Structure of the Vacuum in Light-Front Dynamics
and Emergent Holographic QCD

Guy F. de Téramond

*Universidad de Costa Rica*

Ferrara International School

Niccolò Cabeo 2014

Vacuum and Symmetry Breaking:
From the Quantum to the Cosmos

Ferrara, May 19-23, 2014

In collaboration with Stan Brodsky (SLAC) and Hans G. Dosch (Heidelberg)
νομώ ψυχρόν, νομώ θερμόν, ετεί δε ατόμα καὶ κενόν
(By convention cold, by convention hot, in reality atoms and the void)

Democritus 460 - 370 BC
# Contents

1. Introduction  
2. Dirac Forms of Relativistic Dynamics  
3. Light-Front Dynamics and Light-Front Vacuum  
4. Light-Front Quantization of QCD  
5. Semiclassical Approximation to QCD in the Light Front  
6. Effective Confinement from Underlying Conformal Invariance  
7. Gravity in AdS and Light Front Holographic Mapping  
8. Higher Integer-Spin Wave Equations in AdS Space  
9. Meson Spectrum
10 Higher Half-Integer Spin Wave Equations in AdS Space 34

11 Baryon Spectrum 36

12 Conclusions 38
1 Introduction

- SLAC and DESY 70’s: elementary interactions of quarks and gluons at short distances remarkably well described by QCD

- No understanding of large distance strong dynamics of QCD: how quarks and gluons are confined and how hadrons emerge as asymptotic states

- Euclidean lattice important first-principles numerical simulation of nonperturbative QCD: excitation spectrum of hadrons requires enormous computational complexity beyond ground-state configurations

- Only known analytically tractable treatment in relativistic QFT is perturbation theory: description of confinement from perturbation theory is not possible (Kinoshita-Lee-Nauenberg Theorem)

- Important theoretical goal: find initial analytic approximation to strongly coupled QCD, like Schrödinger or Dirac Eqs in atomic physics, corrected for quantum fluctuations

- Convenient frame-independent Hamiltonian framework for treating bound-states in relativistic theories is light-front (LF) quantization
• Definition of invariant LF vacuum within causal horizon $P^2|0\rangle = 0$

• Simple structure of vacuum allows definition of partonic content of hadron in terms of wavefunctions: quantum-mechanical probabilistic interpretation of hadronic states

• Calculation of matrix elements $\langle P + q|J|P\rangle$ requires boosting the hadronic bound state from $|P\rangle$ to $|P + q\rangle$: no change in particle number from boost in LF (kinematical problem)

• To a first semiclassical approximation one can reduce the strongly correlated multi-parton LF dynamics to an effective one-dim QFTTh, which encodes the conformal symmetry of the classical QCD Lagrangian

• Uniqueness of the form of confining interaction from one dim conformal QFTTh: extension of conformal quantum mechanics to LF bound-state dynamics

• Invariant Hamiltonian equation for bound states similar structure of AdS equations of motion: direct connection of QCD and AdS/CFT possible

• Description of strongly coupled gauge theory in physical space-time using a dual gravity description in a higher dimensional space (holographic)
2 Dirac Forms of Relativistic Dynamics

[Dirac (1949)]

- The Poincaré group is the full symmetry group of any form of relativistic dynamics

\[
\begin{align*}
[P^\mu, P^\nu] &= 0, \\
[M^{\mu\nu}, P^\rho] &= i(g^{\mu\rho} P^\nu - g^{\nu\rho} P^\mu), \\
[M^{\mu\nu}, M^{\rho\sigma}] &= i(g^{\mu\rho} M^{\nu\sigma} - g^{\mu\sigma} M^{\nu\rho} + g^{\nu\sigma} M^{\mu\rho} - g^{\nu\rho} M^{\mu\sigma}),
\end{align*}
\]

- Poincaré generators separated into kinematical and dynamical

- Kinematical generators act along the initial hypersurface where initial conditions are imposed and contain no interactions (leave invariant initial surface)

- Dynamical generators are responsible for evolution of the system and depend on the interactions (map initial surface into another surface)

- Each form has its Hamiltonian and evolve with different time, but results computed in any form should be identical (different parameterizations of space-time)
• **Instant-form**: initial surface defined by $x^0 = 0$

\[ P^0, \mathbf{K} \text{ dynamical, } \mathbf{P}, \mathbf{J} \text{ kinematical} \]

