Simona Donzelli

University of Milan
PhD School XXI cicle

2D IDAPP meeting
Ferrara, may 3-4 2007
Outline

1. **The Cosmic Microwave Background (CMB)**
   - Main features
   - The angular power spectrum

2. **Bias of the power spectrum estimation**
   - Instrumental effects
   - Incomplete sky coverage
The Cosmic Microwave Background (CMB)

Main features
- The angular power spectrum

Figure: The timeline of the Universe
The Cosmic Microwave Background (CMB)

Main features

- Photons coming from the “last scattering surface” ($z_R \sim 1000$, i.e. $t \sim 400,000$ yrs after the Big Bang)
- Black body spectrum at $T = 2.725 \pm 0.001$ K
- Homogeneity and isotropy
- Anisotropy of the order $\Delta T / T = 10^{-5}$
- Linearly polarized at a 10% level
Statistics of the CMB sky

spherical harmonic expansion

\[ T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n}) \]

\[ Q(\hat{n}) \pm iU(\hat{n}) = \sum_{\ell m} a_{\pm 2, \ell m} \mp 2 Y_{\ell m}(\hat{n}) \]

where \( a_{\pm 2, \ell m} = E_{\ell m} \pm iB_{\ell m} \), \((E, B)\) grad and curl component

angular power spectrum

\[ \langle a_{\ell m}^Y \rangle = 0, \quad \langle a_{\ell m}^Y a_{\ell' m'}^* \rangle = \delta_{\ell \ell'} \delta_{mm'} \langle C_{\ell}^{YY'} \rangle, \text{ where } Y, Y' \text{ can be } T, E \text{ or } B \]

if the anisotropies obey Gaussian statistics then the spectra \( C_{\ell}^{TT}, C_{\ell}^{EE}, C_{\ell}^{BB}, C_{\ell}^{TE} \)

describe all the statistical properties, where

\[ C_{\ell}^{YY'} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} (a_{\ell m}^Y a_{\ell' m'}^{Y'}) + c.c. \]
The Cosmic Microwave Background (CMB)
Bias of the power spectrum estimation

Statistics of the CMB sky

**spherical harmonic expansion**

- **temperature** $T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$
- **polarization** $Q(\hat{n}) \pm iU(\hat{n}) = \sum_{\ell m} a_{\mp 2, \ell m} \mp 2 Y_{\ell m}(\hat{n})$

where $a_{\pm 2, \ell m} = E_{\ell m} \pm iB_{\ell m}$, $(E, B$ grad and curl component$)$

**angular power spectrum**

$\langle a_{\ell m}^Y \rangle = 0$, $\langle a_{\ell m}^Y a_{\ell m'}^{Y'} \rangle = \delta_{\ell \ell'} \delta_{mm'} \langle C_{\ell}^{YY'} \rangle$, where $Y, Y'$ can be $T, E$ or $B$

if the anisotropies obey Gaussian statistics then the spectra $C_{\ell}^{TT}, C_{\ell}^{EE}, C_{\ell}^{BB}, C_{\ell}^{TE}$ describe all the statistical properties, where $C_{\ell}^{YY'} = \frac{1}{2\ell + 1} \sum_{m=\ell}^{\ell} (a_{\ell m}^Y a_{\ell m'}^{Y'} + c.c)$
what can you learn from the spectrum?

constrains on CMB power spectrum \( \Rightarrow \) constrains on cosmological parameters

Figure:

- Dependence of CMB power spectrum on some cosmological parameters

(a) Curvature

(b) Dark Energy

(c) Baryons

(d) Matter
The Cosmic Microwave Background (CMB)

Bias of the power spectrum estimation

Main features
The angular power spectrum

Power Spectrum & High Precision Cosmology

state of the art: temperature

Figure: The WMAP 3yr power spectrum
The Cosmic Microwave Background (CMB) Bias of the power spectrum estimation

Power Spectrum & High Precision Cosmology

Main features
The angular power spectrum

state of the art: temperature & polarization

Figure: Plots of signal for TT (black), TE (red), EE (green) for the best fit model. The dashed line for TE indicates areas of anticorrelation.
Bias of the power spectrum estimation in a real experiment

