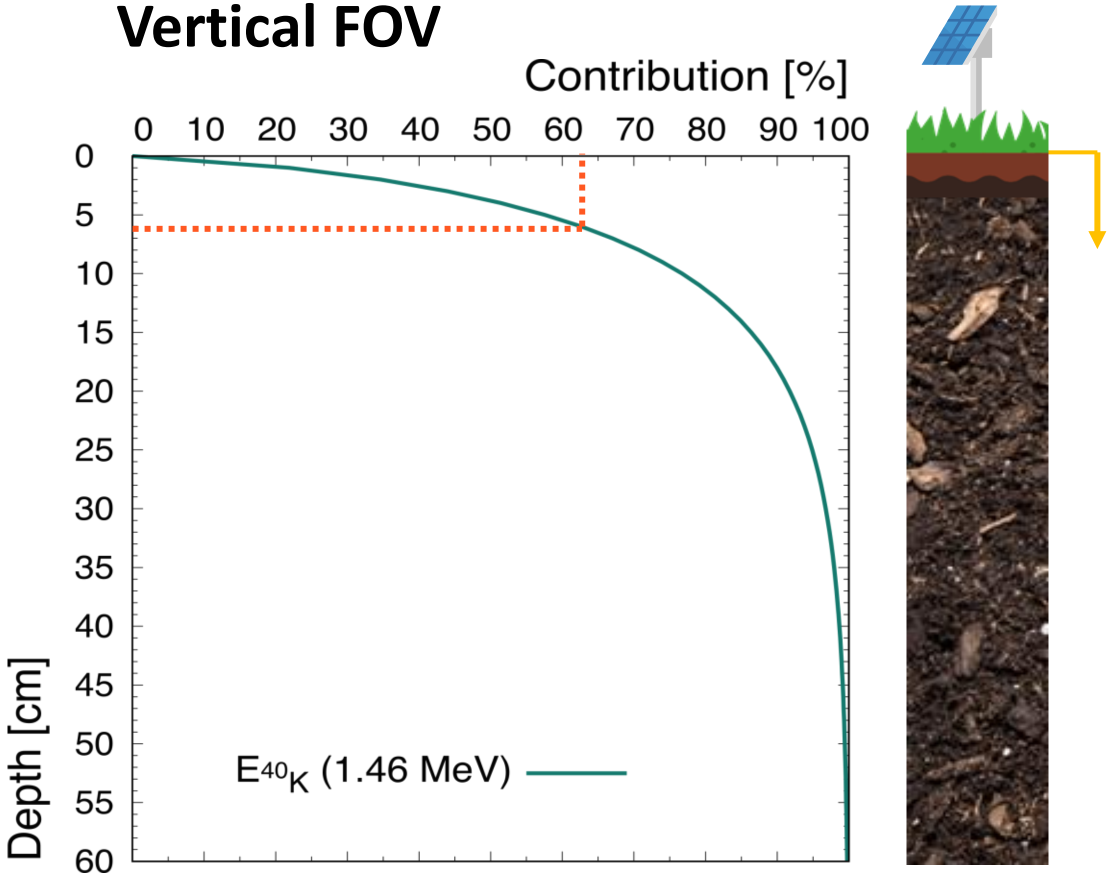
Field of view (FOV) of the γ station

Horizontal FOV



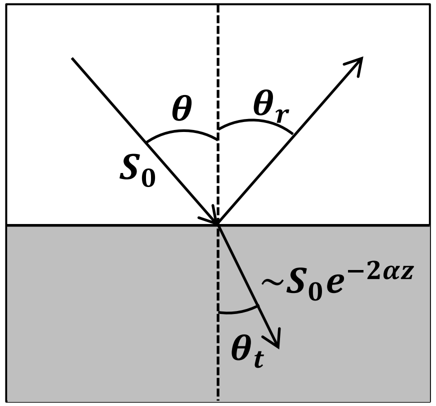
Nearly 95% of ground radioactivity comes from a **~ 25 m** radius area

Vertical FOV



Nearly 63% of ground radioactivity comes from a **~ 6 cm** soil layer

Depth of investigation (DoI) of radar signals

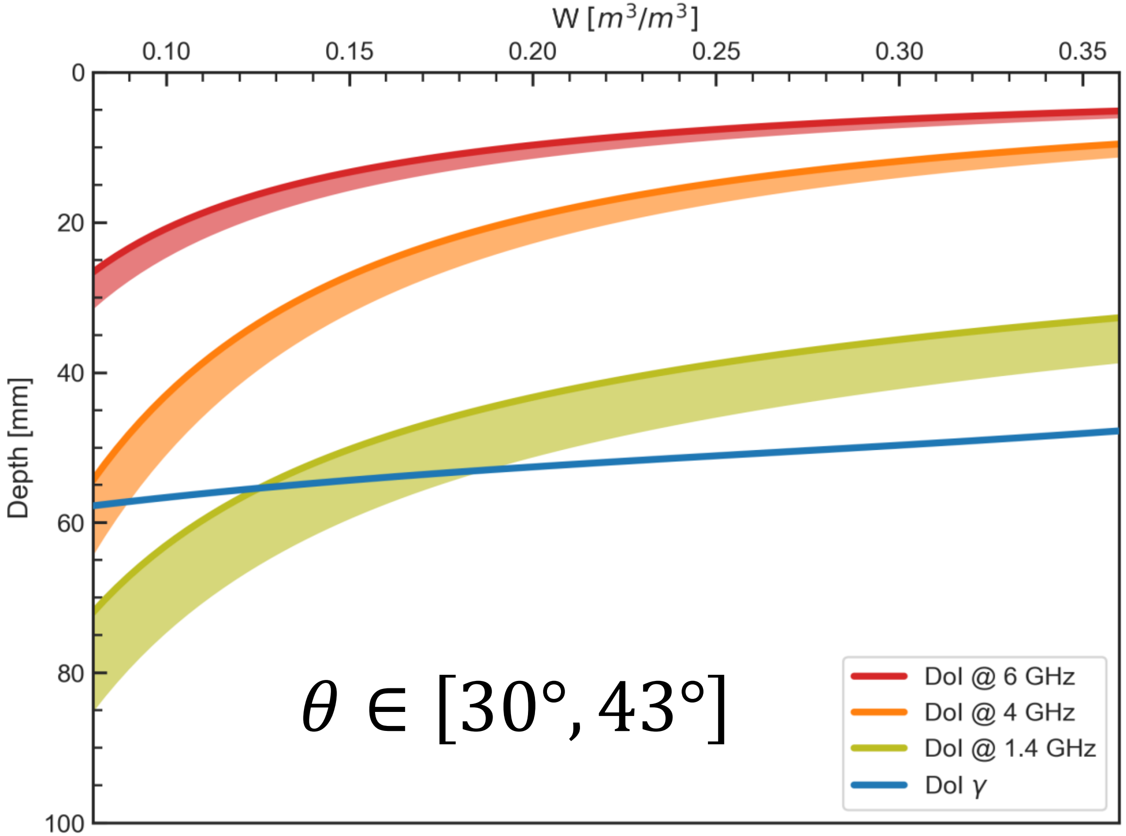


Dielectric constant for lossy media:
$\varepsilon = \varepsilon' + i\varepsilon''$

- The **attenuation constant** $\boldsymbol{\alpha}$ $[m^{-1}]$ is: $\alpha \approx \pi\nu\varepsilon''/\sqrt{\varepsilon'}$
- The depth of investigation δ $[mm]$, corrected for the angle of incidence θ, is

$$\boldsymbol{\delta \approx \frac{c}{2\pi\nu}\frac{\sqrt{\varepsilon'}}{\varepsilon''}\cos\theta}$$

- $\varepsilon', \varepsilon''$ are calculated by empirical **Hallikainen model** (Hallikainen, Ulaby et al. 1985) which is a function of **SM, frequency and soil texture**



The depth δ of the soil layer in SWB is equal to the depth of investigation of microwave signals

SWB to calculate SM

SWB is an integral formula that calculates volumetric soil moisture $SM\ [m^3 m^{-3}]$ in a soil layer of depth $\delta\ [mm]$ at fixed time intervals $\Delta t = t_i - t_{i-1}$ = 1h

$$SM_i = SM_{i-1} + \frac{1}{\delta * \delta_0}\left[(P_i + I_i + CR_i) - (DP_i + RO_i + ET_{c,adj,i})\right]$$

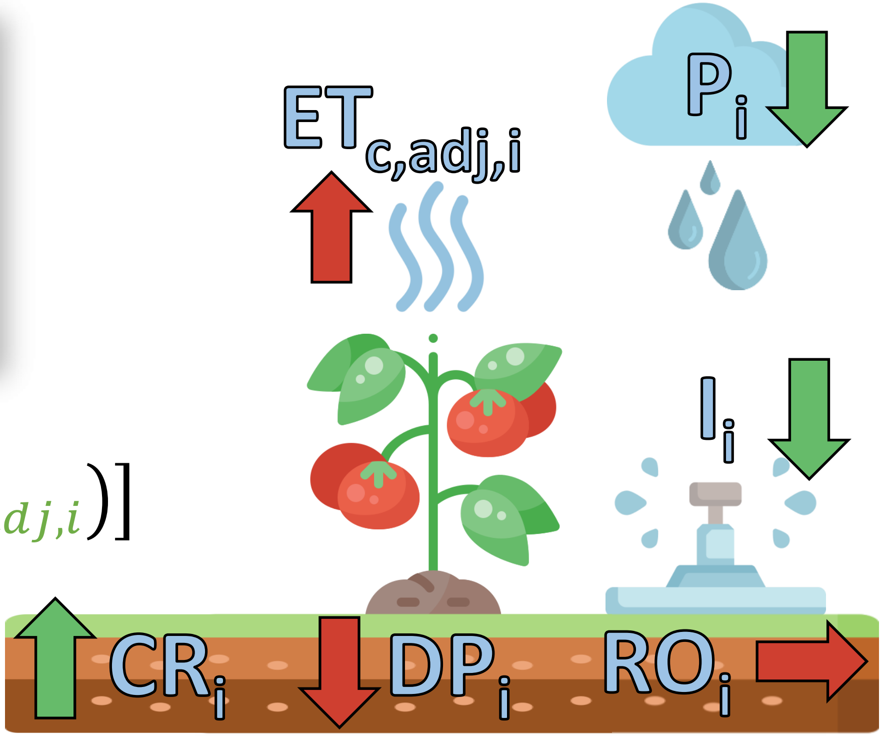
$$ET_{c,adj,i} = ET_{0,i} * K_{c,i} * K_{c,0} * K_{s,i}(\rho_{st}, W_{fc}, W_w)$$

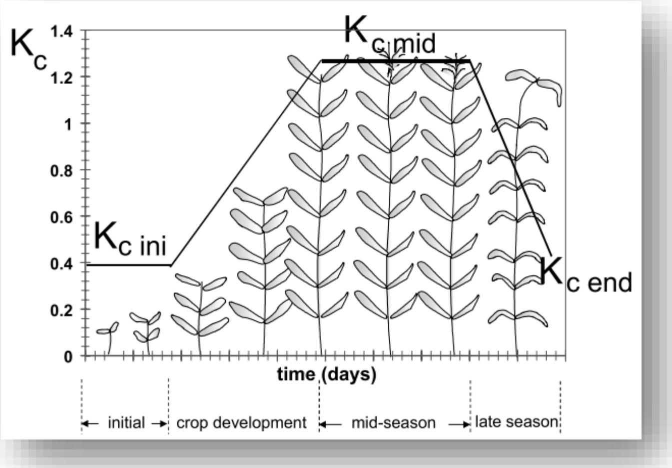
reference ET for hypotetical grass surface

crop coefficient

scaling factor to account for actual field conditions

plant's stress coefficient





CR_i, RO_i are neglected in this study since i) CR_i is capillary rise from the water table which is far below the soil layer considered; ii) RO_i is not negligible on steep terrain only, while the study site is in a flat region

WCM to calculate σ^0 from soil and vegetation

- In WCM the total σ^0 is the incoherent sum of the soil (σ^0_{soil}) and the vegetation (σ^0_{veg}) contributions
- In WCM **vegetation is modeled as a water cloud** (Attema, Ulaby, 1978) of identical, uniformly distributed water particles above a soil layer
- σ^0_{soil} is a linear function of SM by experimental evidence