• **Front-form**: initial surface tangent to the light cone $x^+ = x^0 + x^3 = 0$ \((P^\pm = P^0 \pm P^3)\)

\[ P^-, J^x, J^y \text{ dynamical, } P^+, P_\perp, J^3, \mathbf{K} \text{ kinematical} \]

• **Point-form**: initial surface is the hyperboloid $x^2 = \kappa^2 > 0, x^0 > 0$

\[ P^\mu \text{ dynamical, } M^{\mu\nu} \text{ kinematical} \]
3 Light-Front Dynamics and Light-Front Vacuum

- Hadron with 4-momentum \( P = (P^+, P^-, P_\perp) \), \( P^\pm = P^0 \pm P^3 \), mass-shell relation
  \( P^2 = M^2 \) leads to dispersion relation for LF Hamiltonian \( P^- \)
  \[
P^- = \frac{P^2_\perp + M^2}{P^+}, \quad P^+ > 0
\]

- \( n \)-particle bound state with \( p^2_i = m^2_i \), \( p_i = (p^+_i, p^-_i, p_\perp) \), for each constituent \( i \)
  \[
p^-_i = \frac{p^2_{\perp i} + m^2_i}{p^+_i}, \quad p^+_i > 0
\]

- Longitudinal momentum \( P^+ \) is kinematical: sum of single particle constituents \( p^+_i \) of bound state
  \[
P^+ = \sum_i p^+_i, \quad p^+_i > 0
\]

- LF Hamiltonian \( P^- \) is dynamical: bound-state is arbitrarily off the LF energy shell
  \[
P^- - \sum_i^n p^-_i < 0
\]
• LF evolution given by relativistic Schrödinger-like equation \((x^+ = x^0 + x^3)\) LF time

\[ i \frac{\partial}{\partial x^+} \psi(P) = P^- \psi(P) \]

where

\[ P^- \psi(P) = \frac{P^2_- + M^2}{P^+} \psi(P) \]

• Construct LF invariant Hamiltonian

\[ P^2 = P_\mu P^\mu = P^- P^+ - P^2_- \]

\[ P^2 \psi(P) = M^2 \psi(P) \]

• Since \( P^+ \) and \( P_- \) are kinematical, linear QM evolution in LF Hamiltonian \( P^- \) is maintained

• State \( \psi(P) \) is expanded in multi-particle Fock states \( |n\rangle \) of the free LF Hamiltonian

\[ |\psi\rangle = \sum_n \psi_n |n\rangle, \quad |n\rangle = \{ |uud\rangle, |uudg\rangle, |uudq\rangle, \ldots \} \]

with \( p_i^2 = m_i^2 \), \( p_i = (p_i^+, p_i^-, p_{\perp i}) \), for each constituent \( i \) in state \( n \)

• Fock component \( \psi_n \) is the light-front wave function (LFWF)
**Vacuum in the Front-Form of Dynamics**

- \( P^+ = \sum_i p_i^+ \), \( p_i^+ > 0 \): LF vacuum is the state with \( P^+ = 0 \) and contains no particles: all other states have \( P^+ > 0 \) (usual vacuum bubbles are kinematically forbidden in the front form!)

- Frame independent definition of the vacuum within the causal horizon

\[
P^2|0\rangle = 0
\]

(LF vacuum also has zero quantum numbers and \( P^+ = 0 \))

- LF vacuum is defined at fixed LF time \( x^+ = x^0 + x^3 \) over all \( x^- = x^0 - x^3 \) and \( x_\perp \), the expanse of space that can be observed within the speed of light

- Causality is maintained since LF vacuum only requires information within the causal horizon

- The front form is a natural basis for cosmology: universe observed along the front of a light wave
Vacuum in the Instant-Form of Dynamics

- Unsolved problems concerning the instant vacuum structure of QCD, confinement and chiral symmetry breaking
- In Lattice QCD the structure of the instant-form vacuum is sampled in the Euclidean region
- If monopoles or vortices are removed from the vacuum confinement and chiral symmetry breaking disappear
- Instantons provide a mechanism for symmetry breaking through the Banks-Casher relation
  \[ \langle 0 | \bar{\psi} \psi | 0 \rangle = -\pi \rho(0), \]
  where \( \rho(0) \) is the density or Dirac-zero modes \( D\psi_\lambda = \lambda \psi_\lambda \), but do not lead to confinement
- No convincing proof that continuum QCD is confining in the infrared
- Confinement in continuum QCD is a fundamental unsolved problem in theoretical physics
4 Light-Front Quantization of QCD

