A Novel Intensimetric Technique for Monitoring the Radiative Properties of Sound Fields*

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The development and application of the intensimetric technique for monitoring the radiative properties of sound fields are discussed. The experimental results of three case studies in the areas of room acoustics, musical acoustics, and noise control prove the new technique to be particularly valuable. Special attention is given to the measurement errors of the newly defined component of sound intensity, called oscillating intensity, and to the reliability of the sound intensity meter used. It is shown that the oscillating intensity is almost insensitive to phase errors.

0 INTRODUCTION

In recent years a rigorous theoretical framework for representing the sound energy transfer process inside the acoustic field has been developed [1]–[3]. This scheme depicts the complex phenomenon of energy transport effected by three-dimensional sound waves on the basis of a new kind of analysis of the instantaneous and time-averaged properties of the sound intensity. Roughly speaking, the new scheme splits the sound intensity—the product $pv$ of the sound pressure and the air particle velocity—into two components, called radiating and oscillating intensity according to their time-averaged behavior. The former is responsible for the sound energy, which is propagated to long distances (compared to the wavelength) within the sound field, whereas the latter indicates the presence of energy that just flows, instant by instant, back and forth, passing through the measurement point. Thus the oscillating intensity at a given point does not convey energy far from that point along the power flux streamlines, as the time-averaged radiating (or active) intensity $A$ does, but it confines—averaged over time—part of the sound energy to the region close to that given point. This oscillating energy transfer mechanism can be visualized by means of an intensity ellipsoid which has a well-determined time-averaged orientation in space. Because of this orientation property, the time-averaged oscillating intensity has been called sound intensity polarization. Different from the electromagnetic case, however, the term "polarization" indicates here a spatial property of the sound energy transfer, not a spatial property of the wave propagation (mass polarization) inasmuch as aerial sound waves are of the longitudinal kind. The sound intensity polarization is due to the differences between the time histories of pressure and air particle velocity. Each time the two time histories differ somehow, the transfer of sound energy wanders from the case of perfect radiation, which is modeled by a plane progressive wave where the pressure and velocity time histories are equal. Thus the prototypical sound field, where all of the acoustic energy is bound in the proximity of an arbitrary measurement point, is the plane standing wave. For this simple one-dimensional field all the quantitative information regarding the sound intensity polarization is given by its (scalar) effective value $R$, since all the energy oscillations in space occur along the same axis of the standing wave.

Radiating intensity and oscillating intensity are both detectable and can be evaluated experimentally at any point within a general sound field. The global energy transport can be represented by means of a vector quantity, the time-averaged radiating intensity $A$, and a scalar quantity, the effective value $R$ of the sound intensity polarization which measures the amount of oscillating intensity along all the directions in space. As already mentioned, the full description of the oscillations of sound energy, that is, their amplitude and distribution along the different directions in space, can be achieved...
by the graphical rendering of the intensity ellipsoids. This latter task is beyond the aim of the present engineering report, which is only concerned with the study of the effective value $R$ of intensity polarization.

To support this theory of sound energy fluxes, an adequate intensimetric technique and a special sound intensity meter able to handle six acoustic signals simultaneously in order to determine the value of $R$, have been developed and later applied to three case studies. This work reports about new research findings and implications inferred from the collected experimental data.

1 DEVELOPMENT OF THE INTENSIMETRIC TECHNIQUE

As is known, the starting point of any intensimetric technique consists in capturing and digitizing sound pressure time histories from closely spaced microphones placed within the field under analysis. Following the usual time-domain approach, the data processing can be broken down into three major steps:

1) Digital filtering for frequency band analysis
2) Computation of sound pressure and air particle velocity time histories
3) Computation of standard and newly defined energetic quantities.

The first step involves well-established procedures in digital filtering which are not relevant for the goal of the present study.

The standard algorithm for computing the instantaneous velocity of air particles is based on the finite-difference approximation of Euler's equation which, for each spatial component, reads

\[ v(t) = -\frac{1}{\rho_0 d} \int_{-\infty}^{+\infty} [p_2(\tau) - p_1(\tau)] \, d\tau \tag{1} \]

where $p_1$ and $p_2$ are the acoustic pressures from the microphones, $\rho_0$ is the equilibrium density of air, and $d$ is the spacing between the two microphones. The calculation of the instantaneous sound pressure must be made for the same two points $p_1$ and $p_2$ that were used in Eq. (1), and simply consists of the arithmetic mean of the two channels,

\[ p(t) = \frac{p_1(t) + p_2(t)}{2} \tag{2} \]

The third step is devoted to the time-averaging process and the evaluation of the energy quantities. This step includes the core of the new technique, which is now briefly described.