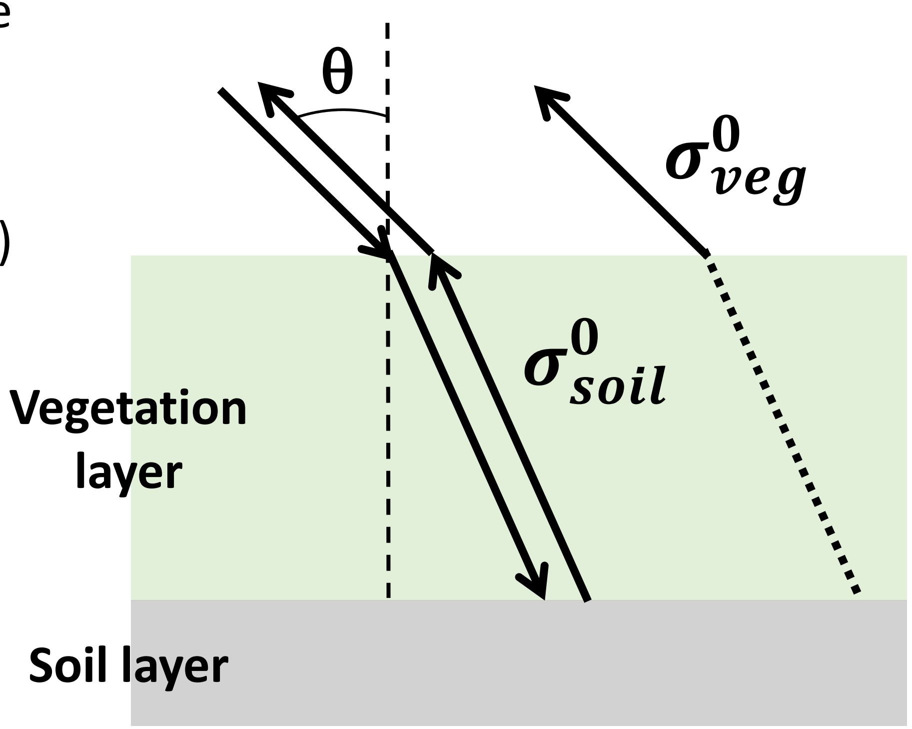
$$\sigma^0_{soil} = C + D * SM$$

- σ^0_{soil} is attenuated by the vegetation layer by a factor γ^2

$$\gamma^2 = e^{-\frac{2*B*NDVI}{\cos\theta}}$$

- Vegetation layer provides a contribution σ^0_{veg}:

$$\sigma^0_{veg} = A * NDVI * \cos\theta\,(1-\gamma^2)$$



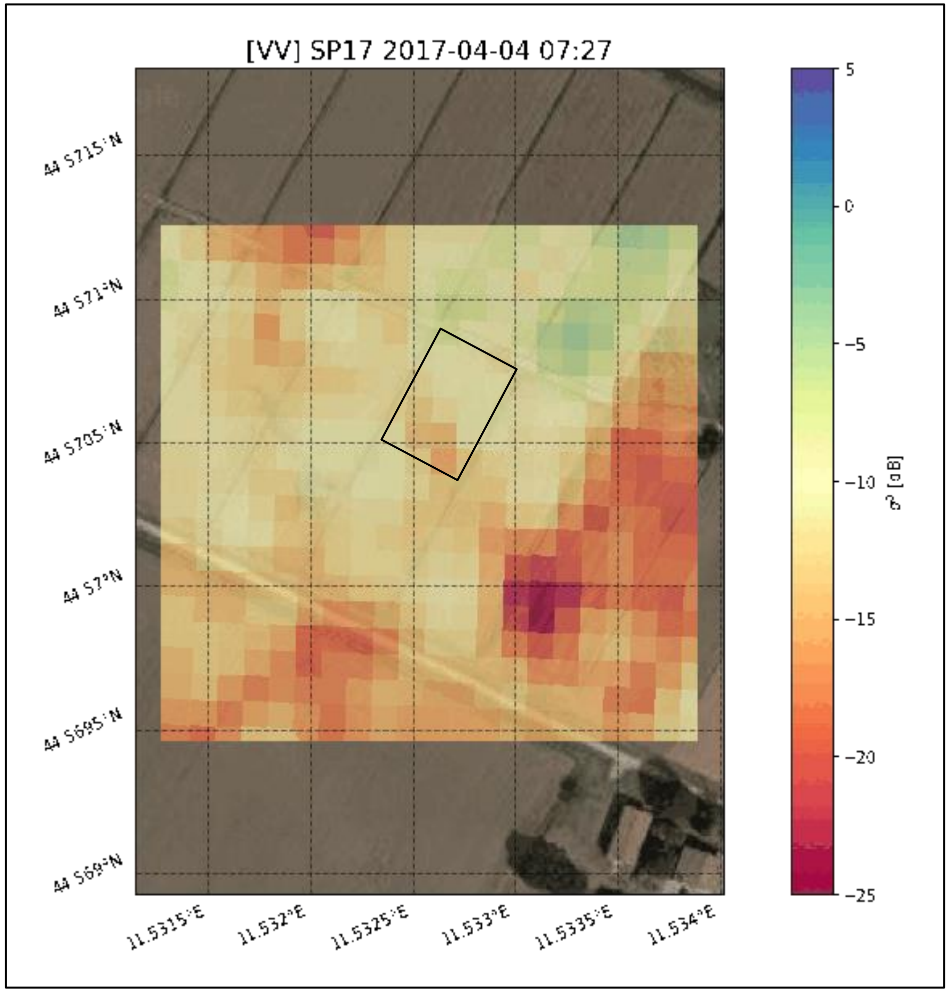
A: scaling factor for vegetation contribution [-]

B: scaling factor in attenuation [-]

C: σ^0 from dry soil, minimum value [dB]

D: σ^0 sensitivity to SM [dB $m^{-3}m^3$]

Area of Interest (AoI) and satellite products processing



AoI of 30 m x 50 m (~ 0.2 ha), field is halved to exclude saturated pixels from electromagnetic shadow due to metal structure

Area of Interest (AoI) and satellite products processing

AoI of 30 m x 50 m (~ 0.2 ha), field is halved to exclude saturated pixels from electromagnetic shadow due to metal structure

Sentinel-1 GRDH images

↓

Co-registration

↓

Average over AoI

↓

Orbit + angle normalization @ 40°

↓

σ^0

Google Earth Engine

Relative orbit	Acquisition time	Direction	Mean incidence angle [°]
168	7 a.m.	descending	31.6
95	7 a.m.	descending	41.6
117	7 p.m.	ascending	37.6

Area of Interest (AoI) and satellite products processing

AoI of 30 m x 50 m (~ 0.2 ha), field is halved to exclude saturated pixels from electromagnetic shadow due to metal structure

Google Earth Engine

Sentinel-1 GRDH images
↓
Co-registration
↓
Average over AoI
↓
Orbit + angle normalization @ 40°
↓
σ^0

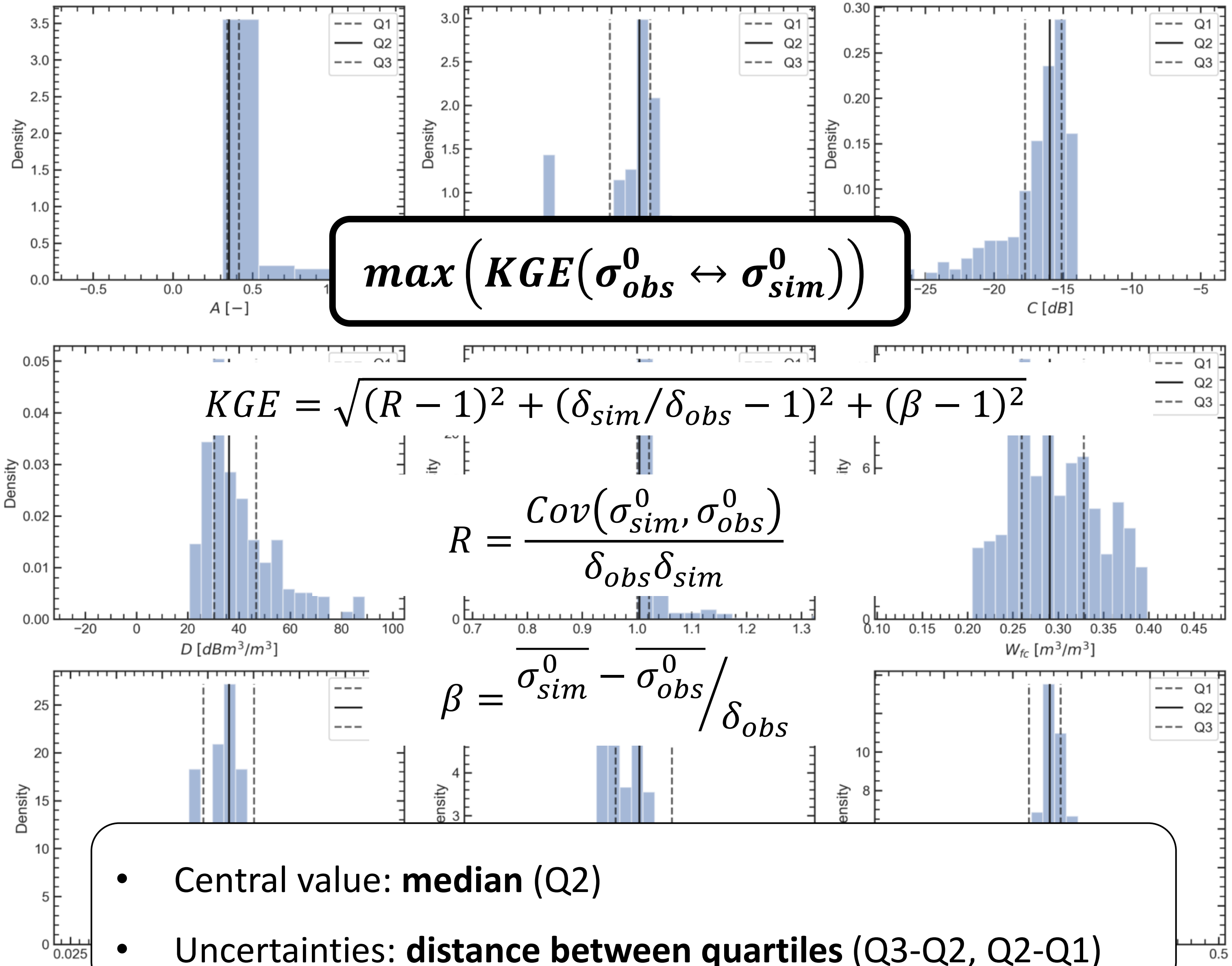
Sentinel-2 BOA images
↓
Co-registration
↓
Cloud masking @ 70%
↓
Average over AoI
↓
NDVI

Period	From - to	Days	S-1 images	S-2 images
T2017	4/4 - 2/11/2017	213	107	27
M2020	3/3 – 31/8/2020	180	80	70

Free bounds (FB) calibration scheme

Free bounds (FB) scheme employs values built upon literary references or physical considerations. All PDFs are supposed uniform.

Quantity	Range	Model
$A\ [-]$	[0, 5]	WCM
$B\ [-]$	[0, 3]	
$C\ [dB]$	[-20, -5]	
$D\ \left[dB\ m^3 m^{-3}\right]$	[10, 100]	
$K_{c,0}\ [-]$	[0, 2]	SWB
$\rho_{st}\ \left[mm\ day^{-1}\right]$	[0.1, 0.8]	
$\delta_0\ [-]$	[1, 2]	
$W_{fc}\left[m^3 m^{-3}\right]$	[0.2, 0.4]	
$W_w\left[m^3 m^{-3}\right]$	[0.07, 0.17]	



$$\boldsymbol{max\left(KGE\left(\sigma^0_{obs} \leftrightarrow \sigma^0_{sim}\right)\right)}$$

$$KGE = \sqrt{(R-1)^2 + (\delta_{sim}/\delta_{obs} - 1)^2 + (\beta - 1)^2}$$

$$R = \frac{Cov\left(\sigma^0_{sim}, \sigma^0_{obs}\right)}{\delta_{obs}\delta_{sim}}$$

$$\beta = \overline{\sigma^0_{sim}} - \overline{\sigma^0_{obs}} \Big/ \delta_{obs}$$

- Central value: **median** (Q2)
- Uncertainties: **distance between quartiles** (Q3-Q2, Q2-Q1)

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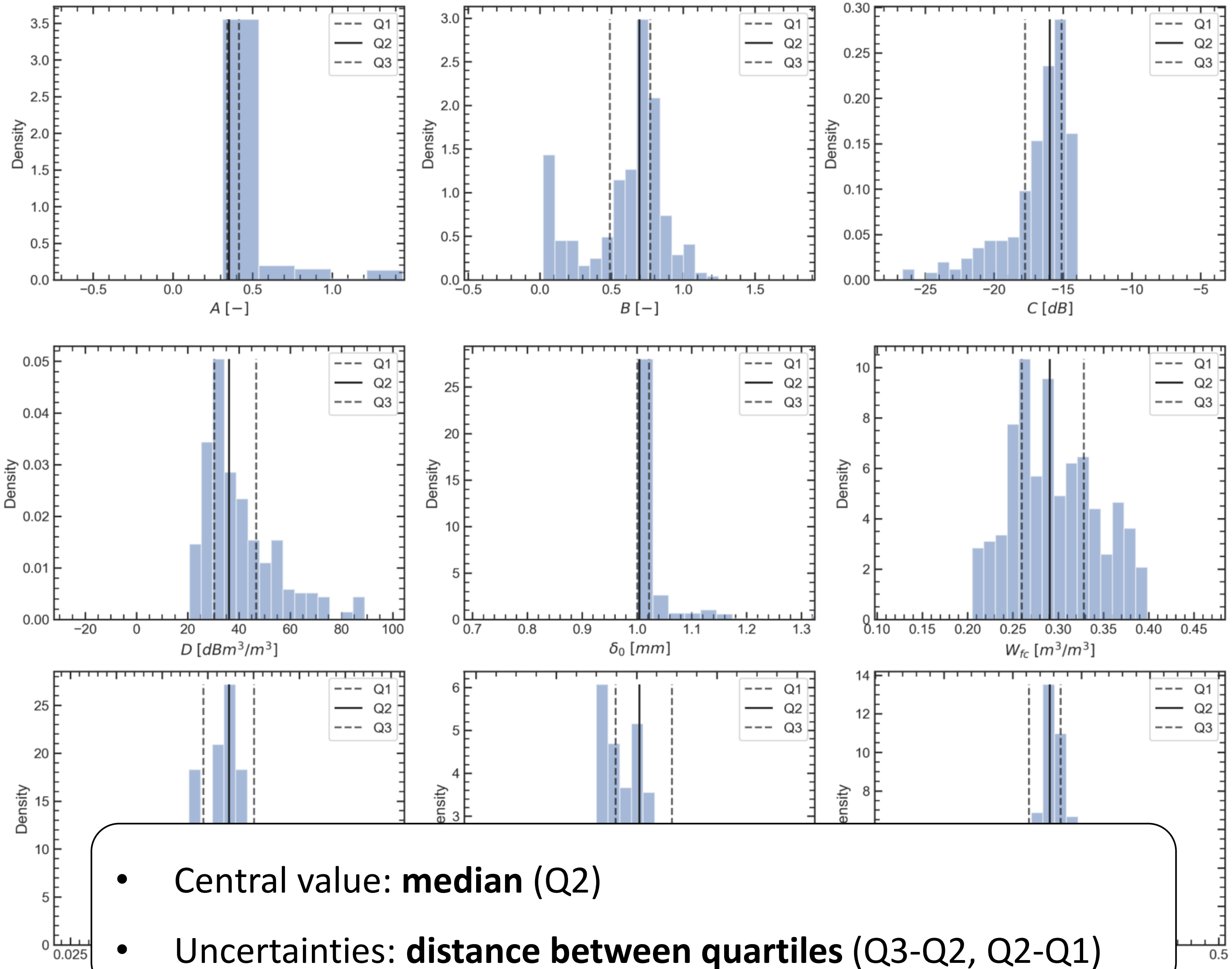