- Start with $SU(3)_C$ QCD Lagrangian
  \[ \mathcal{L}_{\text{QCD}} = \overline{\psi} (i \gamma^\mu D_\mu - m) \psi - \frac{1}{4} G^a_{\mu \nu} G^{a \mu \nu} \]

- Express the hadron four-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields $\psi_+ = \Lambda_\pm \psi$ and $\mathbf{A}_\perp (\Lambda_\pm = \gamma^0 \gamma^\pm)$ quantized in null plane $x^+ = x^0 + x^3 = 0$

  \[
  P^- = \frac{1}{2} \int d x^- d^2 \mathbf{x}_\perp \overline{\psi}_+ \gamma^+ \frac{(i \nabla_\perp)^2 + m^2}{i \partial^+} \psi_+ + \text{interactions}
  \]

  \[
  P^+ = \int d x^- d^2 \mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i \partial^+ \psi_+
  \]

  \[
  \mathbf{P}_\perp = \frac{1}{2} \int d x^- d^2 \mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i \nabla_\perp \psi_+
  \]

  where the integrals are over initial surface $x^+ = 0$, where commutation relation for fields are defined
- Dirac field $\psi_+$ expanded in terms of ladder operators

$$b(q)|0\rangle = d(q)|0\rangle = 0$$

$$\psi_+(x^-, x_\perp) = \sum_\lambda \int_{q^+ > 0} \frac{dq^+}{\sqrt{2q^+}} \frac{d^2q_\perp}{(2\pi)^3} \left[ b_\lambda(q) u_\alpha(q, \lambda) e^{-iq \cdot x} + d_\lambda(q)^\dagger v_\alpha(q, \lambda) e^{iq \cdot x} \right]$$

with

$$\{ b(q), b^\dagger(q') \} = \{ d(q), d^\dagger(q') \} = (2\pi)^3 \delta(q^+ - q'^+) \delta^{(2)}(q_\perp - q'_\perp)$$

$$P^- = \sum_\lambda \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} \left( \frac{q_\perp^2 + m^2}{q^+} \right) b_\lambda(q) b_\lambda(q) + \text{interactions}$$

$$P^+ = \sum_\lambda \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} q^+ b_\lambda(q) b_\lambda(q)$$

$$P_\perp = \sum_\lambda \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} q_\perp b_\lambda(q) b_\lambda(q)$$

where $q^- = \frac{q_\perp^2 + m^2}{q^+}$
• Relative partonic coordinates: \( k_i^+ = x_i P^+ \), \( p_{\perp i} = x_i P_{\perp i} + k_{\perp i} \)

\[
\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n k_{\perp i} = 0
\]

• Fock component \( \psi_n(x_i, k_{\perp i}, \lambda^z_i) \) depends only on relative coordinates: longitudinal momentum fraction \( x_i = k_i^+ / P^+ \), transverse momentum \( k_{\perp i} \) and spin \( \lambda^z_i \)

• LFWF \( \psi_n \) is frame independent

• Momentum conservation: \( P^+ = \sum_{i=1}^n k_i^+ \), \( k_i^+ > 0 \), but LFWF represents a bound-state which is off the LF energy shell, \( P^- - \sum_{i=1}^n k_i^- < 0 \)

• Express off-shell dependence of bound-state in terms of invariant variable (invariant mass of constituents)

\[
M_n^2 = (k_1 + k_2 + \cdots + k_n)^2 = \left( \sum_{i=1}^n k_i^\mu \right)^2 = \sum_i \frac{k_{\perp i}^2 + m_i^2}{x_i}
\]

• \( M^2 - M_n^2 \) is the measure of the off-energy shell: key variable which controls the bound state
5 Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

- Effective reduction of strongly coupled LF multiparticle dynamics to a 1-dim QFT with no particle creation and absorption: LF quantum mechanics!

