Hadronic Form Factor Models Within the Gauge/Gravity Correspondence

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Electromagnetic form factors of hadrons

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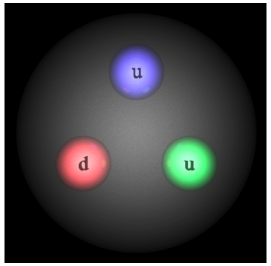
1 Introduction

Internal Structure of the Proton

- High Energy (20 GeV) scattering at SLAC (1969) revealed the internal structure of the proton
- Deep inelastic scattering experiments (1967-1973): Bjorken and Feynman partons identified with Gell-Mann and Zweig quarks

- Interactions of quarks and gluons at high energies are well described by Quantum Chromodynamics (QCD) a remarkable generalization of QED

Quantum Chromodynamics (QCD)



- QCD fundamental theory of quarks and gluons
- QCD Lagrangian follows from the gauge invariance of the theory

$$\psi(x) \to e^{i\alpha^a(x)T^a}\psi(x), \qquad \left[T^a, T^b\right] = i f_{abc} T^c$$

- Find QCD Lagrangian

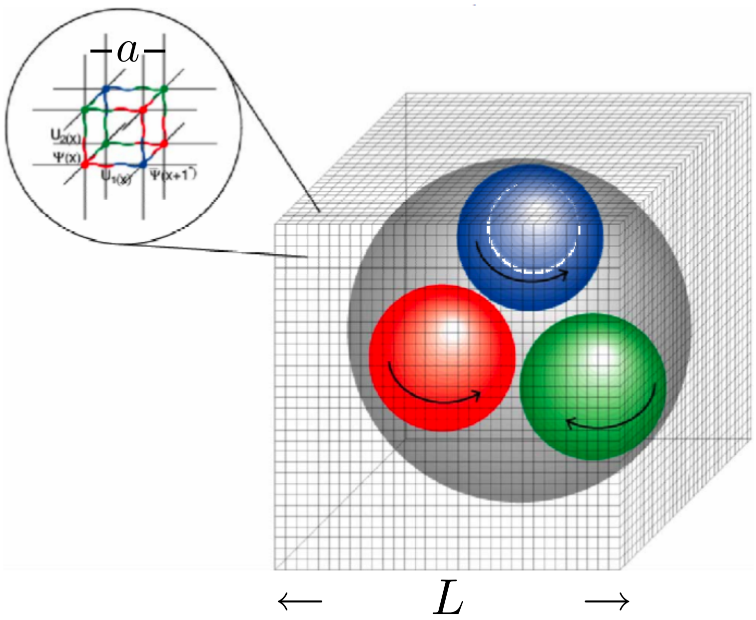
$$\mathcal{L}_{\rm QCD} = -\frac{1}{4g^2}\mathrm{Tr}\left(G^{\mu\nu}G_{\mu\nu}\right) + i\overline{\psi}D_\mu\gamma^\mu\psi + m\overline{\psi}\psi$$

where $D_\mu = \partial_\mu - igT^a A^a_\mu, \;\; G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f_{abc}A^b_\mu A^c_\nu$

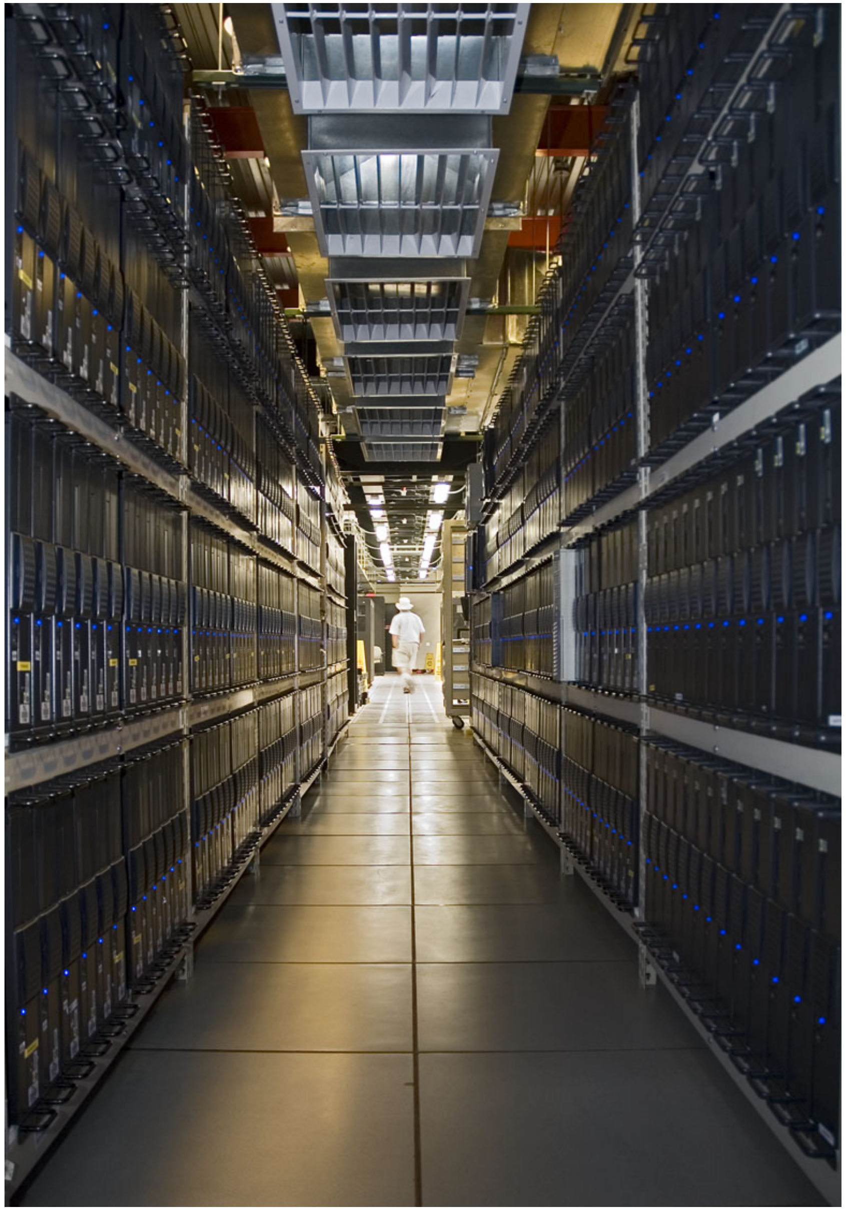
- Quarks and gluons interactions from color charge, but ... gluons also interact with each other: strongly coupled non-abelian gauge theory $\longrightarrow$ color confinement
- Most challenging problem of strong interaction dynamics: determine the composition of hadrons in terms of their fundamental QCD quark and gluon degrees of freedom

Lattice QCD

- Lattice numerical simulations at the teraflop/sec scale (resolution $\sim L/a$)
- Sums over quark paths with billions of dimensions
- LQCD (2009) $>$ 1 petaflop/sec



- Dynamical properties in Minkowski space-time not amenable to Euclidean lattice computations
- Example: Time-like nucleon form factors



General Theory of Relativity

- Space curvature determined by the mass-energy present following Einstein's equations

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu}}_{\text{geometry}} = \kappa \underbrace{T_{\mu\nu}}_{\text{mater}}$$

