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2aAAc11. Active playback of acoustic quadraphonic sound events

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The reconstruction into a cinema hall or a smaller room, of a sound event which has been previously recorded in a different acoustical environment is an interesting and still open acoustical problem. A new method for hi-fi multi-channel audio playback based on the general solution of the acoustical inverse problem is here proposed. This is implemented as a general feed-forward redundant control system where the number of acoustical signals to be reconstructed is greater than the number of the control signals feeding one or more loudspeakers working as active boundaries of the virtual playback room. This way an optimal and stable solution via a least square approach is obtained. This control system can be implemented even for complex configurations thanks to acoustic quadraphony: the application of sound intensimetry to audio technology recently developed within the IST-2-511316-IP European project denominated IP-RACINE. After a short explanation of the model theory, the experimental application to the simplest case of 1-D confined field is here presented and some experimentally obtained results are shown.

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Active Playback of Acoustic Quadraphonic Sound Events

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INTRODUCTION

A typical situation arising when listening to musical recordings is that sounds originally generated in large halls are reproduced in small rooms, for instance into parlors, drawing or living rooms. Apart from the technique used for the recording process: mono, stereo or multichannel, the acoustics of the small room always act as a filter on the reproduced sound. Even if recorded sound events could contain all exact acoustic information about the original space, the reproduced sound event will include the added acoustics of the listening space, which is usually not desirable from the point of view of the hi-fi process. This unwanted acoustics can thus be considered as an error that should be minimized during the playback process and to this aim an adaptive control approach can be followed. Recently, the so called "sound field reproduction" techniques for spatial sound – which also the one here presented belongs to – have been very clearly framed and referenced as an introduction in [10]. In that paper anyway the adaptive control has been focused to the wave field synthesis (WFS) approach while in this one the active control is applied to "Acoustic Quadraphony" [11] i.e. the application of sound intensimetry to audio recording and reproduction processes. For the sake of experimental simplicity, in the following the case of 1-D acoustic field the will be treated.
The here developed model regards the virtual reconstruction of any 1-D sound event previously recorded at a single point[1] of a real acoustic environment. This recorded sound event is assumed as the original sound to be reproduced [3,5]. The reconstruction process is implemented over a virtual domain by means of loudspeakers which are thought as the virtual walls of the playback environment fed by proper voltage signals processed according to the general inverse solution of the below stated differential problem.
In the following the sound $\Phi(x,\omega)=\mathcal{F}(\phi(x,t))\in\mathbb{C}$ is represented by the Fourier transform

$$\mathcal{F}(\cdot)\equiv(2\pi)^{-1/2}\int_{-\infty}^{+\infty}e^{i\omega t}(\cdot)\,dt$$

of the *kinetic or velocity potential* $\phi(x,t)\in\mathbb{R}$ which is related, according to the quadraphonic approach [1] to the sound pressure and air particle velocity fields, respectively indicated as p and v, through the well known relations:

$$\begin{cases} p(x,t)=-\rho_0\,\phi_t(x,t) \\ v(x,t)=\phi_x(x,t) \end{cases} \qquad (1)$$

A basic definition of the velocity potential field and a derivation of the wave equation in term of it can be found in [6] while a complete account of the Fourier transform method in theoretical Physics is given in [7].

THEORETICAL MODEL AND CONTROL SCHEME FOR 1-D FIELDS

The 1-D differential problem for the reproduced field can be formulated from the mathematical point of view using a homogeneous d'Alembert wave equation whose solution given in terms of the velocity potential fulfils boundary conditions at two points $x=0$ and $x=l$ together with the initial conditions for $t=0$.

[1] This limitation will allow only the implementation of a "perspective" audio quad-playback. The "holographic" playback needs at least the recording of two sound events at two distinct points.

The boundary conditions are modeled as distributions y characterizing the linear relation between acoustic pressure and velocity forced by an external (non-acoustical) power source.