First the sound pressure level (SPL) is obtained from $p(t)$ by squaring and time averaging to get $\langle p^2 \rangle$ and then reporting the square root of this value using the standard dB re $2 \times 10^{-5}$ Pa scale. The vector quantity $A = \langle pv \rangle$—usually called active intensity—is similarly obtained by the time-averaging process of the instantaneous sound intensity $j(t) = p(t) \cdot v(t)$.

Then the computation of the time-averaged oscillating intensity is performed. This is a twofold algorithm. First the sound intensity $j(t)$ is decomposed into its radiating and oscillating instantaneous contributions $a(t)$ and $r(t)$, respectively, which are defined as

\[ a(t) := \frac{P^2 \langle pv \rangle}{\langle p^2 \rangle} \tag{3} \]

\[ r(t) := j(t) - a(t) = \frac{\langle p^2 \rangle pv - \langle p^2 \rangle \langle pv \rangle}{\langle p^2 \rangle} \tag{4} \]

Second the computation of the time-averaged value of the oscillating intensity is performed. This is not a simple process. From the given definition of $r$, in fact, the first statistical moment always vanishes, that is, $\langle r \rangle \equiv 0$. This important feature of the new energy analysis prevents the representation of the time-averaged oscillating intensity by a vector quantity. Thus, differently from $a(t)$, the nonvanal time-averaged behavior of the oscillating intensity is expressed in terms of the second-order statistical moment getting a suitable measure of oscillations of sound energy. This quantity can be written as

\[ r^2 = \sqrt{2 \langle r \circ r \rangle} \], \text{where} \langle \circ \rangle \text{stands for the tensor multiplication of two vectors.} \]

from this definition, whose details can be found in [2], the effective values $R$ of $r$ are obtained,

\[ R = \sqrt{2 \langle r^2 \rangle} \tag{5} \]

where $r^2$ stands now for the usual scalar multiplication of two vectors.

2 RELIABILITY AND ERROR ESTIMATE OF THE MEASURING PROCESS

The reliability of the developed intensity meter has been tested in accordance with international standards [4] by measuring the pressure intensity index $\delta_{pa}$. This indicator is the measured difference of sound pressure and active intensity levels when the two microphones are exposed to exactly the same instantaneous sound pressure field. An ideal measurement system using Eq. (1) would indicate no active sound intensity. The measurement has been carried out in the frequency range of 50–5000 Hz by means of a B&K-UA0914 acoustic calibrator fed with a white-noise source B&K-ZI0055. The sound intensity probe was assembled with a pair of phase-matched B&K-4181 microphones. Fig. 1 compares the $\delta_{pa}$ values obtained with the standard class I and II curves for the spacers used. Globally the intensity meter meets the class I requirements except for the 5-kHz band, which is the upper frequency limit of the B&K-UA0914.

It is known from the literature that many sources of error can affect the measurement of the sound intensity, some of them fundamental while others are due to technical limitations. An account of the most important measurement problems that can arise when dealing with sound intensity can be found in [5]. The measurement of the oscillating intensity, however, needs a special...
treatment due to its nonstandard definition. In the following, experimental and theoretical aspects will be dealt with together, since their mutual influence is the major concern of the measuring procedure.

The more insidious sources of errors of any intensimetric technique are of the systematic kind. They are principally caused both by the fundamental error due to the finite-difference approximation in the computation of the air particle velocity [using Eq. (1)] and by the technical limitations due to the different phase responses of the two microphones assembled in the sound intensity probe. The former can be reduced by correctly selecting the microphone spacing for the upper measurement frequency. The difference in phase response between the microphones cannot be completely eliminated and it becomes critical when the difference of the true "field phases" between the two points where the sound transducers are placed is small (a few degrees). In this situation, the extraneous phase shift introduced by the microphones can introduce a significant error in the intensity estimate when the probe spacing is very small with respect to the wavelength. An even more critical situation arises when—indeed of the wavelength—sound intensity measurements have to be carried out within a plane standing-wave field, since in this case the true field phase difference theoretically is vanishing everywhere and consequently the extraneous phase-shift information due to the microphones simply reveals the systematic error of the intensimetric system in use.