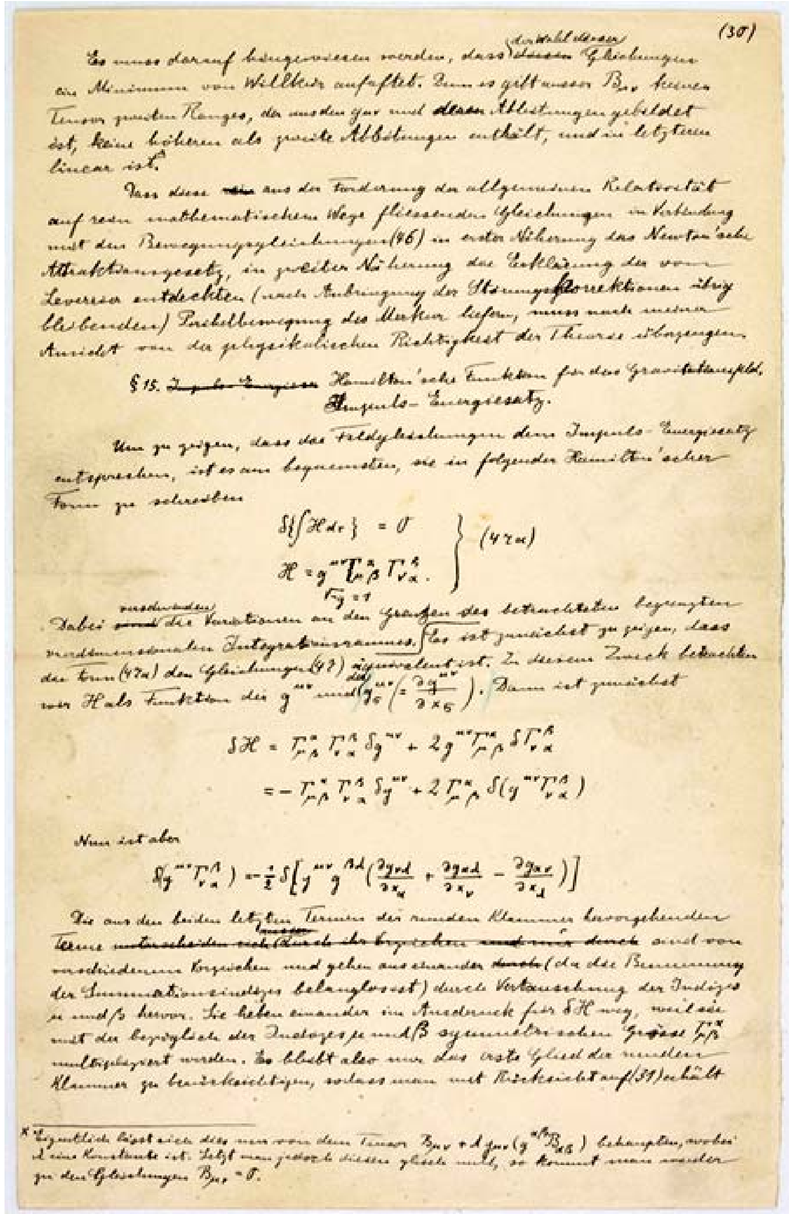
$R_{\mu\nu}$ Ricci tensor , R space curvature

$g_{\mu\nu}$ metric tensor ($ds^2 = g_{\mu\nu} dx^\mu dx^\nu$)

$T_{\mu\nu}$ energy-momentum tensor

$\kappa = 8\pi G_N / c^4$

- Matter curves space and space determines how matter moves



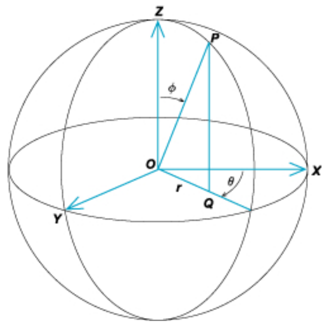
Annalen der Physik **49** (1916) p. 30

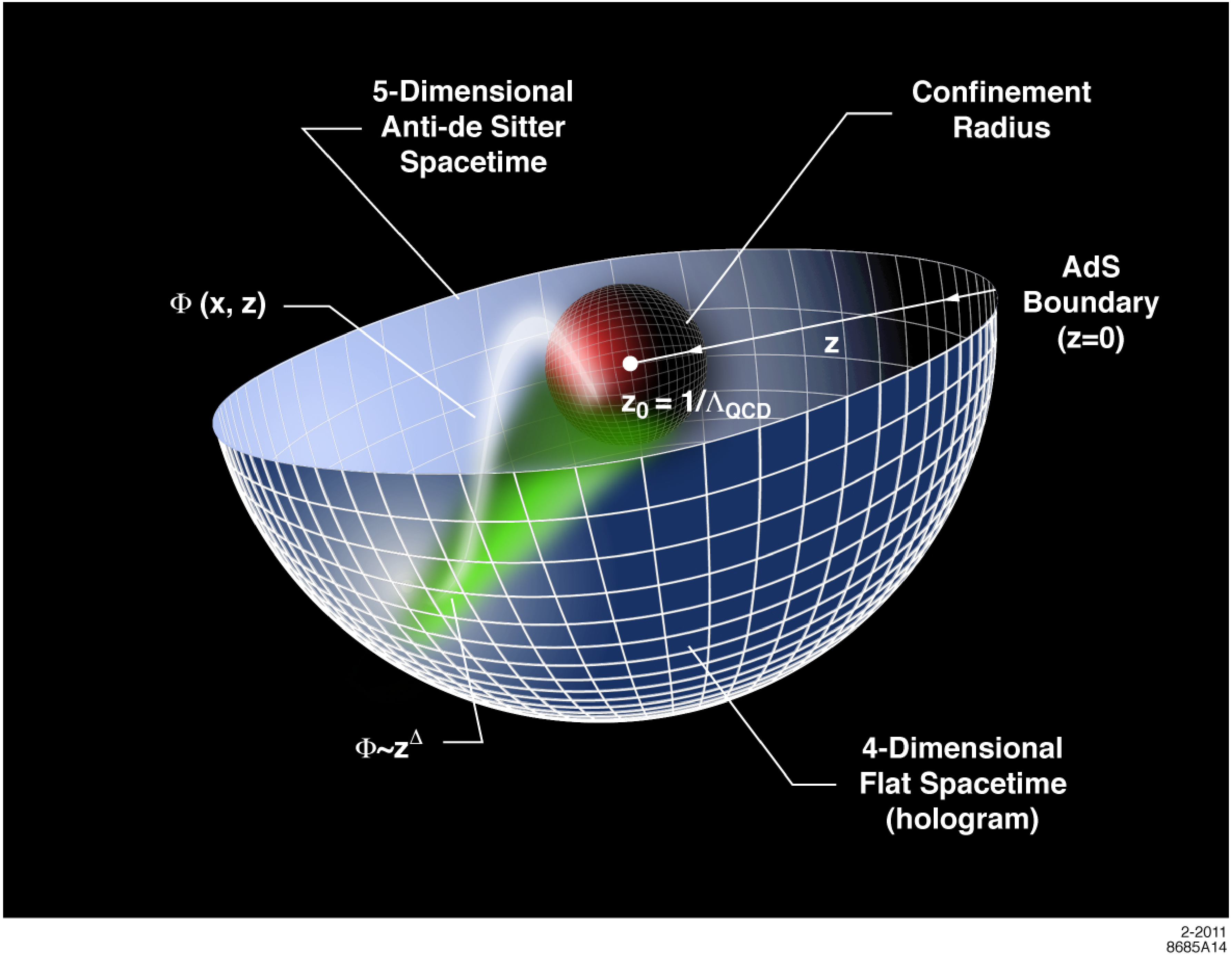
Gauge/Gravity Correspondence and QCD

- Recent developments inspired by the AdS/CFT correspondence [Maldacena (1998)] between gravity in AdS space and conformal field theories in physical space-time have led to an analytical semiclassical approximation to light-front QCD, which provides physical insights into its non-perturbative dynamics
- Description of strongly coupled gauge theory using a dual gravity description!
- Strings describe spin-J extended objects (no quarks). QCD degrees of freedom are pointlike particles and hadrons have orbital angular momentum: how can they be related?
- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS$_5$, a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

Isometry group: most general group of transformations which leave invariant the distance between two points Ej: $S^N \sim O(N+1)$