**the pseudo power spectrum** $\tilde{C}_\ell$

the direct spherical harmonics transform $\tilde{C}_\ell$ of an anisotropies map is biased by:
Bias of the power spectrum estimation in a real experiment

The pseudo power spectrum $\tilde{C}_\ell$

The direct spherical harmonics transform $\tilde{C}_\ell$ of an anisotropies map is biased by:

- **instrumental effects**
  - beam function of the antenna
  - observation and data analysis processing
  - instrumental noise

- **incomplete sky coverage**
  - mode-mode coupling of the spherical harmonics
  - $E \rightarrow B$ polarizations mode mixing
Bias of the power spectrum estimation in a real experiment

The pseudo power spectrum $\tilde{C}_\ell$

The direct spherical harmonics transform $\tilde{C}_\ell$ of an anisotropies map is biased by:

- instrumental effects
  - beam function of the antenna
  - observation and data analysis processing
  - instrumental noise

- incomplete sky coverage
  - mode-mode coupling of the spherical harmonics
  - $E - B$ polarizations mode mixing
Bias of the power spectrum estimation in a real experiment

The pseudo power spectrum $\tilde{C}_\ell$

The direct spherical harmonics transform $\tilde{C}_\ell$ of an anisotropies map is biased by:

- instrumental effects
  - beam function of the antenna
  - observation and data analysis processing
  - instrumental noise

- incomplete sky coverage
  - mode-mode coupling of the spherical harmonics
  - $E - B$ polarizations mode mixing
Bias of the power spectrum estimation in a real experiment

The pseudo power spectrum $\tilde{C}_\ell$

The direct spherical harmonics transform $\tilde{C}_\ell$ of an anisotropies map is biased by:

- instrumental effects
  - beam function of the antenna
  - observation and data analysis processing
  - instrumental noise
- incomplete sky coverage
  - mode-mode coupling of the spherical harmonics
  - $E - B$ polarizations mode mixing
Bias of the power spectrum estimation in a real experiment

The pseudo power spectrum $\tilde{C}_\ell$

The direct spherical harmonics transform $\tilde{C}_\ell$ of an anisotropies map is biased by:

- instrumental effects
  - beam function of the antenna
  - observation and data analysis processing
  - instrumental noise
- incomplete sky coverage
  - mode-mode coupling of the spherical harmonics
  - $E - B$ polarizations mode mixing
Debiasing from the instrumental effects

the model

In a full-sky hypothesis:

\[
\langle \tilde{C}_\ell \rangle = F_\ell B_\ell^2 \langle C_\ell \rangle + \langle \tilde{N}_\ell \rangle
\]

contributions

- window function \( B_\ell^2 \): beam and pixelization smoothing effects
- transfer function \( F_\ell \): effects of the data analysis (filtering, scanning strategy, etc.)
- instrumental noise \( \langle N_\ell \rangle \): average noise power spectrum
Debiasing from the instrumental effects

the model

In a full-sky hypothesis:

$$\langle \tilde{C}_\ell \rangle = F_\ell B^2_\ell \langle C_\ell \rangle + \langle \tilde{N}_\ell \rangle$$

contributions

- window function $B^2_\ell$: beam and pixelization smoothing effects
- transfer function $F_\ell$: effects of the data analysis (filtering, scanning strategy, etc.)
- instrumental noise $\langle N_\ell \rangle$: average noise power spectrum
Debiasing from the instrumental effects

The model

\[ \langle \tilde{C}_\ell \rangle = F_\ell B_\ell^2 \langle C_\ell \rangle + \langle \tilde{N}_\ell \rangle \quad \text{(full-sky)} \]

estimation of the contributions

- window function \( B_\ell^2 \): study of the antenna beam pattern
calibrations with Monte Carlo (MC) simulations:
  - transfer function \( F_\ell = \langle \tilde{C}_\ell \rangle \langle C_\ell^{th} \rangle^{-1} (B_\ell^2)^{-1} \), where \( C_\ell^{th} \) is an arbitrary theoretical p.s. and \( \langle \tilde{C}_\ell \rangle \) is obtained from “observations” of \( N^{MC} \) full-sky and noise-free theoretical sky realizations.
  - instrumental noise \( \langle \tilde{N}_\ell \rangle_{MC} \): \( N^{MC} \) simulations of the noise contained in the data