In the following an error estimate on the effective value 
 of the oscillating intensity will be developed, and the roles of systematic (that is, bias) and random errors will be pointed out during the error propagation procedure.

The error estimate 
 is easily obtained considering that the definition of 
 is based on the fundamental first-order statistical property 
 = 0. So the first-order approximation to the error 
 can be written as

$$
\delta R = \sqrt{2} \sum_{i=1}^{3} \delta r_i,
$$

where the subscript 
, taking the values 1, 2, and 3 stands for the three orthogonal spatial directions. This expression allows to evaluate 
 in terms of the estimates 
. Eq. (4) involves the subtraction of two terms both containing the product 
, which is, by far, the quantity most affected by phase errors. The crucial point in the derivation is to make sure that, while calculating 
, the errors of bias associated with the product 
 have the same algebraic sign in both terms of the subtraction. Thus bias errors completely cancel during the calculation. The final result of the estimate (whose calculation details are reported in the Appendix) does not contain any bias contribution and can be simply expressed as

$$
\delta R = 4 \sqrt{2} \sum_{i=1}^{3} \epsilon \rho^2 A_i.
$$

The term 
 is the relative error on the time-averaged squared sound pressure 
, which can be reduced by a careful choice of the integration time and by taking into account the sensitivity differences between the two microphones. 
 is the error-free ith component of active intensity 
. Following Eq. (6) an important property of 
 can be inferred—unlike active intensity, oscillating intensity is almost insensitive to phase errors. Thus this quantity has a high measurement precision, even in sound field conditions that are usually regarded as unfavorable for measuring active intensity (that is, in the near field of a sound source or in a very reverberant environment).

3 APPLICATIONS

The intensimetric technique and the related measurement instrument have been used in studies of three dif-

![Fig. 1. Plot of residual pressure intensity index \( \delta R \) measured by intensity meter compared with reference values for class I and II instruments according to IEC 1043 [4]. Measurements were taken with acoustical calibrator B&K-UA0914 fed by a white-noise source B&K-Zi0055. Upper frequency limit of calibrator is 5 kHz.](image-url)
different acoustical problems in the fields of musical acoustics, noise control, and room acoustics.

As a first case study, the one-dimensional sound field of an organ pipe has been investigated more extensively than in previous works [6], [7]. The second application examines the sound field generated by a moped through the combined mapping of radiating and oscillating intensities. This kind of monitoring gives a more complete description of the sound energy transfer than usual intensity maps [3]. The last case, a room acoustics application, consists of octave-band frequency monitoring of the locally confined energy (intensity polarization) at one stall of the Teatro Comunale in Ferrara, Italy. This last application was preceded by tests of the intensimetric apparatus by means of measures within a duct equipped with different terminations.

3.1 Musical Acoustics: Thorough Intensimetric Analysis of the Sound Field of an Organ Pipe

3.1.1 Measurement Setup

The measurements were carried out under steady sound field conditions along the axis and in front of the mouth of a square open wooden organ pipe 120 mm wide and 1800 mm long. The pipe was connected to a blowing apparatus that was able to maintain steady vibrations of the air column inside the pipe, which generates the typical majestic sound of the organ. The inner sound field is built up by the superposition of nearly standing waves having a harmonic spectrum whose fundamental frequency is 82.5 Hz, corresponding to the E2 of the tempered musical scale. On one side along the pipe axis, 44 holes were made on 40-mm centers. A special sound intensity probe assembled with two B&K 4135 microphones in side-by-side configuration and a 40-mm spacer was inserted in specific pairs of adjacent holes. The measurements were made at 33 points, each 80 mm apart. The first group of 22 points was inside the pipe, whereas the remaining 11 points were outside the pipe’s upper end. The holes not housing the probe were sealed by screws. A set of 11 points was also measured in front of the pipe’s mouth on a straight line perpendicular to the pipe axis, in order to compare the radiative properties of the two openings. The measured frequency range was 50–630 Hz. The probe reversal technique [8] was adopted to minimize phase errors in radiating intensity. As is known, this procedure consists of making two series of measurements at the same positions but reversing the probe orientation between measurements. The final results can be written in linear units as

$$\begin{align*}
A & = \frac{A_I - A_{II}}{2}, \\
R & = \frac{R_I + R_{II}}{2}
\end{align*}$$

where the subscripts I and II refer to the two sets of measurements and the minus sign in the formula for $A$ takes into account the behavior of this vectorial quantity when the probe orientation is inverted. Even though the measured $R$ proved to be robust against phase errors, averaging was adopted for consistency with the other quantity.