Dimension of isometry group of AdS$_{d+1}$ is $\frac{(d+1)(d+2)}{2}$




5-Dimensional
Anti-de Sitter
Spacetime
Confinement
Radius
AdS
Boundary
(z=0)
Φ (x, z)
z
$z_0 = 1/\Lambda_{QCD}$
Φ~z$^\Delta$
4-Dimensional
Flat Spacetime
(hologram)
2-2011
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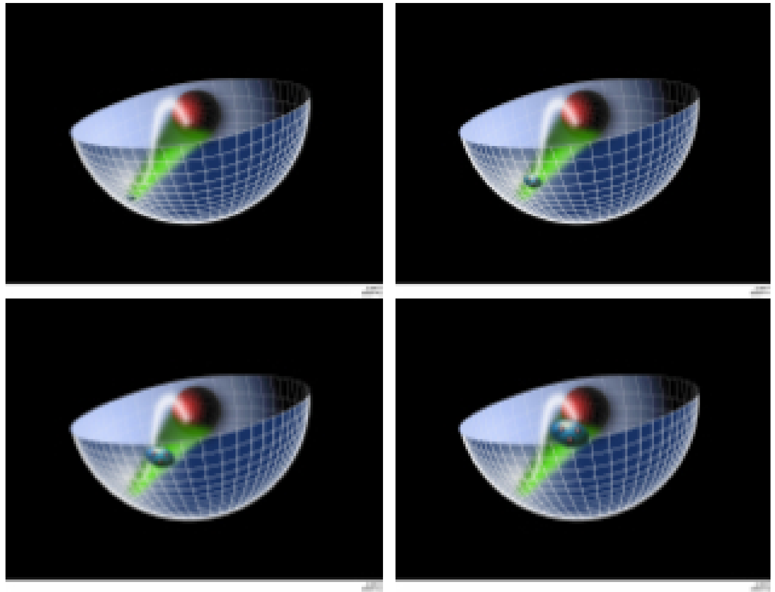
- AdS$_5$ metric:

$$\underbrace{ds^2}_{L_{\rm AdS}} = \frac{R^2}{z^2}\big(\underbrace{\eta_{\mu\nu}dx^\mu dx^\nu}_{L_{\rm Minkowski}} - dz^2\big)$$



- A distance $L_{\rm AdS}$ shrinks by a warp factor z/R as observed in Minkowski space $(dz = 0)$:

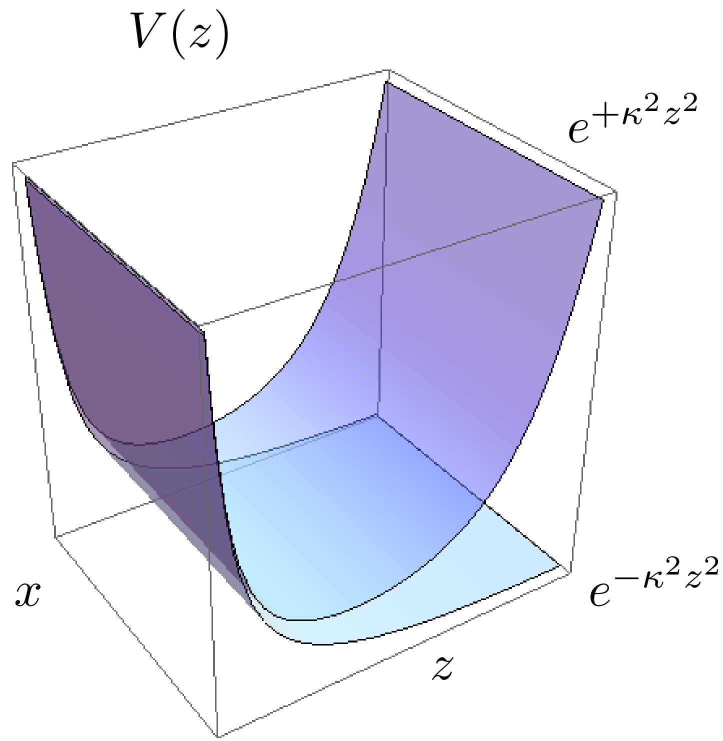
$$L_{\rm Minkowski} \sim \frac{z}{R}\, L_{\rm AdS}$$

- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \to \lambda x^\mu$, $z \to \lambda z$, the variable z acts like a scaling variable in Minkowski space

- Short distances $x_\mu x^\mu \to 0$ maps to UV conformal AdS$_5$ boundary $z \to 0$

- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{\rm QCD}$ maps to large IR region of AdS$_5$, $z \sim 1/\Lambda_{\rm QCD}$, thus there is a maximum separation of quarks and a maximum value of z

- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS



- Nonconformal metric dual to a confining gauge theory

$$ds^2 = \frac{R^2}{z^2}\, e^{2A(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $A(z) \to 0$ at small z for geometries which are asymptotically AdS_5

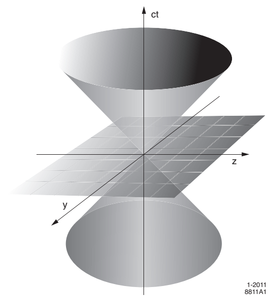
- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R\, \frac{e^{A(z)}}{z}$$

- Consider warp factor $\exp(\pm \kappa^2 z^2)$
- Plus solution $e^{\kappa^2 z^2}$: $V(z)$ increases exponentially confining any object to distances $\langle z \rangle \sim 1/\kappa$
- Minus solution $e^{-\kappa^2 z^2}$: does not provides area law for the Wilson loop

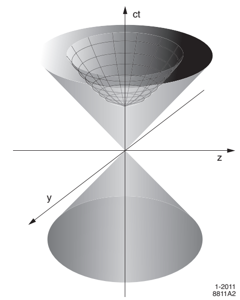
2 Light Front Dynamics

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results
- Forms of Relativistic Dynamics: dynamical *vs.* kinematical generators [Dirac (1949)]
- *Instant form*: hypersurface defined by $t = 0$, the familiar one



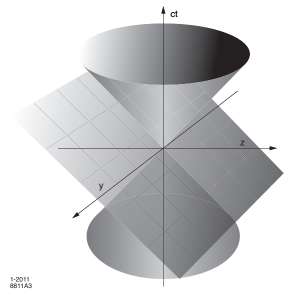
$$H,\ \mathbf{K}\ \text{ dynamical}, \qquad \mathbf{L},\ \mathbf{P}\ \text{ kinematical}$$

- *Point form*: hypersurface is an hyperboloid



$$P^{\mu}\ \text{ dynamical}, \qquad M^{\mu\nu}\ \text{ kinematical}$$

- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$



$$P^{-},\ L^{x},\ L^{y}\ \text{ dynamical}, \qquad P^{+},\ \mathbf{P}_{\perp},\ L^{z},\ \mathbf{K}\ \text{ kinematical}$$

- LF quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of partonic content of a hadron

- Calculation of a matrix elements $\langle P+q|J|P\rangle$ requires boosting the hadronic bound state from $|P\rangle$ to $|P+q\rangle$: extremely complicated in the instant form, whereas $\mathbf{K}$ is trivial in the LF

- Invariant Hamiltonian equation for bound states similar structure of AdS equations of motion: direct connection of QCD and AdS/CFT possible

- LF coordinates

  $x^+ = x^0 + x^3$ light-front time

  $x^- = x^0 - x^3$ longitudinal space variable

  $k^+ = k^0 + k^3$ longitudinal momentum ($k^+ > 0$)

  $k^- = k^0 - k^3$ light-front energy

  $$k \cdot x = \frac{1}{2}\left(k^+ x^- + k^- x^+\right) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

  On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$

Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Compute $\mathcal{M}^2$ from LF Lorentz invariant Hamiltonian equation for the relativistic bound state

$$P_\mu P^\mu |\psi(P)\rangle = \left(P^- P^+ - \mathbf{P}_\perp^2\right) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

- In terms of $n-1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$,

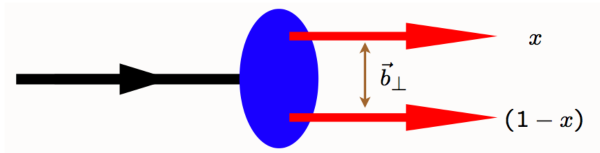
$$\mathcal{M}^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \psi_n^*(x_i, \mathbf{b}_{\perp i}) \sum_\ell \left(\frac{-\nabla^2_{\mathbf{b}_{\perp \ell}} + m_\ell^2}{x_q} \right) \psi_n(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

- Relevant variable in the limit of zero quark masses (dual to the invariant mass)

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$

the x-weighted transverse impact coordinate of the spectator system (x active quark)

- Semiclassical approximation to light-front QCD does not account for particle creation and absorption
- For a two-parton system $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$