- Central problem is derivation of effective interaction which acts only on the valence sector: express higher Fock states as functionals of the lower ones

- Advantage: Fock space not truncated and symmetries of the Lagrangian preserved

[H. C. Pauli, EPJ, C7, 289 (1999)]

- Compute $M^2$ from hadronic matrix element

\[
\langle \psi(P') | P\mu P^\mu | \psi(P) \rangle = M^2 \langle \psi(P') | \psi(P) \rangle
\]

- Find

\[
M^2 = \sum_n \int [dx_i] [d^2 k_{\perp i}] \sum_q \left( \frac{k_{\perp q}^2 + m_q^2}{x_q} \right) |\psi_n(x_i, k_{\perp i})|^2 + \text{interactions}
\]

with phase space normalization

\[
\sum_n \int [dx_i] [d^2 k_{\perp i}] |\psi_n(x_i, k_{\perp i})|^2 = 1
\]
• In terms of $n - 1$ independent transverse impact coordinates $b_{\perp j}, j = 1, 2, \ldots, n - 1,$

$$M^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2 b_{\perp j} \psi_n^*(x_i, b_{\perp i}) \sum_q \left( -\nabla_{b_{\perp q}}^2 + m_q^2 \right) x_q \psi_n(x_i, b_{\perp i}) + \text{interactions}$$

with normalization

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2 b_{\perp j} |\psi_n(x_j, b_{\perp j})|^2 = 1.$$ 

• Semiclassical approximation

$$\psi_n(k_1, k_2, \ldots, k_n) \to \phi_n(\left( k_1 + k_2 + \cdots + k_n \right)^2), \quad m_q \to 0$$

$$M_n^2 = \sum_i \frac{k_{\perp i}^2 + m_i^2}{x_i}$$

• For a two-parton system in the $m_q \to 0$ 

$$M_{qq}^2 = \frac{k_{\perp}^2}{x(1-x)}$$

• Conjugate invariant variable in transverse impact space is

$$\zeta^2 = x(1-x)b_{\perp}^2$$
• To first approximation LF dynamics depend only on the invariant variable $\zeta$, and dynamical properties are encoded in the hadronic mode $\phi(\zeta)$

$$\psi(x, \zeta, \varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}},$$

where we factor out the longitudinal $X(x)$ and orbital kinematical dependence from LFWF $\psi$

• Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple ($L = L^z$)

$$M^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta),$$

where effective potential $U$ includes all interaction terms upon integration of the higher Fock states

• LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = M^2 |\phi\rangle$ is a LF wave equation for $\phi$

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} \right) \phi(\zeta) + U(\zeta) \phi(\zeta) = M^2 \phi(\zeta)$$

• Relativistic and frame-independent LF Schrödinger equation: $U$ is instantaneous in LF time

• Critical value $L = 0$ corresponds to lowest possible stable solution, the ground state of the LF invariant Hamiltonian $P^2$
• Compare invariant mass in the instant-form in the hadron center-of-mass system $P = 0$,

$$M_{q\bar{q}}^2 = 4 m_q^2 + 4p^2$$

with the invariant mass in the front-form in the constituent rest frame, $k_q + k_{\bar{q}} = 0$

$$M_{q\bar{q}}^2 = \frac{k_\perp^2 + m_q^2}{x(1-x)}$$

obtain

$$U = V^2 + 2 \sqrt{p^2 + m_q^2} V + 2 V \sqrt{p^2 + m_q^2}$$

where $p_\perp^2 = \frac{k_\perp^2}{4x(1-x)}$, $p_3 = \frac{m_q(x-1/2)}{\sqrt{x(1-x)}}$, and $V$ is the effective potential in the instant-form

• For small quark masses a linear instant-form potential $V$ implies a harmonic front-form potential $U$ and thus linear Regge trajectories

6 Effective Confinement from Underlying Conformal Invariance

[S. J. Brodsky, GdT and H.G. Dosch, PLB 729, 3 (2014)]

- Incorporate in a 1-dim QFT – as an effective theory, the fundamental conformal symmetry of the 4-dim classical QCD Lagrangian in the limit of massless quarks

- Invariance properties of 1-dim field theory under the full conformal group from dAFF action
  [V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A 34, 569 (1976)]

\[ S = \frac{1}{2} \int dt \left( \dot{Q}^2 - \frac{g}{Q^2} \right) \]

where \( g \) is a dimensionless number (Casimir operator which depends on the representation)