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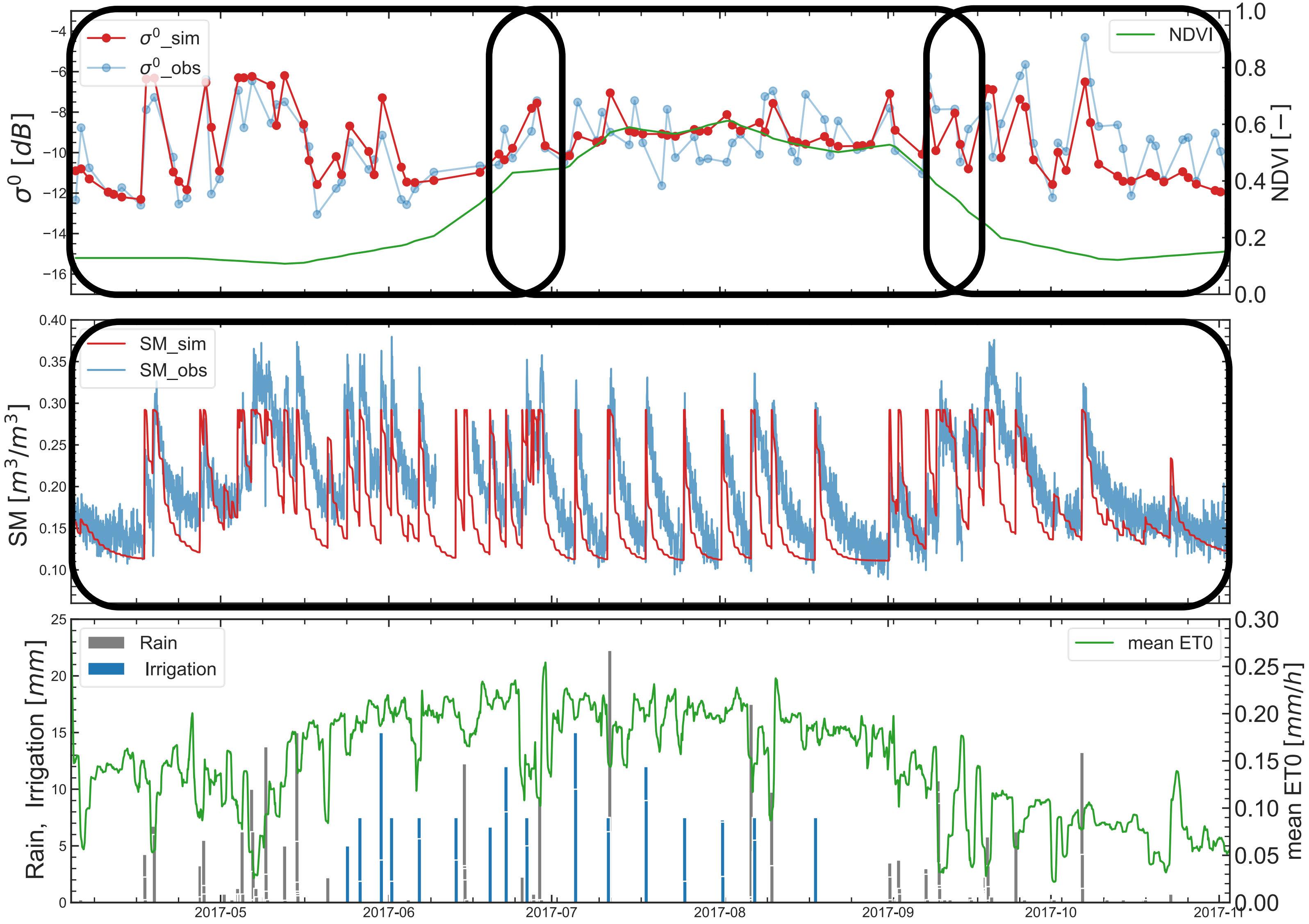
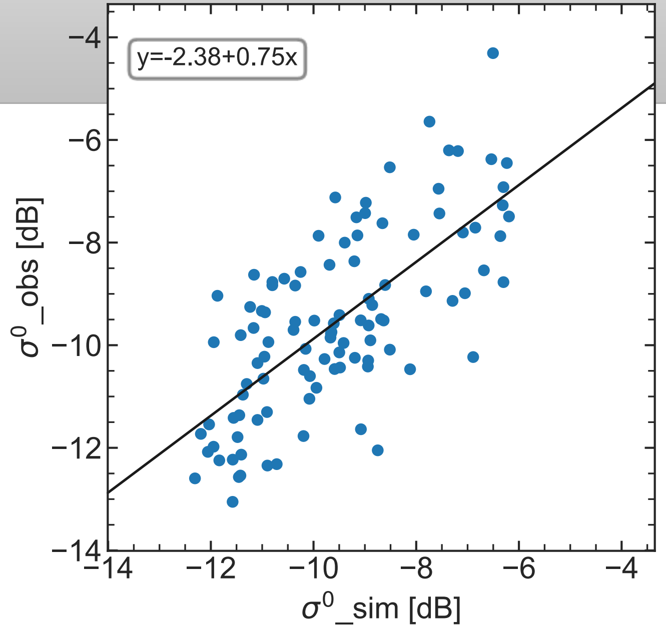
FB T2017 – Results

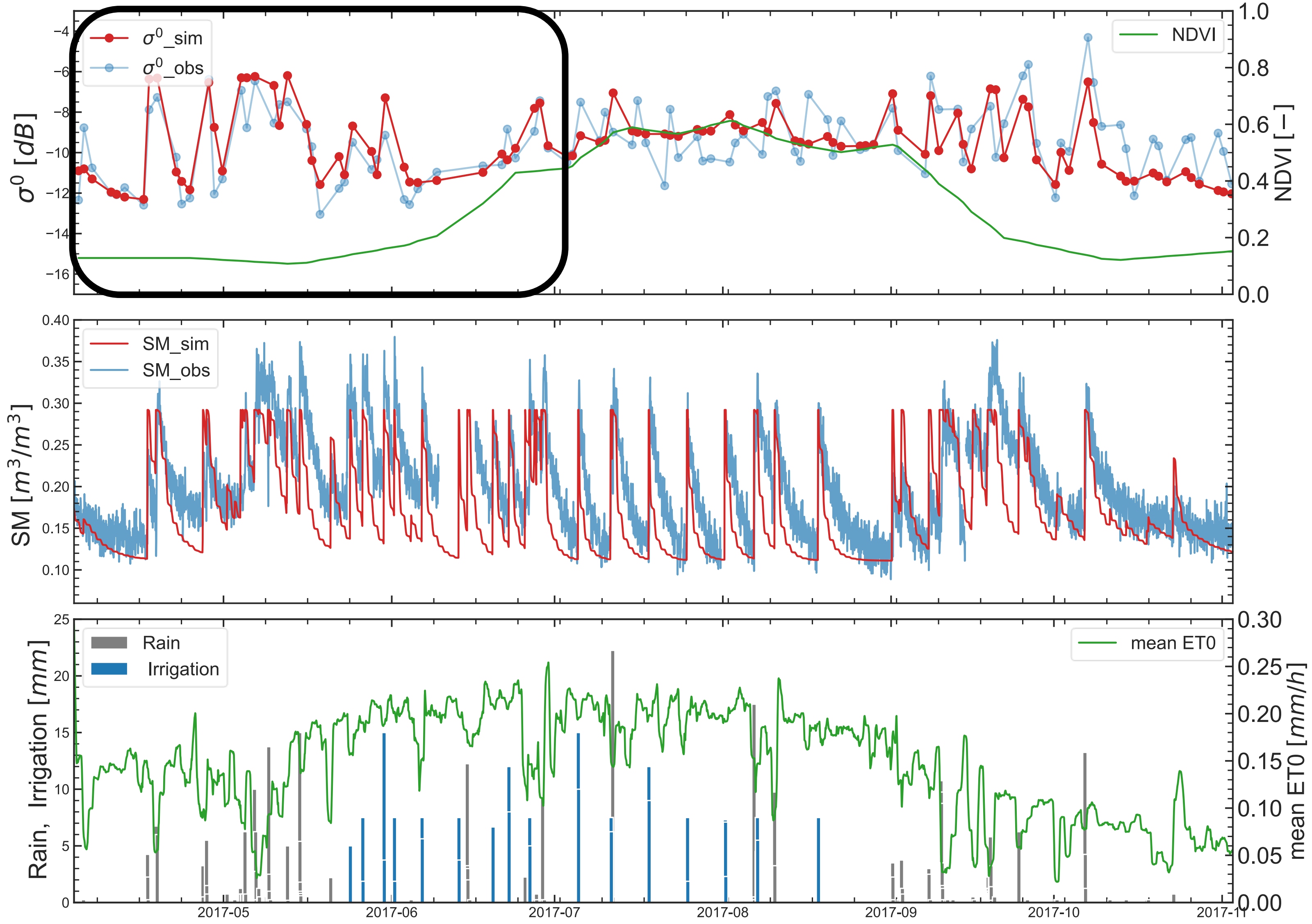


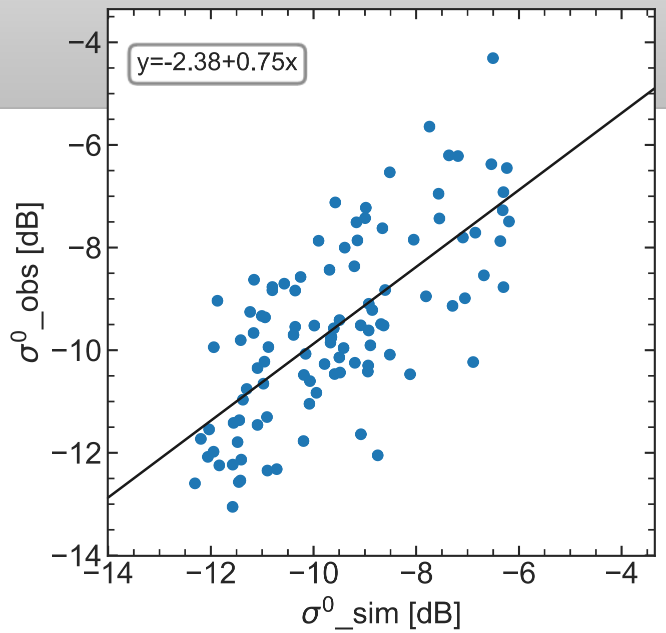


	KGE	**R**	**R^2**	**bias**
σ^0 [dB]	0.70	0.70	0.49	-0.01
SM [m^3m^{-3}]	0.68	0.72	0.52	-0.03

- SM overall well modeled
- calculated SM dries out faster than observations due to thinner soil layer

FB T2017 – Results





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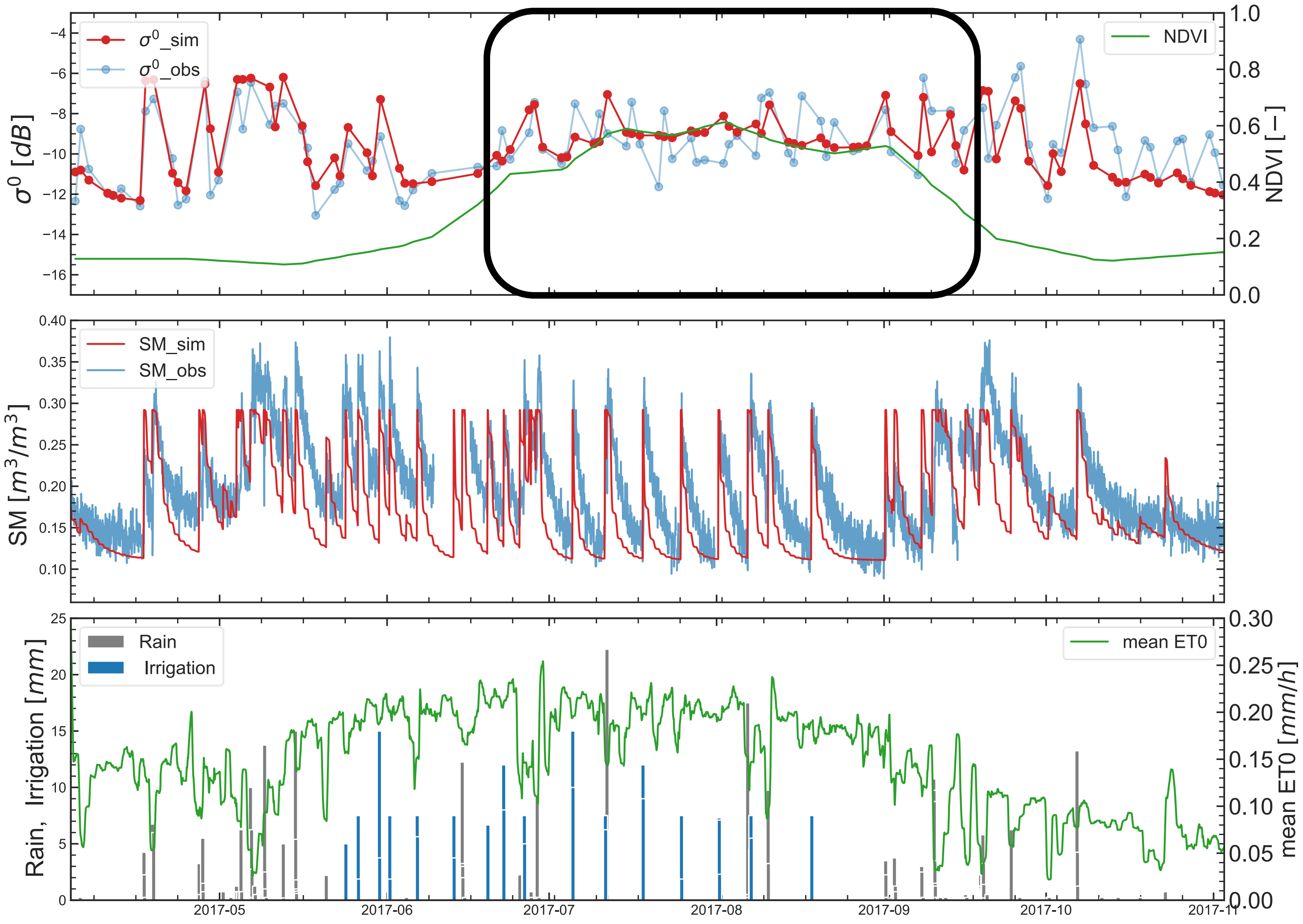
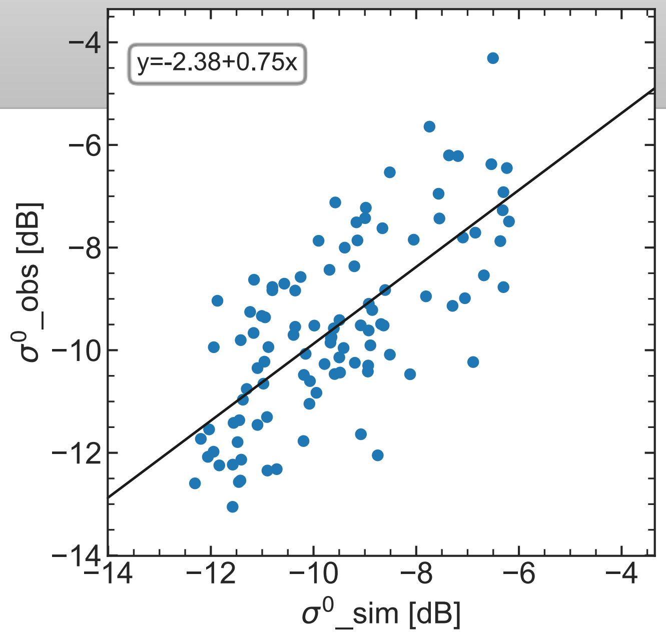
σ^0 [dB]	0.70	0.70	0.49	-0.01
SM [m^3m^{-3}]	0.68	0.72	0.52	-0.03