- To first approximation LF dynamics depend only on the invariant variable ζ, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iL^z \varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular φ, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$ (P^+, $\mathbf{P}_\perp$, J_z commute with P^-)

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple ($L = |L^z|$)

$$\mathcal{M}^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)$$

where the confining forces from the interaction terms are summed up in the effective potential $U(\zeta)$

- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\Bigg(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement} \Bigg) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n-massless partons at transverse impact separation ζ within the hadron at equal light-front time

3 Light-Front Holographic Mapping of Wave Equations

Higher Spin Modes in AdS Space

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)
- Spin-J in AdS represented by totally symmetric rank J tensor field $\Phi_{M_1 \cdots M_J}$
- Action for spin-J field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ $\left(x^M = (x^\mu, z) \right)$

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \Big(g^{NN'} g^{M_1 M_1'} \cdots g^{M_J M_J'} D_N \Phi_{M_1 \cdots M_J} D_{N'} \Phi_{M_1' \cdots M_J'} \\ - \mu^2 g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1 \cdots M_J} \Phi_{M_1' \cdots M_J'} + \cdots \Big)$$

where D_M is the covariant derivative which includes parallel transport

- Physical hadron has plane-wave and polarization indices along $3+1$ physical coordinates

$$\Phi_P(x, z)_{\mu_1 \cdots \mu_J} = e^{-iP \cdot x} \Phi(z)_{\mu_1 \cdots \mu_J}, \quad \Phi_{z \mu_2 \cdots \mu_J} = \cdots = \Phi_{\mu_1 \mu_2 \cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu = M^2$

- Construct effective action in terms of spin-J modes Φ_J with only physical degrees of freedom

  [H. G. Dosch, S. J. Brodsky and GdT (in preparation)]

- Introduce fields with tangent indices

$$\hat{\Phi}_{A_1 A_2 \cdots A_J} = e^{M_1}_{A_1} e^{M_2}_{A_2} \cdots e^{M_J}_{A_J} \, \Phi_{M_1 M_2 \cdots M_J} = \left(\frac{z}{R}\right)^J \Phi_{A_1 A_2 \cdots A_J}$$

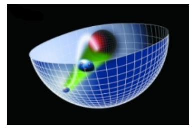
- Find effective action

$$S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \Big(g^{NN'} \eta^{\mu_1 \mu'_1} \cdots \eta^{\mu_J \mu'_J} \partial_N \hat{\Phi}_{\mu_1 \cdots \mu_J} \partial_{N'} \hat{\Phi}_{\mu'_1 \cdots \mu'_J} - \mu^2 \eta^{\mu_1 \mu'_1} \cdots \eta^{\mu_J \mu'_J} \hat{\Phi}_{\mu_1 \cdots \mu_J} \hat{\Phi}_{\mu'_1 \cdots \mu'_J} \Big)$$

  upon μ-rescaling

- Variation of the action gives AdS wave equation for spin-J mode $\Phi_J = \Phi_{\mu_1 \cdots \mu_J}$

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi}(z)}{z^{d-1-2J}} \partial_z \right) + \left(\frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)}$$



with $\hat{\Phi}_J(z) = (z/R)^J \Phi_J(z)$ and all indices along 3+1

Dual QCD Light-Front Wave Equation

$$\boxed{z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle}$$

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Upon substitution $z \rightarrow \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right)+\left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z)=\mathcal{M}^2\Phi_J(z)$$

find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right)\phi_J(\zeta)=M^2\phi_J(\zeta)$$

with

$$U(\zeta)=\frac{1}{2}\varphi''(z)+\frac{1}{4}\varphi'(z)^2+\frac{2J-3}{2z}\varphi'(z)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

- Scaling dimension τ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

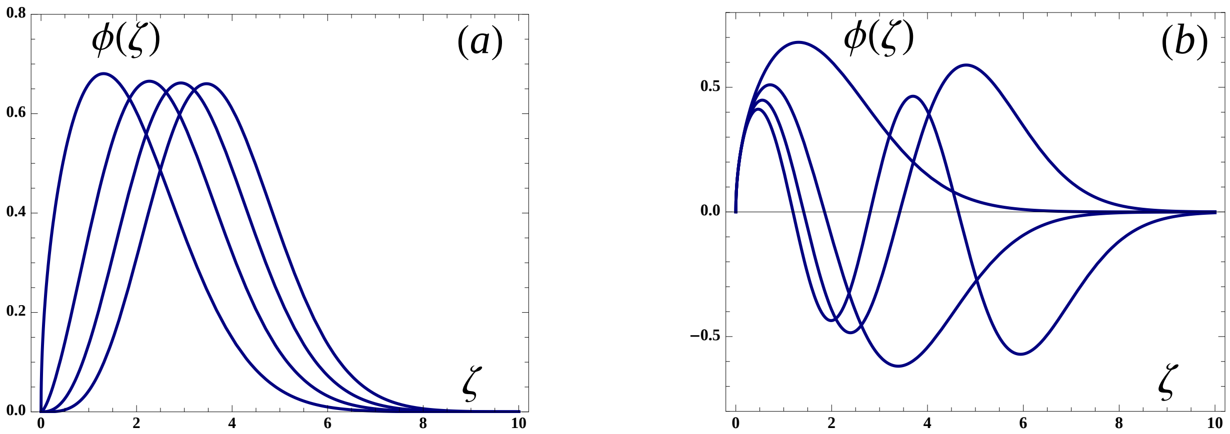
Bosonic Modes and Meson Spectrum

Soft wall model: linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]

- Dilaton $\varphi(z) = +\kappa^2 z^2$ (Minkowski metrics), $\varphi(z) = -\kappa^2 z^2$ (Euclidean metrics)
- Effective potential: $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$
- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta \, |\phi(z)^2| = 1$

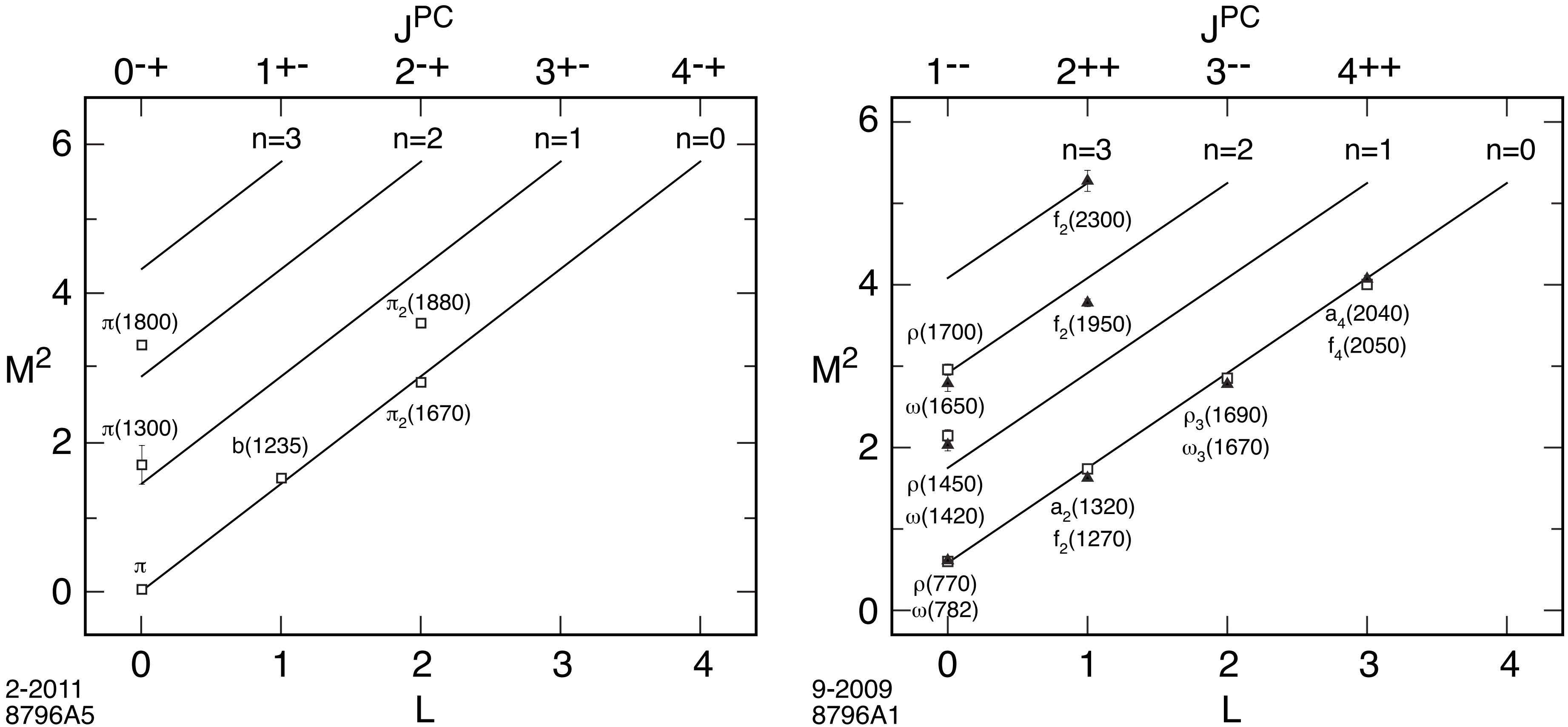
$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues $\mathcal{M}^2_{n,L,S} = 4\kappa^2 \,(n + L + S/2)$