3.1.2 Experimental Results and Discussion

The aim of these measurements was to give the complete sound intensity mapping of the combined inner and outer sound fields on the axis of an open organ pipe and of the sound field in front of the pipe’s mouth. Using the radiating and oscillating sound intensities $A$ and $R$, respectively [3], it is possible to compare, on physical grounds, the actual radiation characteristics of the sound emitted from different parts of a source and to distinguish between the so-called acoustic near field ($|A| < R$) and the far field ($|A| > R$). Furthermore this application served to test the influence of the phase error on the value of $R$.

In Fig. 2 the overall levels of $A$ and $R$ along the pipe’s

![Fig. 2. Plot of overall values of oscillating intensity effective value $R$ and active intensity $A$ along axis of open organ pipe. Mouth is located at the origin of the axes, and the last 11 points on the right are outside the upper end of the pipe.](image-url)
axial are shown on a logarithmic scale, with the origin of the plot located at the mouth. The first 22 points are inside the pipe and show that in this region \( R \) has a much higher level than \( A \), as expected for a standing-wave pattern. Moreover the two maxima of oscillating intensity coincide with points where kinetic and potential energy densities have the same values (see also [6]). When one of the two kinds of energy falls, that is, the respective acoustic variable \((p \text{ or } v)\) shows a node, then the other one cannot oscillate at all. When passing from the last inner point to the outer region, which is located above the pipe's top, one finds that the levels of \( A \) and \( R \) drop considerably (drastically for the latter quantity). The decrease continues while moving upward, with pronounced but different slopes for the two quantities. It is very interesting to inspect the resulting intersection point of \( A \) and \( R \), which is located at nearly 850 mm from the upper end. This point physically separates the near field of the pipe, where most of the sound energy is confined, and the far field, where, conversely, the energy is chiefly radiated. Next the overall values of the two quantities \( R \) and \( A \) were measured in front of the pipe's mouth and are presented in Fig. 3, with the origin coinciding again with the location of the mouth. Even if the levels at the first point of the plot are comparable with the respective levels for an equal distance from the upper end of the pipe (see Fig. 2), there is a major difference between the two openings as far as the balance between \( R \) and \( A \) is concerned. In fact, in this second case the intersection point between the two intensities occurs at a distance that is about one-half the 850 mm reported for the upper end. This finding implies that the pipe's mouth is more radiative than the upper termination, because for the former the passage from near to far field occurs closer to the pipe termination.

Adoption of the probe reversal technique gave the opportunity to test the sensitivity of \( R \) to phase errors. The plot in Fig. 4 combines the results of the two sets of measurements of \( R \) (at the same points as in Fig. 2) for the two probe orientations, forward \((R_0)\) and reverse \((R_m)\). One can see easily that globally there are only slight differences between the forward and reverse sets of measurements. Only at two points situated at the midlength of the pipe was a discrepancy found, but there the velocity of the air particles has indeed a node and the measurement procedure as a whole is very critical. The rest of the plot shows a very good agreement between the two sets, thus confirming the stated immunity of \( R \) from the bias errors caused by phase mismatch between the microphones (see Appendix).

### 3.2 Noise Control: Sound Intensity Mapping of a Moped

#### 3.2.1 Measurement Setup

This second application is the sound intensity mapping of the noise emitted by a stationary and idling moped in a free outdoor environment. The mapping was obtained from a series of measurements spanned over a grid on one side of the moped whose engine was kept at a low speed. As shown in Fig. 5, the grid was made up of 48 points, eight along the length \((x)\) direction and six along the height \((y)\) direction of the moped. The distance between the grid nodes was 100 mm along the \(x\) axis and 50 mm along the \(y\) axis. The orthogonal \(z\) axis was directed toward the moped, intersecting its longitudinal section at 150 mm away from the grid plane. The first row of the grid was located at a height of 150 mm from the ground. A three-dimensional sound intensity probe assembled with three pairs of B&K 4181 phase-matched microphones and 50-mm spacers was used. The frequency analysis was performed in the range of 50–1000 Hz.