- Spring: well modeled with wide dynamics due to SM
- low NDVI → σ^0_{soil} dominates

FB T2017 – Results

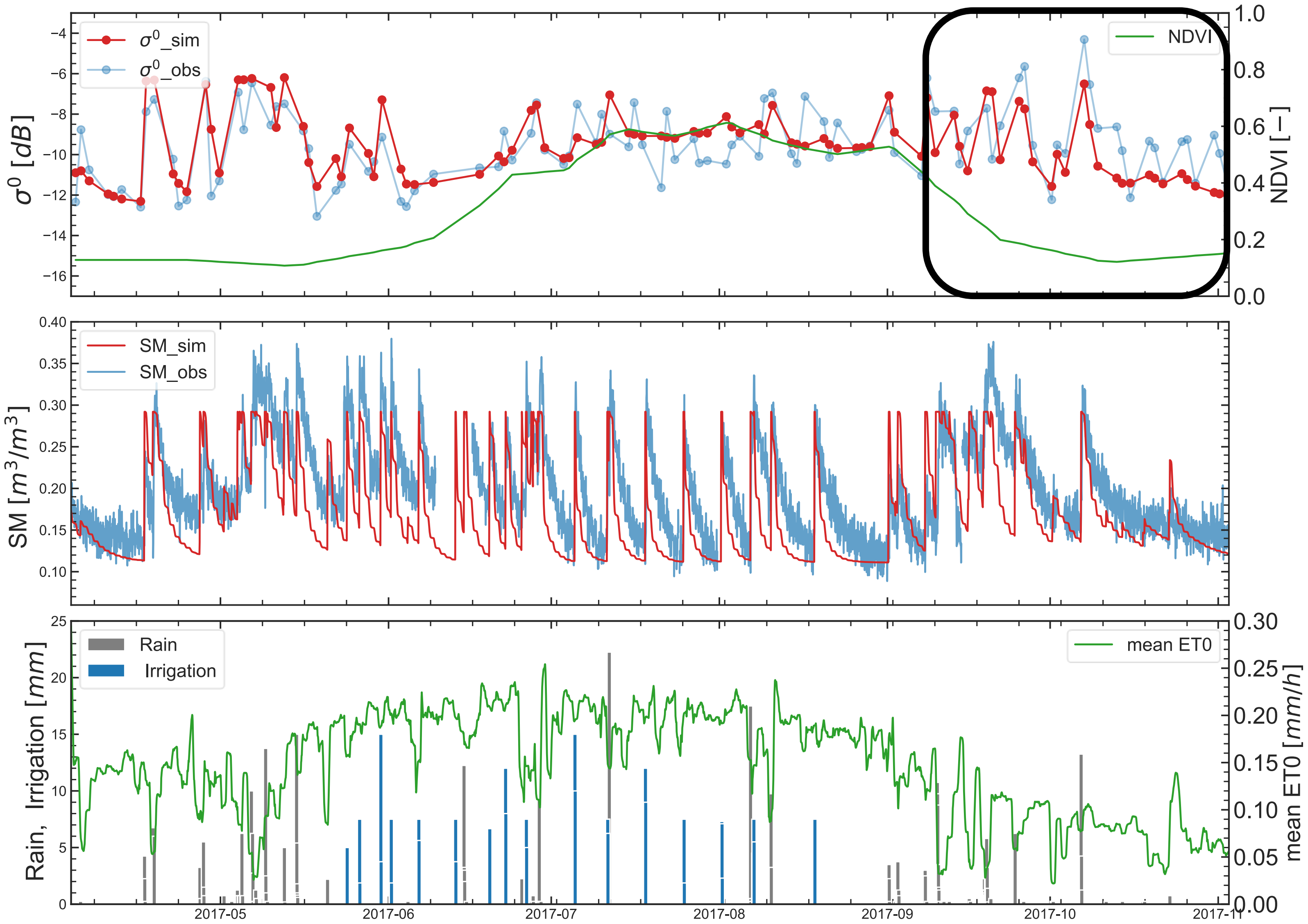


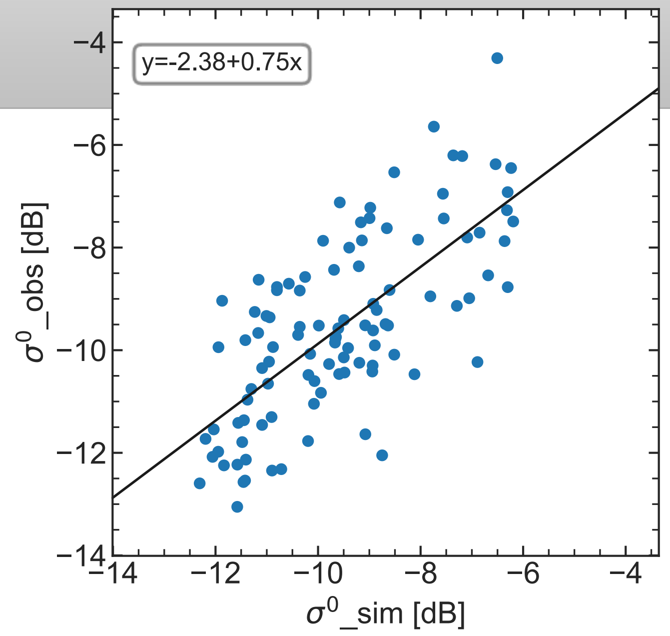


	KGE	R	R^2	bias
σ^0 [dB]	0.70	0.70	0.49	-0.01
SM [m^3m^{-3}]	0.68	0.72	0.52	-0.03

- Summer: dumped dynamics due to high NDVI
- high NDVI → σ_{veg}^0 dominates

FB T2017 – Results

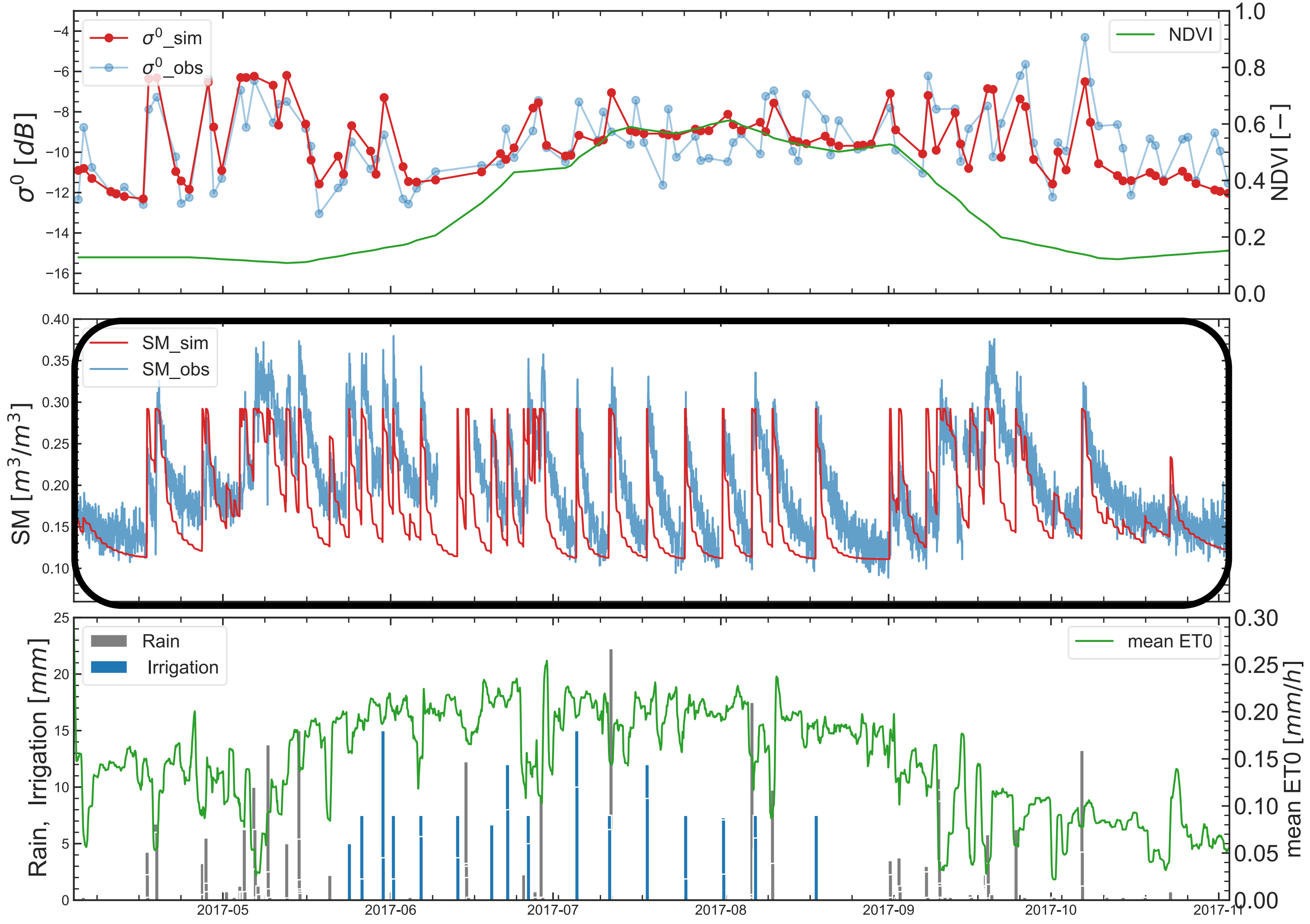


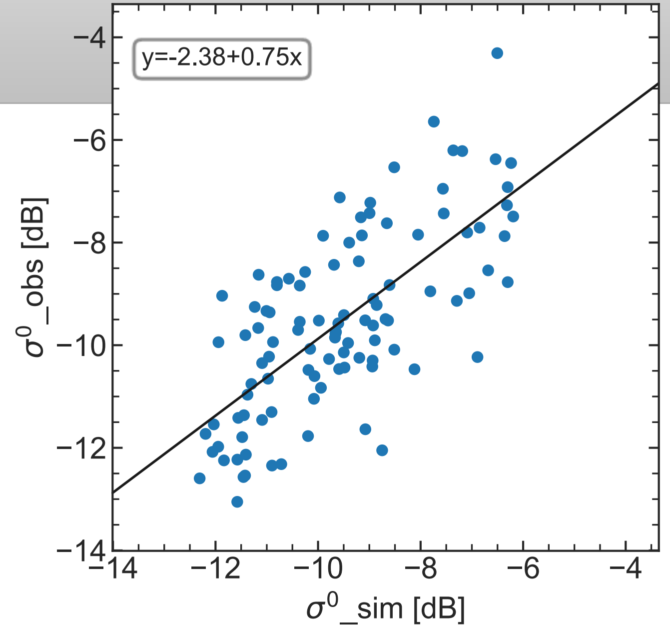


	KGE	R	R^2	bias
σ^0 [dB]	0.70	0.70	0.49	-0.01
SM [m^3m^{-3}]	0.68	0.72	0.52	-0.03

- Fall: some discrepancies probably due to changes in soil roughness
- small NDVI → σ^0_{soil} dominates

FB T2017 – Results





	KGE	R	R²	bias
σ^0 [dB]	0.70	0.70	0.49	-0.01
SM [m^3m^{-3}]	0.68	0.72	0.52	-0.03

- SM overall well modeled
- calculated SM dries out faster than observations due to thinner soil layer

FB T2017 – Discussion

	$A\ [-]$	$B\ [-]$	$C\ [dB]$	$D\ [dBm^3/m^3]$	$\delta_0\ [mm]$	$W_{fc}\ [m^3/m^3]$	$W_w\ [m^3/m^3]$	$\rho_{st}\ [mm/h]$	$K_{c0}\ [-]$
Cal/fix	cal	cal	cal	cal	cal	cal	cal	cal	cal
Bounds	[0.0, 5.0]	[0.0, 3.0]	[-30.0, -5.0]	[10.0, 100.0]	[1.0, 2.0]	[0.2, 0.4]	[0.07, 0.17]	[0.1, 0.8]	[0.0, 2.0]
Median	0.35	0.7	-16.0	36.1	1.004	0.29	0.11	0.21	0.25
Err	[0.01, 0.05]	[0.2, 0.1]	[1.8, 0.9]	[5.7, 10.6]	[0.003 0.018]	[0.03, 0.04]	[0.01, 0.02]	[0.06 0.07]	[0.03, 0.01]