LFWFs $\phi_{n,L}(\zeta)$ in physical space time for dilaton $\exp(\kappa^2 z^2)$: a) orbital modes and b) radial modes

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

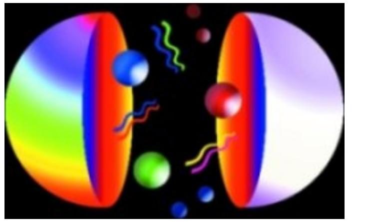


Regge trajectories for the π ($\kappa = 0.6$ GeV) and the $I = 1$ ρ-meson and $I = 0$ ω-meson families ($\kappa = 0.54$ GeV)

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

- Eigenvalues

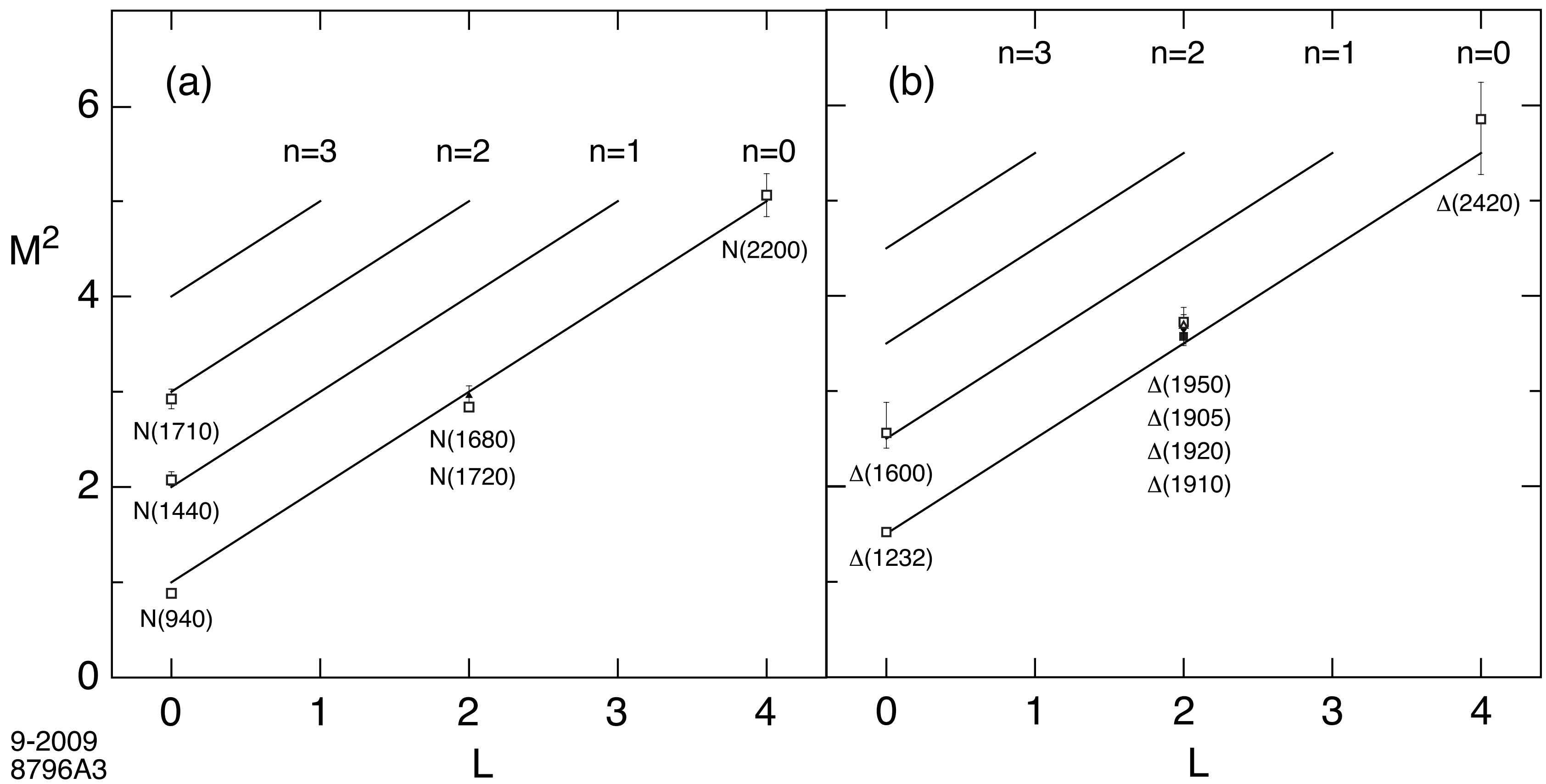
$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Same multiplicity of states for mesons and baryons!

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$



Regge trajectories for positive parity N and Δ baryon families ($\kappa = 0.5$ GeV)

4 Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006)], PRD **77**, 056007 (2008)]

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle \psi(P') | J^\mu | \psi(P) \rangle = (P + P')^\mu F(Q^2)$$

where $Q = P' - P$ and $J^\mu = e_q \bar{q} \gamma^\mu q$

- EM hadronic matrix element in AdS space from coupling of external EM field propagating in AdS with extended mode $\Phi(x, z)$

$$\int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} A^M(x, z) \Phi^*_{P'}(x, z) \overleftrightarrow{\partial}_M \Phi_P(x, z)$$
$$\sim (2\pi)^4 \delta^4 \left(P' - P\right) \epsilon_\mu \left(P + P'\right)^\mu F(Q^2)$$

- How to recover hard pointlike scattering at large Q out of soft collision of extended objects?
[Polchinski and Strassler (2002)]

- Mapping of J^+ elements at fixed light-front time: $\Phi_P(z) \;\Leftrightarrow\; |\psi(P)\rangle$

- Electromagnetic probe polarized along Minkowski coordinates, $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} V(Q,z), \quad A_z = 0$$

- Propagation of external current inside AdS space described by the 'free' AdS wave equation

$$\left[z^2 \partial_z^2 - z\,\partial_z - z^2 Q^2\right] V(Q,z) = 0$$

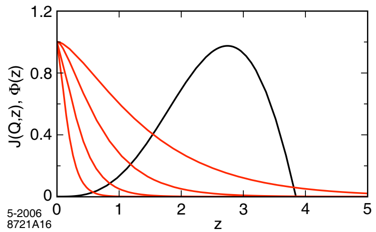
- Solution $\;V(Q,z) = zQK_1(zQ)$

- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

$$\Phi_P(x,z) = e^{-iP\cdot x}\,\Phi(z), \qquad \Phi(z) \to z^\tau, \quad z \to 0$$

- Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons Φ_P and $\Phi_{P'}$, with the non-normalizable mode $V(Q,z)$ dual to external source [Polchinski and Strassler (2002)].