- The equation of motion

\[ \ddot{Q} - \frac{g}{Q^3} = 0 \]

and the generator of evolution in \( t \), the Hamiltonian

\[ H = \frac{1}{2} \left( \dot{Q}^2 + \frac{g}{Q^2} \right) \]

follow from the dAFF action
Absence of dimensional constants implies that the action

\[ S = \frac{1}{2} \int dt \left( \dot{Q}^2 - \frac{g}{Q^2} \right) \]

is invariant under a larger group of transformations, the general conformal group

\[ t' = \frac{\alpha t + \beta}{\gamma t + \delta}, \quad Q'(t') = \frac{Q(t)}{\gamma t + \delta} \]

with \( \alpha \delta - \beta \gamma = 1 \)

Applying Noether’s theorem one obtains 3 conserved operators

I. Translations in \( t \): \( H = \frac{1}{2} \left( \dot{Q}^2 + \frac{g}{Q^2} \right) \),

II. Dilatations: \( D = \frac{1}{2} \left( \dot{Q}^2 + \frac{g}{Q^2} \right) t - \frac{1}{4} \left( \dot{QQ} + QQ \right) \),

III. Special conformal transformations: \( K = \frac{1}{2} \left( \dot{Q}^2 + \frac{g}{Q^2} \right) t^2 - \frac{1}{2} \left( \dot{QQ} + QQ \right) t + \frac{1}{2} Q^2 \),
• Using canonical commutation relations  
\[ [Q(t), \dot{Q}(t)] = i \quad \text{find} \]
\[ [H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK, \]
the algebra of the generators of the conformal group \( Conf(R^1) \)

• Introduce the linear combinations
\[ J_{12}^1 = \frac{1}{2} \left( \frac{1}{a} K + aH \right), \quad J_{01}^1 = \frac{1}{2} \left( \frac{1}{a} K - aH \right), \quad J_{02} = D, \]
where \( a \) has dimension \( t \) since \( H \) and \( K \) have different dimensions

• Generators \( J \) have commutation relations
\[ [J_{12}^1, J_{01}^1] = iJ_{02}, \quad [J_{12}^1, J_{02}] = -iJ_{01}^1, \quad [J_{01}^1, J_{02}] = -iJ_{12}^1, \]
the algebra of \( SO(2, 1) \)

• \( J_{0i}, \ i = 1, 2, \) boost in space direction \( i \) and \( J_{12} \) rotation in the \( (1,2) \) plane

• \( J_{12} \) is compact and has thus discrete spectrum with normalizable eigenfunctions and a ground state

• The relation between the generators of conformal group and generators of \( SO(2, 1) \) suggests that the scale \( a \) may play a fundamental role
• dAFF construct a new generator as a superposition of the 3 constants of motion

\[ G = uH + vD + wK \]

and introduce new time variable \( \tau \) and field operator \( q(\tau) \)

\[ d\tau = \frac{dt}{u + vt + wt^2}, \quad q(\tau) = \frac{Q(t)}{[u + vt + wt^2]^{1/2}} \]

• Find usual quantum mechanical evolution for time \( \tau \)

\[ G|\psi(\tau)\rangle = i \frac{d}{d\tau} |\psi(\tau)\rangle \]

\[ i [G, q(\tau)] = \frac{dq(\tau)}{d\tau} \]

and usual equal-time quantization \( [q(t), \dot{q}(t)] = i \)
• In terms of $\tau$ and $q(\tau)$

\[
S = \frac{1}{2} \int dt \left( \dot{Q}^2 - \frac{g}{Q^2} \right) \\
= \frac{1}{2} \int d\tau \left( \dot{q}^2 - \frac{g}{q^2} - \frac{4u\omega - v^2}{4} q^2 \right) + \text{surface term}
\]

Action is conformal invariant up to a surface term!

• The corresponding Hamiltonian

\[
G = \frac{1}{2} \left( \dot{q}^2 + \frac{g}{q^2} + \frac{4u\omega - v^2}{4} q^2 \right)
\]

is a compact operator for

\[
\frac{4u\omega - v^2}{4} > 0
\]

• Scale appears in the Hamiltonian without affecting the conformal invariance of the action!
• Evolution in $\tau$ in terms of original field operator $Q(t)$ at $t = 0$

\[
G(Q, \dot{Q}) = \frac{1}{2} u \left( \dot{Q}^2 + \frac{g}{Q^2} \right) - \frac{1}{4} v \left( Q \dot{Q} + \dot{Q} Q \right) + \frac{1}{2} wQ^2
\]

\[
= uH + vD + wK,
\]

• The Schrödinger picture follows from the representation of $Q$ and $P = \dot{Q}$

\[
Q \rightarrow x, \quad \dot{Q} \rightarrow -i \frac{d}{dx}
\]