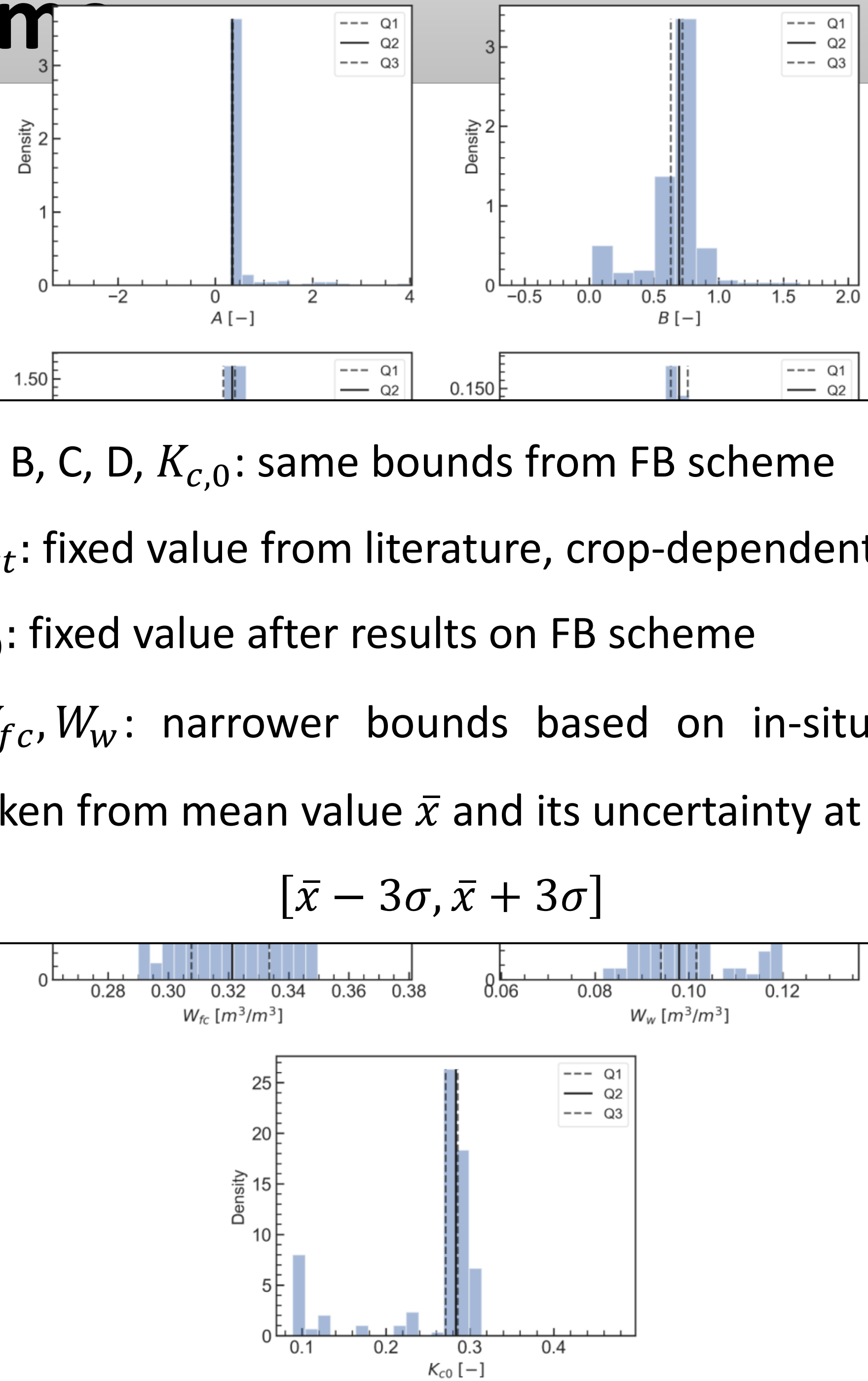
	KGE	**R**	**R^2**	**bias**
σ^0 [dB]	0.70	0.70	0.49	-0.01
SM [m^3m^{-3}]	0.68	0.72	0.52	-0.03

- Good results are obtained on KGE for both σ^0 and SM (KGE > 0.6) with very low bias
- Parameters have high uncertainties (up to ~ 30%)
- Parameter δ_0 will be fixed to $\delta_0 = 1$ (curve at 6 GHz in DoI plot)
- Parameter ρ_{st} has high uncertainty and saturates near its lower boundary: can be fixed to its reference value from the literature

Physical bounds (PB) calibration scheme

Physical bounds (PB) scheme employs bounds that are narrower than in the FB calibration and are based on in-situ data.

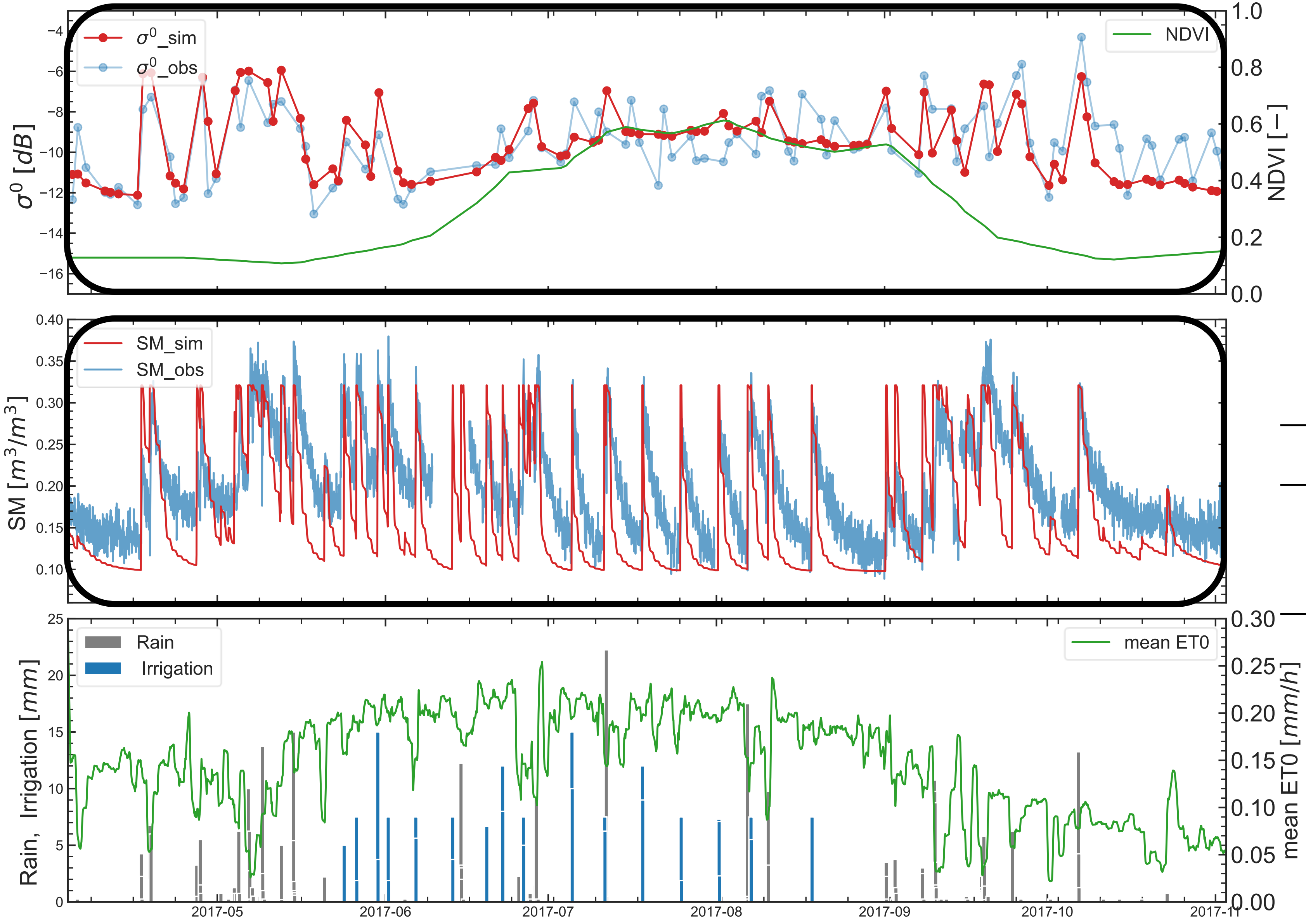
Quantity	Central value	Range
$\boldsymbol{A}\ [-]$		[0, 5]
$\boldsymbol{B}\ [-]$		[0, 3]
$\boldsymbol{C}\ [\boldsymbol{dB}]$		[-20, -5]
$\boldsymbol{D}\left[\boldsymbol{dB\ m^3 m^{-3}}\right]$		[10, 100]
$\boldsymbol{K_{c,0}}\ [-]$		[0, 2]
$\boldsymbol{\rho_{st}}\left[\boldsymbol{mm\ day^{-1}}\right]$	Crop-dependent	/
$\boldsymbol{\delta_0}\ [-]$	1	/
$\boldsymbol{W_{fc}}\left[\boldsymbol{m^3 m^{-3}}\right]$	0.32 ± 0.01	[0.29, 0.35]
$\boldsymbol{W_w}\left[\boldsymbol{m^3 m^{-3}}\right]$	0.09 ± 0.01	[0.06, 0.12]

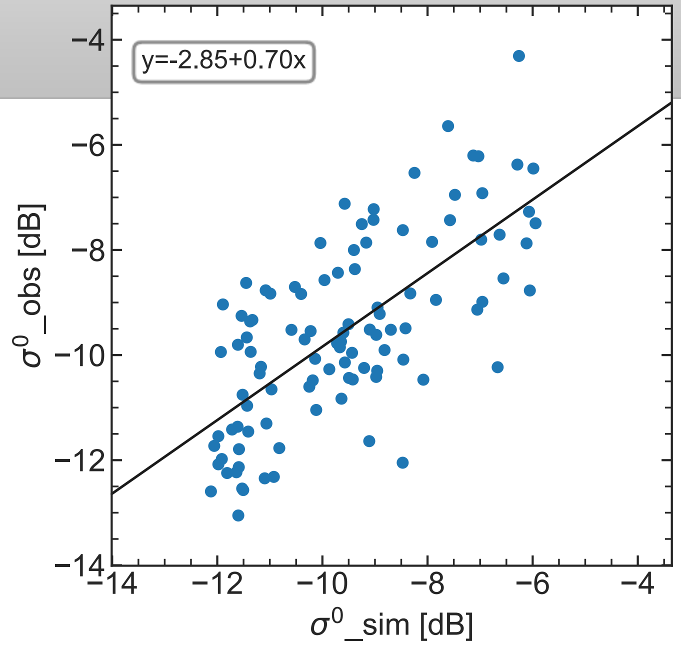


- A, B, C, D, $K_{c,0}$: same bounds from FB scheme
- ρ_{st}: fixed value from literature, crop-dependent
- δ_0: fixed value after results on FB scheme
- W_{fc}, W_w: narrower bounds based on in-situ data, taken from mean value $\bar{x}$ and its uncertainty at 1σ

$$[\bar{x} - 3\sigma, \bar{x} + 3\sigma]$$

PB T2017 – Results





	KGE	R	R^2	bias
σ^0 [dB]	0.69	0.69	0.48	-0.03
SM [m^3m^{-3}]	0.58	0.72	0.52	-0.04

- Same behavior as for FB
- Slight lower performance for KGE on SM due to higher bias and faster drying curves

PB T2017 – Discussion

	$A\ [-]$	$B\ [-]$	$C\ [dB]$	$D\ [dBm^3/m^3]$	$\delta_0\ [mm]$	$W_{fc}\ [m^3/m^3]$	$W_w\ [m^3/m^3]$	$\rho_{st}\ [mm/h]$	$K_{c0}\ [-]$
Cal/fix	cal	cal	cal	cal	fix	cal	cal	fix	cal
Bounds	[0.0, 5.0]	[0.0, 3.0]	[-30.0, -5.0]	[10.0, 100.0]	['/', '/']	[0.29, 0.35]	[0.06, 0.12]	['/', '/']	[0.0, 2.0]
Median	0.347	0.69	-14.5	29.2	1	0.32	0.098	0.4	0.28
Err	[0.004, 0.012]	[0.07, 0.03]	[0.2, 0.1]	[1.5, 1.6]	/	[0.01, 0.01]	[0.004, 0.004]	/	[0.01, 0.0]

M2020

	$A\ [-]$	$B\ [-]$	$C\ [dB]$	$D\ [dBm^3/m^3]$	$\delta_0\ [mm]$	$W_{fc}\ [m^3/m^3]$	$W_w\ [m^3/m^3]$	$\rho_{st}\ [mm/h]$	$K_{c0}\ [-]$
Cal/fix	cal	cal	cal	cal	fix	cal	cal	fix	cal
Bounds	[0.0, 5.0]	[0.0, 3.0]	[-30.0, -5.0]	[10.0, 100.0]	['/', '/']	[0.29, 0.35]	[0.06, 0.12]	['/', '/']	[0.0, 2.0]
Median	0.22	0.6	-17.4	33.0	1	0.31	0.10	0.5	0.9
Err	[0.04, 0.19]	[0.4, 0.7]	[0.5, 0.3]	[4.4, 3.6]	/	[0.01, 0.01]	[0.01, 0.01]	/	[0.3, 0.0]