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, V(Q,z)\, \Phi_J^2(z) \to \left(\frac{1}{Q^2}\right)^{\tau-1}$$



At large Q important contribution to the integral from $z \sim 1/Q$ where $\Phi \sim z^\tau$ and power-law point-like scaling is recovered [Polchinski and Susskind (2001)]

- QCD Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\Big(i\mathbf{q}_\perp \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k}\Big) \left|\psi_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

- Consider a two-quark π^+ Fock state $|u\overline{d}\rangle$ with $e_u = \frac{2}{3}$ and $e_{\overline{d}} = \frac{1}{3}$

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_\perp e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp (1-x)} \left|\psi_{u\bar{d}/\pi}(x, \mathbf{b}_\perp)\right|^2$$

with normalization $F_\pi^+(q=0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left|\psi_{u\bar{d}/\pi}(x, \zeta)\right|^2$$

where $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$

- Compare with electromagnetic FF in AdS space [Polchinski and Strassler (2002)]

$$F(Q^2) = R^3 \int \frac{dz}{z^3} \, V(Q,z) \Phi^2_{\pi^+}(z)$$

where $V(Q,z) = zQK_1(zQ)$

- Use the integral representation

$$V(Q,z) = \int_0^1 dx \, J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right)$$

- Compare with electromagnetic FF in LF QCD for arbitrary Q. Expressions can be matched only if LFWF is factorized

$$\psi(x,\zeta,\varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

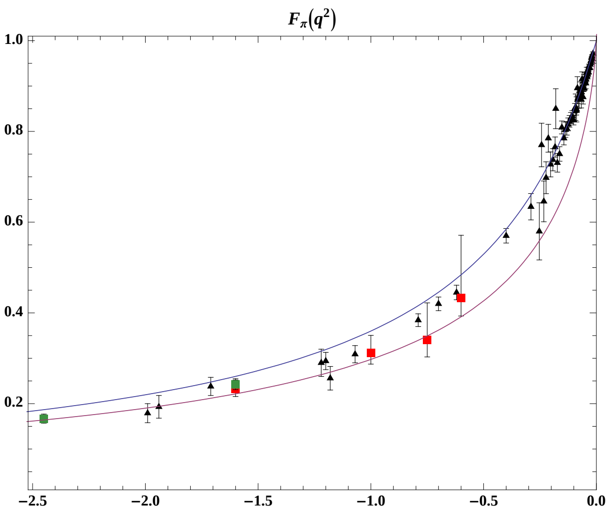
$$X(x) = \sqrt{x(1-x)}, \qquad \phi(\zeta) = \left(\frac{\zeta}{R}\right)^{-3/2} \Phi(\zeta), \qquad z \to \zeta$$

- Form factor in soft-wall model expressed as $\tau - 1$ product of poles along vector radial trajectory (twist $\tau = N + L$) [Brodsky and GdT, Phys.Rev. D77 (2008) 056007]

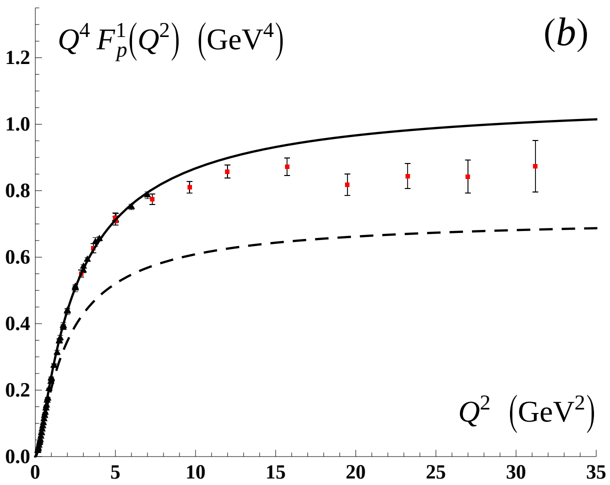
$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{\tau-2}}^2}\right)}$$

where ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n + 1/2)$ [negative SL (dashed line) $\to$ positive TL dilaton (continuous)]

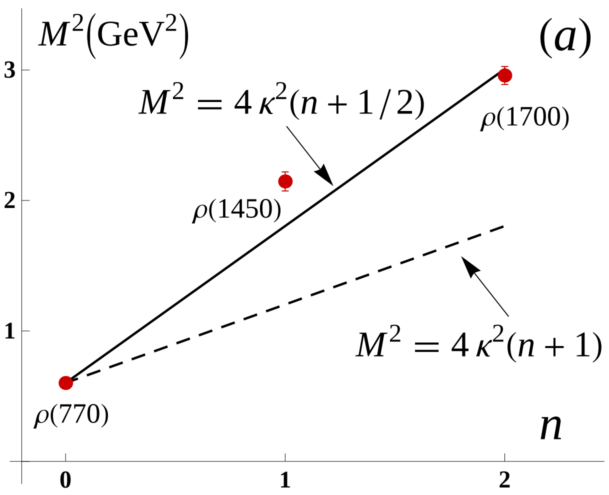
- Correct scaling incorporated in the model
- "Free current" $V(Q, z) = zQK_1(zQ) \to$ infinite radius (mauve), no pole structure in time-like region
- "Dressed current" non-perturbative sum of an infinite number of terms $\to$ finite radius (blue)



Pion form factor ($\tau = 2$)



Proton form factor ($\tau = 3$)



VM radial trayectories

Nucleon Form Factors

- Light Front Holographic Approach [Brodsky and GdT]
- EM hadronic matrix element in AdS space from non-local coupling of external EM field in AdS with fermionic mode $\Psi_P(x,z)$

$$\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \overline{\Psi}_P(x,z)\, e^A_M \Gamma_A A^M(x,z) \Psi_P(x,z)$$
$$\sim (2\pi)^4 \delta^4 \left(P' - P\right) \epsilon_\mu \langle \psi(P'), \sigma' | J^\mu | \psi(P), \sigma \rangle$$

- Effective AdS/QCD model: additional term in the 5-dim action

  [Abidin and Carlson, Phys. Rev. D79, 115003 (2009)]

$$\eta \int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \overline{\Psi}\, e^A_M e^B_N \left[\Gamma_A, \Gamma_B\right] F^{MN} \Psi$$

  Couplings η determined by static quantities

- Generalized Parton Distributions in gauge/gravity duals

  [Vega, Schmidt, Gutsche and Lyubovitskij, Phys.Rev. D83 (2011) 036001]

  [Nishio and Watari, arXiv:1105.290]

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q,z)\Psi_+^2(z)$$

- Nucleon AdS wave function

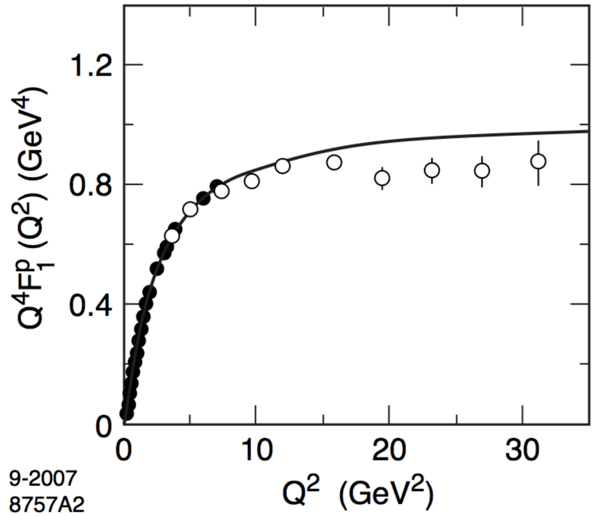
$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2}\sqrt{\frac{2n!}{(n+L)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^2(z) = 1$$

- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\kappa^2 z^2 x/(1-x)}$$



- Find

$$\boxed{F_1^p(Q^2) = \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}}$$

Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_1{}^p_{N \to N^*}(Q^2) = R^4 \int \frac{dz}{z^4} \, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