• dAFF Conformal Quantum Mechanics

• Schrödinger wave equation determines evolution of bound states in terms of the variable $\tau$

\[
i \frac{\partial}{\partial \tau} \psi(x, \tau) = H_\tau \left( x, -i \frac{d}{dx} \right) \psi(x, \tau)
\]

• dAFF Hamiltonian $G$

\[
G = \frac{1}{2} u \left( - \frac{d^2}{dx^2} + \frac{g}{x^2} \right) + \frac{i}{4} v \left( x \frac{d}{dx} + \frac{d}{dx} x \right) + \frac{1}{2} w x^2
\]
• Compare dAFF Hamiltonian

\[ G = \frac{1}{2} u \left( -\frac{d^2}{dx^2} + \frac{g}{x^2} \right) + \frac{i}{4} v \left( x \frac{d}{dx} + \frac{d}{dx} x \right) + \frac{1}{2} wx^2 \]

with LF Hamiltonian for \( u = 2, \ v = 0, \)

\[ P^2 = -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \]

• \( x \) is identified with the LF variable \( \zeta: \ x = \zeta \)

• Casimir \( g \) with the LF orbital angular momentum \( L: \ g = L^2 - \frac{1}{4} \)

• \( w = 2\lambda^2 \sim \frac{1}{a^2} \) fixes effective LF confining interaction to quadratic \( \lambda^2 \zeta^2 \) dependence

\[ U \sim \lambda^2 \zeta^2 \]

• Also, dAFF evolution variable \( \tau \) is identified with LF time \( \tau = x^+/P^+ \)
7 Gravity in AdS and Light Front Holographic Mapping

\[ R_{ijklm} = -\frac{1}{R^2} (g_{inlj} g_{km} - g_{nj} g_{ikm}) \]

- Why is AdS space important?
  AdS\(_5\) is space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

- Isomorphism of \(SO(4,2)\) group of conformal transformations with generators \(P^\mu, M^{\mu\nu}, K^\mu, D\) with the group of isometries of AdS\(_5\)

- AdS\(_5\) metric \(x^M = (x^\mu, z)\):
  \[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \]

- Since the AdS metric is invariant under a dilatation of all coordinates \(x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z\), the variable \(z\) acts like a scaling variable in Minkowski space

- Short distances \(x_\mu x^\mu \rightarrow 0\) map to UV conformal AdS\(_5\) boundary \(z \rightarrow 0\)

- Large confinement dimensions \(x_\mu x^\mu \sim 1/\Lambda_{QCD}^2\) map to IR region of AdS\(_5\), \(z \sim 1/\Lambda_{QCD}\), thus AdS geometry has to be modified at large \(z\) to include the scale of strong interactions
8 Higher Integer-Spin Wave Equations in AdS Space

[GdT, H.G. Dosch and S. J. Brodsky, PRD 87, 075004 (2013)]

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)

- Integer spin-$J$ fields in AdS conveniently described by tensor field $\Phi_{N_1 \cdots N_J}$ with effective action

\[
S_{\text{eff}} = \int d^d x \, dz \, \sqrt{|g|} \, e^{\varphi(z)} \left( g^{N_1 N_1'} \cdots g^{N_J N_J'} \left( g^{MM'} D_M \Phi^*_{N_1 \cdots N_J} D_{M'} \Phi_{N_1' \cdots N_J'} - \mu_{\text{eff}}^2(z) \Phi^*_{N_1 \cdots N_J} \Phi_{N_1' \cdots N_J'} \right) \right)
\]

$D_M$ is the covariant derivative which includes affine connection and dilaton $\varphi(z)$ breaks conformality

- Effective mass $\mu_{\text{eff}}(z)$ is determined by precise mapping to light-front physics

- Non-trivial geometry of pure AdS encodes the kinematics and the additional deformations of AdS encode the dynamics, including confinement
• Physical hadron has plane-wave and polarization indices along 3+1 physical coordinates and a profile wavefunction \( \Phi(z) \) along holographic variable \( z \)

\[
\Phi_P(x, z)_{\mu_1 \cdots \mu_J} = e^{i P \cdot x} \Phi(z)_{\mu_1 \cdots \mu_J}, \quad \Phi_{z\mu_2 \cdots \mu_J} = \cdots = \Phi_{\mu_1 \mu_2 \cdots \mu_J} = 0
\]

with four-momentum \( P_\mu \) and invariant hadronic mass \( P_\mu P^\mu = M^2 \)