⇒ an accurate knowledge on the noise characteristics is required!
Debiasing from the instrumental effects

\[ \langle \tilde{C}_\ell \rangle = F_\ell B^2_\ell \langle C_\ell \rangle + \langle \tilde{N}_\ell \rangle \quad \text{(full-sky)} \]

- **the model**
  - window function \( B^2_\ell \): study of the antenna beam pattern
  - transfer function \( F_\ell = \langle \tilde{C}_\ell \rangle \langle C^{th}_\ell \rangle^{-1} (B^2_\ell)^{-1} \), where \( C^{th}_\ell \) is an arbitrary theoretical p.s. and \( \langle \tilde{C}_\ell \rangle \) is obtained from “observations” of \( N^{MC} \) full-sky and noise-free theoretical sky realizations.
  - instrumental noise \( \langle \tilde{N}_\ell \rangle_{MC} \): \( N^{MC} \) simulations of the noise contained in the data

  \[ \Rightarrow \text{an accurate knowledge on the noise characteristics is required!} \]
Debiasing from the instrumental effects

the model

\[ \langle \tilde{C}_\ell \rangle = F_\ell B^2_\ell \langle C_\ell \rangle + \langle \tilde{N}_\ell \rangle \quad \text{(full-sky)} \]

estimation of the contributions

- window function \( B^2_\ell \): study of the antenna beam pattern
- calibrations with Monte Carlo (MC) simulations:
  - transfer function \( F_\ell = \langle \tilde{C}_\ell s \rangle \langle C^{th}_\ell \rangle^{-1} (B^2_\ell)^{-1} \), where \( C^{th}_\ell \) is an arbitrary theoretical p.s. and \( \langle \tilde{C}_\ell s \rangle \) is obtained from “observations” of \( N^{MC} \) full-sky and noise-free theoretical sky realizations.
  - instrumental noise \( \langle \tilde{N}_\ell \rangle_{MC} \): \( N^{MC} \) simulations of the noise contained in the data

⇒ an accurate knowledge on the noise characteristics is required!
Debiasing from the instrumental effects

The Cosmic Microwave Background (CMB)
Bias of the power spectrum estimation

Instrumental effects
Incomplete sky coverage

Debiasing from the instrumental effects

the model

\[ \langle \tilde{C}_\ell \rangle = F_\ell B^2_\ell \langle C_\ell \rangle + \langle \tilde{N}_\ell \rangle \quad \text{(full-sky)} \]

estimation of the contributions

- window function \( B^2_\ell \): study of the antenna beam pattern
calibrations with Monte Carlo (MC) simulations:
  - transfer function \( F_\ell = \langle \tilde{C}_\ell s \rangle \langle C^{th}_\ell \rangle^{-1} (B^2_\ell)^{-1} \), where \( C^{th}_\ell \) is an arbitrary theoretical p.s. and \( \langle \tilde{C}_\ell s \rangle \) is obtained from “observations” of \( N^{MC} \) full-sky and noise-free theoretical sky realizations.

- instrumental noise \( \langle \tilde{N}_\ell \rangle_{MC} \): \( N^{MC} \) simulations of the noise contained in the data

\[ \Rightarrow \text{an accurate knowledge on the noise characteristics is required!} \]
Debiasing from the instrumental effects

The model

\[ \langle \tilde{C}_\ell \rangle = F_\ell B^2_\ell \langle C_\ell \rangle + \langle \tilde{N}_\ell \rangle \quad \text{(full-sky)} \]

estimation of the contributions

- window function \( B^2_\ell \): study of the antenna beam pattern
- calibrations with Monte Carlo (MC) simulations:
  - transfer function \( F_\ell = \langle \tilde{C}_{\ell s} \rangle \langle C^{th}_\ell \rangle^{-1} (B^2_\ell)^{-1} \), where \( C^{th}_\ell \) is an arbitrary theoretical p.s. and \( \langle \tilde{C}_{\ell s} \rangle \) is obtained from “observations” of \( N^{MC} \) full-sky and noise-free theoretical sky realizations.
  - instrumental noise \( \langle \tilde{N}_\ell \rangle_{MC} \): \( N^{MC} \) simulations of the noise contained in the data