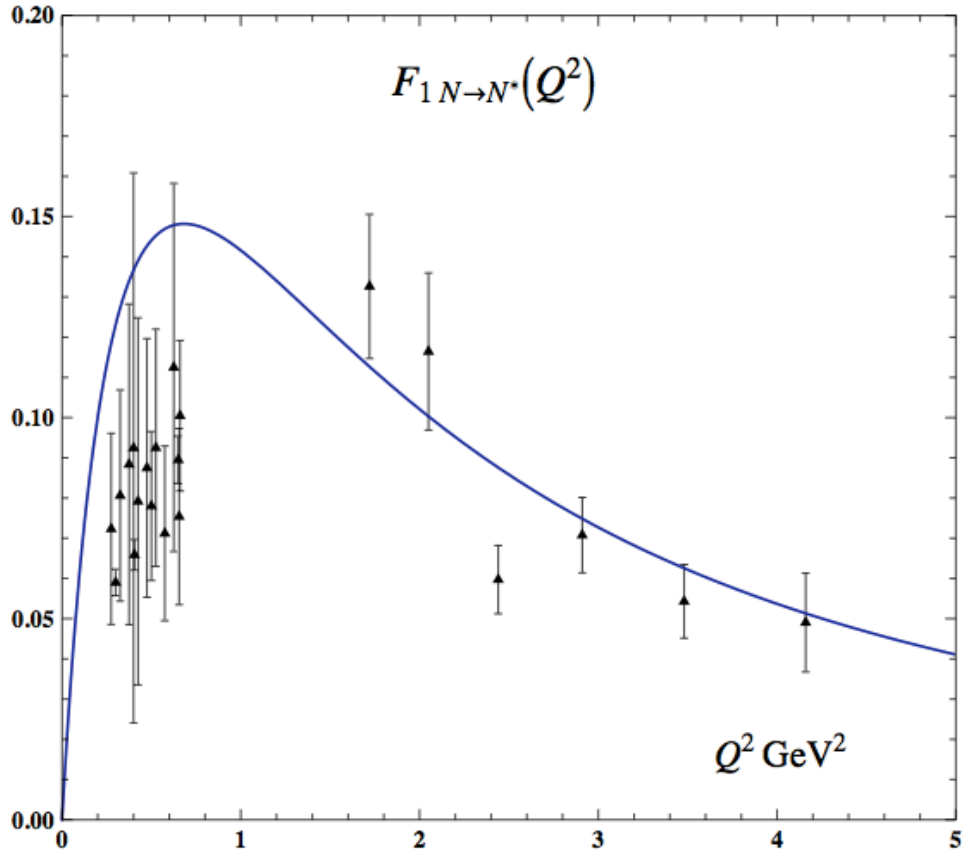
- Orthonormality of Laguerre functions $\left(F_1{}^p_{N \to N^*}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$\boxed{F_1{}^p_{N \to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}}$$

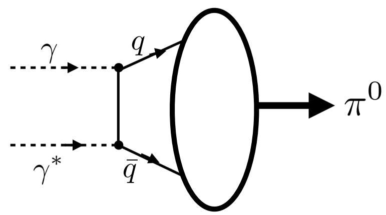
with $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n + 1/2)$



Data from I. Aznauryan, *et al.* CLAS (2009)

Meson Transition Form-Factors

[S. J. Brodsky, Fu-Guang Cao and GdT, arXiv:1005.39XX]



- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz\, \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$
$$\sim (2\pi)^4 \delta^{(4)}\,(p_\pi + q - k)\, F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q)(p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Take $\quad A_z \propto \Phi_\pi(z)/z, \quad \Phi_\pi(z) = \sqrt{2}\,\kappa\, z^2 e^{-\kappa^2 z^2/2},$

- Find $\quad \left(\phi(x) = \sqrt{3} f_\pi x(1-x), \quad f_\pi = \kappa/\sqrt{2}\pi\right)$

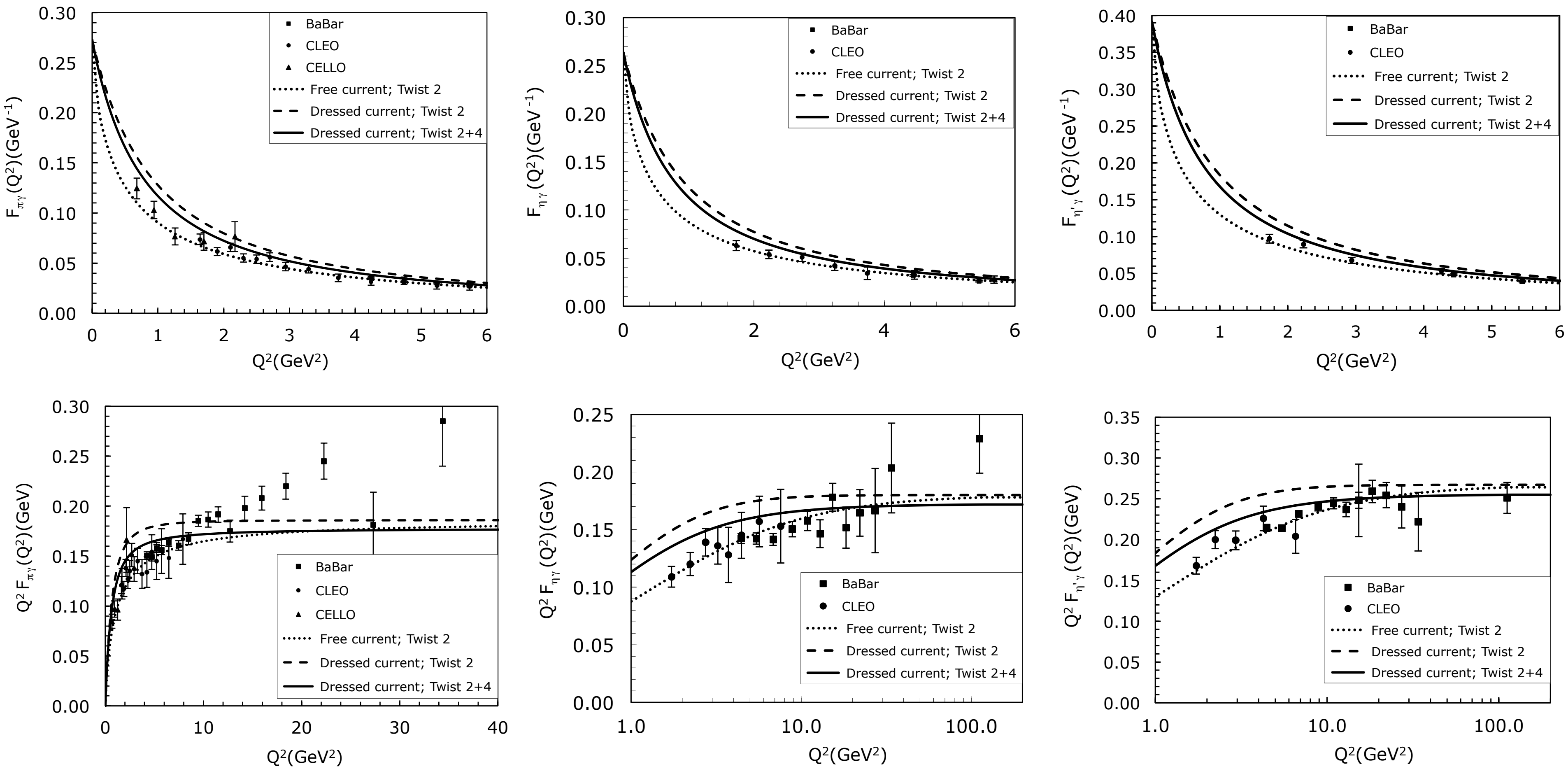
$$\boxed{Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[1 - e^{Q^2(1-x)/4\pi^2 f_\pi^2\, x}\right]}$$

normalized to the asymptotic DA [Musatov and Radyushkin (1997)]

- Large Q^2 TFF is identical to first principles asymptotic QCD result $Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi$

[Lepage and Brodsky (1980)]

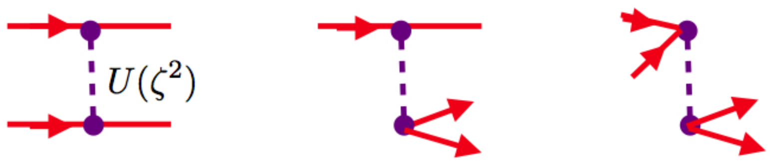
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA