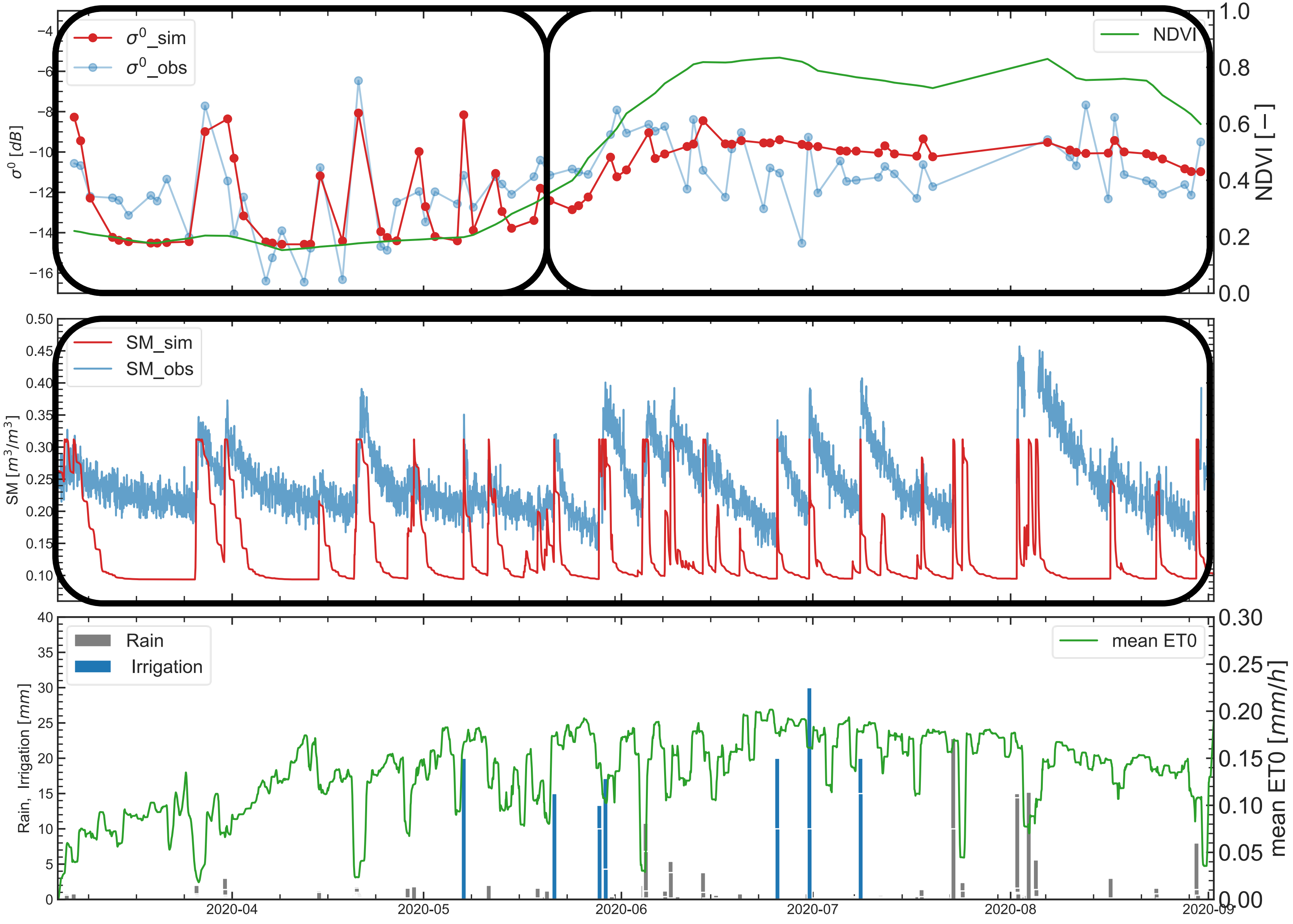
- Good results are obtained on KGE for both σ^0 and SM (KGE > 0.6) with very low bias
- Results are similar with those obtained by FB scheme, but **parameters have smaller % uncertainties**
- 5 (A, B, D, W_{fc}, W_w) out of 7 parameters are **compatible** (in the limits of uncertainties) in the two years
- A, B depend on the vegetation descriptor used (NDVI) and not on the crop type, nor field or period
- $C, K_{c,0}$ must be calibrated each year/period
  - C is the dry soil σ^0 and depends on soil roughness which changes after plowing practices for different crops
  - $K_{c,0}$ is the crop coefficient and depends crop type and frequency of wetting events

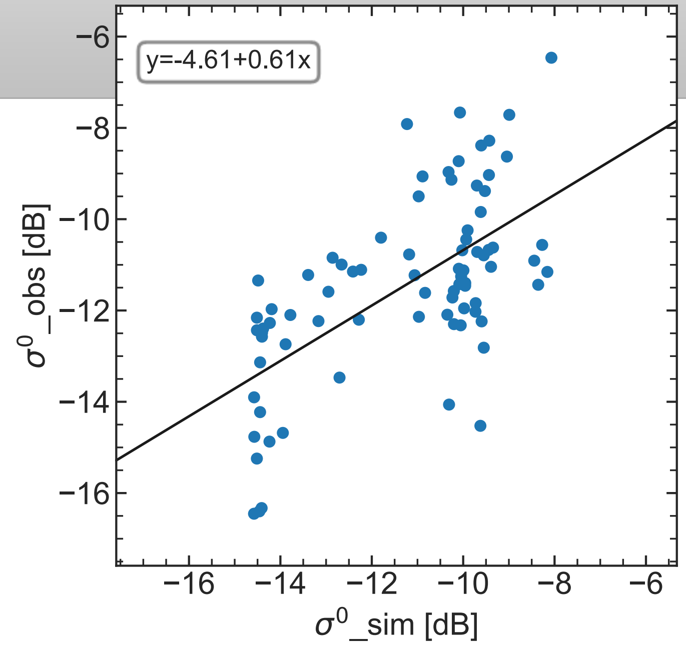
Conclusions

- For the **first time SWB has been used to provide SM inputs in the WCM**, leading to a new parametrization of the WCM that doesn't require in-situ SM measurements, but only simple weather data and soil texture
- **Depth of the soil layer under study is a function of SM** and scales as the depth of investigation of radar signals
- Best performance (KGE = 0.69) are obtained with the "**physical bounds**" calibration scheme on a tomato field in 2017
- Out of 9 parameters, **only 2 ($\boldsymbol{C}, \boldsymbol{K_{c,0}}$)** (empirical) need year-by-year calibration for the same field
- Parameters which are fixed for the same test site: A, B, D (empirical), δ_0 (empirical) depends on signal frequency and is fixed = 1, W_{fc}, W_w (physical observables) depend on site, are calibrated and are constrained by their in-situ measurement, ρ_{st} (physical observable) depends on crop and is fixed to its reference value from the literature
- Future perspectives…
  - use a vegetation index which is a better proxy for water content in the vegetation than the NDVI
  - implement time-dependent percolation contribution in SWB
  - test on grasslands, since WCM was developed to describe grass
  - study the impact of soil roughness

Thank you for your attention!

PB M2020 – Results

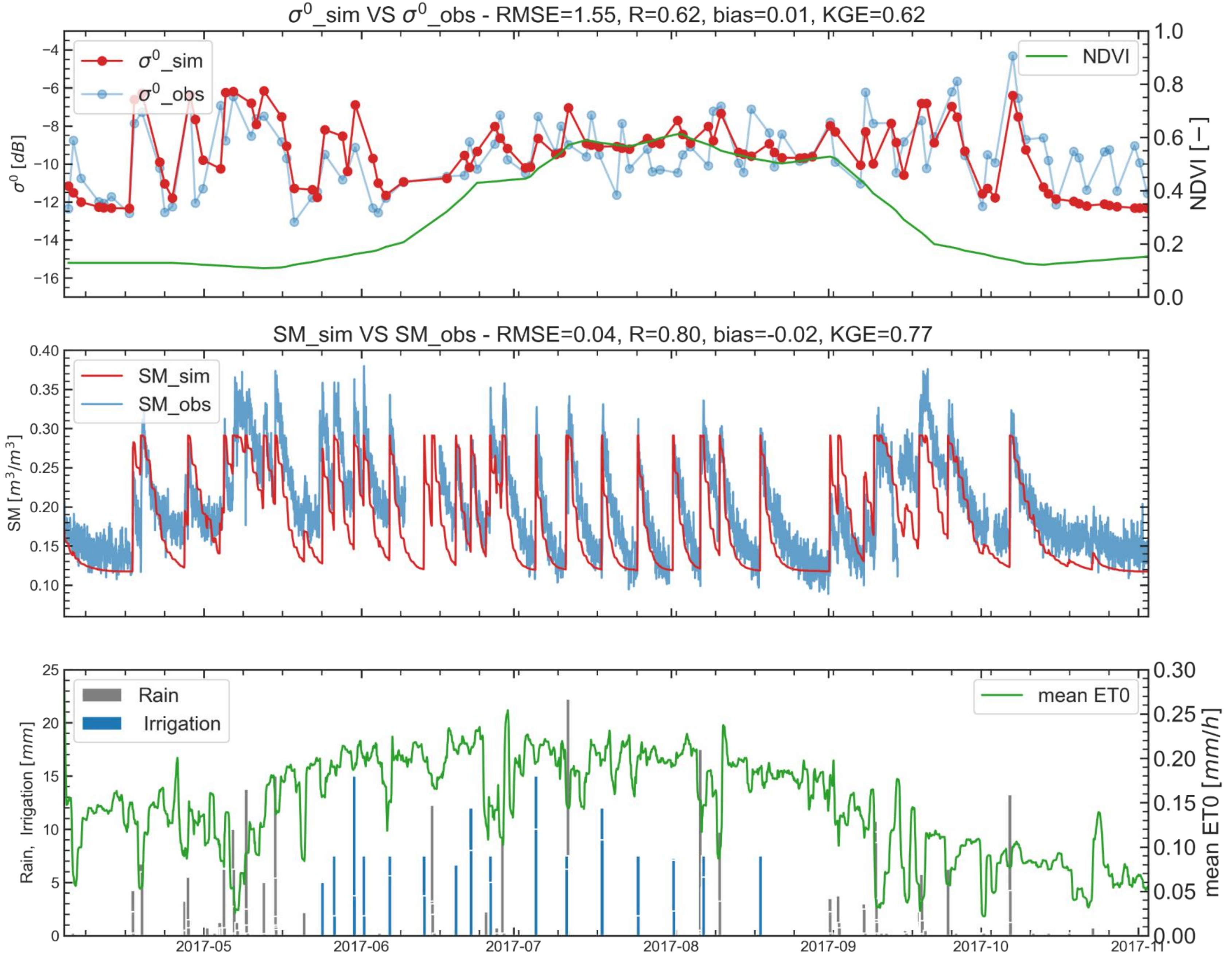




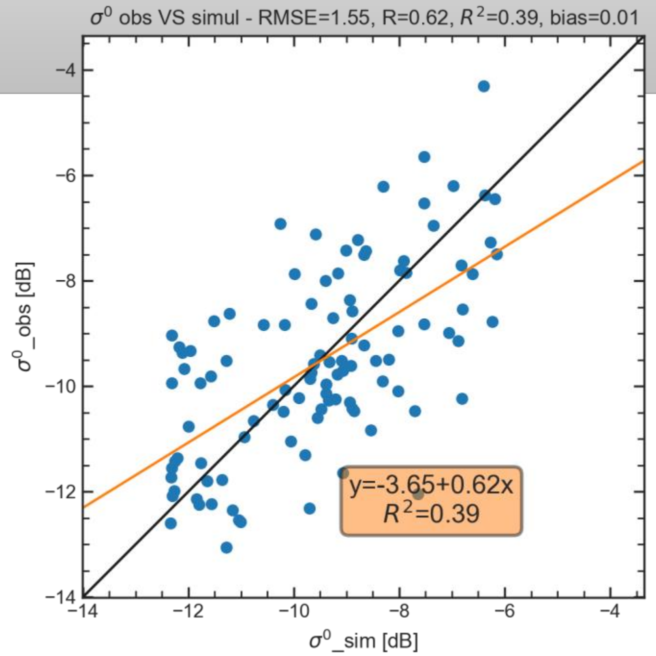
	KGE	R	R^2	bias
σ^0 [dB]	0.62	0.63	0.40	0.16
SM [m^3m^{-3}]	0.24	0.42	0.18	-0.11

- SM poorly modeled
- wilting point not compatible with observations
- calculated SM reduces too fast

PB T2017 @ 1.4 GHz – Results


σ^0_sim VS σ^0_obs - RMSE=1.55, R=0.62, bias=0.01, KGE=0.62

SM_sim VS SM_obs - RMSE=0.04, R=0.80, bias=-0.02, KGE=0.77

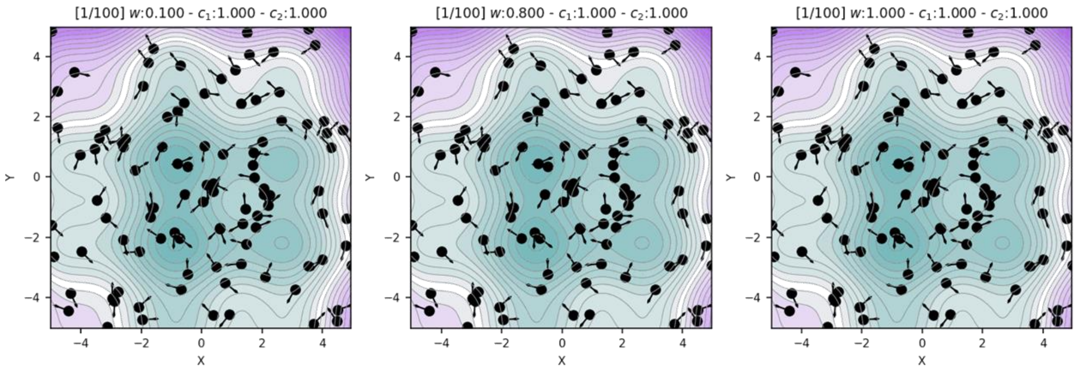

σ^0 obs VS simul - RMSE=1.55, R=0.62, R^2=0.39, bias=0.01

- Performance on σ^0 is good
- Performance on SM is much better and drying curves are much similar (due to similar soil layer investigated? ← this is my guess and hypothesis (SEE DOI PLOT slide 7)

Optimizer (PSO) and cost function (KGE)

OPTIMIZER: Particle Swarm Optimization (PSO)

- Social optimization algorithm based on a swarm of interacting particles.
- Implementation by library **`pyswarms`**, using a routine depending on hyperparameters w (inertia), c_1 (cognitive parameter), c_2 (social parameter)



Hyperparameter	w	c_1	c_2
Initial value	0.6	2.05	2.05
Variational strategy	exponential decay	linear variation	linear variation

COST FUNCTION: Kling-Gupta Efficiency (KGE)

$$KGE = 1 - \sqrt{(R-1)^2 + (\varepsilon - 1)^2 + (\beta - 1)^2}$$

- $\boldsymbol{R}$ **= Pearson's correlation coefficient** of simulated (x_{sim}) and observed (x_{obs}) values
- $\boldsymbol{\varepsilon}$ **= relative variability**
- $\boldsymbol{\beta}$ **= bias** = difference between the means normalized by the standard deviation of the observed data,