#### 3.2.2 Experimental Results and Discussion

Fig. 6 shows the \( P_{\text{rms}} \) contour map and Fig. 7 the contour map for \( |A| \). In Fig. 8 a combination of the

![Fig. 3. Plot of overall values of \( R \) and \( A \) in front of mouth of pipe, on an axis perpendicular to the pipe. Mouth is located at the origin of the axes.](image-url)
vector field $A$ and the effective value $R$ is presented. This representation gives a synthetic description of energy fluxes for this complex sound source. In each figure linear units are used.

The first remark on the data regards the values of $|A|$ and $R$, which, though represented using the same chromatic scale, actually range between very different values. The averaged oscillating intensity always has values that are much higher than those of the radiating intensity, indicating, as was pointed out in the first application, that the measurement grid is located within a region where the efficiency of sound radiation is poor and most of the sound energy is confined.

The graphics show also how the different parts of the source contribute to the emission of sound energy. The maxima of $P_{m}$ are localized at grid points corresponding to mechanical parts and accessories which are especially noisy, such as the transmission box, the chain connection, the muffler junction, and its termination. Whereas for three of the former points there is a match between maxima of $|A|$ and $R$, in the case of the mufflers' termination this match gets lost. So it is also possible to characterize the parts of a large and complex source with respect to their major or minor attitude to radiate or store sound energy, depending on the behavior of $|A|$ and $R$ found at the corresponding positions.

### 3.3 Room Acoustics: How the Intensimetric Technique Accounts for the Reverberation of Sound

#### 3.3.1 Measurement Setups

For this application two experimental setups were used. In the first setup the measuring apparatus was tested under strictly controlled experimental conditions. The equipment consisted of a 4-m-long Plexiglas duct with square section ($0.28 \times 0.28 \text{ m}^2$), one end of which was plugged with a loudspeaker for generating the stimulus and the other was equipped with different termina-

![Fig. 4](image1.png)

**Fig. 4.** Combined plot of the two sets of $R$ values obtained with direct and reversed probe orientations, indicated as $R_1$ and $R_2$, at the same points as in Fig. 2.

![Fig. 5](image2.png)

**Fig. 5.** Combined plot of moped and of $8 \times 6$ measurement grid traced for sound intensity mapping. Points on grid are spaced 100g mm along $x$ axis and 50 mm along $y$ axis.
tions: a foam rubber layer, an open window (open termination), and an aluminum plate.

The second setup was prepared inside the Teatro Comunale in Ferrara, Italy. This is an horseshoe-shaped theater built with rings of boxes one atop the other, and capped by a gallery. The signal output section consisted of a pseudorandom generator and a Norsonik™ dodecahedric source placed in a central position on the stage. The measurements in the duct are summarized in Table 1, where overall values of 200 to 5000 Hz are reported.

As can be seen from Table 1, changing the termination as in the noise-control study was used. In this case the x axis pointed toward the source, the z axis pointed upward, and the y axis completed the orthogonal frame. The frequency range analyzed inside the theater was 50–1250 Hz.

3.3.2 Experimental Results and Discussion

The measurements in the duct are summarized in Table 1, where overall values of 200 to 5000 Hz are reported.

The measurement point was at one stall about 7 m from the back of the hall and at a height of 1.3 m above the floor. The same three-dimensional sound intensity probe was used.

![Contour plot of Pressure](image1)

![Contour plot of Active Intensity](image2)

![Combined Contour Plot](image3)

**Fig. 6.** Contour plot of $P_{\text{rms}}$ level over grid points described in Fig. 5.

**Fig. 7.** Contour plot of active intensity of $|A|$ over grid points described in Fig. 5.

**Fig. 8.** Combined contour plot of oscillating intensity effective value $R$ and vector plot of active intensity $A$ over grid points described in Fig. 5. Arrows are rescaled for better rendering.
while the stimulus amplitude is kept fixed greatly affects the sound energy transfer process inside the duct. In particular, the more "reflective" the termination, the more oscillating energy is confined inside the duct. This means that the near field of a source radiating in a reverberant environment is either shrunk or expanded by the conditions experienced by the field at the boundaries of the enclosure.