\( \Rightarrow \) an accurate knowledge on the noise characteristics is required!
A new approach: cross power spectrum (CrossSpect)

**the idea**

I consider the maps of two *independent* detectors A e B

\[
\hat{C}_{\ell}^{AB} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a^A_{\ell m} a^B_{\ell m}^* \\
\implies \hat{C}_{\ell}^{AB} = B^A_\ell B^B_\ell F^{AB}_\ell \langle C_{\ell}^{AB} \rangle + \langle N_{\ell}^{AB} \rangle
\]

**estimation of the contributions**

- window function \( B^A_\ell e B^B_\ell \)
- transfer function \( F^{AB}_\ell = \sqrt{F^A_\ell F^B_\ell} \)
- instrumental noise \(\to\) is not correlated: \( \langle n^A_{\ell m} n^{B*}_{\ell m'} \rangle = 0 \)
A new approach: cross power spectrum (CrossSpect)

**the idea**

I consider the maps of two *independent* detectors A and B

\[
\hat{C}_{\ell}^{AB} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell m}^A a_{\ell m}^B
\]

\[\Rightarrow \quad \hat{C}_{\ell}^{AB} = B_{\ell}^A B_{\ell}^B F_{\ell}^{AB} \langle C_{\ell}^{AB} \rangle + \langle N_{\ell}^{AB} \rangle\]

**estimation of the contributions**

- Window function \(B_{\ell}^A, B_{\ell}^B\)
- Transfer function \(F_{\ell}^{AB} = \sqrt{F_{\ell}^A F_{\ell}^B}\)
- Instrumental noise → is not correlated: \(\langle n_{\ell m}^A n_{\ell m'}^B \rangle = 0\)
A new approach: cross power spectrum (CrossSpect)

the idea

I consider the maps of two independent detectors $A$ and $B$

\[
\hat{C}_{\ell}^{AB} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell m}^A a_{\ell m}^B \\
\Rightarrow \hat{C}_{\ell}^{AB} = B_{\ell}^A B_{\ell}^B F_{\ell}^{AB} \langle C_{\ell}^{AB} \rangle + \langle N_{\ell}^{AB} \rangle
\]

estimation of the contributions

- window function $B_{\ell}^A$ e $B_{\ell}^B$
- transfer function $F_{\ell}^{AB} = \sqrt{F_{\ell}^A F_{\ell}^B}$
- instrumental noise $\rightarrow$ is not correlated: $\langle n_{\ell m}^A n_{\ell, m'}^B \rangle = 0$
A new approach: cross power spectrum (CrossSpect)

The idea

I consider the maps of two independent detectors A and B

\[
\tilde{C}_{\ell}^{AB} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell m}^A a_{\ell m}^B \ast \\
\Rightarrow \quad \tilde{C}_{\ell}^{AB} = B_{\ell}^A B_{\ell}^B F_{\ell}^{AB} \langle C_{\ell}^{AB} \rangle + \langle N_{\ell}^{AB} \rangle
\]

Estimation of the contributions

- window function \( B_{\ell}^A \) e \( B_{\ell}^B \)
- transfer function \( F_{\ell}^{AB} = \sqrt{F_{\ell}^A F_{\ell}^B} \)
- instrumental noise → is not correlated: \( \langle n_{\ell m}^A n_{\ell, m'}^B \rangle = 0 \)
The Cosmic Microwave Background (CMB) Bias of the power spectrum estimation

A new approach: cross power spectrum (CrossSpect)

**the idea**

I consider the maps of two *independent* detectors A and B

\[
\tilde{C}_{\ell}^{AB} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} a_{\ell m}^A a_{\ell m}^B
\]

\[\Rightarrow \tilde{C}_{\ell}^{AB} = B_{\ell}^A B_{\ell}^B F_{\ell}^{AB} \langle C_{\ell}^{AB} \rangle + \langle N_{\ell}^{AB} \rangle\]