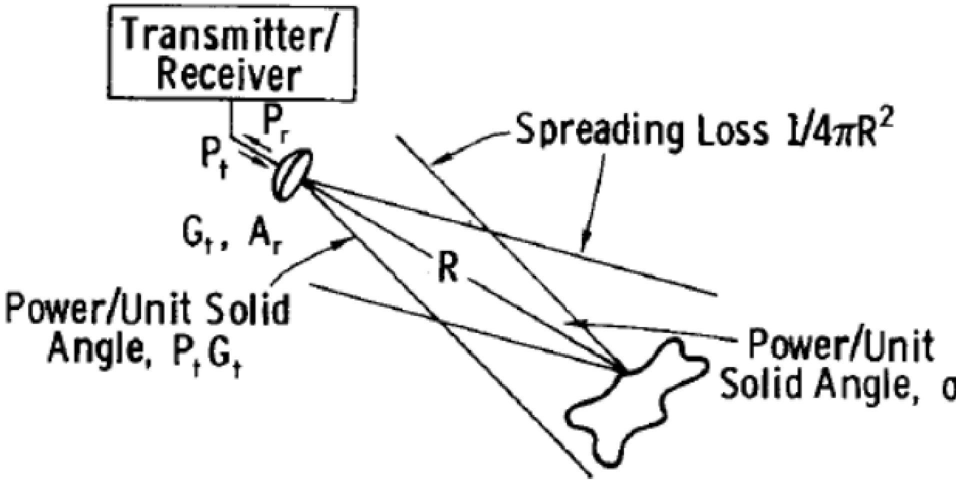
$$R = \frac{Cov(x_{sim}, x_{obs})}{\sigma_{obs}\sigma_{sim}}$$

$$\varepsilon = \sigma_{sim}/\sigma_{obs}$$

$$\beta = (\overline{x_{sim}} - \overline{x_{obs}})/\sigma_{obs}$$

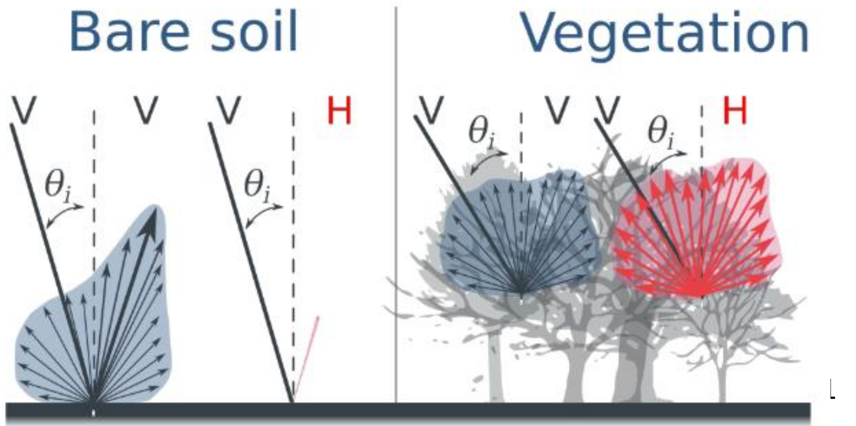
Backscattering (σ^0) and scattering mechanisms

- Radar backscattering coefficient $\boldsymbol{\sigma^0}$ [dB] is the radar cross section $\sigma\ [\mathrm{m}^2]$ (RCS) normalized onto the ground range surface of incidence and can is calculated by the radar equation or as a function of the scattered L_s and incident L_i radiance ($L\ [erg\ s^{-1}cm^{-2}sr^{-1}]$)
- The scattered radiance is given by the **sum** of the contribution of surface scattering from bare soil, volume scattering from vegetation and interactions between the two, $\sigma^0 = \sigma^0_{soil} + \sigma^0_{veg} + \sigma^0_{inter}$
- σ^0 from bare soil is affected by both soil moisture (through soil reflectivity), texture and roughness; σ^0 from vegetation is affected by the vegetation water content and vegetation structure; σ^0 contributions change for different polarizations (VV or VH)



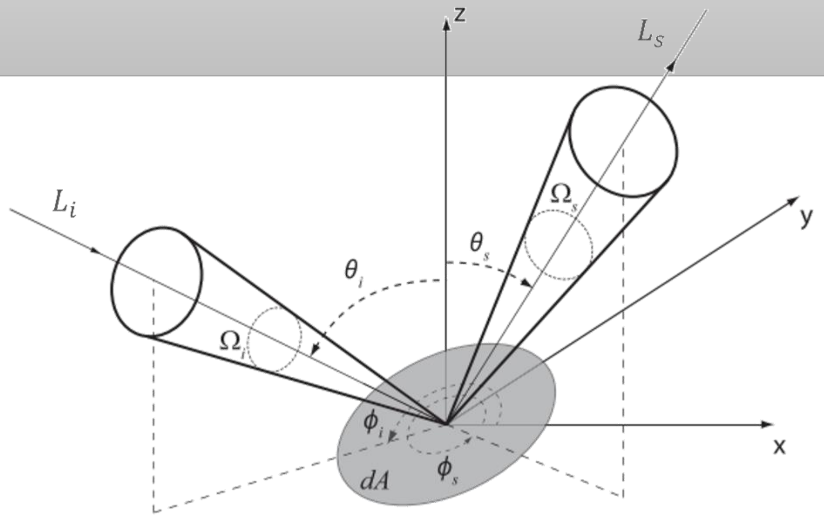
$$P_r = \frac{P_t G_t A_r}{(4\pi)^2 R^4}\ (\sigma^0 A^*)$$

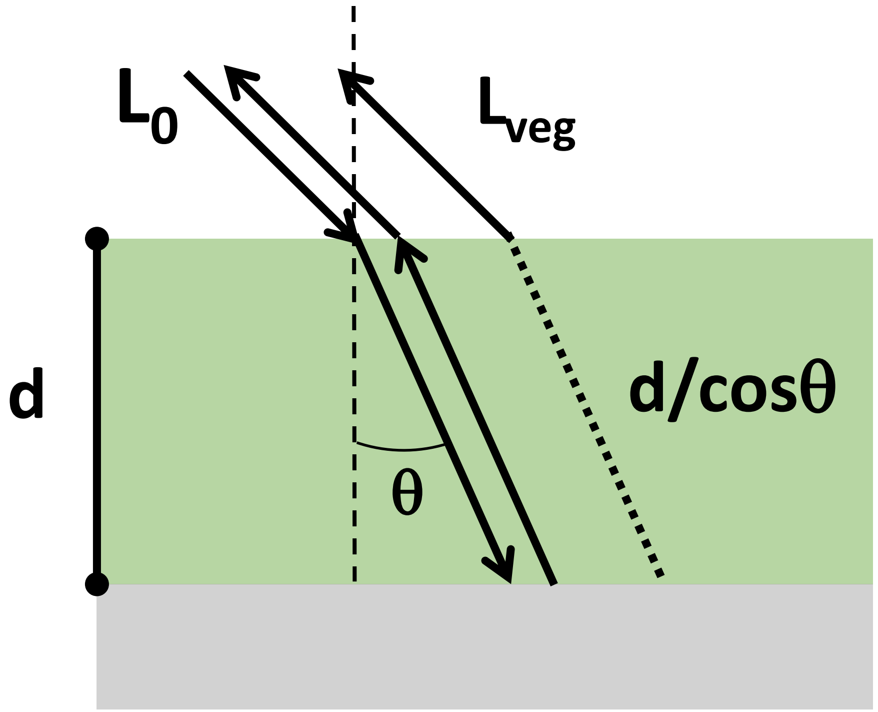
$$\sigma^0 \propto \frac{L_s}{L_i}$$



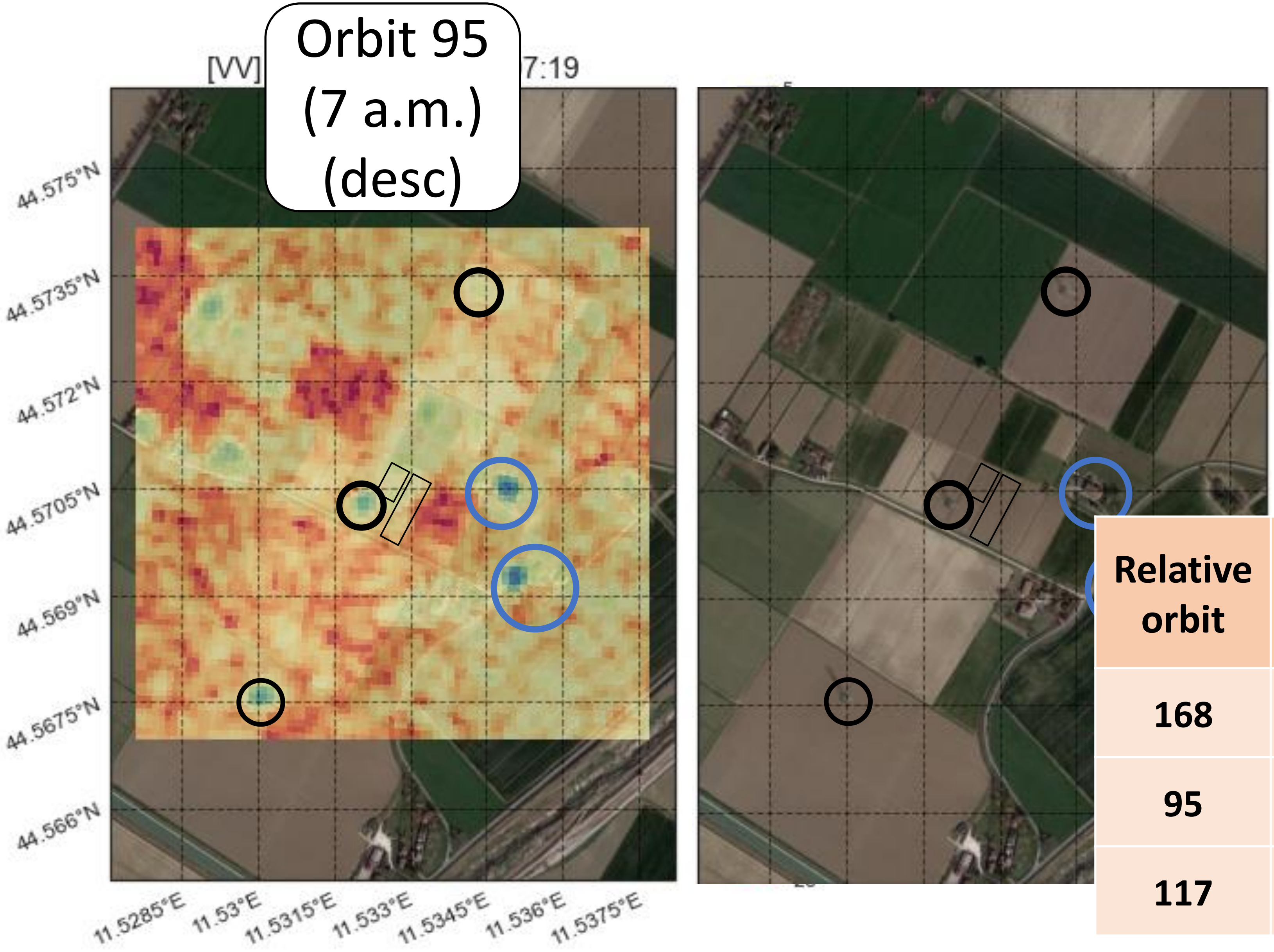
σ^0 from vegetation

- σ^0 can be calculated from radiance derived from radiated transfer theory: $\sigma^0(\theta_i, \phi_i) = 4\pi \frac{L_s(\theta_i,\phi_i+\pi)}{L_i(\pi-\theta_i,\phi_i)} cos\,\theta$
- $L_{veg}(0) = \int_o^{d/\cos\theta} \kappa_s L_0 e^{-2\kappa_e z} dz = \frac{\kappa_s}{2\kappa_e} L_0 \cos\theta\,(1-\gamma^2) = \frac{\omega}{2} L_0 (1-\gamma^2)\cos\theta$
- where $\gamma^2 = e^{-\frac{2\tau}{\cos\theta}}$ and τ is vegetation optical depth $\tau = \kappa_e d$
- $P(\theta, \phi, \theta', \phi') = P_{Ray}(\Theta) = \frac{3}{4}(1+\cos^2\Theta) \xrightarrow{\Theta=\pi} \frac{3}{2}$
- $\sigma^0_{veg} \propto \frac{L_s}{L_0} = \frac{L_{veg}}{L_0} = \frac{3\omega}{4}\cos\theta\,(1-\gamma^2)$





Maps of region 800 m x 800 m: who sees the pylon?