The acoustic system can thus be modeled as follows

$$\begin{cases} \phi_{xx}(x,t) - \frac{1}{c^2}\phi_{tt}(x,t) = 0 & ; \; x \in [0,l] \\ -\phi_x(x,t) + \left(\frac{y}{\sqrt{2\pi}} * \rho_0\phi_t\right)(x,t) = k(x,t) & ; \; x = 0 \\ \phi_x(x,t) + \left(\frac{y}{\sqrt{2\pi}} * \rho_0\phi_t\right)(x,t) = k(x,t) & ; \; x = l \\ \phi(x,0) = \phi_t(x,0) = 0 & ; \; t = 0 \end{cases} \tag{2}$$

where the symbol $*$ stands for time-convolution operation. It is worthwhile to notice here that while the wave equation has to be solved for the velocity potential – a physically non-observable quantity – the boundary conditions are given instead in terms of observable quantities: i.e. the particle velocity and the sound pressure (see Eqs. (1)). This means that the quantity y appearing in the spatial boundary condition must act at the boundary surface like a convolution operator transforming the sound pressure into the air particle velocity.

The term k represents a *causal* constraint for the air particle velocity which can be thought as due to an action external to the field (for instance the diaphragm vibration of a loudspeaker). The physical meaning of temporal boundary conditions looks more obvious: sound waves always start from an equilibrium condition defined in terms of kinetic potential and sound pressure. The same differential problem can thus be rewritten in the frequency domain as

$$\begin{cases} \Phi''(x,\omega) + \frac{\omega^2}{c^2}\Phi(x,\omega) = 0 & ; \; x \in [0,l] \\ -\Phi'(0,\omega) + i\omega\rho_0 Y_0(\omega)\Phi(0,\omega) = K_0(\omega) \\ \Phi'(l,\omega) + i\omega\rho_0 Y_l(\omega)\Phi(l,\omega) = K_l(\omega) \\ \int \Phi(x,\omega)\mathrm{d}\omega = \int \omega\Phi(x,\omega)\mathrm{d}\omega = 0 \end{cases} \tag{3}$$

where primes indicate the derivative with respect to x and $Y(\omega) = \mathcal{F}(y(t))$ is an admittance-like quantity having physical dimensions of $[M^{-1}][L^2][T]$ acting on the Fourier transformed velocity potential field.

A well known technique for solving system of equations like (3) is based on the knowledge of the associated Green Function which is defined by the solution of the system[2]

$$\begin{cases} -\left[\frac{\partial^2}{\partial x^2} + \left(\frac{\omega}{c}\right)^2\right] G(x,x_0,\omega) = \frac{1}{\sqrt{2\pi}}\delta(x - x_0) \; ; \\ \qquad\qquad ; \; x, x_0 \in [0,l] \\ -G'(0,x_0,\omega) + i\omega\rho_0 Y_0(\omega) G(0,x_0,\omega) = 0 \\ G'(l,x_0,\omega) + i\omega\rho_0 Y_l(\omega) G(l,x_0,\omega) = 0 \\ \int G(x,x_0,\omega)\mathrm{d}\omega = \int \omega G(x,x_0,\omega)\mathrm{d}\omega = 0 \end{cases} \tag{4}$$

where x_0 is the excitation point and x the receiving point where the system impulse response (Green's function) is measured. In fact, once the Green's function or *fundamental solution* of the acoustic system is known, the solution

[2] A clear and synthetic introduction to the application of the Green's Function in Acoustics is given in [8]. A complete treatment of this topic can be found in [9] .

$\Phi(x_0,\omega)=\mathcal{F}(\phi(x_0,t))\in\mathbb{C}$ satisfying the given boundary conditions can be written as a surface integral extended all over the boundary of a product of the Green's functions G and the source functions K. In the 1-D case, this reduces to

$$\Phi(x_0,\omega)=\sqrt{2\pi}[K_0(\omega)G(0,x_0,\omega)+K_l(\omega)G(l,x_0,\omega)]\in\mathbb{C} \tag{5}$$

From (5) it is clear that the excitation point x_0 of the Green function becomes now the receiving point where a listener can hear the virtual sound

$$\phi_B(x_0,t)=\mathcal{F}^{-1}(\Phi_B(x_0,\omega))\in\mathbb{R}$$

generated by the synchronous active boundaries sources $K_{0,l}(\omega)$ located respectively in $x=0$ and $x=l$.