**estimation of the contributions**

- window function \(B_{\ell}^A \) and \(B_{\ell}^B\)
- transfer function \(F_{\ell}^{AB} = \sqrt{F_{\ell}^A F_{\ell}^B}\)
- instrumental noise \(\rightarrow\) is not correlated: \(\langle n_{\ell m}^A n_{\ell m'}^B \rangle = 0\)

\[\Rightarrow \text{ADVANTAGE: } \langle N_{\ell}^{AB} \rangle = 0!\]
CrossSpect: fullsky results

Figure: PLANCK 70 GHz - white noise – auto vs cross spectrum
CrossSpect: fullsky results

Figure: PLANCK 70 GHz - white noise
Outline

1. The Cosmic Microwave Background (CMB)
   - Main features
   - The angular power spectrum

2. Bias of the power spectrum estimation
   - Instrumental effects
   - Incomplete sky coverage
The Cosmic Microwave Background (CMB) Bias of the power spectrum estimation

Incomplete sky coverage

Instrumental effects

mode-mode coupling

the problem

\[ T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n}) \]

\[ Q(\hat{n}) \pm iU(\hat{n}) = \sum_{lm} a_{\mp 2,lm} \mp 2 Y_{lm}(\hat{n}) \]

where \( a_{\pm 2,lm} = E_{lm} \pm iB_{lm} \)

if I introduce a weight function \( w(\hat{n}) \)

\[ \tilde{T}_{lm} = \int d\hat{n} w^T(\hat{n}) T(\hat{n}) Y_{lm}^*(\hat{n}) \]

\[ \tilde{E}_{lm} = \frac{1}{2} \int d\hat{n} w^P(\hat{n}) \left[ Q(\hat{n}) \left( 2 Y_{lm}^*(\hat{n}) + -2 Y_{lm}^*(\hat{n}) \right) \right. \]

\[ \left. -i U(\hat{n}) \left( 2 Y_{lm}^*(\hat{n}) - -2 Y_{lm}^*(\hat{n}) \right) \right] \]

\[ \tilde{B}_{lm} = -\frac{1}{2} \int d\hat{n} w^P(\hat{n}) \left[ U(\hat{n}) \left( 2 Y_{lm}^*(\hat{n}) + -2 Y_{lm}^*(\hat{n}) \right) \right. \]

\[ \left. + i Q(\hat{n}) \left( 2 Y_{lm}^*(\hat{n}) - -2 Y_{lm}^*(\hat{n}) \right) \right] . \]
The Cosmic Microwave Background (CMB)

Bias of the power spectrum estimation

Instrumental effects
Incomplete sky coverage

mode-mode coupling

The problem

expanding $w$: $w(\hat{n}) = \sum_{\ell m} w_{\ell m} Y_{\ell m}(\hat{n})$

\[
\tilde{T}_{lm} = \sum_{l',m',l'',m''} w_{l',m',l''}^T T_{l',m'} \int d\hat{n} Y_{l',m'}(\hat{n}) Y_{l'',m''}(\hat{n}) Y_{lm}^*(\hat{n})
\]

\[
\tilde{E}_{lm} = \frac{1}{2} \sum_{l',m',l'',m''} w_{l',m',l''}^P \left[ E_{l',m'} \int d\hat{n} Y_{l'',m''}(\hat{n}) \left( 2Y_{l',m'}(\hat{n}) 2Y_{lm}^*(\hat{n}) + -2Y_{l',m'}(\hat{n}) -2Y_{lm}^*(\hat{n}) \right) \right]
\]

\[
+iB_{lm} \int d\hat{n} Y_{l'',m''}(\hat{n}) \left( 2Y_{l',m'}(\hat{n}) 2Y_{lm}^*(\hat{n}) - -2Y_{l',m'}(\hat{n}) -2Y_{lm}^*(\hat{n}) \right) \right]
\]

\[
\tilde{B}_{lm} = \frac{1}{2} \sum_{l',m',l'',m''} w_{l',m',l''}^P \left[ B_{l',m'} \int d\hat{n} Y_{l'',m''}(\hat{n}) \left( 2Y_{l',m'}(\hat{n}) 2Y_{lm}^*(\hat{n}) + -2Y_{l',m'}(\hat{n}) -2Y_{lm}^*(\hat{n}) \right) \right]
\]

\[
-IE_{lm} \int d\hat{n} Y_{l'',m''}(\hat{n}) \left( 2Y_{l',m'}(\hat{n}) 2Y_{lm}^*(\hat{n}) - -2Y_{l',m'}(\hat{n}) -2Y_{lm}^*(\hat{n}) \right) \right].
\]
The Cosmic Microwave Background (CMB) Bias of the power spectrum estimation Instrumental effects Incomplete sky coverage