• Further simplification by using a local Lorentz frame with tangent indices

• Variation of the action gives AdS wave equation for spin-\( J \) field

\[
\left[ -\frac{z^{d-1-2J}}{e^\varphi(z)} \partial_z \left( \frac{e^\varphi(z)}{z^{d-1-2J}} \partial_z \right) + \left( \frac{m R}{z} \right)^2 \right] \Phi_J = M^2 \Phi_J
\]

with

\[
(m R)^2 = (\mu_{\text{eff}}(z) R)^2 - J z \varphi'(z) + J(d - J + 1)
\]

and the kinematical constraints

\[
\eta^{\mu\nu} P_\mu \epsilon_{\nu_2 \cdots \nu_J} = 0, \quad \eta^{\mu\nu} \epsilon_{\mu\nu_3 \cdots \nu_J} = 0.
\]

• Kinematical constrains in the LF imply that \( m \) must be a constant

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D 85, 076003 (2012)]
Light-Front Mapping

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

- Upon substitution $\Phi_J(z) \sim z^{(d-1)/2-J} e^{-\varphi(z)/2} \phi_J(z)$ and $z \to \zeta = \sqrt{x(1-x)} b$ in AdS WE

$$\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left( \frac{m_R}{z} \right)^2 \right] \phi_J(z) = M^2 \phi_J(z)$$

we find LFWE ($d = 4$)

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2} \varphi''(\zeta) + \frac{1}{4} \varphi'(\zeta)^2 + \frac{2J - 3}{2z} \varphi'(\zeta)$$

and

$$(m_R)^2 = -(2 - J)^2 + L^2$$

- Unmodified AdS equations correspond to the kinetic energy terms of the partons inside a hadron
- Interaction terms in the QCD Lagrangian build the effective confining potential $U(\zeta)$ and correspond to the truncation of AdS space in an effective dual gravity approximation
- AdS Breitenlohner-Freedman bound $(m_R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
9 Meson Spectrum

- Dilaton profile in the dual gravity model determined from one-dim QFT/h (dAFF)
  \[ \varphi(z) = \lambda z^2, \quad \lambda^2 = w/2 \]

- Effective potential:
  \[ U = \lambda^2 \zeta^2 + 2\lambda (J - 1) \]

- LFWE
  \[ \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \lambda^2 \zeta^2 + 2\lambda (J - 1) \right) \phi_J(\zeta) = M_J^2 \phi_J(\zeta) \]

- Normalized eigenfunctions
  \[ \langle \phi|\phi \rangle = \int d\zeta \phi^2(z) = 1 \]

  \[ \phi_{n,L}(\zeta) = |\lambda|^{(1+L)/2} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-|\lambda|\zeta^2/2} L_n^L(|\lambda|\zeta^2) \]

- Eigenvalues for \( \lambda > 0 \)
  \[ M_{n,J,L}^2 = 4\lambda \left( n + \frac{J + L}{2} \right) \]

- \( \lambda < 0 \) incompatible with LF constituent interpretation
LFWFs $\phi_{n,L}(\zeta)$ in physical space-time: (L) orbital modes and (R) radial modes
Results easily extended to light quarks masses (Ex: $K$-mesons)


First order perturbation in the quark masses

$$\Delta M^2 = \langle \psi | \sum_a m_a^2 / x_a | \psi \rangle$$

Holographic LFWF with quark masses


$$\psi(x, \zeta) \sim \sqrt{x(1-x)} e^{-\frac{1}{2\lambda} \left( \frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right)} e^{-\frac{1}{2} \lambda \zeta^2}$$

Ex: Description of diffractive vector meson production at HERA

[J. R. Forshaw and R. Sandapen, PRL 109, 081601 (2012)]

For the $K^*$

$$M^2_{n, L, S} = M^2_{K^\pm} + 4\lambda \left( n + \frac{J + L}{2} \right)$$

Effective quark masses from reduction of higher Fock states as functionals of the valence state:

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$
Orbital and radial excitations for vector mesons for $\sqrt{\lambda} = 0.54$ GeV

- Linear Regge trajectories, a massless pion and relation between the $\rho$ and $a_1$ mass $M_{a_1}/M_\rho = \sqrt{2}$ usually obtained from Weinberg sum rules described by LF harmonic confinement model
10 Higher Half-Integer Spin Wave Equations in AdS Space