Suppose now that you have previously recorded the original sound $\phi_A(t)=\mathcal{F}^{-1}(\Phi_A(\omega))\in\mathbb{R}$ at a generic point within an actual environment denoted by the subscript A and that you want to reproduce it at the point x_0 of a virtual playback room B. Of course if you are expecting that the virtual listeners hear exactly the sound that you have recorded, then the following condition

$$\phi_B(x_0,t)=\phi_A(t) \tag{6}$$

has to be fulfilled. In fact if we rewrite formally Eq. (5) as

$$\Phi_B(x_0,\omega)=\sqrt{2\pi}[K_0(\omega)G(0,x_0,\omega)+K_l(\omega)G(l,x_0,\omega)]\in\mathbb{C} \tag{7}$$

where

$$\begin{aligned}K_0(\omega)&=\frac{1}{2\sqrt{2\pi}}G^{-1}(0,x_0,\omega)\Phi_A(\omega)\\ K_l(\omega)&=\frac{1}{2\sqrt{2\pi}}G^{-1}(l,x_0,\omega)\Phi_A(\omega)\end{aligned} \tag{8}$$

and $G^{-1}=1/G$, the condition (6) is clearly verified.

In conclusion, if we want to reproduce into a playback room B a virtual acoustic environment starting from a sound event actually recorded in any original environment A, we have to drive the active boundaries at $x=0$ and $x=l$ of the room B with the synchronous signals given by expressions (8). These signals have to be considered as discrete sources of kinetic potential acting as a constraint for the air particle velocity.

Let now see how this model can be interpreted from the experimental point of view and implemented in practice.

Let's start rewriting the solution (5) in terms of the two acoustical observables p and v using the relation (1), so obtaining for the air particle velocity and sound pressure the following 2×2 system

$$\begin{cases}P(x_0,\omega)=\mathcal{F}(-\rho_0\phi_t(x_0,t))=\\ =-i\omega\rho_0(2\pi)^{-\frac{1}{2}}\left[K_0(\omega)G(0,x_0,\omega)+K_l(\omega)G(l,x_0,\omega)\right];\\ \rho_0cV(x_0,\omega)=\mathcal{F}(\rho_0c\phi_x(x_0,t))=\\ =\rho_0c(2\pi)^{-\frac{1}{2}}\left[K_0(\omega)G_x(0,x_0,\omega)+K_l(\omega)G_x(l,x_0,\omega)\right];\end{cases} \tag{9}$$

where, in order to have the same units in both equations, the second one has been multiplied by $\rho_0 c$.
The system (9) can be rewritten in general form by a matrix equation, as

$$\mathbf{A}\cdot\mathbf{z}=\mathbf{b} \tag{10}$$

or explicitly

$$\begin{aligned} a_{11}z_1 + \quad a_{12}z_2 &= b_1 \\ a_{21}z_1 + \quad a_{22}z_2 &= b_2 \end{aligned} \tag{11}$$

where the number of loudspeakers generating the boundary velocity constraints z_i, $i = 1,2$ at $\{x_1 = 0,\ x_2 = l\}$ in the playback 1-D room, has been assumed to be $N = 2$. Thus the $N \times N$ square matrix $\mathbf{A}$ and the right-hand side quantities $\mathbf{b}$ defined over the complex number field $\mathbb{C}$ are

$$\mathbf{A} \equiv (2\pi)^{-\frac{1}{2}} \rho_0 \begin{bmatrix} -i\omega G_1 & -i\omega G_2 \\ c\partial_x G_1 & c\partial_x G_2 \end{bmatrix}; \quad \mathbf{b} \equiv \begin{bmatrix} P \\ \rho_0 c V \end{bmatrix}; \quad \mathbf{z} \equiv \begin{bmatrix} K_0 \\ K_l \end{bmatrix} \tag{12}$$

where $G_1 \equiv G(0, x_0, \omega)$, $G_2 \equiv G(l, x_0, \omega)$. Clearly, provided that $\mathbf{A}$ is not a singular matrix, a unique solution exists for the system (11) giving of course the two expressions (8). It is important to consider here that this solution relies on the hypothesis that the right playback configuration for reproducing the original sound event at the single sweet spot x_0 is given by 2 loudspeakers located at $x = 0$ and $x = l$. This choice however, may be not so convenient because of the experimental errors inherent in the known quantities. In fact the main target of our application is to fit a linear mathematical model to measurements obtained from experiments, with the goal of extracting predictions from the measurements and reducing the effect of measurement errors in the reproduction of sound. It is thus better to choose more equations than unknowns and then looking for a *pseudo-solution* following the least square method of Gauss.

In our experimental case study for instance we'll try to reproduce the original 1-D sound event $\mathbf{b}$ (represented by a 2×1 vector but in general, in the 3-D case, represented by an $M \equiv 4m \times 1$ column vector) with only 1 loudspeaker at a single sweet spot ($m = 1$) or using 2 loudspeakers but controlling 2 sweet spots during the playback.