BaBar
CLEO
CELLO
Free current; Twist 2
Dressed current; Twist 2
Dressed current; Twist 2+4
$F_{\pi\gamma}(Q^2)(GeV^{-1})$
$F_{\eta\gamma}(Q^2)(GeV^{-1})$
$F_{\eta'\gamma}(Q^2)(GeV^{-1})$
$Q^2 F_{\pi\gamma}(Q^2)(GeV)$
$Q^2 F_{\eta\gamma}(Q^2)(GeV)$
$Q^2 F_{\eta'\gamma}(Q^2)(GeV)$
$Q^2(GeV^2)$

5 Confinement Interaction and Higher Fock States

[S. J. Brodsky and GdT (in progress)]

- Is the AdS/QCD confinement interaction responsible for quark pair creation?
- Only interaction in AdS/QCD is the confinement potential
- In QFT the resulting LF interaction is a 4-point effective interaction wich leads to $qq \to qq$, $q \to qq\bar{q}$, $q\bar{q} \to q\bar{q}$ and $\bar{q} \to \bar{q}q\bar{q}$



- Create Fock states with extra quark-antiquark pairs.
- No mixing with $q\bar{q}g$ Fock states (no dynamical gluons)
- Explain the dominance of quark interchange in large angle elastic scattering

  [C. White *et al.* Phys. Rev D **49**, 58 (1994)

Space and Time-Like Pion Form Factor

[GdT and S. J. Brodsky, arXiv:1010.1204]

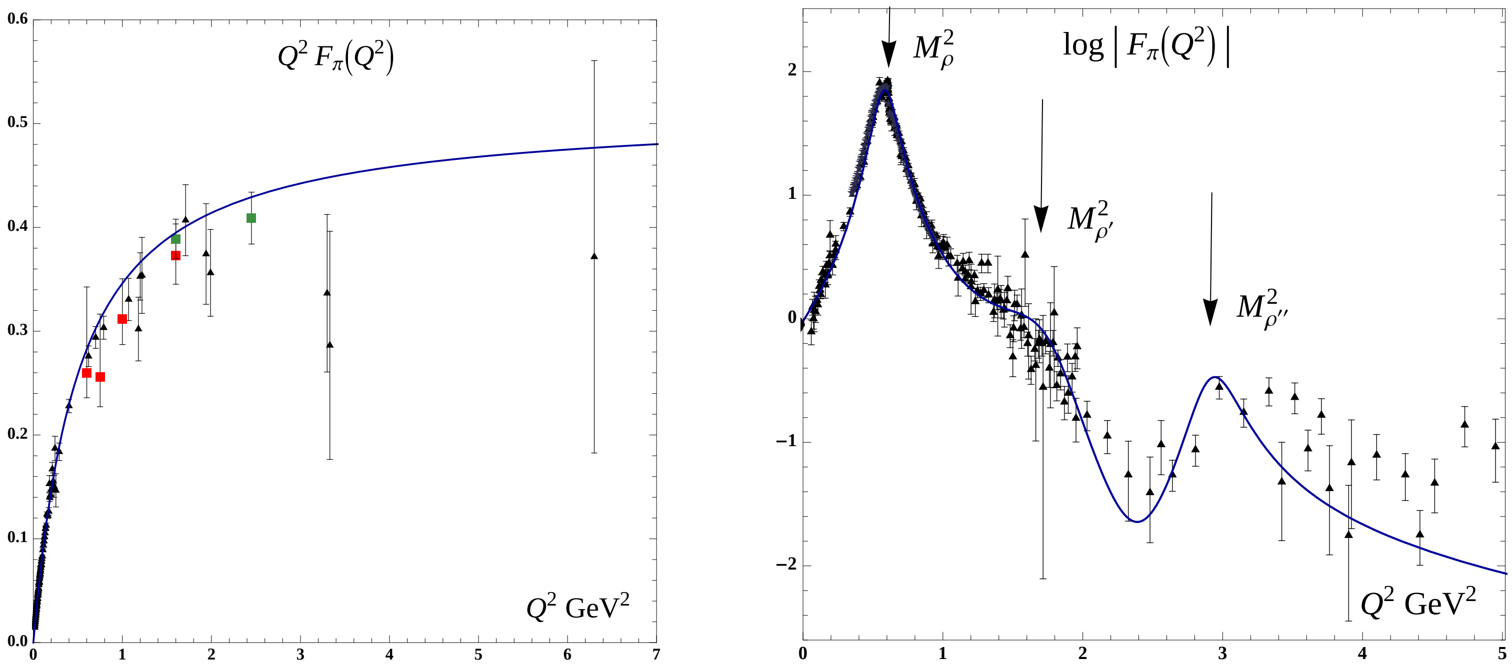
- Higher Fock components in pion LFWF

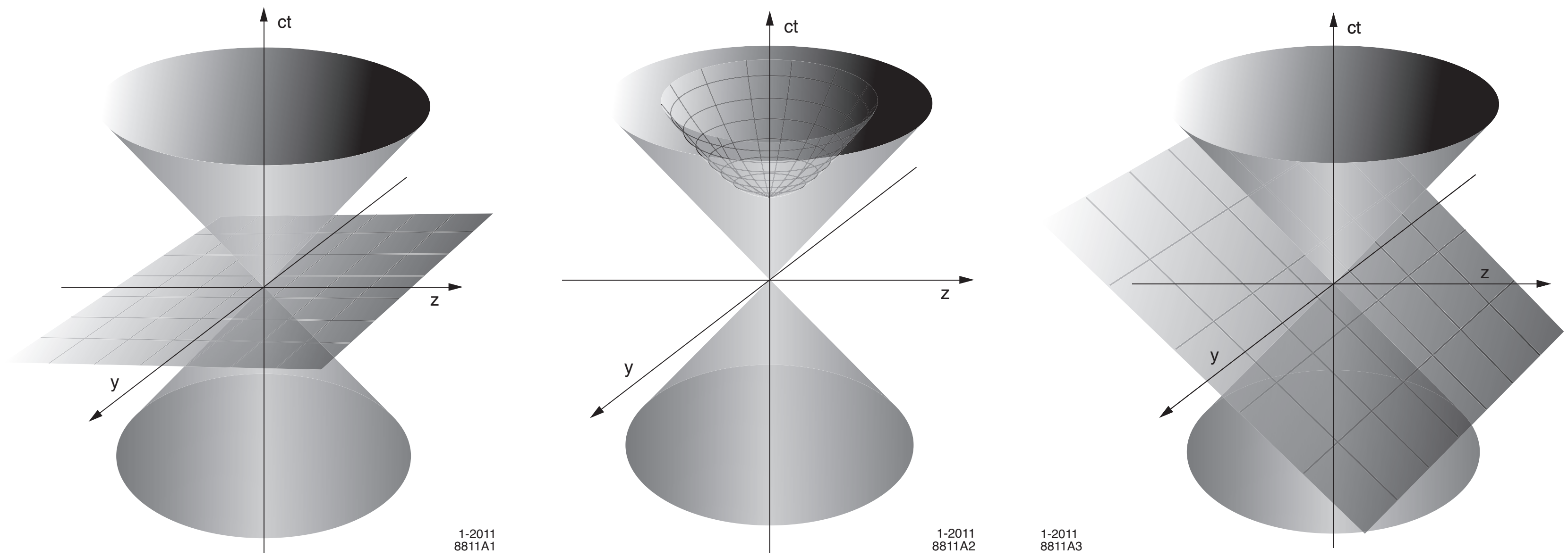
$$|\pi\rangle = \psi_{q\bar{q}/\pi} |q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}/\pi} |q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

corresponding to interpolating operators $\mathcal{O} = \overline{\psi}\gamma^+\gamma^5\psi$ and $\mathcal{O} = \overline{\psi}\gamma^+\gamma^5\psi\psi\overline{\psi}$

- Expansion of LFWF up to twist 4

$\kappa = 0.54 \text{ GeV}, \Gamma_\rho = 130, \; \Gamma_{\rho'} = 400, \; \Gamma_{\rho''} = 300 \text{ MeV}, P_{q\bar{q}q\bar{q}} = 13\%$





“ Working with a front is a process that is unfamiliar to physicists. But still I feel that the mathematical simplification that it introduces is all-important. I consider the method to be promising and have recently been making an extensive study of it. It offers new opportunities, while the familiar instant form seems to be played out ”

P.A.M. Dirac (1977)