[J. Polchinski and M. J. Strassler, JHEP 0305, 012 (2003)]
[GdT and S. J. Brodsky, PRL 94, 201601 (2005)]
[GdT, H.G. Dosch and S. J. Brodsky, PRD 87, 075004 (2013)]

- The gauge/gravity duality can give important insights into the strongly coupled dynamics of nucleons using simple analytical methods: analytical exploration of systematics of light-baryon resonances
- Extension of holographic ideas to spin-$\frac{1}{2}$ (and higher half-integral $J$) hadrons by considering wave equations for Rarita-Schwinger spinor fields in AdS space and their mapping to light-front physics
- LF clustering decomposition of invariant variable $\zeta$: same multiplicity of states for mesons and baryons
  \[ \zeta = \sqrt{\frac{x}{1-x}} \sum_{j=1}^{n-1} x_j b_{\perp j} \] (spectators vs active quark)
- But in contrast with mesons there is important degeneracy of states along a given Regge trajectory for a given $L$: no spin-orbit coupling

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D 85, 076003 (2012)]
• Half-integer spin $J = T + \frac{1}{2}$ conveniently represented by RS spinor $[\Psi_{N_1 \ldots N_T}]_\alpha$ with effective AdS action

$$S_{\text{eff}} = \frac{1}{2} \int d^d x \, dz \, \sqrt{|g|} \, g^{N_1 N_1'} \ldots g^{N_T N_T'}$$

$$\left[ \bar{\Psi}_{N_1 \ldots N_T} \left( i \Gamma^A e^M_A D_M - \mu - U(z) \right) \Psi_{N_1' \ldots N_T'} + \text{h.c.} \right]$$

where the covariant derivative $D_M$ includes the affine connection and the spin connection

• $e^A_M$ is the vielbein and $\Gamma^A$ tangent space Dirac matrices $\{ \Gamma^A, \Gamma^B \} = \eta^{AB}$

• LF mapping $z \rightarrow \zeta$ find coupled LFWE

$$- \frac{d}{d\zeta} \psi_- - \frac{\nu + \frac{1}{2}}{\zeta} \psi_- - V(\zeta) \psi_- = M \psi_+$$

$$\frac{d}{d\zeta} \psi_+ - \frac{\nu + \frac{1}{2}}{\zeta} \psi_+ - V(\zeta) \psi_+ = M \psi_-$$

provided that $|\mu R| = \nu + \frac{1}{2}$ and

$$V(\zeta) = \frac{R}{\zeta} U(\zeta)$$

a $J$-independent potential – No spin-orbit coupling along a given trajectory!

11 Baryon Spectrum

- Choose linear potential \( V = \lambda \zeta, \quad \lambda > 0 \) to satisfy dAFF

- Eigenfunctions
  \[ \psi_+ (\zeta) \sim \zeta^{\frac{1}{2} + \nu} e^{-\lambda \zeta^2 / 2} L_\nu (\lambda \zeta^2), \quad \psi_- (\zeta) \sim \zeta^{\frac{3}{2} + \nu} e^{-\lambda \zeta^2 / 2} L_{\nu + 1} (\lambda \zeta^2) \]

- Eigenvalues: \( M^2 = 4\lambda (n + \nu + 1) \)

- Lowest possible state \( n = 0 \) and \( \nu = 0 \): orbital excitations \( \nu = 0, 1, 2 \cdots = L \)

- \( L \) is the relative LF angular momentum between the active quark and spectator cluster

- \( \nu \) depends on internal spin and parity

  The assignment

  \[
  \begin{array}{c|cc}
  & S = \frac{1}{2} & S = \frac{3}{2} \\
  \hline
  P = + & \nu = L & \nu = L + \frac{1}{2} \\
  P = - & \nu = L + \frac{1}{2} & \nu = L + 1 \\
  \end{array}
  \]

  describes the full light baryon orbital and radial excitation spectrum
Baryon orbital and radial excitations for $\sqrt{\lambda} = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas)
12 Conclusions

We have described a threefold connection between a one-dimensional QFT semiclassical approximation to light-front QCD with gravity in a higher dimensional AdS space, and the constraints imposed by the invariance properties under the full conformal group in one dimension. This provides a new insight into the physics underlying the QCD vacuum, confinement, chiral invariance and the QCD mass scale.
Thank you!