These two canonical $m = 1,\ N = 1$ and $m = 2,\ N = 2$, 1-D playback configurations ($M = 2m$ in the 1-D case) have been experimentally checked and preliminary results will be reported in section 3.

In the following section we'll see how this kind of feed-forward redundant ($M > N$) control can be implemented.

THE LEAST SQUARE METHOD IN 1-D

As known [12], provided that $\det(\mathbf{A}^* \cdot \mathbf{A}) \neq 0$, the pseudo-solution of an over-determined linear system of equations is given by[3]

$$\tilde{\mathbf{z}} = (\mathbf{A}^* \cdot \mathbf{A})^{-1} \cdot (\mathbf{A}^* \cdot \mathbf{b}) \tag{13}$$

minimizing the norm

$$\|\mathbf{A} \cdot \mathbf{z} - \mathbf{b}\|. \tag{14}$$

For the 1-D, $m = 1$, $N = 1$ case, we simply have

$$\begin{aligned} a_{11}z_1 &= b_1 \\ a_{21}z_1 &= b_2 \end{aligned}; \quad \mathbf{A} = \frac{\rho_0}{\sqrt{2\pi}} \begin{bmatrix} -i\omega G_1 \\ c\,\partial_x G_1 \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} P \\ \rho_0 c V \end{bmatrix} \tag{15}$$

giving the following pseudo solution

[3] The notation $\mathbf{A}^*$ means the complex conjugated of the transposed of matrix $\mathbf{A}$. $\mathbf{A}^*$ is usually called the *adjoint* of $\mathbf{A}$

$$\tilde{z}_1(\omega) = \frac{\sqrt{2\pi}\left(\overline{\partial_x G_1}\Big|_{x_0} V(\rho_0 c)^2 + i\omega\rho_0 \bar{G}_1 P\right)}{(\rho_0 c)^2 \left(\dfrac{\omega^2}{c^2}|G_1|^2 + \left|\partial_x G_1\big|_{x_0}\right|^2\right)} \in \mathbb{C} \tag{16}$$

where the bar over $G_1 \equiv G(0, x_0, \omega)$ stands for the complex conjugation operation. Eq. (16) can be rewritten in terms of the pressure and velocity propagators measured from the playback boundary at $x = 0$ to the virtual sweet spot at x_0 as

$$\tilde{z}_1(\omega) = \frac{\sqrt{2\pi}\left(\rho^2{}_0 c^2 \bar{G}_v V - \bar{G}_p P\right)}{\left(\left|G_p\right|^2 + \rho_0{}^2 c^2 \left|G_v\right|^2\right)} \tag{17}$$

having the physical dimension of a length so representing in the frequency domain the vibration amplitude of the loudspeaker diaphragm. It is clear that the acoustic velocity constraint at boundaries (i.e. the term $k(x,t)$ appearing in (2) can be obtained from (17) by Fourier anti-transformation of $\tilde{z}_1(\omega)$.

The $m = 2,\ N = 2$ case is a bit more complicated, so the problem will be outlined here, while the detailed solution will be given in appendix **A** . Only "perspective" quad-playback is here considered (see footnote 1): this means that a single sound event recorded in the original environment will be reproduced at two different sweet spots in the virtual room. In this configuration the pseudo-solution $\tilde{\mathbf{z}}$ is obviously a 2×1 vector, the known terms consist of a 4×1 vector where the recorded pressure and velocity signals are cycled once and the 4×2 **A** matrix contains the acoustic pressure and air vibration velocity propagators measured between each one of the two sweet spots and each one of the active boundaries, for a total of eight elements. Explicitly, we have:

$$\begin{array}{rcl} a_{11}z_1 + a_{12}z_2 &=& b_1 \\ a_{21}z_1 + a_{22}z_2 &=& b_2 \\ a_{31}z_1 + a_{32}z_2 &=& b_3 \\ a_{41}z_1 + a_{42}z_2 &=& b_4 \end{array};$$

$$\mathbf{A} = \frac{\rho_0}{\sqrt{2\pi}} \begin{bmatrix} -i\omega G_{11} & -i\omega G_{12} \\ c\,\partial_x G_{11} & c\,\partial_x G_{12} \\ -i\omega G_{21} & -i\omega G_{22} \\ c\,\partial_x G_{21} & c\,\partial_x G_{22} \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} P \\ \rho_0 c V \\ P \\ \rho_0 c V \end{bmatrix} \tag{18}$$

where the firs index in G_{ij} refers to the sweet spot while the second to the boundary.