**mode-mode coupling**

we have

\[ \int d\hat{n} \, N_{l}^{*} (\hat{n})_{N} \, Y_{l}^{m} (\hat{n})_{N'} \, Y_{l'}^{m'} (\hat{n})_{N''} \, Y_{l''}^{m''} (\hat{n}) = \]

\[ (-1)^{N+m} \left[ \frac{(2l + 1)(2l' + 1)(2l'' + 1)}{4\pi} \right]^{1/2} \begin{pmatrix} l & l' & l'' \\ -N & N' & N'' \end{pmatrix} \begin{pmatrix} l & l' & l'' \\ -m & m' & m'' \end{pmatrix} \]

we can compute

\[
\begin{pmatrix}
\tilde{c}_{TT}^{l}
\tilde{c}_{EE}^{l}
\tilde{c}_{BB}^{l}
\tilde{c}_{TE}^{l}
\tilde{c}_{TB}^{l}
\tilde{c}_{EB}^{l}
\end{pmatrix} = \begin{pmatrix}
M_{TT}^{l',l''}
M_{EE}^{l',l''}
M_{BB}^{l',l''}
M_{TE}^{l',l''}
M_{TB}^{l',l''}
M_{EB}^{l',l''}
\end{pmatrix} \begin{pmatrix}
c_{TT}^{l'}
c_{EE}^{l'}
c_{BB}^{l'}
c_{TE}^{l'}
c_{TB}^{l'}
c_{EB}^{l'}
\end{pmatrix}.
\]
### Mode-mode coupling

The Cosmic Microwave Background (CMB) bias of the power spectrum estimation

**Instrumental effects**

**Incomplete sky coverage**

#### Mode-mode coupling kernel

\[
M_{l'll'}^{TT,TT} = \frac{(2l' + 1)}{4\pi} \sum_{l''} W_{l''}^{TT} \left( \begin{array}{ccc} l & l' & l'' \\ 0 & 0 & 0 \end{array} \right)^2
\]

\[
M_{l'll'}^{TE,TE} = M_{l'll'}^{TB,TB}
\]

\[
M_{l'll'}^{EE,EE} = M_{l'll'}^{BB,EE}
\]

\[
M_{l'll'}^{EE,EE} = \frac{(2l' + 1)}{16\pi} \sum_{l''} W_{l''}^{PP} \left[ \left( \begin{array}{ccc} l & l' & l'' \\ -2 & 2 & 0 \end{array} \right) + \left( \begin{array}{ccc} l & l' & l'' \\ 2 & -2 & 0 \end{array} \right) \right]
\]

\[
\times \left[ \left( \begin{array}{ccc} l & l' & l'' \\ -2 & 2 & 0 \end{array} \right) + \left( \begin{array}{ccc} l & l' & l'' \\ 2 & -2 & 0 \end{array} \right) \right]
\]

\[
M_{l'll'}^{EE,BB} = M_{l'll'}^{BB,EE}
\]

\[
M_{l'll'}^{EE,BB} = \frac{(2l' + 1)}{16\pi} \sum_{l''} W_{l''}^{PP} \left[ \left( \begin{array}{ccc} l & l' & l'' \\ -2 & 2 & 0 \end{array} \right) - \left( \begin{array}{ccc} l & l' & l'' \\ 2 & -2 & 0 \end{array} \right) \right]
\]

\[
\times \left[ \left( \begin{array}{ccc} l & l' & l'' \\ -2 & 2 & 0 \end{array} \right) - \left( \begin{array}{ccc} l & l' & l'' \\ 2 & -2 & 0 \end{array} \right) \right]
\]