This same behavior has been verified in the theater, where the boundary conditions are made up by a very complicated mixture of absorbing and reflective materials. The overall values measured using the theater setup are reported in Table 2, using the same logarithmic scale as Table 1. As was proven experimentally in both previous applications, the far field of any kind of complex source becomes more radiative in free-field conditions as the distance of the measuring point from the source is increased. In the theater setup the source and the measurement point are far apart (more than 20 m), so one might expect a prominence of the radiating intensity compared to the oscillating intensity, if there is considerable absorption at the boundaries. The measurement results reported in Table 2 show that the magnitude of the oscillating intensity is greater than the radiating intensity, suggesting that the presence of the enclosure changes the energy transport in the sound field radically. The oscillating flow at the measurement point is in fact due to the superposing of the wavefronts that impinge on each other, those coming directly from the different parts of a large complex source as well as those coming back from the hall boundaries during sound reverberation. In other words, as expected from theoretical wave treatment of room acoustics, a reverberant environment is characterized by the high density of the vibrations of the air, which form a myriad of standing-wave patterns, thus localizing the sound energy.

Fig. 9 shows the frequency distribution of the measured quantities $|A|$, $R$, and $P_{\text{rms}}$ over the octave bands in the appropriate logarithmic scale. Regarding the oscillating intensity, it can be observed that while the central frequency of the band decreases, $R$ increases. This indicates that the lower modes store more sound energy in the field than the higher ones, owing to the inseparable effects of geometry and absorption. Finally it can be noted that the well-known dip-seat effect occurring at 125 Hz for the sound pressure does not influence the values of $R$.

4 SUMMARY

In this work the intensimetric technique has been established, which is based on a new and rigorous energetic analysis of the sound field, and a suitable intensity

<table>
<thead>
<tr>
<th>Table 1. Overall values inside duct at 1 m from termination.</th>
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<tbody>
<tr>
<td>Termination</td>
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<tr>
<td>Foam rubber</td>
</tr>
<tr>
<td>Open window</td>
</tr>
<tr>
<td>Aluminum panel</td>
</tr>
</tbody>
</table>

Fig. 9. Frequency distribution of $|A|$, $R$, and $P_{\text{rms}}$ measured in a central stall (row 13) inside Teatro Comunale in Ferrara, Italy. Each quantity is normalized to its own standard reference value.
meter supporting it has been developed. The reliability of the instrument has been proven experimentally by the standard measure of the residual pressure intensity index. Special attention from both the theoretical and the experimental points of view was devoted to the measurement error on the effective R value of the oscillating intensity.

The new technique has been applied to three case studies in the different areas of musical acoustics, noise control, and room acoustics, and new research findings and implications for each case were presented.

5 REFERENCES


APPENDIX
ESTIMATE OF ERROR ON R

A precise estimate of the error δR is easily obtained if the contributions of random and systematic (that is, bias) errors to the intensimetric quantities are separated while making the derivation. The basic difference between the two types of errors is that bias errors always act in the same direction in the measurement of j and a, that is, these two quantities are always underestimated or overestimated by the measurement process. Since the measurement of R involves the subtraction of a from j, the introduced bias, which has the same sign for both quantities, is virtually eliminated by the algebraic structure of the calculation. This is illustrated in the following. First the global statistical error \( \langle \delta r \rangle \), which is made up of both bias and random contributions, is to be estimated from

\[
\delta r_i = \left| \frac{\partial r_i}{\partial j} \right| \delta j_i + \left| \frac{\partial r_i}{\partial a} \right| \delta a_i .
\]

After evaluating the derivatives and taking the time average, the formula reduces to

\[
\langle \delta r \rangle = \sqrt{2} (\langle \delta j_i \rangle + \langle \delta a_i \rangle) .
\]  

The next step is the computation of the error in \( A_i \). From the definition of the instantaneous radiating intensity [Eq. (3)] one can estimate that

\[
\frac{\delta a_i}{A_i} = \frac{\delta p^2}{p^2} + \frac{\delta A_i}{A_i} + \frac{\delta (p^2)}{p^2} .
\]