The last step in our model is to find the voltage signal to be used for feeding the control loudspeaker in order to satisfy Eq. (17). This task can be accomplished in practice with a black box model of any loudspeaker and characterizing its electro-mechanical behaviour by means of a frequency independent voltage-displacement factor. Of course the air is here considered as a pure resistive load for the electromechanical power. Following this hypothesis the driving voltage signal can then be considered to be well approximated by

$$s(\omega) = \Upsilon(\omega)\tilde{z}_1(\omega) \tag{19}$$

where Υ is the loudspeaker conversion factor from displacement to voltage.

EXPERIMENTAL CHECKING OF THE MODEL

In order to check the experimental validity of the model, four 1-D rooms consisting in a 4 m long tube with a loudspeaker at one of its terminations, have been set up (see Figure 1). The original sound event has been recorded in a room having a foam rubber termination (FIGURE 2). Three virtual environments have been setup for quad-playback: a first room with an open termination (FIGURE 3), a second one with a wooden baffle (FIGURE 4) termination and a third one equipped with a second loudspeaker (FIGURE 5), so that both the ends of the tube were constrained with loudspeakers diaphragms.

FIGURE 1. The one-dimensional environment

The following materials and instrumentation has been set up for simulating the different 1-D environments where the quad-sound event has been recorded and reproduced.

- A square section Plexiglas tube ($4\times0.28\times0.28$) m. It is a good approximation of a one- dimensional field till a cut-off frequency $f_c \simeq 610\,\text{Hz}$.
- Two subwoofer Monacor® Carpower, 25 cm diameter, chosen for their good response from 50 Hz up to about 2 kHz
- One power amplifier Alesis® RA 150 to drive the loudspeakers.
- A MOTU® Traveler digital I/O audio device.
- A Personal computer with Matlab® software.
- A Microflown® Match Size p-v axial probe with its own signal conditioner.

All the pressure-velocity impulse responses were recorded with the sine sweep technique, using a signal in the [50, 650] Hz range. In the $N=1, M=1$ case the recording/reconstructing point is about 1.50 m far from the sound source, in the $N=2, M=2$ case the sweet spots are 1.50 m and 1.70 m far from it.

FIGURE 2. Foam rubber termination

We first examine the one-loudspeaker/one-sweet-spot configuration: a single sound event is recorded in the 1-D environment of Fig.1 (i.e. the tube closed at one side by foam rubber) and virtually reproduced in other two different environments (i.e. the tube with open termination, and the tube terminated with a wooden baffle).

FIGURE 3. Open termination

FIGURE 4. Wooden termination

FIGURE 5. Loudspeaker termination and p-v probe

In this two case studies the redundancy of the system is 2×1 (two equations/one unknown), and the solution is analytically given by Eq. (16). In FIGURE 6 and FIGURE 7 the *playback error functions*, given by

$$\frac{\left\|\mathbf{A}\tilde{\mathbf{z}}_X - \mathbf{b}\right\|}{\left\|\mathbf{b}\right\|} \equiv \frac{\left\|\tilde{\mathbf{b}}_X - \mathbf{b}\right\|}{\left\|\mathbf{b}\right\|} \equiv \frac{\varepsilon}{\left\|\mathbf{b}\right\|} \tag{20}$$

are plotted.