**Where**

\[
W_{l'}^{ab} = \sum_{lm} w_{lm}^a w_{lm}^{b*}
\]
The Cosmic Microwave Background (CMB)

Bias of the power spectrum estimation

Instrumental effects

Incomplete sky coverage

Kernel $M_{\ell \ell'}$ computation

Wigner 3-j symbol

having $\binom{l_1 \ l_2 \ l_3 \ 
m_1 \ m_2 \ m_3}{\ell_1 \ \ell_2 \ \ell_3}$

- it's not null only if (triangle condition):
  
  $$|l_1 - l_2| \leq l_3 \leq l_1 + l_2 \quad \text{e} \quad m_1 + m_2 + m_3 = 0$$

- symmetry property:

  $$(-1)^{l_1 + l_2 + l_3} \binom{l_1 \ l_2 \ l_3 \ 
m_1 \ m_2 \ m_3}{\ell_1 \ \ell_2 \ \ell_3} = \binom{\ell_2 \ l_1 \ l_3 \ 
m_2 \ m_1 \ m_3}{\ell_2 \ \ell_1 \ \ell_3}$$
Kernel $M_{\ell \ell'}$ computation

**the code**

for $l_1 = 2, l_{\text{max}}$
for $l_2 = 2, l_1$
  for $l_3 = |l_1 - l_2|, l_1 + l_2$
  if $l_1 + l_2 + l_3 = \text{even}$
    \[
    M_{l_1,l_2}^{TT} = \mathcal{W}_{l_3}^{TT} \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right)^2
    \]
    \[
    M_{l_1,l_2}^{TP} = \mathcal{W}_{l_3}^{TP} 2 \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ -2 & 2 & 0 \end{array} \right)
    \]
    \[
    M_{l_1,l_2}^{PT} = \mathcal{W}_{l_3}^{PT} 2 \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ -2 & 2 & 0 \end{array} \right)
    \]
    \[
    M_{l_1,l_2}^{PP} = \mathcal{W}_{l_3}^{PP} 4 \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ -2 & 2 & 0 \end{array} \right)^2
    \]
  else $l_1 + l_2 + l_3 = \text{odd}$
    \[
    M_{l_1,l_2}^{P\text{P\text{e\text{a\text{a\text{k}}}}}} = \mathcal{W}_{l_3}^{PP} 4 \left( \begin{array}{ccc} l_1 & l_2 & l_3 \\ -2 & 2 & 0 \end{array} \right)^2
    \]
mode-mode coupling kernel

Figure: Binned kernel and its inverse (absolute values, with green color indicating the negative elements) for an elliptically top hat sky window
The Cosmic Microwave Background (CMB) Bias of the power spectrum estimation

Instrumental effects

Incomplete sky coverage

Decoupling

**computation of the power spectrum**

\[
\langle \tilde{C}_\ell \rangle = \sum_{\ell'} M_{\ell\ell'} F_{\ell'} B_{\ell}^2 \langle C_{\ell'} \rangle ,
\]

I consider \( C_\ell \equiv \ell (\ell + 1) C_\ell / 2\pi \).
for a set of bins \( n_{\text{bins}}, \ell_{\text{min}}^{(b)} < \ell^{(b)} < \ell_{\text{min}}^{(b+1)} \), we can define the operator

\[
P_{b\ell} = \begin{cases} 
\frac{1}{2\pi} \frac{\ell (\ell + 1)}{\ell_{\text{min}}^{(b+1)} - \ell_{\text{min}}^{(b)}} & \text{if } 2 \leq \ell_{\text{min}}^{(b)} \leq \ell < \ell_{\text{min}}^{(b+1)} \\
0 & \text{otherwise}
\end{cases}
\]

and the *binned* power spectrum is \( C_b = P_{b\ell} C_\ell \). The reciprocal operator is:

\[
Q_{\ell b} = \begin{cases} 
\frac{2\pi}{\ell (\ell + 1)} & \text{se } 2 \leq \ell_{\text{min}}^{(b)} \leq \ell < \ell_{\text{min}}^{(b+1)} \\
0 & \text{otherwise}.
\end{cases}
\]