Multiplying both sides of the inequality by \( A_i \) and exploiting the time average, one obtains

\[
\langle \delta a_i \rangle = \delta A_i + 2A_i \langle \delta (p^2)/p^2 \rangle .
\]  

If we indicate the relative errors in pressure and intensity by \( e_p = \delta (p^2)/p^2 \) and \( e_A = \delta A/A \), the final result for an expression of the global error in \( R \) is achieved by inserting the preceding relation into Eq. (8),

\[
\delta R = 2\sqrt{2} \sum_{i=1}^{3} (e_A + e_p) A_i .
\]

Let us separate the errors into random and bias explicitly,

\[
\delta R = \delta R_{ran} + \delta R_{bias}
\]

\[
\delta j_i = \delta j_{i,ran} + \delta j_{i,bias}
\]

\[
\delta a_i = \delta a_{i,ran} + \delta a_{i,bias}
\]

\[
\delta p^2 = \delta p^2_{ran}
\]

where the last equation states the noninfluence of phase errors in the value of \( p^2 \). The different contributions affect the global estimate differently, as seen by inserting their expressions

\[
\langle \delta a_{i,ran} \rangle = \langle \delta j_{i,ran} \rangle + 2e_p A_i
\]

\[
\langle \delta a_{i,bias} \rangle = \langle \delta j_{i,bias} \rangle
\]

in the formula for the error \( \delta R \). Here Eq. (13) is simply Eq. (10), but rewritten for the random errors, whereas Eq. (14) includes only the bias error coming from the product \( pv \). Separating random and bias errors in the
estimate of the oscillating intensity gives

\[ \delta R_{\text{ran}} = 2\sqrt{2} \sum_{i=1}^{3} \left( \frac{\delta i_{\text{ran}}}{j} \right) A_i \]  

(15)

\[ \delta R_{\text{bias}} = 2\sqrt{2} \sum_{i=1}^{3} \left( \frac{\delta i_{\text{bias}}}{j} - \frac{\delta a_i_{\text{bias}}}{\alpha_i} \right) A_i \approx 0 \]  

(16)

Let us now first analyze \( \delta R_{\text{ran}} \). Since the random error in \( j \) is of the same order as the random error in \( p^2 \) (provided that the integration time satisfies the so-called \( BT \) rule\(^1\)), the two random errors can be joined into one single term with the substitution \( \epsilon_{R_{\text{ran}}} \approx \epsilon_{p^2} \). Moreover, the bias error \( \delta R_{\text{bias}} \) tends to zero because of the compensation introduced by the algebraic definition of the oscillating intensity, as explained. Thus the final formula reads

\[ \delta R = \delta R_{\text{ran}} = 4\sqrt{2} \sum_{i=1}^{3} \epsilon_{p^2} A_i \]  

(17)

This conclusion shows that the error in the oscillating intensity is essentially due to amplitude errors, since phase errors are minimized by the structure of the measurement procedure, and moreover it tends to zero when the field presents negligible values of the active intensity. The noninfluence of phase errors on the measurement of the oscillating intensity is an extremely valuable feature of this quantity and has also been validated experimentally, as shown in Fig. 4.

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### THE AUTHORS

Nicola Prodi was born in Reggio Emilia, Italy, in 1971 and graduated in applied physics from the University of Bologna. His education included studies as a professional oboe player and a singer at the Musical Institute of Reggio Emilia. This was followed by work experience at the Cemector-C.N.R. Institute of Ferrara, Italy, and stages at the Technical University of Denmark, Lyngby, and the Institut für Technische Akustik in Aachen, Germany. He is currently enrolled as a Ph.D. candidate in technical physics at the Engineering Department of the University of Ferrara. Email address: prodi@ing.unife.it.

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Since 1996, Dr. Stanzial’s primary research activity has been devoted to the unified physical interpretation of the acoustic energy-momentum tensor in the Minkowski 4-dimensional space. Since 1998 he has headed the Interuniversity Centre for Acoustics and Musical Research (CIARM) and recently he joined the Scuola di San Giorgio Foundation in Venice, as director of the Musical Acoustics Laboratory granted by CNR. Email address: acustica@cini.it.

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