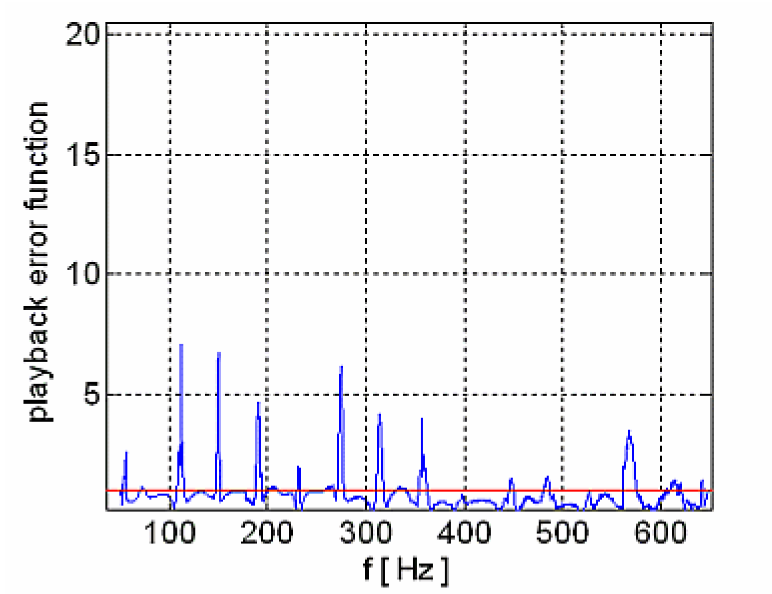

FIGURE 6. Playback error function for the open termination. The red line stands for a relative error equal to 1 (100%).

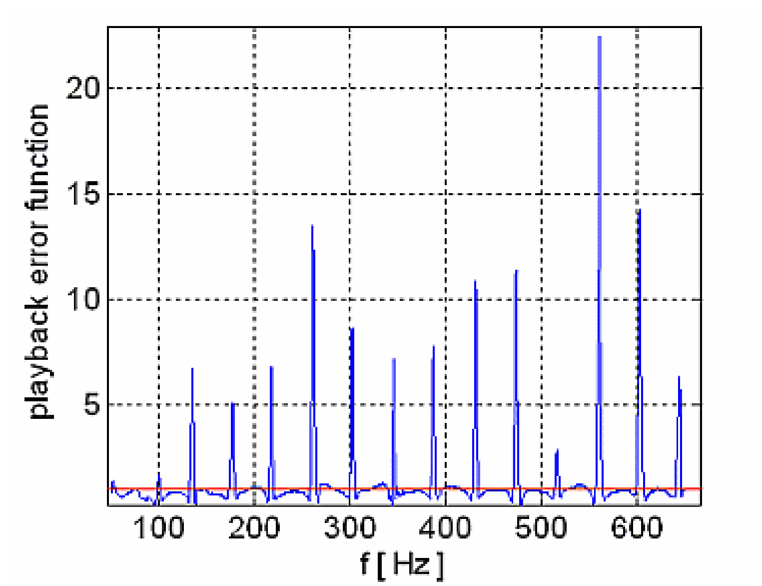

FIGURE 7. Playback error function for the wooden termination.

These obtained results indicate how far from a hypothetical exact solution the optimal one $\tilde{z}_1$ actually is. The graphics show that the pseudo-solution obtained for virtual reconstruction is a good one, apart from those frequencies corresponding to the virtual environment modal resonances. Thus, as expected, the playback error function correctly foresees how much the virtual sound approximates the original one at the proper modal frequencies of virtual environments. It is worthwhile to be remarked here that the model has been checked in the worst possible condition: trying to reproduce a sound event recorded in a dead room into a more reverberant one.

The same behavior appears also in the $N = 2$, $m = 2$ configuration (see FIGURE 8). In this case, the playback error function is much bigger than those previously reported, showing very high and sharp peaks corresponding to the tube odd resonances. This implies that the two-loudspeakers virtual room behaves like a one-side open, one-side closed tube.

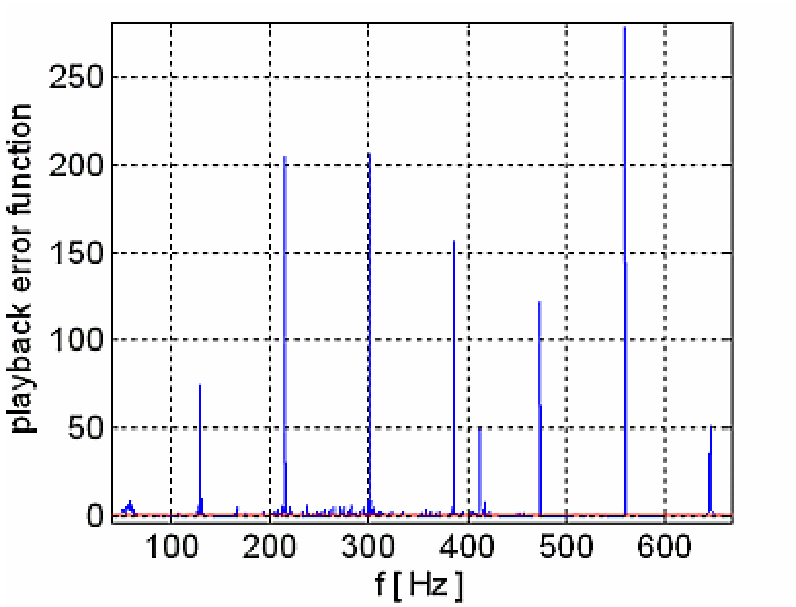

FIGURE 8. Relative error function for a two-loudspeakers virtual room.