Inverting we obtain:

\[
\langle C_b \rangle = K^{-1}_{bb'} P_{b',\ell} \langle \tilde{C}_\ell \rangle ,
\]

where

\[
K_{bb'} = P_{b\ell} M_{\ell\ell'} F_{\ell'} B_{\ell}^2 Q_{\ell b'} .
\]
CrossSpect: cut-sky results

**Figure:** input vs output. Error bars evaluated from MC simulations.

Simona Donzelli  
CMB Angular Power Spectrum estimation
conclusions and future

conclusions

- the CrossSpect code is able to debias the power spectrum from the main "observational" effects without requiring noise estimation
- actually it’s one of the code candidates for the future PLANCK power spectrum estimation: testing ongoing on simulations with increasing numbers on biasing effects

possible developing

- take into account the correlated component of the noise
- estimate the error bars analytically
conclusions and future

conclusions

- the CrossSpect code is able to debiase the power spectrum from the main “observational” effects without requiring noise estimation.
- actually it’s one of the code candidates for the future PLANCK power spectrum estimation: testing ongoing on simulations with increasing numbers on biasing effects.

possible developing

- take into account the correlated component of the noise.
- estimate the error bars analytically.
conclusions and future

conclusions

- the CrossSpect code is able to debiase the power spectrum from the main "observational" effects without requiring noise estimation
- actually it’s one of the code candidated for the future PLANCK power spectrum estimation: testing ongoing on simulations with increasing numbers on biasing effects

possible developing

- take into account the correlated component of the noise
- estimate the error bars analytically
conclusions and future

conclusions

- the CrossSpect code is able to debias the power spectrum from the main “observational” effects without requiring noise estimation
- actually it’s one of the code candidates for the future PLANCK power spectrum estimation: testing ongoing on simulations with increasing numbers on biasing effects

possible developing

- take into account the correlated component of the noise
- estimate the error bars analytically
conclusions and future

**conclusions**

- the CrossSpect code is able to debias the power spectrum from the main “observational” effects without requiring noise estimation
- actually it’s one of the code candidates for the future PLANCK power spectrum estimation: testing ongoing on simulations with increasing numbers on biasing effects

**possible developing**

- take into account the correlated component of the noise
- estimate the error bars analytically
CrossSpect: the core of the code

\[
\begin{align*}
tt[l] & \quad + = \quad 2(alm1_T(l, m) \, re \, alm2_T(l, m) \, re + alm1_T(l, m) \, im \, alm2_T(l, m) \, im); \\
gg[l] & \quad + = \quad 2(alm1_G(l, m) \, re \, alm2_G(l, m) \, re + alm1_G(l, m) \, im \, alm2_G(l, m) \, im); \\
cc[l] & \quad + = \quad 2(alm1_C(l, m) \, re \, alm2_C(l, m) \, re + alm1_C(l, m) \, im \, alm2_C(l, m) \, im); \\
tg[l] & \quad + = \quad (alm1_T(l, m) \, re \, alm2_G(l, m) \, re + alm1_T(l, m) \, im \, alm2_G(l, m) \, im \\
& \quad + alm1_G(l, m) \, re \, alm2_T(l, m) \, re + alm1_G(l, m) \, im \, alm2_T(l, m) \, im); \\
tc[l] & \quad + = \quad (alm1_T(l, m) \, re \, alm2_C(l, m) \, re + alm1_T(l, m) \, im \, alm2_C(l, m) \, im \\
& \quad + alm1_C(l, m) \, re \, alm2_T(l, m) \, re + alm1_C(l, m) \, im \, alm2_T(l, m) \, im); \\
gc[l] & \quad + = \quad (alm1_G(l, m) \, re \, alm2_C(l, m) \, re + alm1_G(l, m) \, re \, alm2_G(l, m) \, re \\
& \quad + alm1_G(l, m) \, im \, alm2_C(l, m) \, im + alm1_C(l, m) \, im \, alm2_G(l, m) \, im); 
\end{align*}
\]
The Cosmic Microwave Background (CMB) Bias of the power spectrum estimation

Instrumental effects
Incomplete sky coverage

CrossSpect: primi risultati

Figure: 70 GHz - white noise – distribuzione $C_{\ell}^{MC}$

Simona Donzelli  CMB Angular Power Spectrum estimation