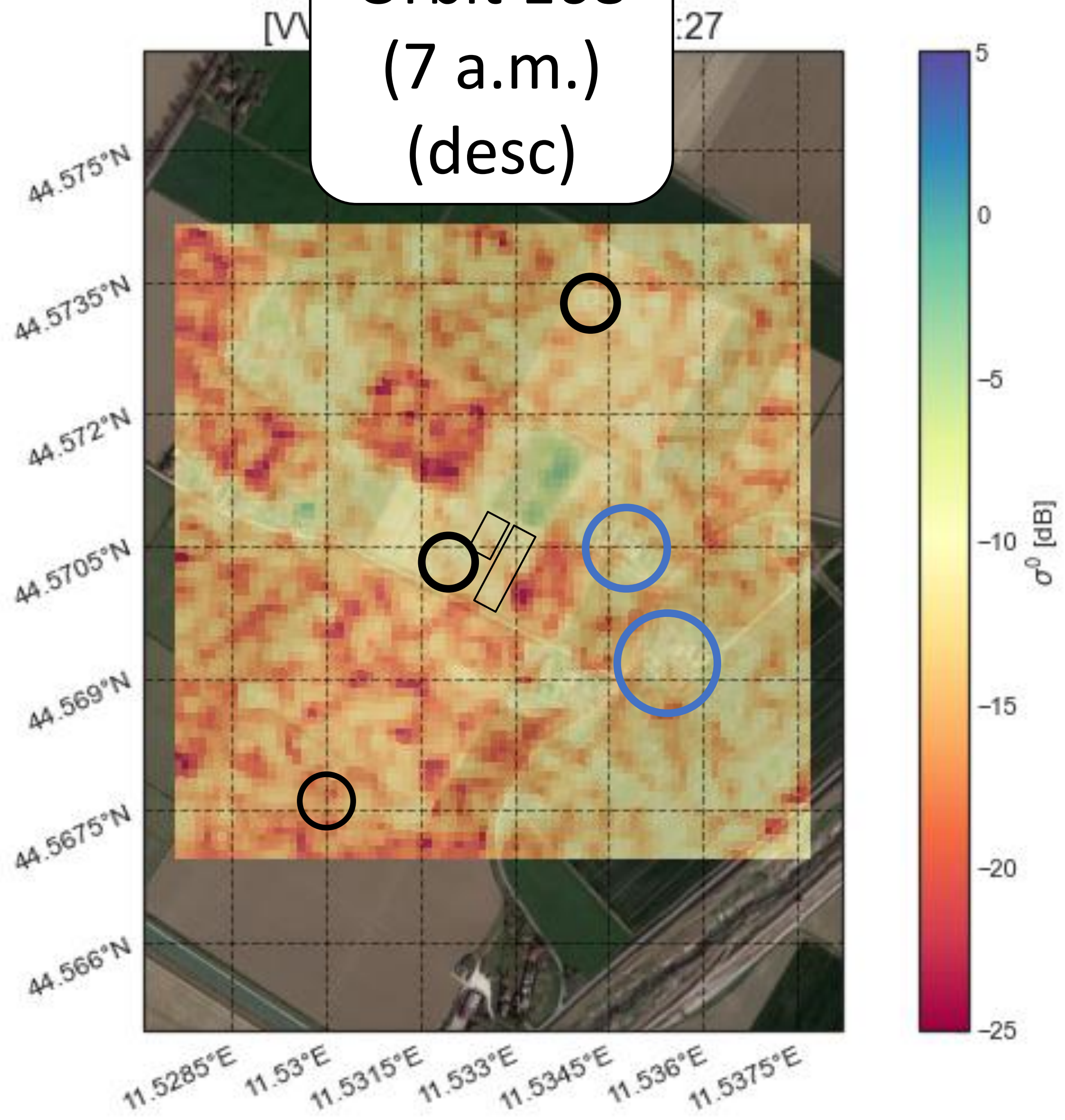


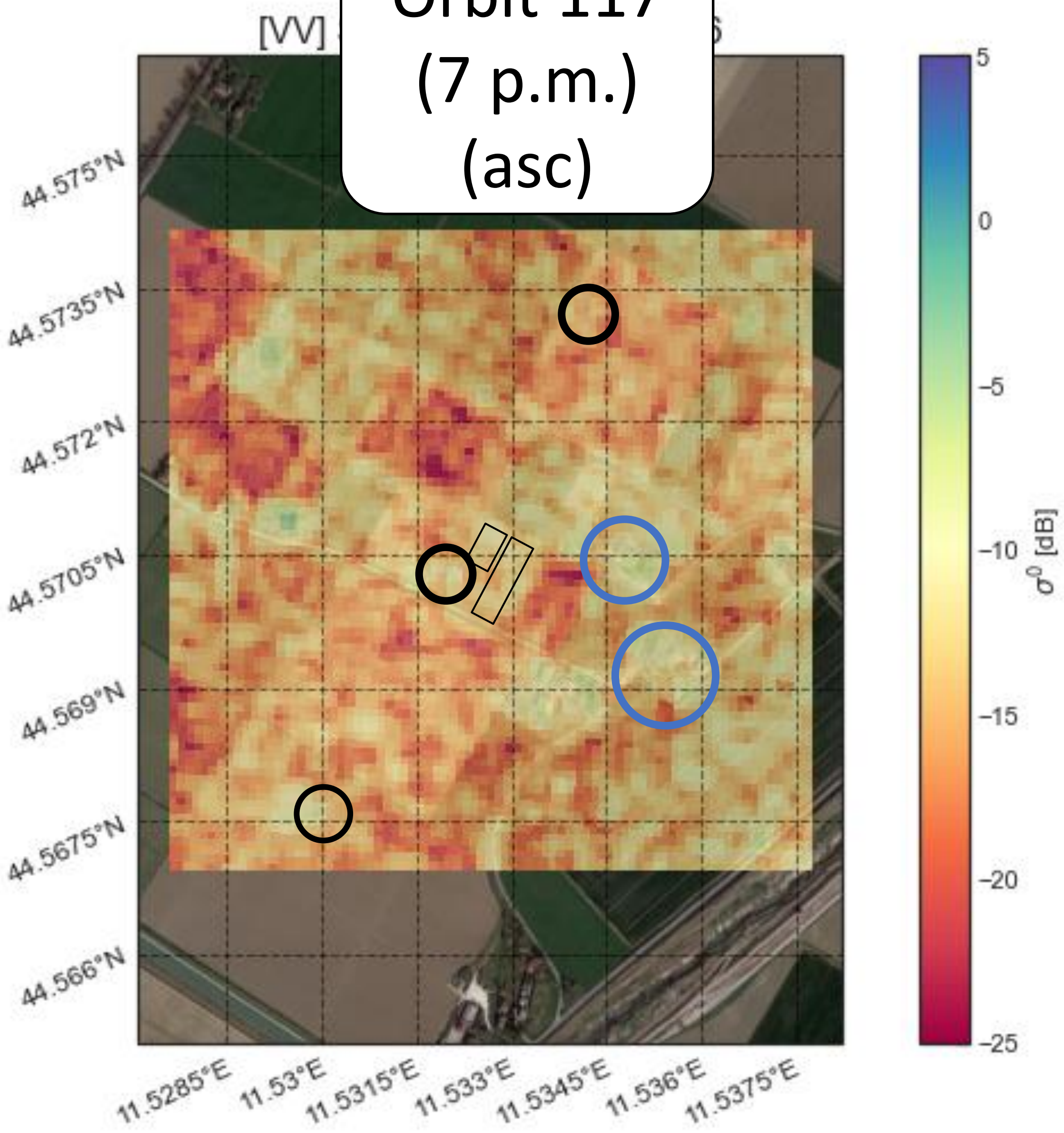
Orbit 95
(7 a.m.)
(desc)

- **Electricity pylons,** **houses and buildings** are seen in images of orbit 95.
- Pylons and buildings are not clearly seen in images of other orbits.

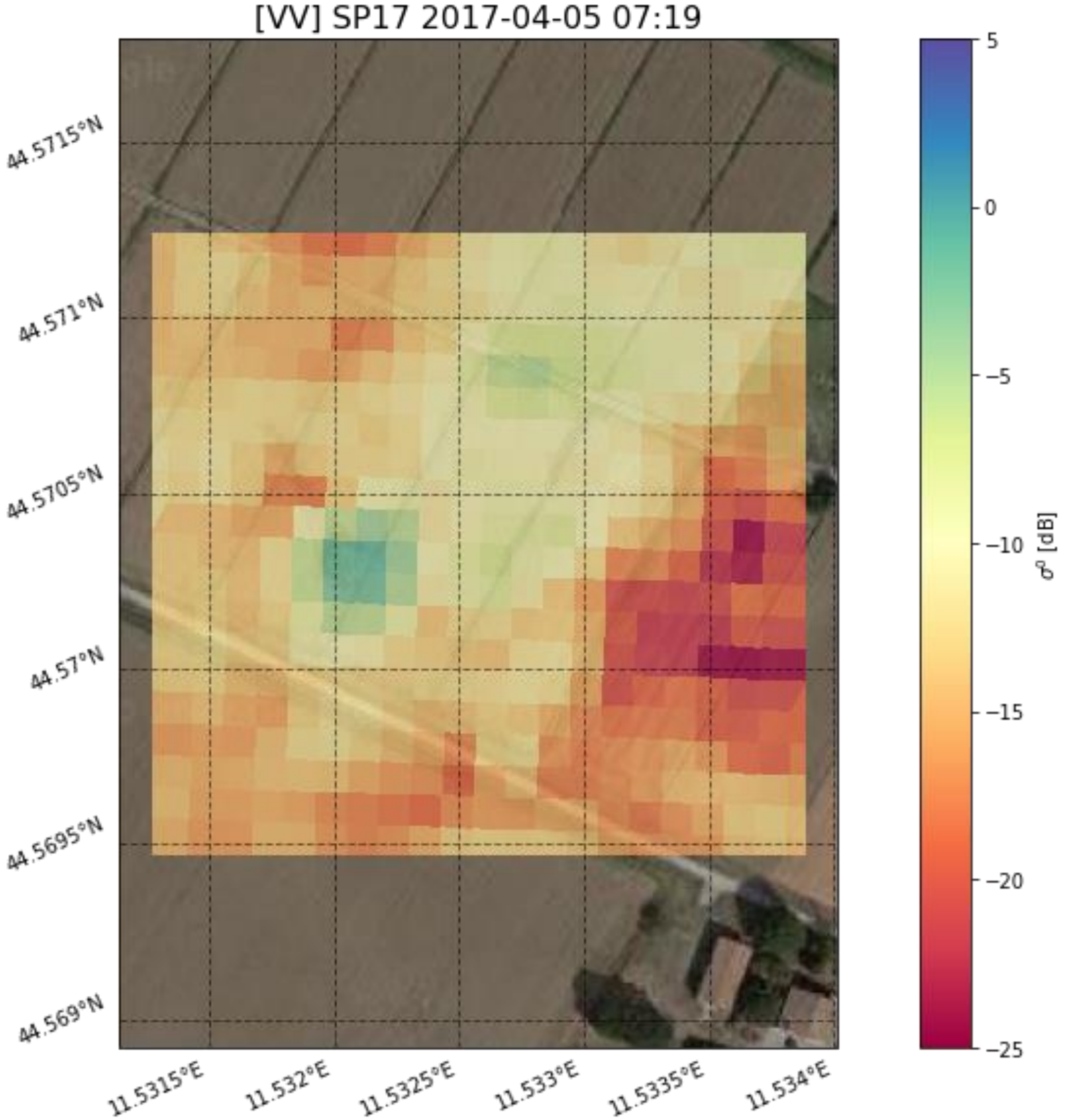
Relative orbit	Acquisition time	Direction	Mean incidence angle [°]
168	7 a.m.	descending	31.6
95	7 a.m.	descending	41.6
117	7 p.m.	ascending	37.6

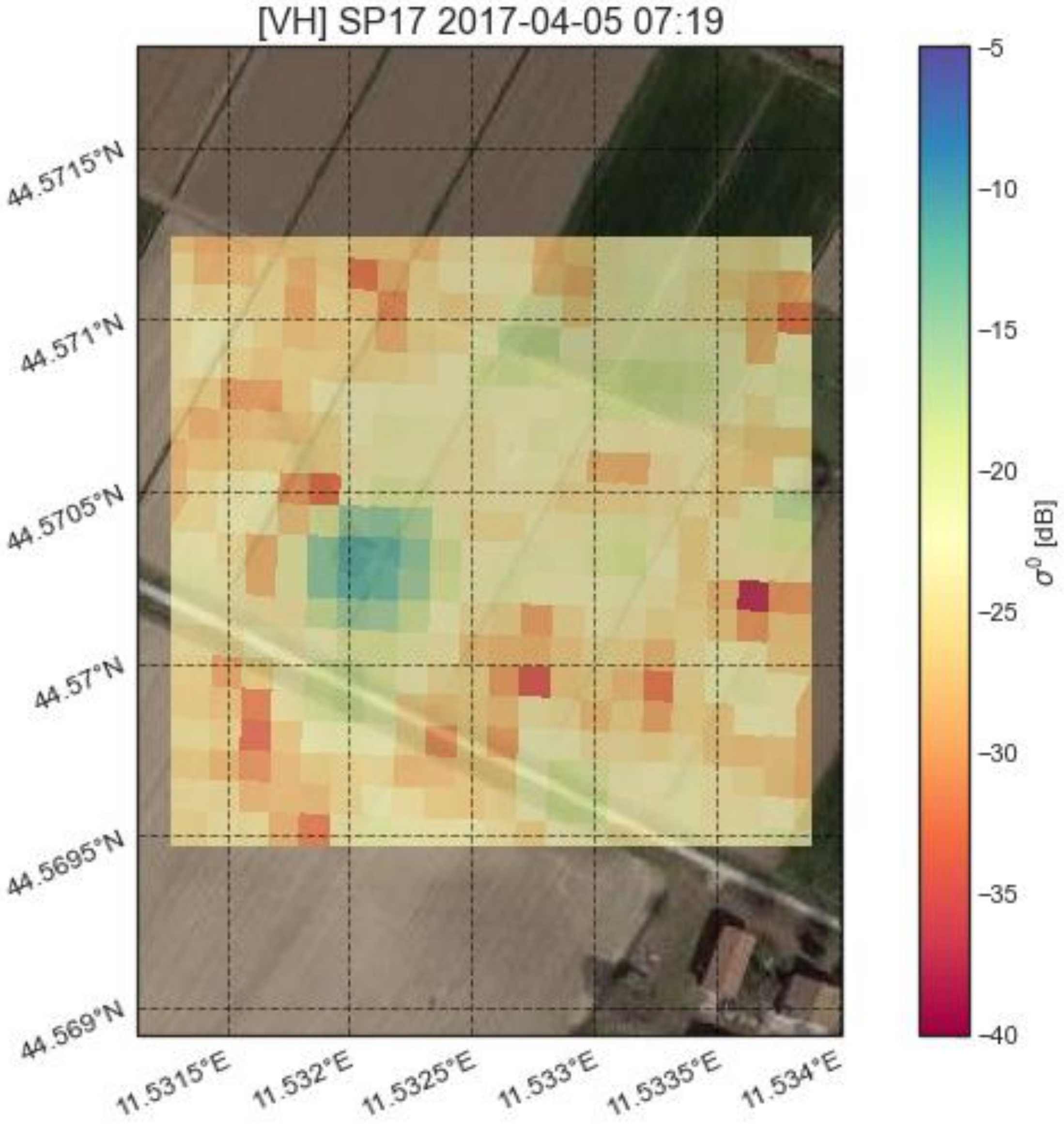
Maps of region 800 m x 800 m: who doesn't see the pylon?


Orbit 168
(7 a.m.)
(desc)


Orbit 117
(7 p.m.)
(asc)

Electricity pylon in both VV, VH polarization, orbit 95





SWB overview

INPUTS

PARAMETERS

$$S_i = S_{i-1} + P_i + I_i - \frac{ET_{0,i}*K_{c,i}*(S_{i-1}-W_w*W_{max})}{1-\rho_{st}-0.04\left(5-ET_{0,i}*K_{c,i}\right)}$$

$I_i = W_{fc} * W_{max} - S_{i-1}$ if $S_{i-1} < \left(1 - \rho_{st} + 0.04(5 - ET_{0,i} * K_{c,i})\right) * W_{fc} * W_{max}$

$PS_i = S_i - W_{fc} * W_{max}$ if $S_i > W_{fc} * W_{max}$

- S_i = soil moisture [mm]
- P_i = precipitation [mm]
- I_i = irrigation [mm]
- $ET_{0,i}$ = potential evapotranspiration [-]

- $K_{c,i}$ = crop coefficient [-]
- W_w = wilting point [m^{3}m^{-3}]
- W_{fc} = field capacity [m^{3}m^{-3}]
- W_{max} = soil layer depth [mm]
- ρ_{st} = standard potential depletion fraction [-]