CONCLUSIONS AND FURTHER DEVELOPMENTS

A theoretical model to solve the inverse acoustic problem based on the *a priori* knowledge of a quadraphonic sound event and the actual measurement of the acoustic pressure and velocity propagators into the playback room have been physically interpreted and experimentally checked in the 1-D case.
The obtained results show that the implemented redundant feed-forward active control for the quad-playback process should works fine in dead or anechoic environments while playback errors are expected to increase with increasing reverberation. [2,4].
Further experimental analysis are in progress for a better understanding of the two-loudspeakers virtual room physical behavior.

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A. APPENDIX: Explicit Formulas for Two Loudspeakers Configuration

Provided that $\det(\mathbf{A}) \neq 0$, the explicit solution for the $N = 2$, $m = 2$ problem (Eq. (18)) can be written as follows:

$$\tilde{\mathbf{z}} = \frac{\rho_0 c}{\sqrt{2\pi} D} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix};$$

$$D = \frac{(\rho_0 c)^4}{4\pi^2}\left[\left(\left|\frac{\omega}{c}G_{11}\right|^2 + |G'_{11}|^2 + \left|\frac{\omega}{c}G_{21}\right|^2 + |G'_{21}|^2\right)\left(\left|\frac{\omega}{c}G_{12}\right|^2 + |G'_{12}|^2 + \left|\frac{\omega}{c}G_{22}\right|^2 + |G'_{22}|^2\right) - \right.$$

$$\left. - \left(\left(\frac{\omega}{c}\right)^2 \bar{G}_{11}G_{12} + \bar{G}'_{11}G'_{12} + \left(\frac{\omega}{c}\right)^2 \bar{G}_{21}G_{22} + \bar{G}'_{21}G'_{22}\right)\left(\left(\frac{\omega}{c}\right)^2 \bar{G}_{12}G_{11} + \bar{G}'_{12}G'_{11} + \left(\frac{\omega}{c}\right)^2 \bar{G}_{22}G_{21} + \bar{G}'_{22}G'_{21}\right)\right];$$

$$x_1 = \left[\left|\frac{\omega}{c}G_{12}\right|^2 + |G'_{12}|^2 + \left|\frac{\omega}{c}G_{22}\right|^2 + |G'_{22}|^2\right]\left[\frac{i\omega}{c}\left(\bar{G}_{11} + \bar{G}_{21}\right)P + \left(\bar{G}'_{11} + \bar{G}'_{21}\right)V\right] -$$

$$- \left[\left(\frac{\omega}{c}\right)^2 \bar{G}_{11}G_{12} + \bar{G}'_{11}G'_{12} + \left(\frac{\omega}{c}\right)^2 \bar{G}_{21}G_{22} + \bar{G}'_{21}G'_{22}\right]\left[\frac{i\omega}{c}\left(\bar{G}_{12} + \bar{G}_{22}\right)P + \left(\bar{G}'_{12} + \bar{G}'_{22}\right)V\right];$$

$$x_2 = \left[\left|\frac{\omega}{c}G_{11}\right|^2 + |G'_{11}|^2 + \left|\frac{\omega}{c}G_{21}\right|^2 + |G'_{21}|^2\right]\left[\frac{i\omega}{c}\left(\bar{G}_{12} + \bar{G}_{22}\right)P + \left(\bar{G}'_{12} + \bar{G}'_{22}\right)V\right] -$$

$$- \left[\left(\frac{\omega}{c}\right)^2 \bar{G}_{21}G_{22} + \bar{G}'_{21}G'_{22} + \left(\frac{\omega}{c}\right)^2 \bar{G}_{12}G_{11} + \bar{G}'_{12}G'_{21}\right]\left[\frac{i\omega}{c}\left(\bar{G}_{11} + \bar{G}_{21}\right)P + \left(\bar{G}'_{11} + \bar{G}'_{21}\right)V\right];$$

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